Unbinned measurements in global SMEFT fits from machine learning

DIS2022

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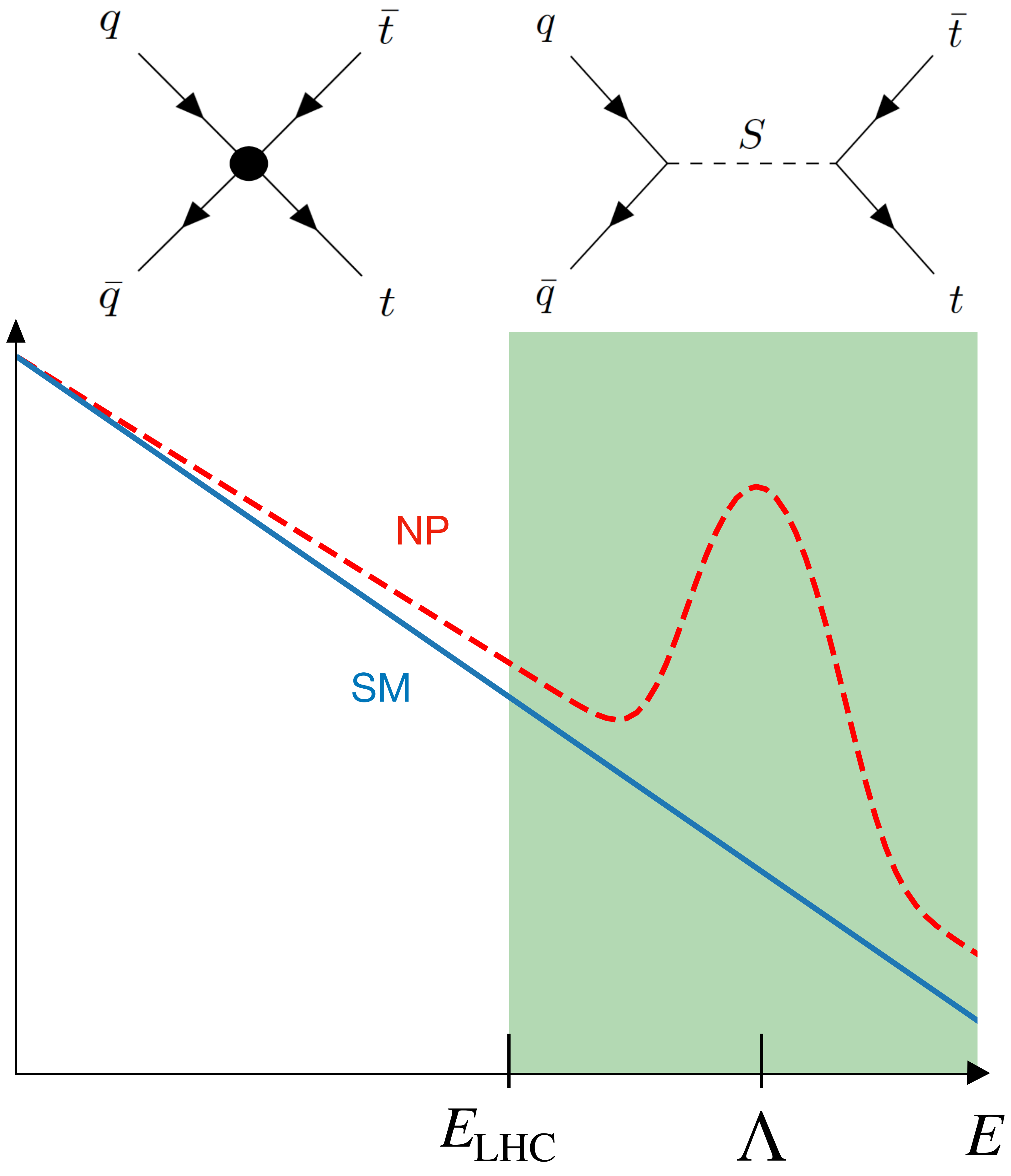
Jaco ter Hoeve

Work done in collaboration with R. Gomez Ambrosio, M. Madigan, J. Rojo, V. Sanz

The Standard Model Effective Field Theory

- Systematic parameterisation of the **theory space** close to the Standard Model
- Study the **fingerprints** of NP at low energies through higher dimensional operators
- Assumes the **SM field content** and **gauge symmetries**

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \sum_i^{N_{d6}} \frac{c_i}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_i^{N_{d8}} \frac{b_i}{\Lambda^4}\mathcal{O}_i^{(8)} + \dots$$



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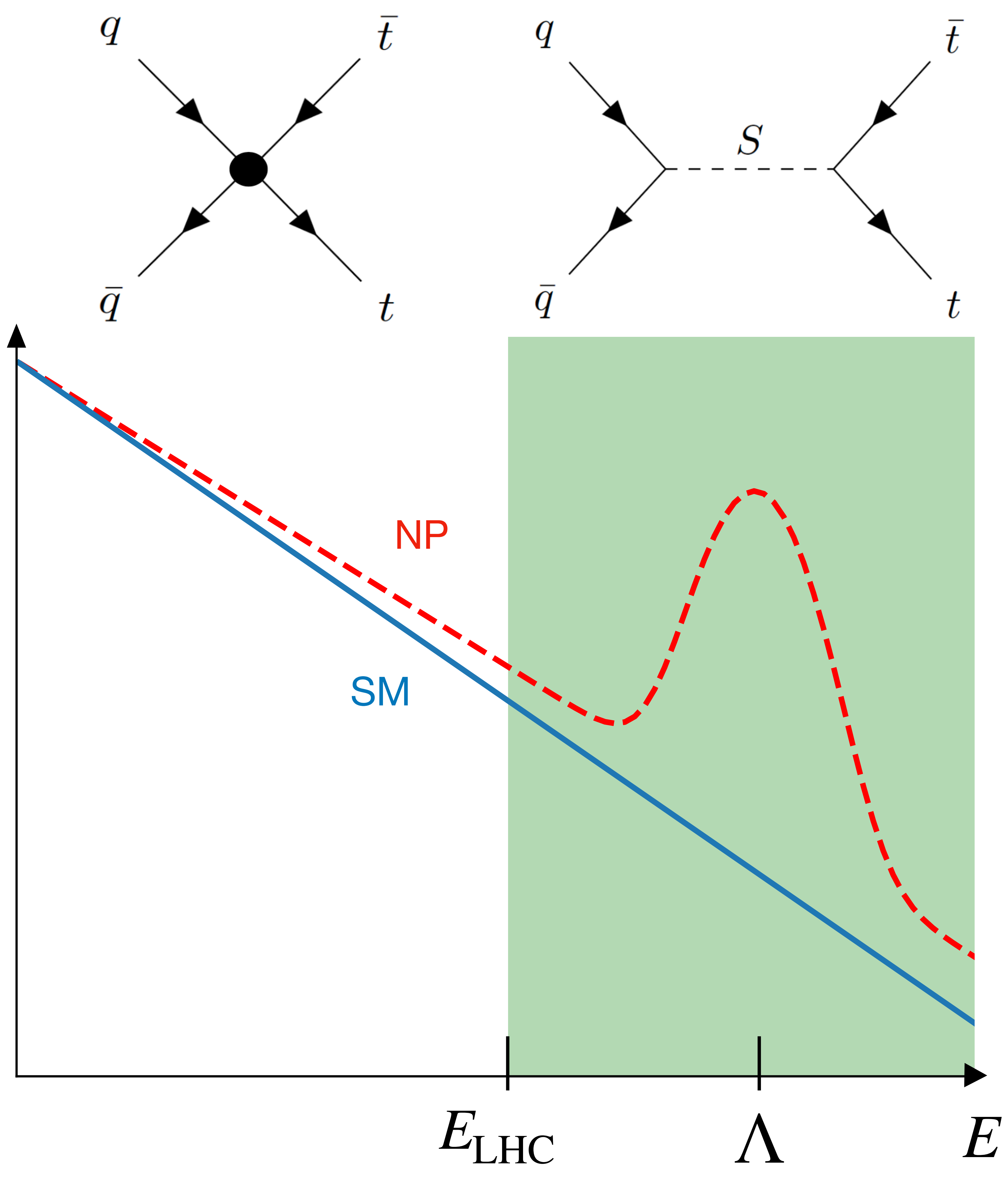
$$\sigma = \sigma_{\mathrm{SM}} + \underbrace{\sum_{i}^{N_{d6}} \frac{c_i}{\Lambda^2}\kappa_i}_{\text{Interference}} + \underbrace{\sum_{i,j}^{N_{d6}} \frac{c_i c_j}{\Lambda^4}\tilde{\kappa}_{ij}}_{\text{Quadratic corrections}} + \ldots$$



So what then are global SMEFT fits?

- A global SMEFT analysis needs to explore a **huge** parameter space (2499 at dim 6)
- Studies the intricate interplay between the **top, Higgs** and **diboson** sectors
- Requires state-of-the-art theory calculations in both the SM (NLO QCD + NNLO K-factors) and EFT

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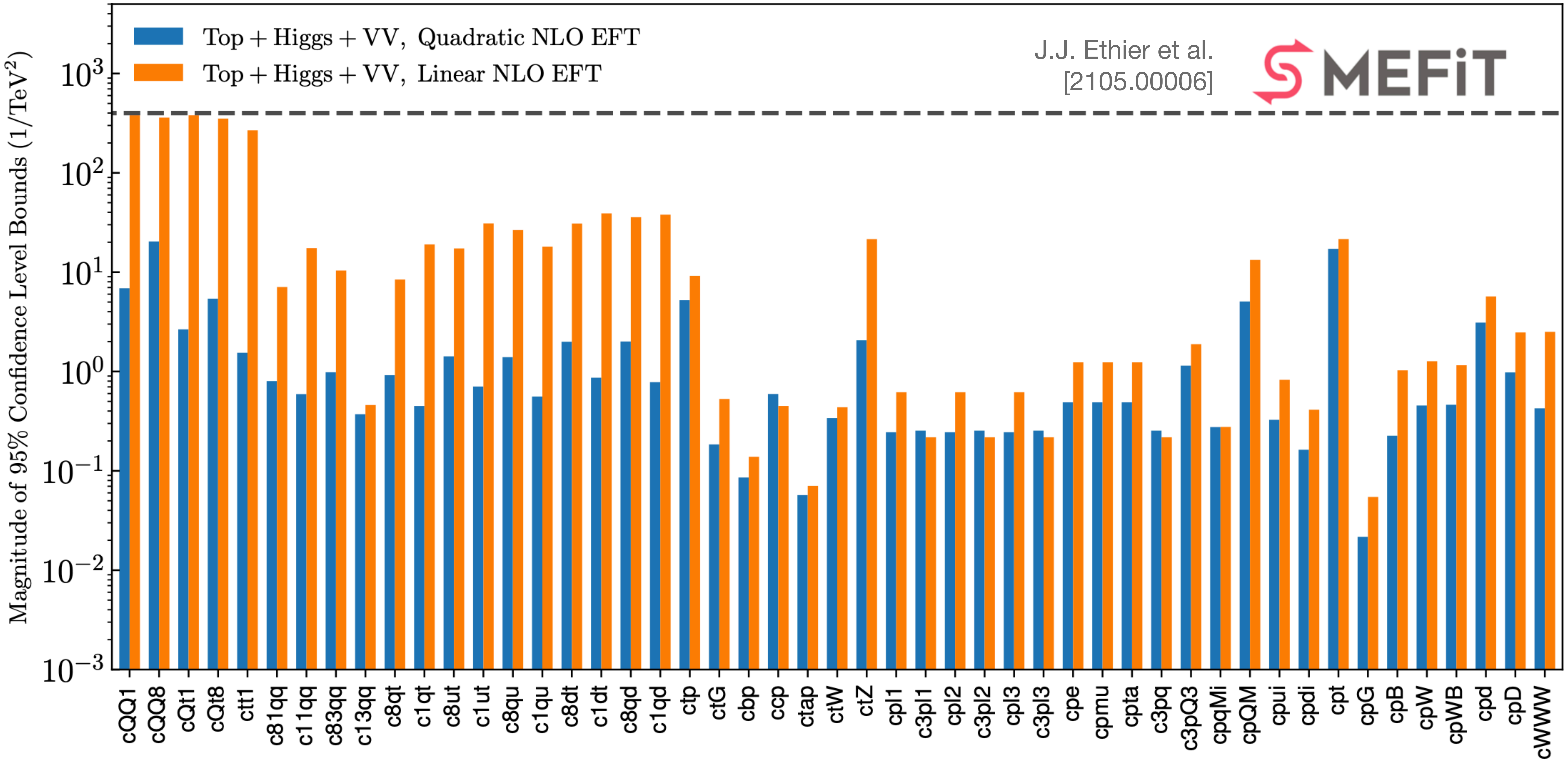

J.J. Ethier et al. [2105.00006]

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- Based on unfolded cross sections **not tailored** for EFT studies!

Can we extend global SMEFT fits with ML based observables?

Statistically optimal observables from ML

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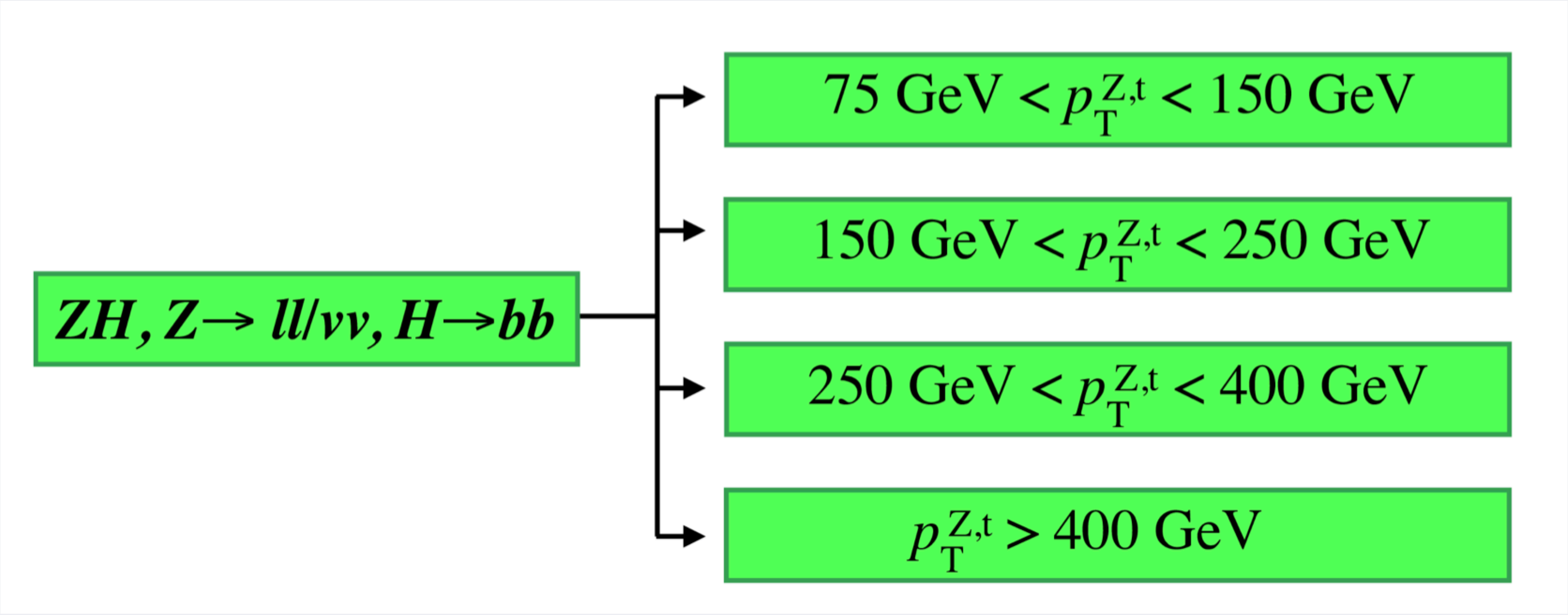
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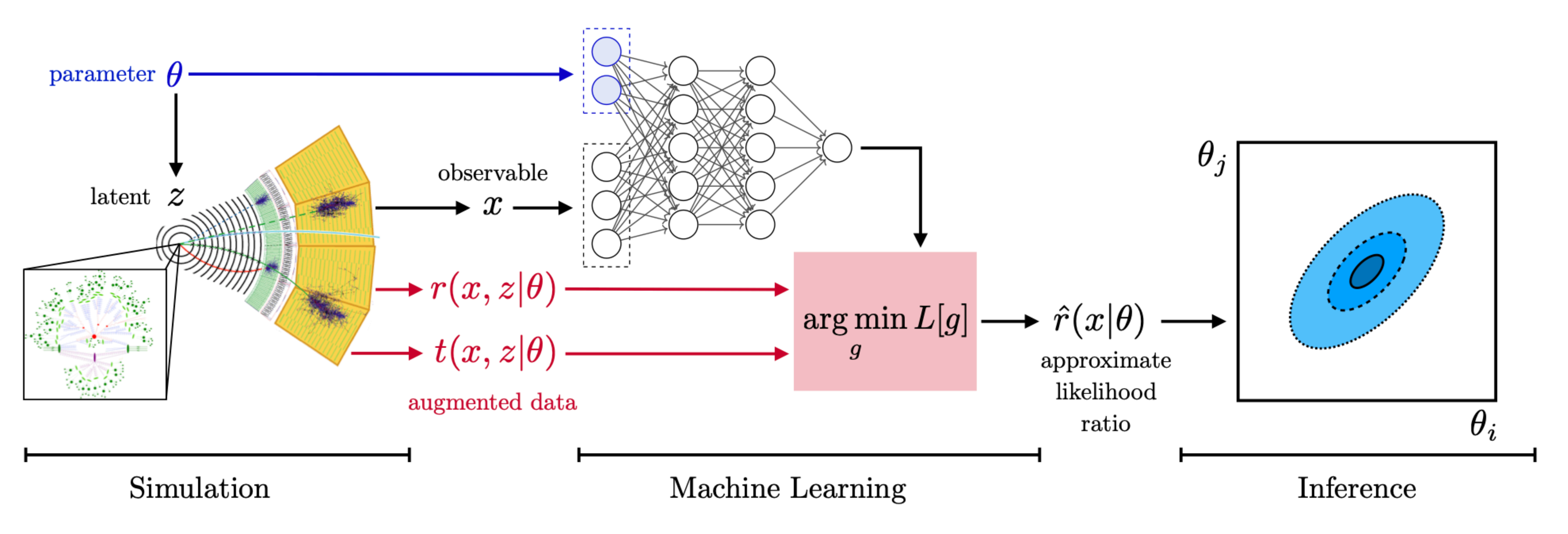
Goal: deploy unbinned measurements to determine the optimal sensitivity using ML techniques

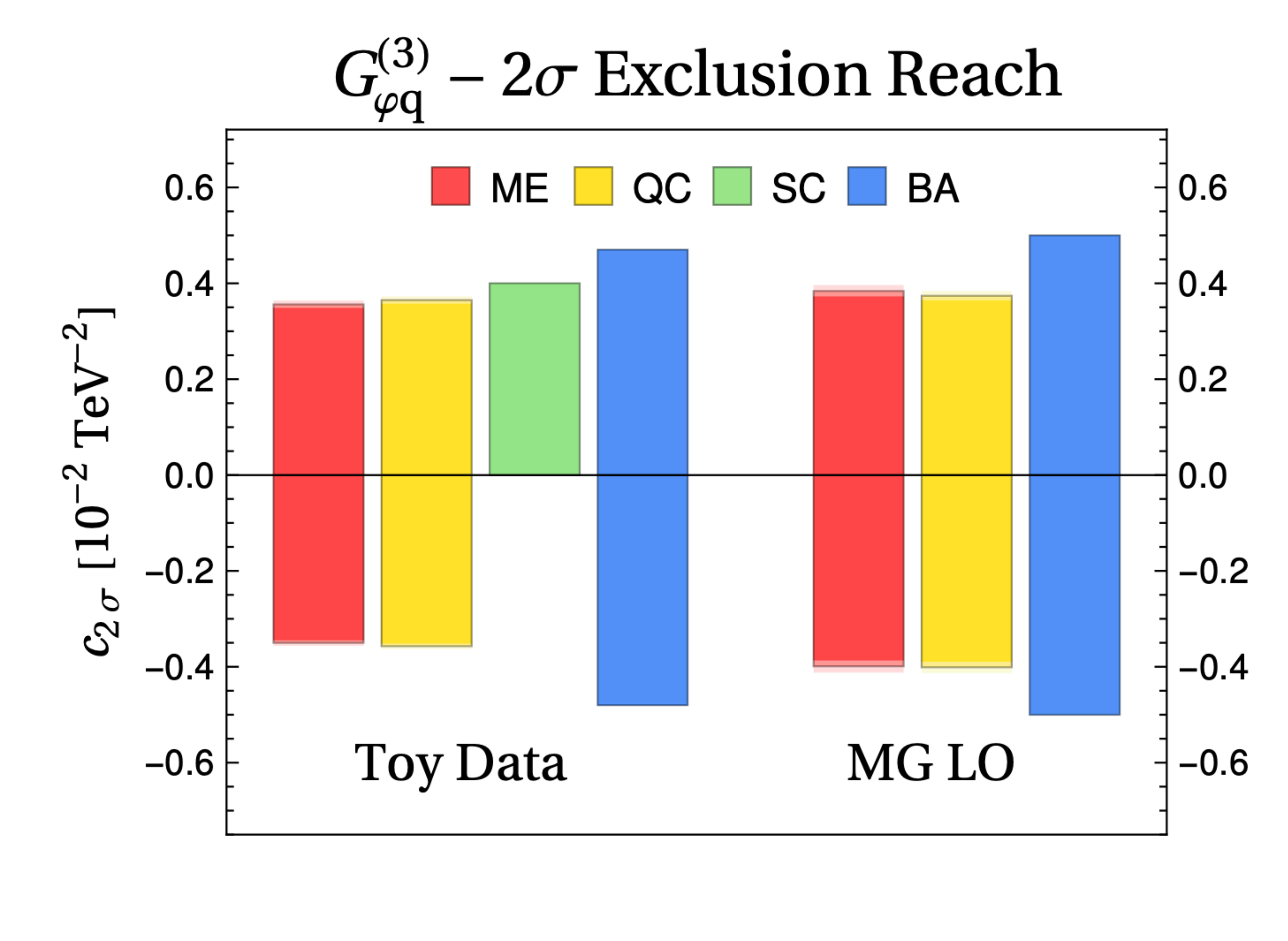
Useful diagnosis tool!

Related work

- The **likelihood ratio** as central object
- Parameterise the likelihood ratio with **Neural Networks**
- Current studies are limited to only a **small number of EFT coefficients**



J. Brehmer et al. [2010.06439]



S. Chen, A. Glioti, G. Panico, A. Wulzer [2007.10356]

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Carry out EFT analysis with **different** variants of the **same** measurement

$$\log \mathscr{L}_{\text{binned}}(c) = -\frac{1}{2}\sum_{i=1}^{n_{\text{bins}}} \frac{(n_i - \nu_i)^2}{\nu_i}$$

Binned Gaussian Likelihood

$$\log \mathscr{L}_{\text{binned}}(c) = \sum_{i=1}^{n_{\text{bins}}} n_i \log \nu_i(c) - \nu_i(c)$$

Binned Poissonian Likelihood

Unbinned extended likelihood

$$\log \mathscr{L}_{\text{unbinned}}(\nu, c) = -\,\nu(c) + \sum_{i=1}^{N} \log \nu(c) g(x_i, c) \qquad g(x_i, c) \equiv \frac{1}{\sigma(X, c)}\frac{d\sigma(x, c)}{dx}$$

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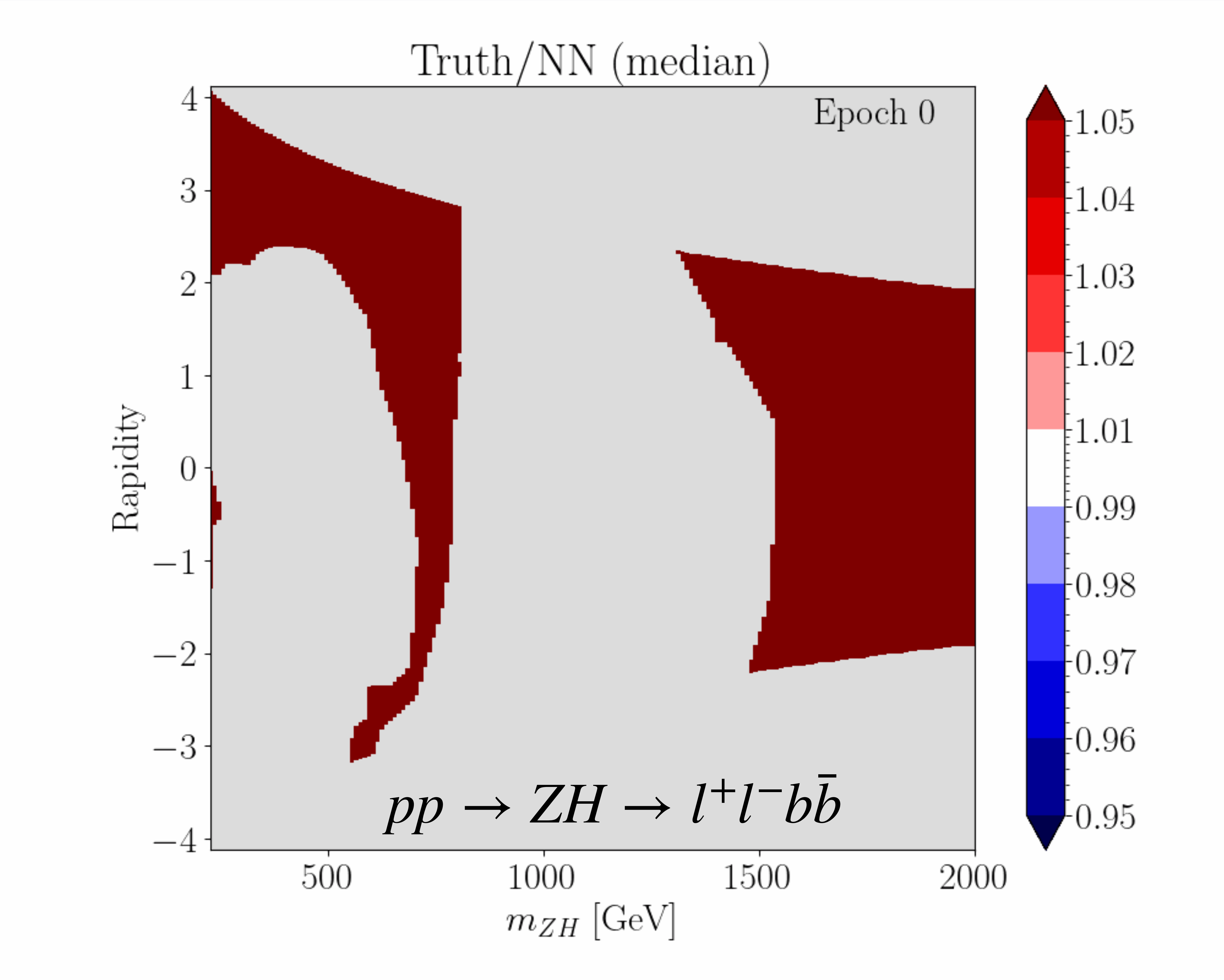
We consider processes that are dominated by statistical uncertainties, adding systematics is WIP

Statistically optimal observables from ML

Dependence of cross-section* on all (independent) kinematic variables and all EFT coefficients

$$g(x_i, c) \equiv \frac{1}{\sigma(X, c)} \frac{d\sigma(x, c)}{dx}$$

is parameterised by a feed-forward NN trained to Monte-Carlo simulations, **benchmarked** with analytical calculations



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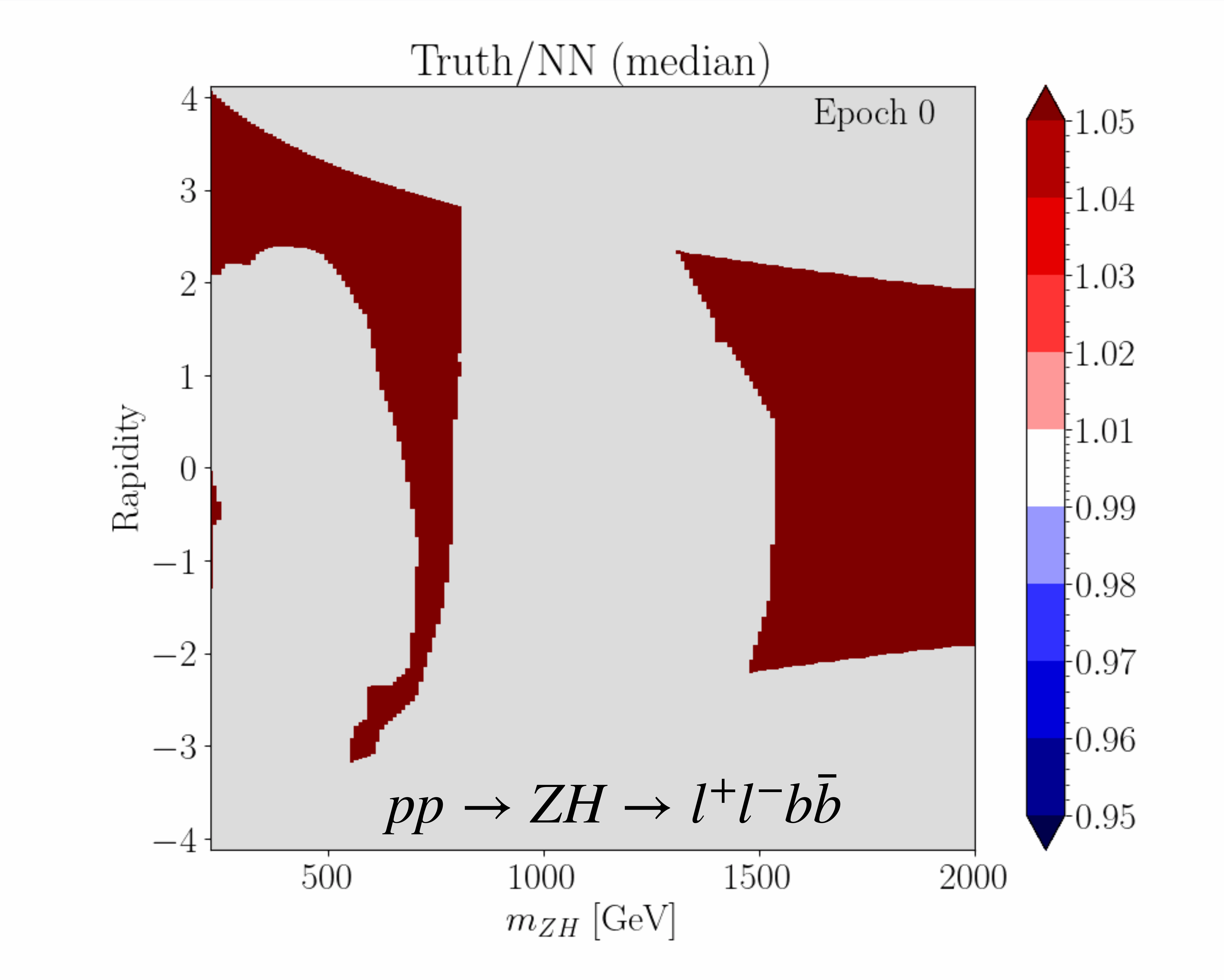

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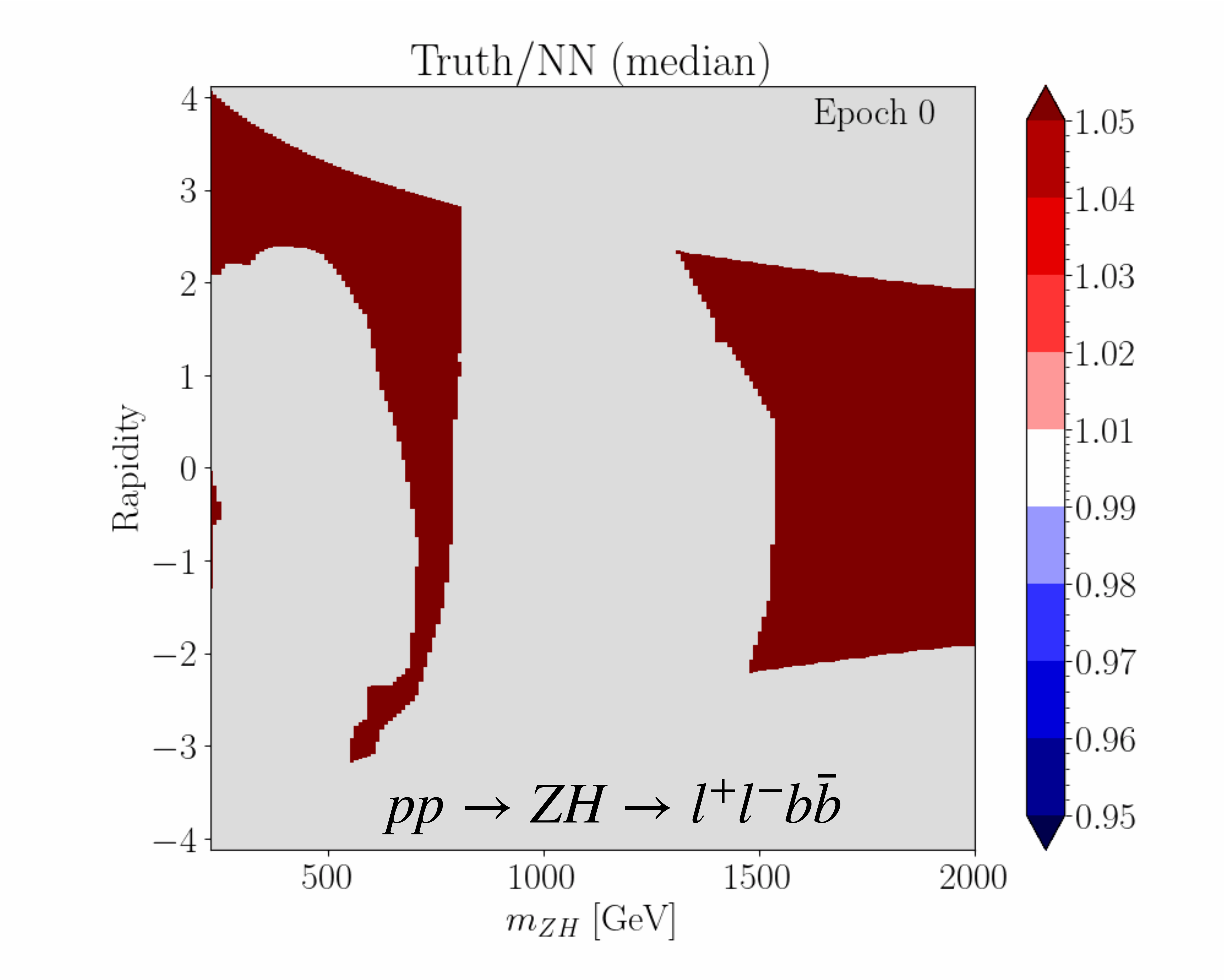

Challenge: correctly describing tails of distributions (low stats)

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Training formalism

- Finding the likelihood ratio between hypotheses H_0 and H_1 can be formulated in terms of an **optimisation problem** on two balanced training sets S_0 and S_1

$$\boxed{L[f(x)] = -\sum_{e\in S_0} w_e \log(1-f(x_e)) - \sum_{e\in S_1} w_e \log f(x_e)}$$

EFT events

SM events

$$x_e = \{p_T, Y, m_{ZH}, \ldots\}$$

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- Equivalent formulations exist, e.g. in terms of a quadratic loss

S. Chen, A. Glioti, G. Panico, A. Wulzer [2007.10356]

Training formalism

- Exploit the polynomial structure of the EFT coefficients to separate the learning problem

$$\boxed{\frac{d\sigma_0}{d\sigma_1} = 1 + c \cdot n_\alpha}$$

1. Linear term

$$\boxed{\frac{d\sigma_0}{d\sigma_1} = 1 + c \cdot n_\alpha + c^2 n_\beta}$$

2. Quadratic term

$$\boxed{\frac{d\sigma_0}{d\sigma_1} = \ldots + c_1 c_2 n_\gamma}$$

3. Cross term

- Cross section **positivity** can be enforced through either Lagrange multipliers or a final ReLU

Scaling behaviour

Our method scales efficiently with the number of EFT coefficients and can be parallelised

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Trained in parallel

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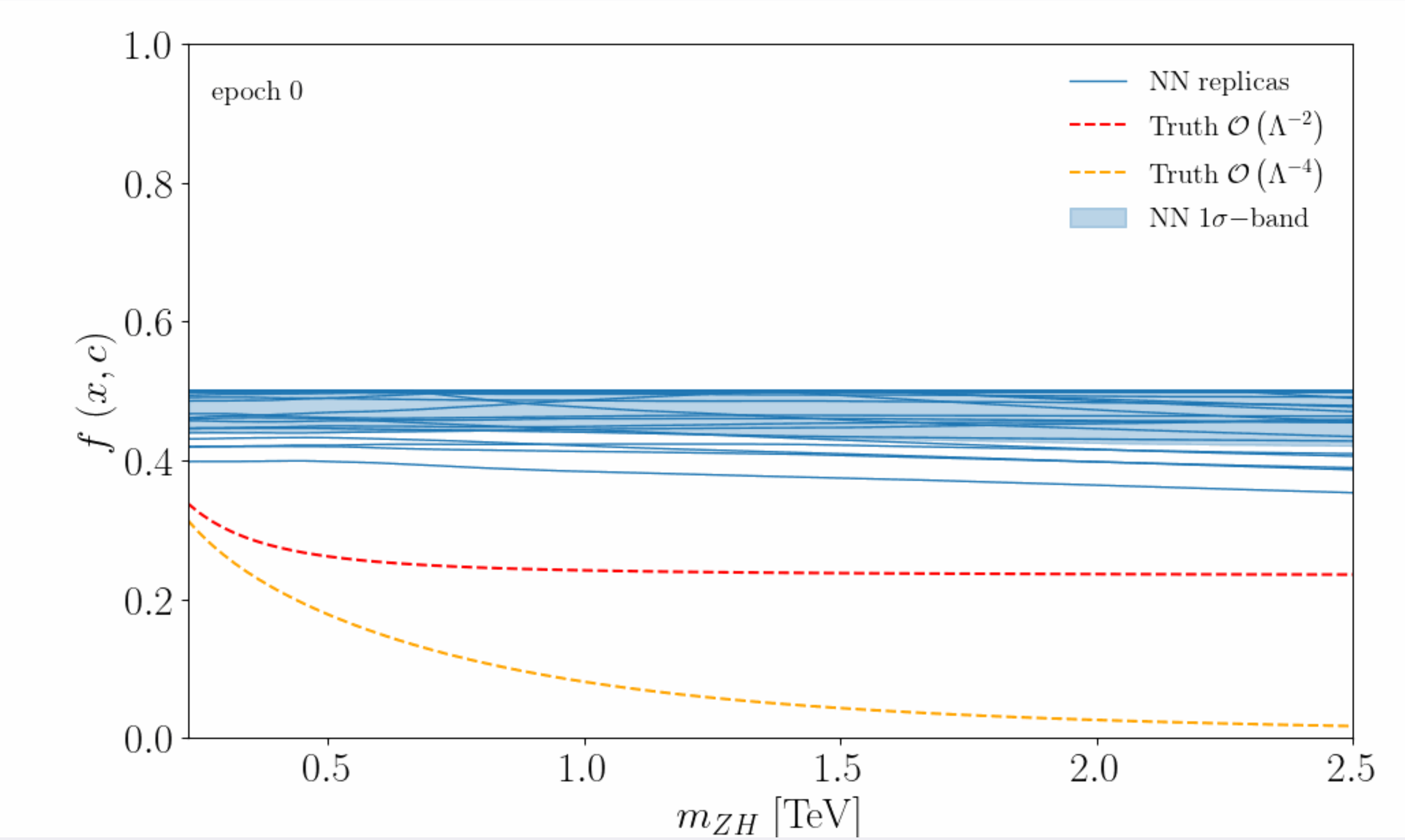
The Monte Carlo replica method

We train a collection of 50 NN instances on independent MC datasets to estimate the **NN uncertainty**



Training settings

- LO QCD MC dataset (100K)
- PyTorch Implementation
- Validation loss is monitored to avoid overfitting
- Training takes ~30 min per core (1 replica/core)
- 5 hidden layers (30 units)
- ReLU activation functions

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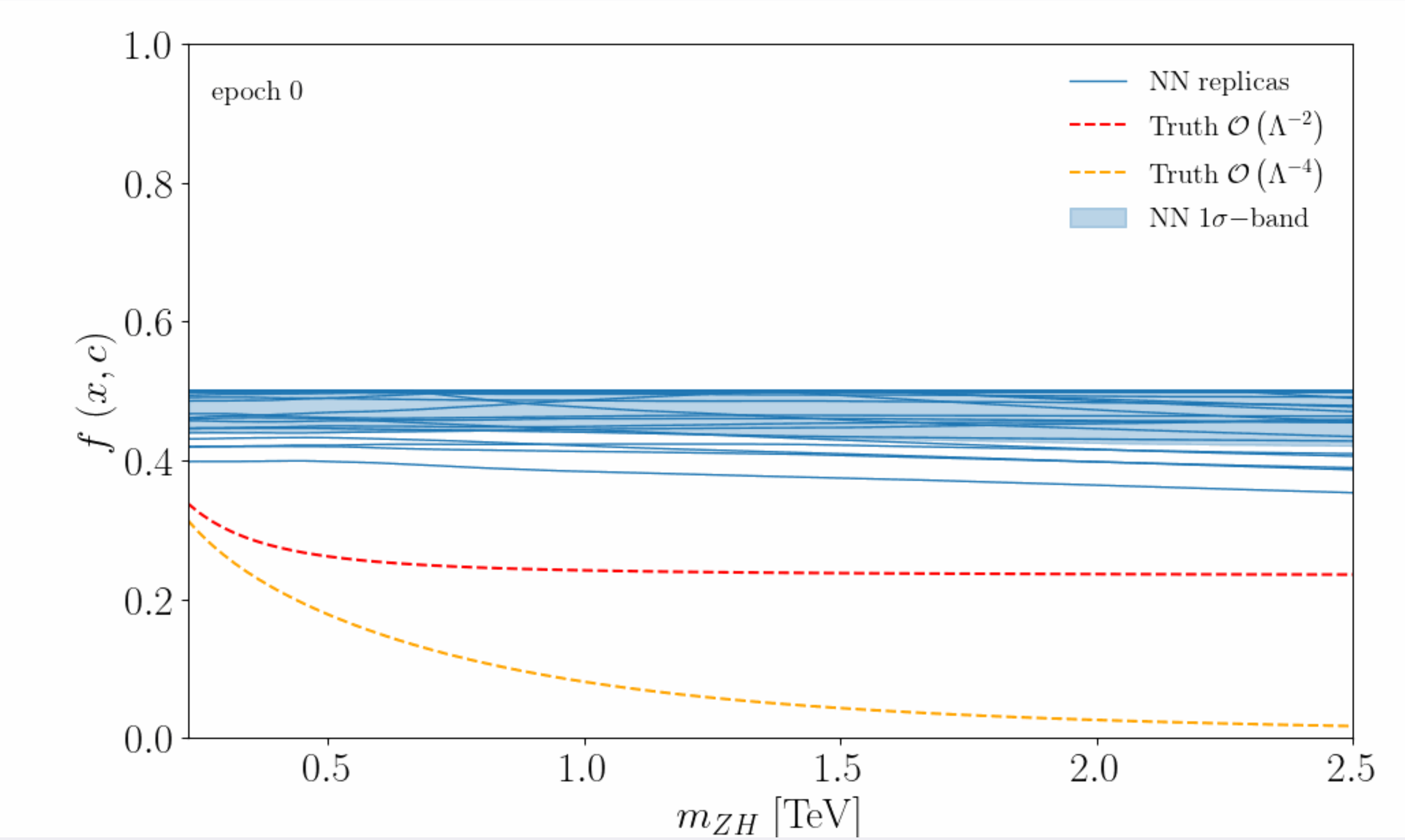

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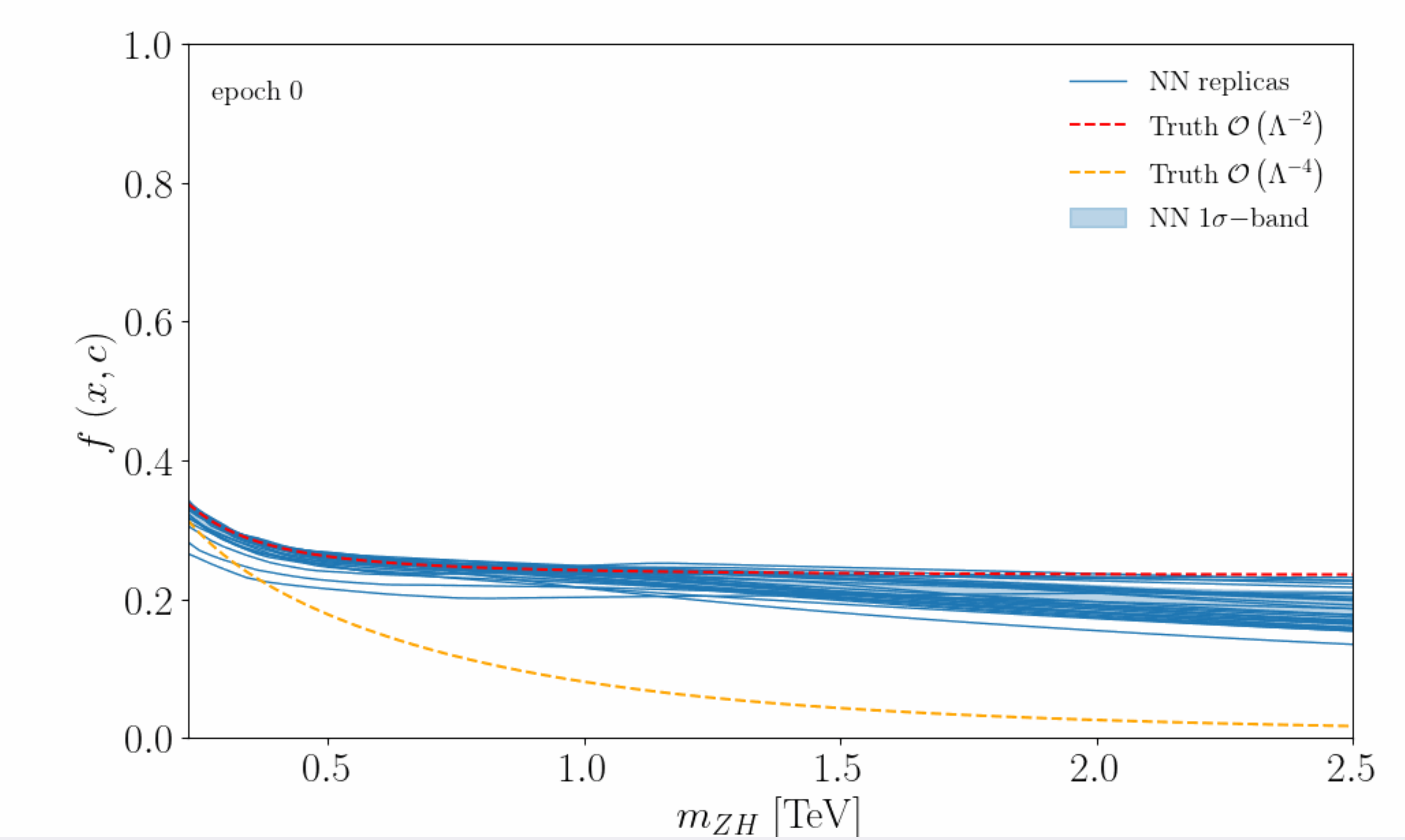

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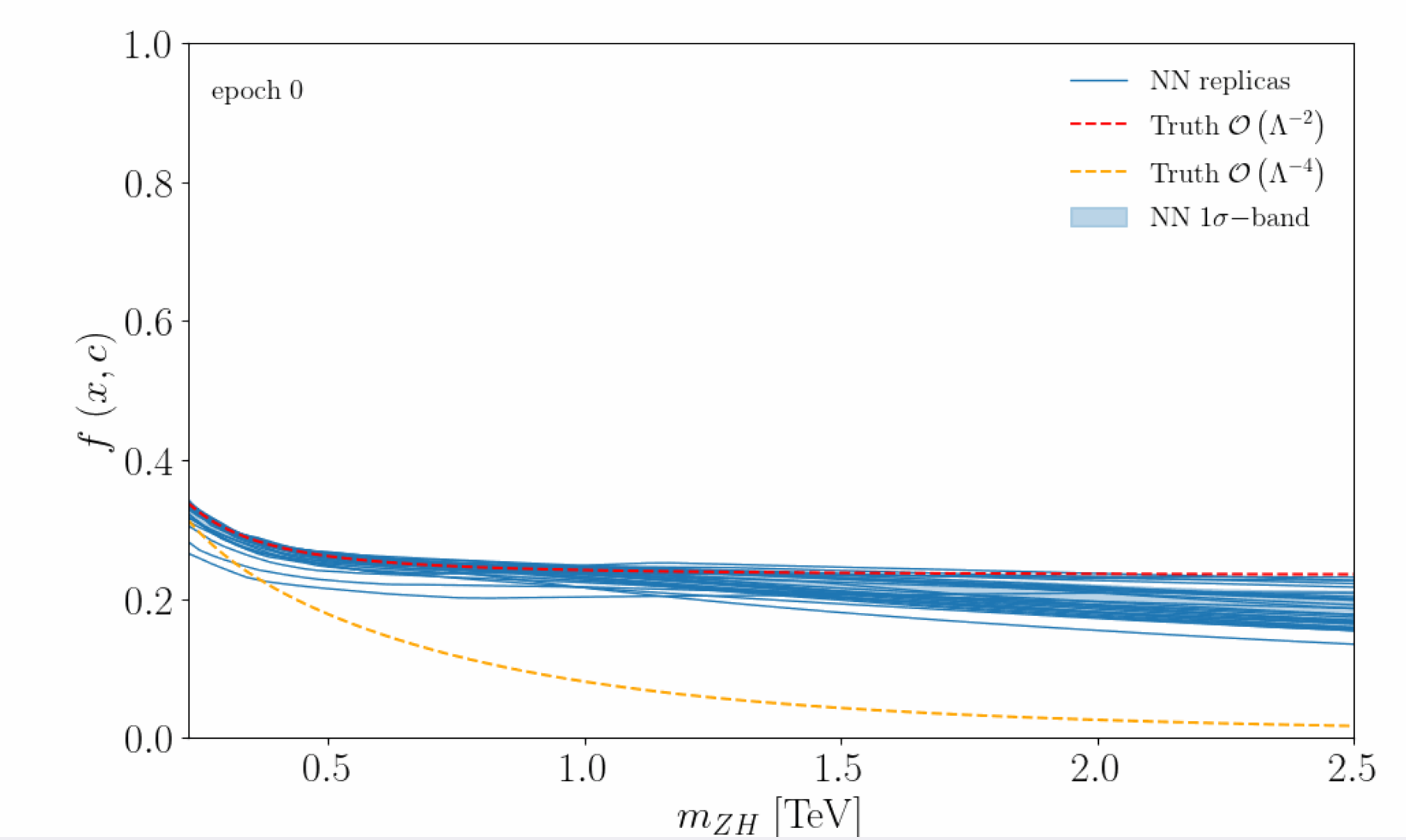

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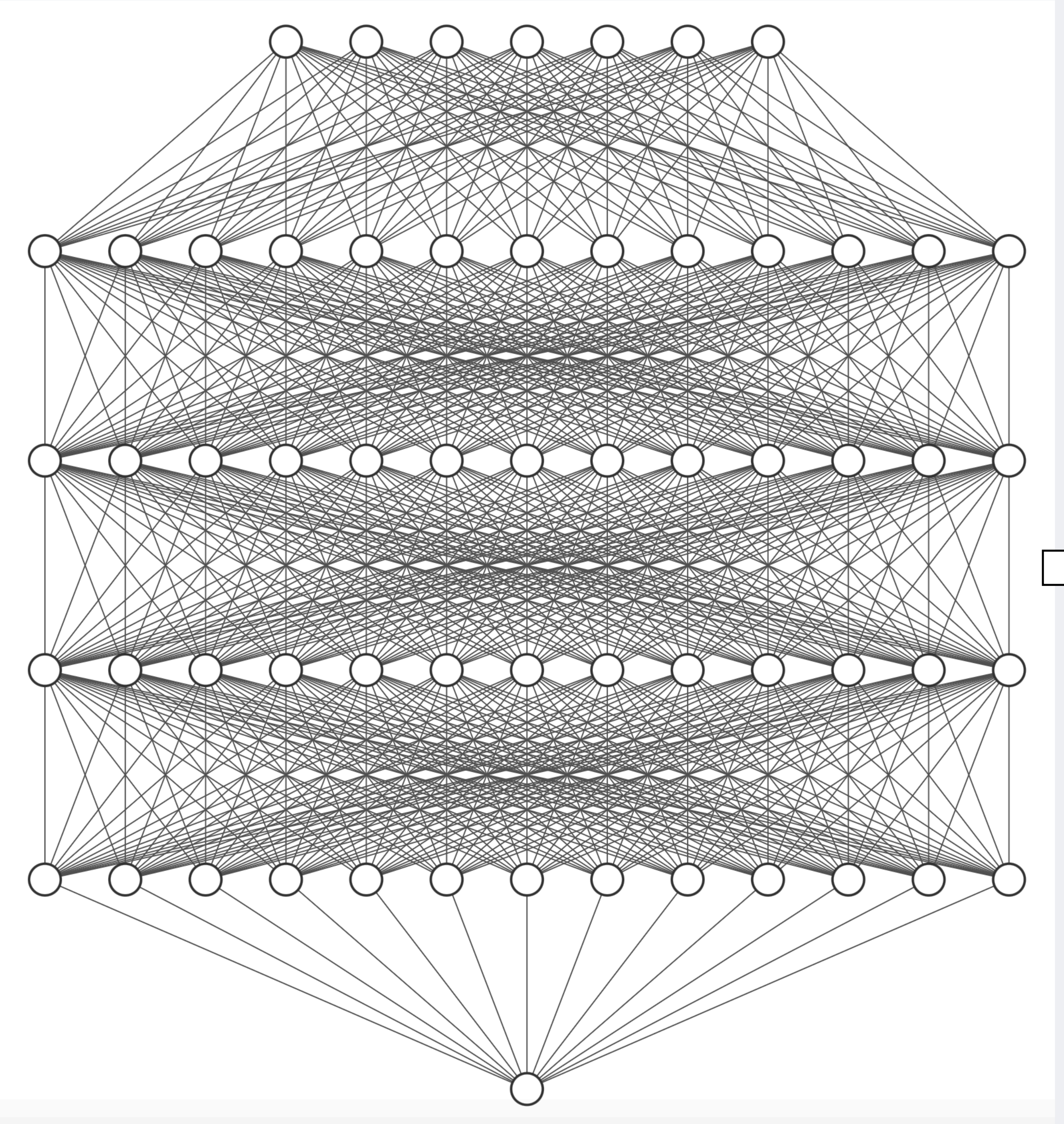

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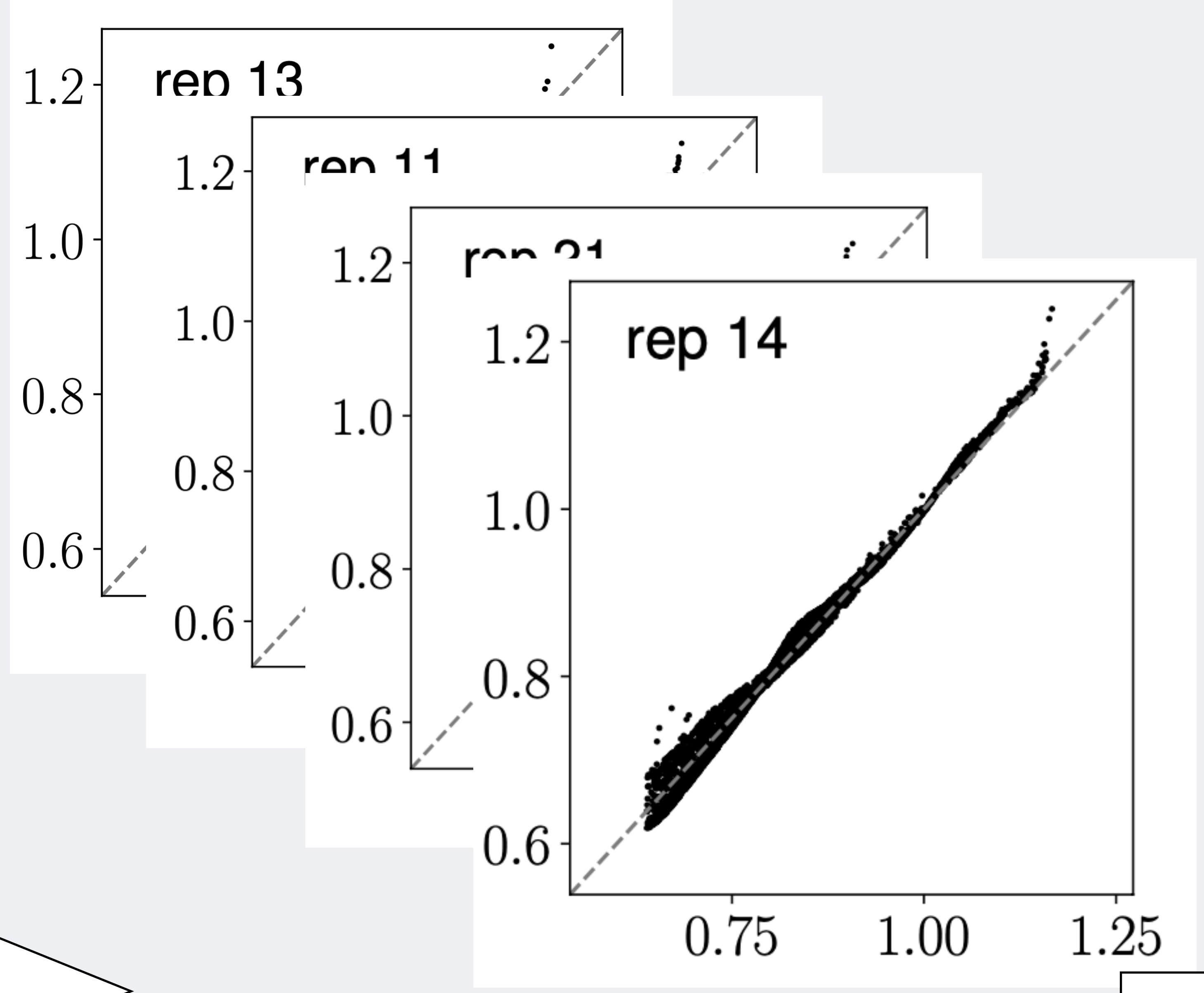
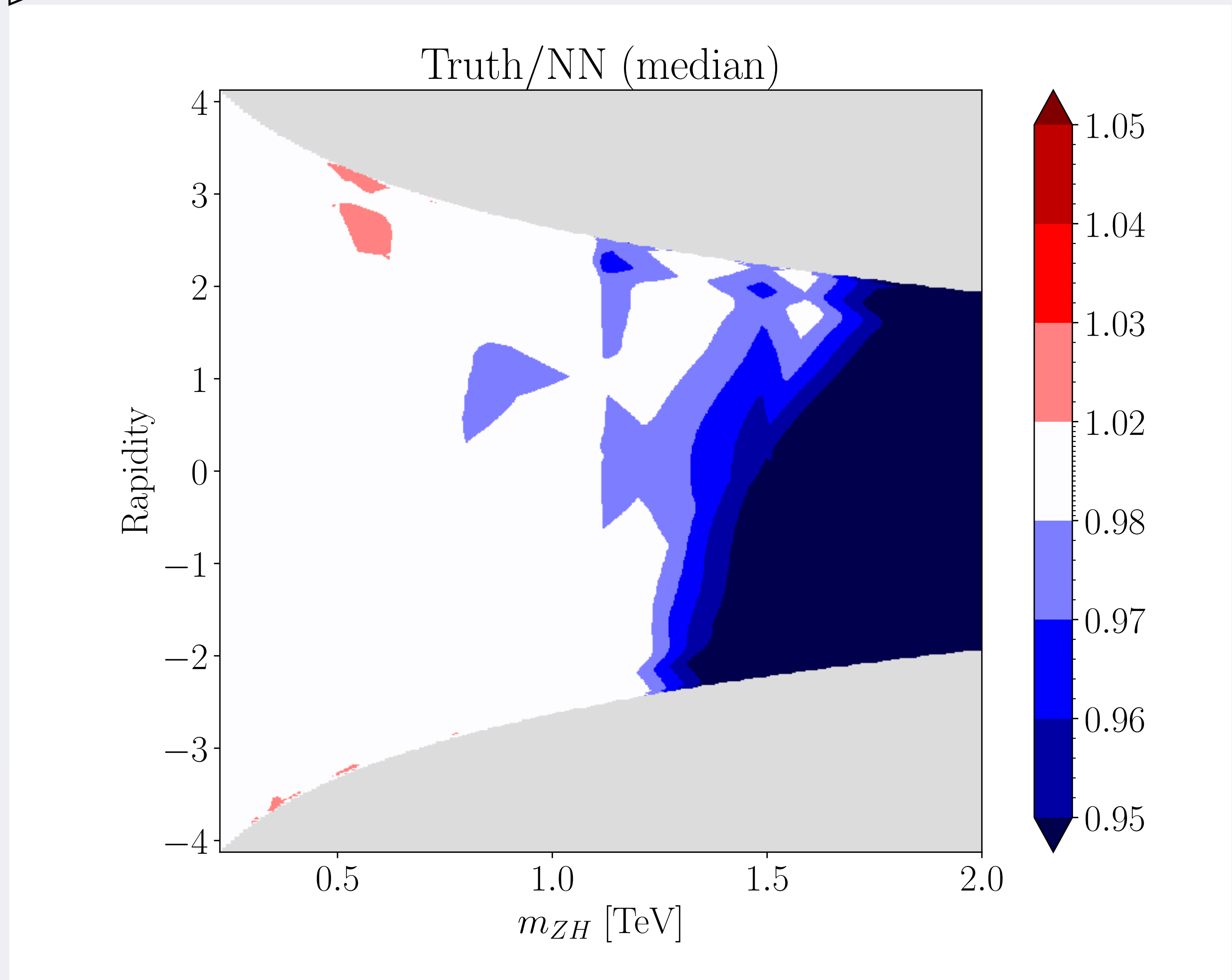
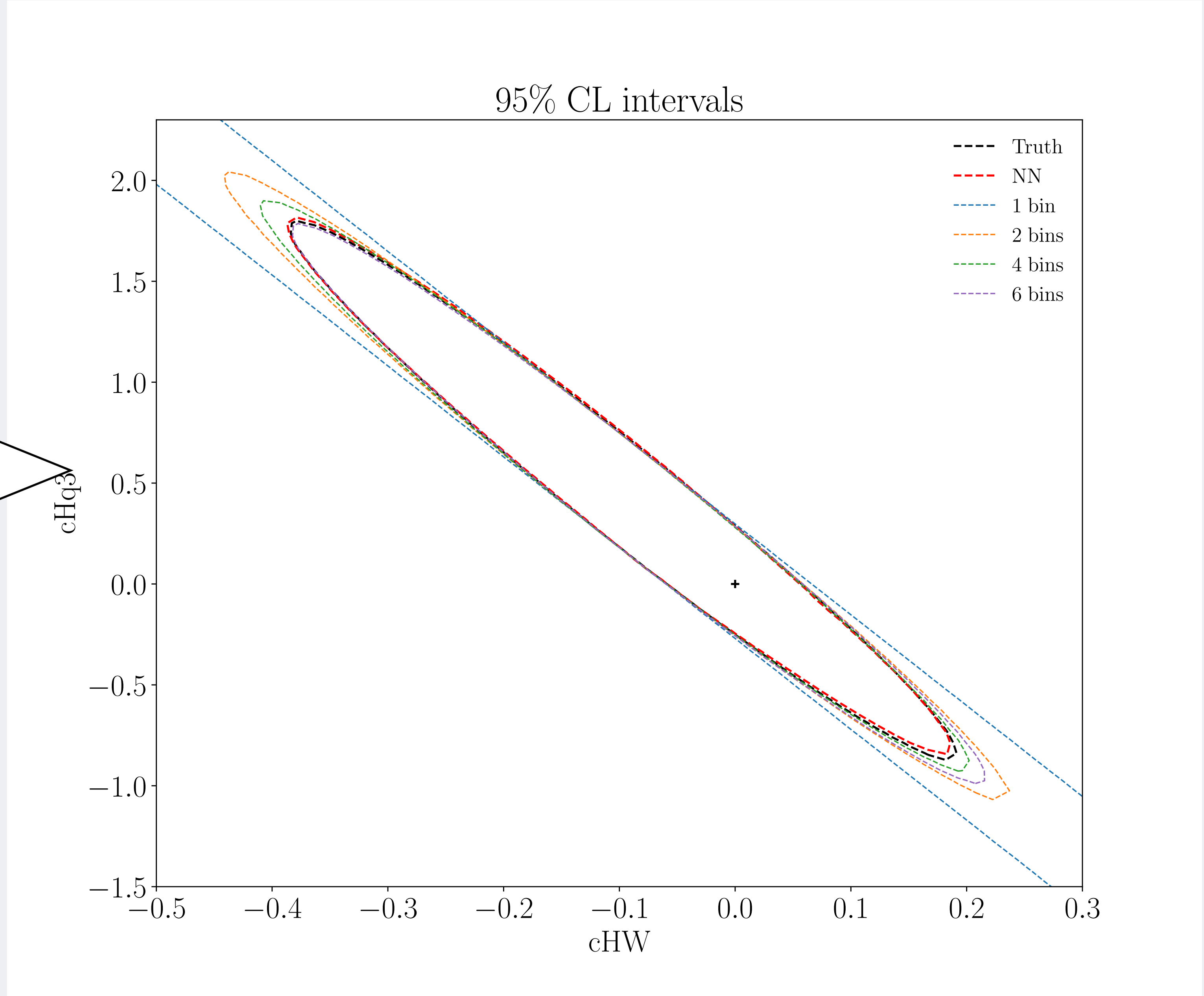




Inference



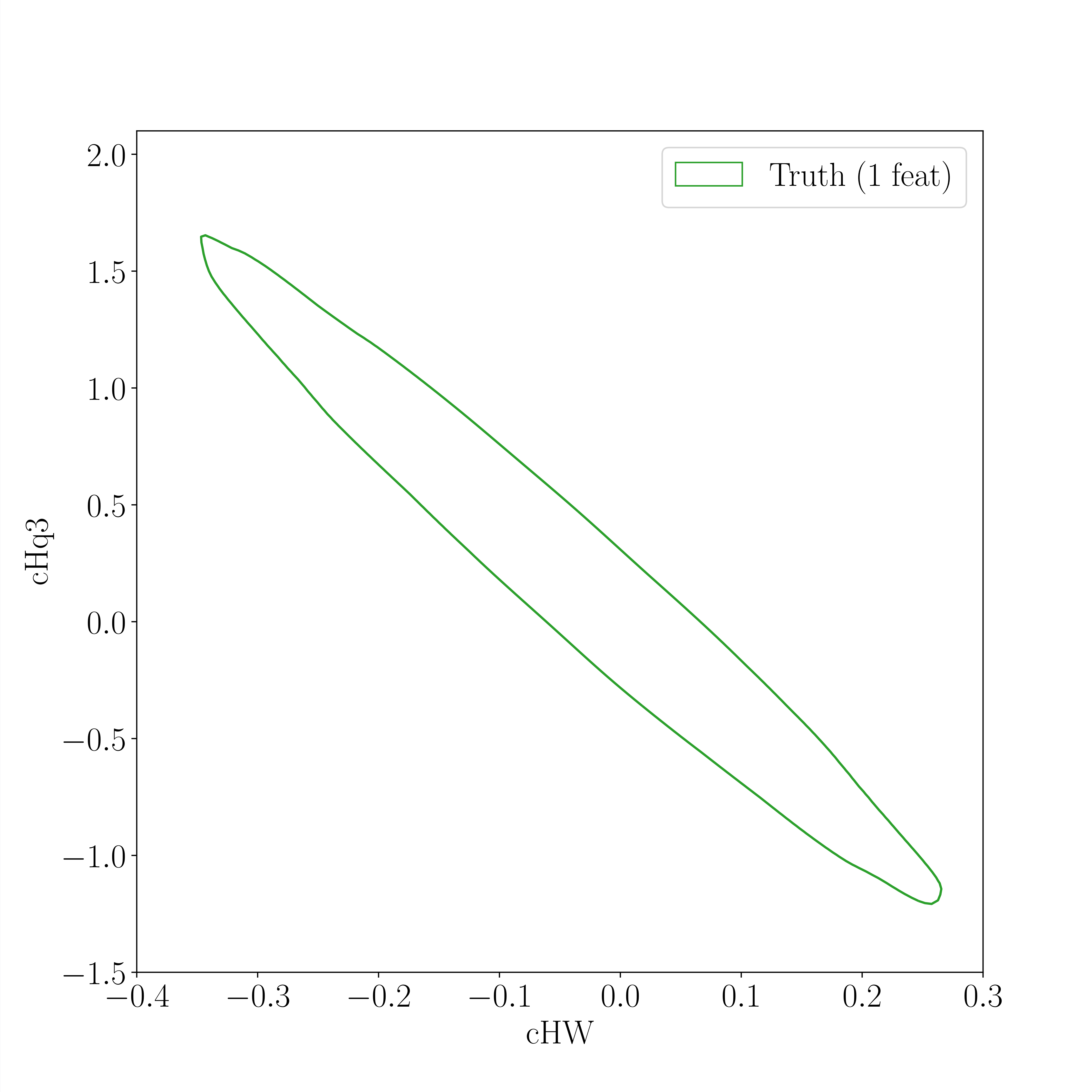
Profile Likelihood Ratio

$$q_c = 2\left[\nu(c) - \nu(\hat{c}) - \sum_i \log \frac{d\sigma(x_i, c)}{d\sigma(x_i, \hat{c})}\right]$$

Results

- Use **Nested Sampling** to draw posterior samples F. Feroz et al. [1306.2144]

- A ground truth analysis with a combination of either m_{ZH} , Y or p_T^Z performs best with **all features** included



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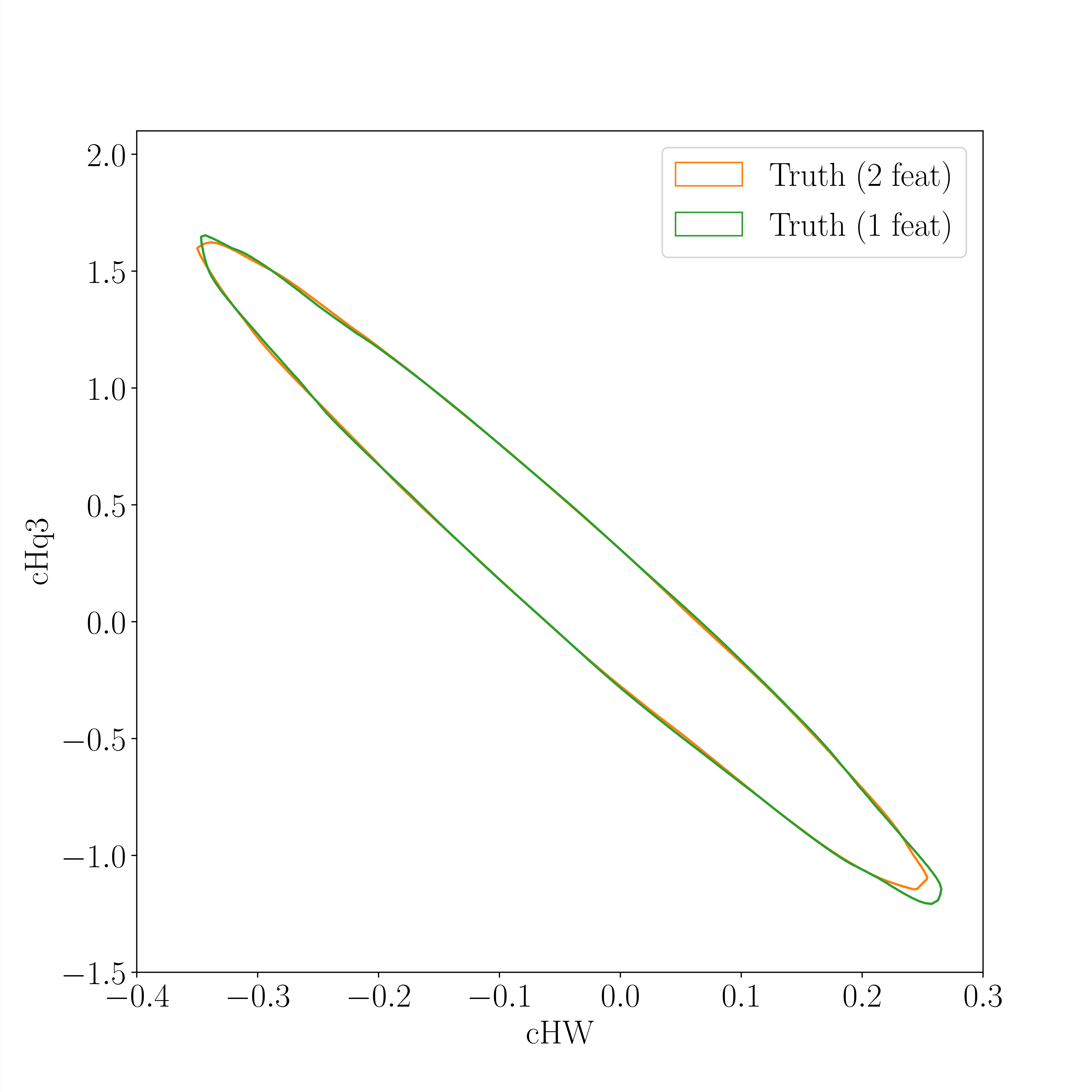

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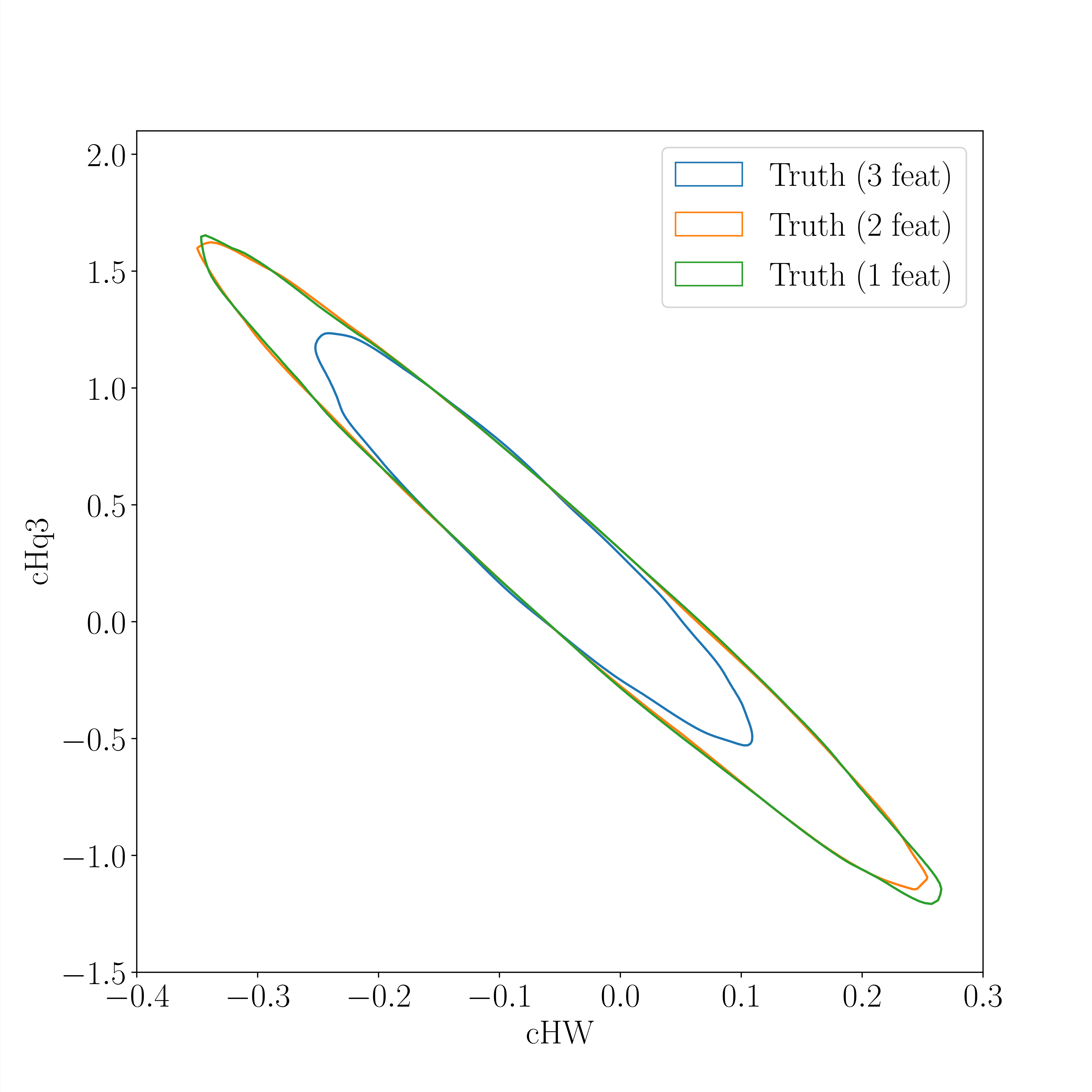

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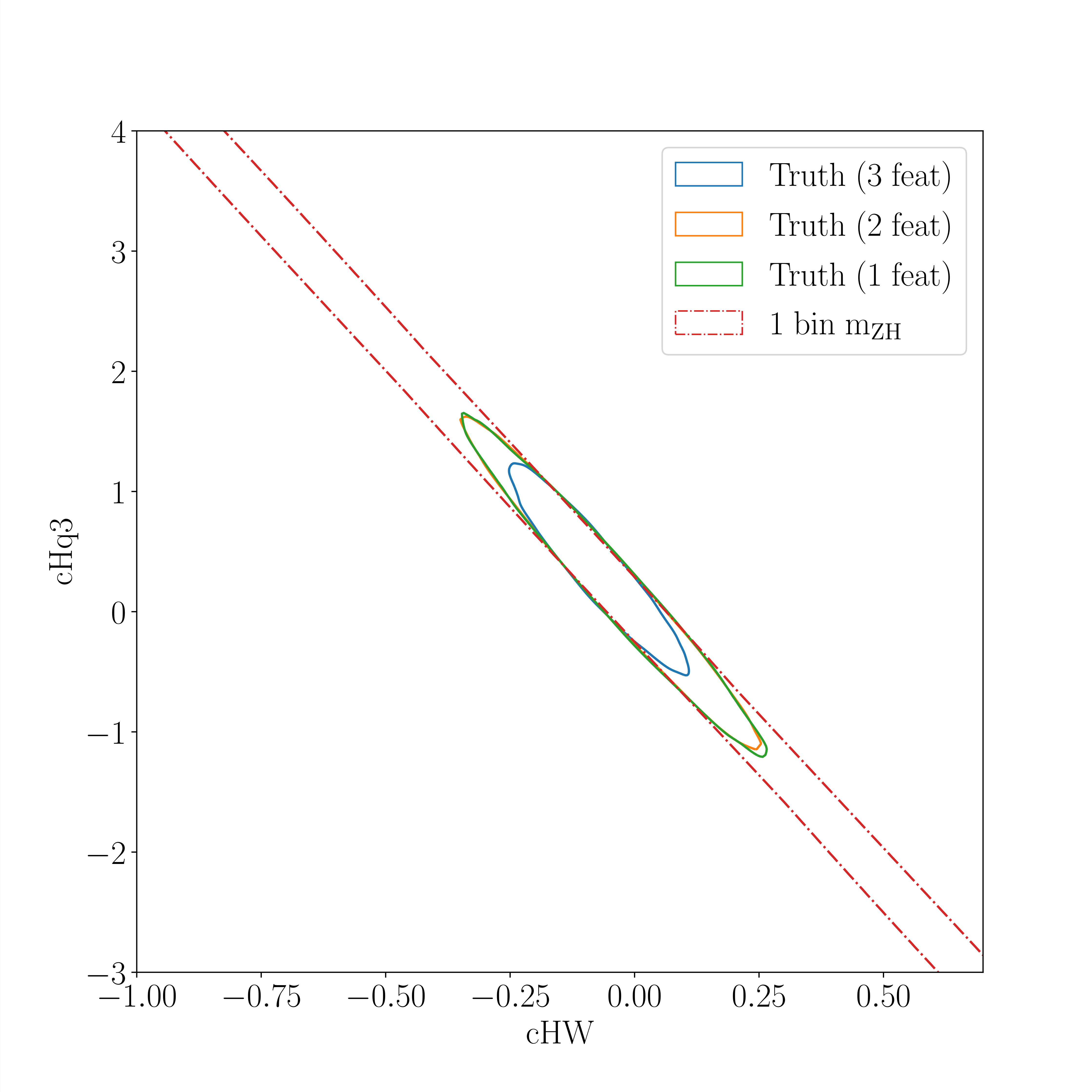

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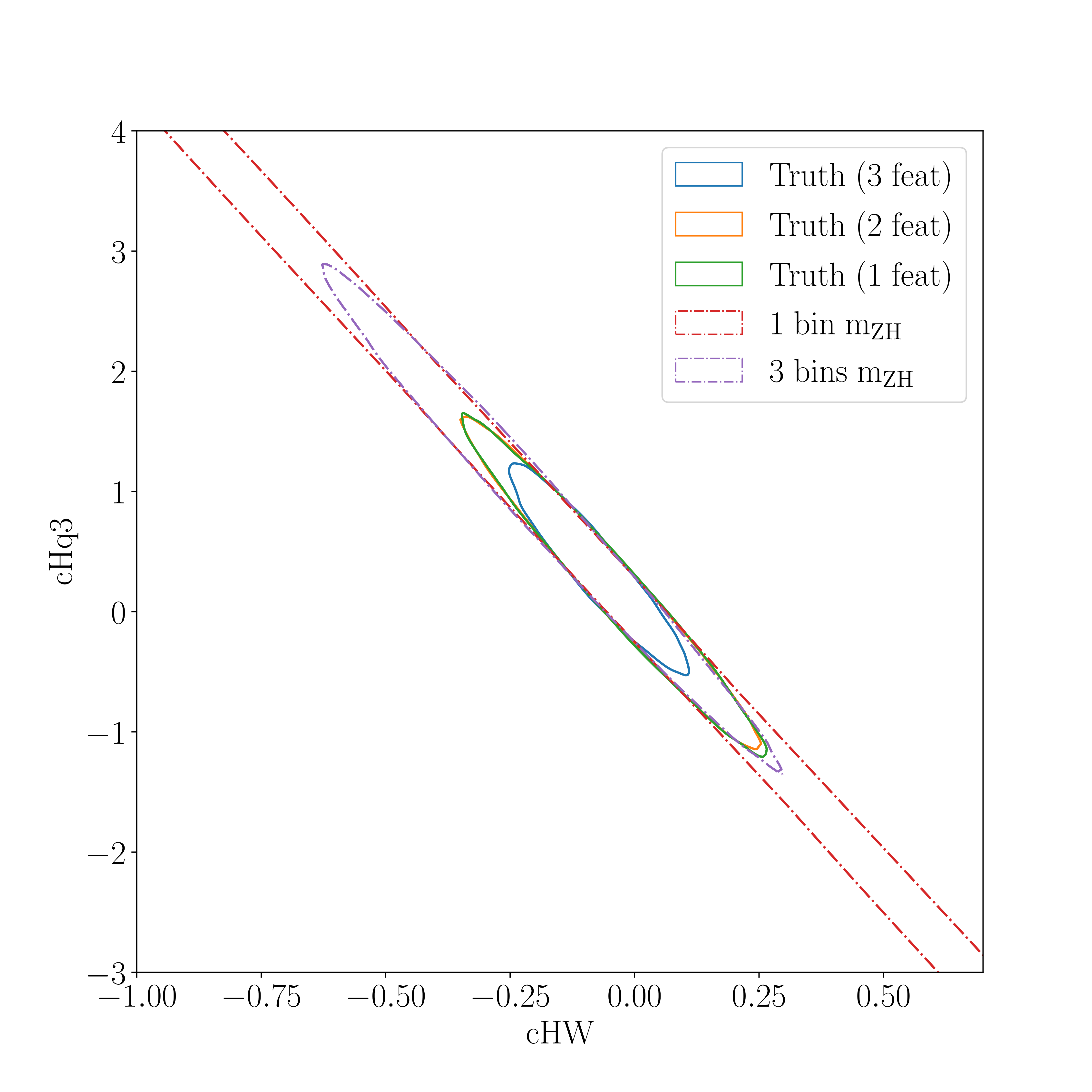

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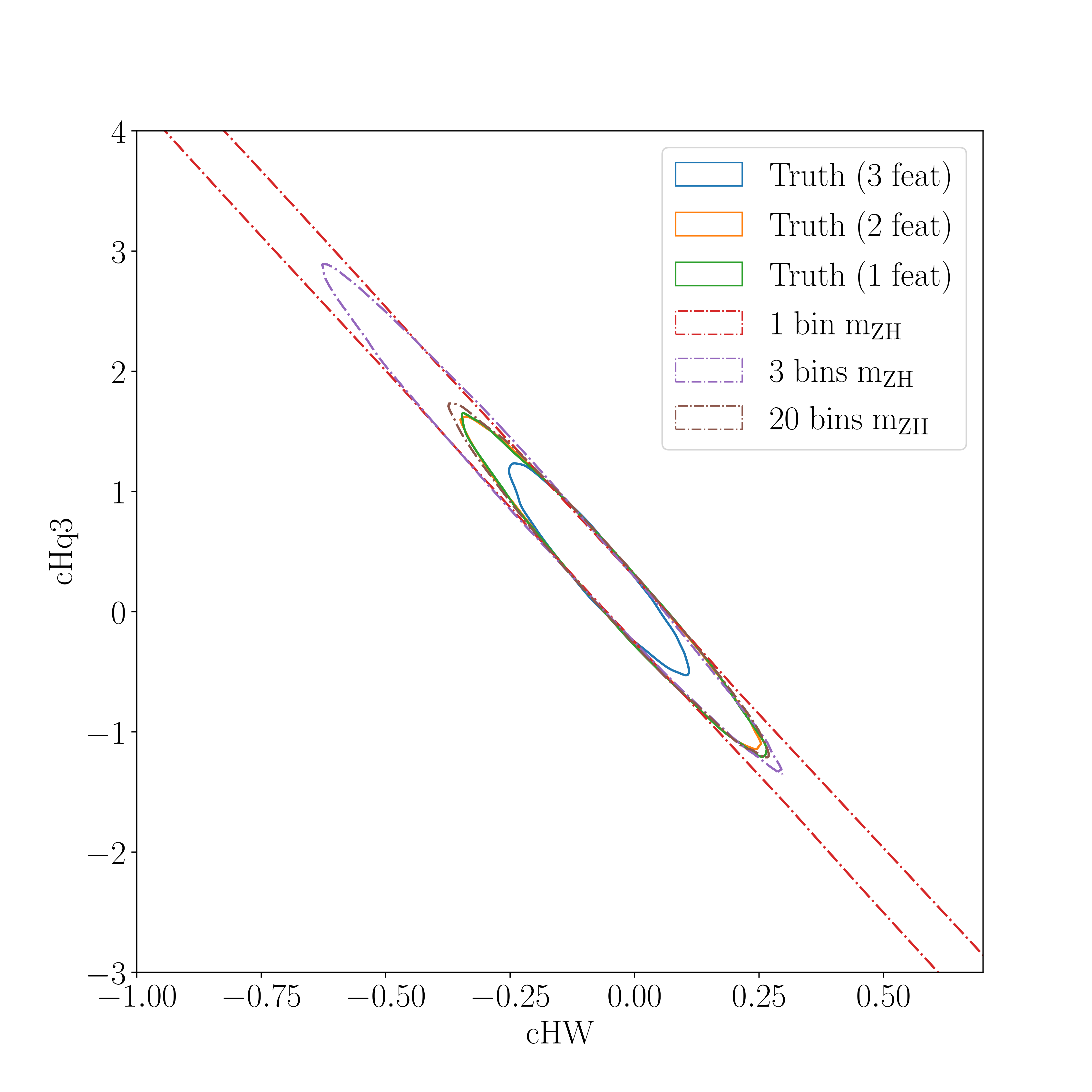

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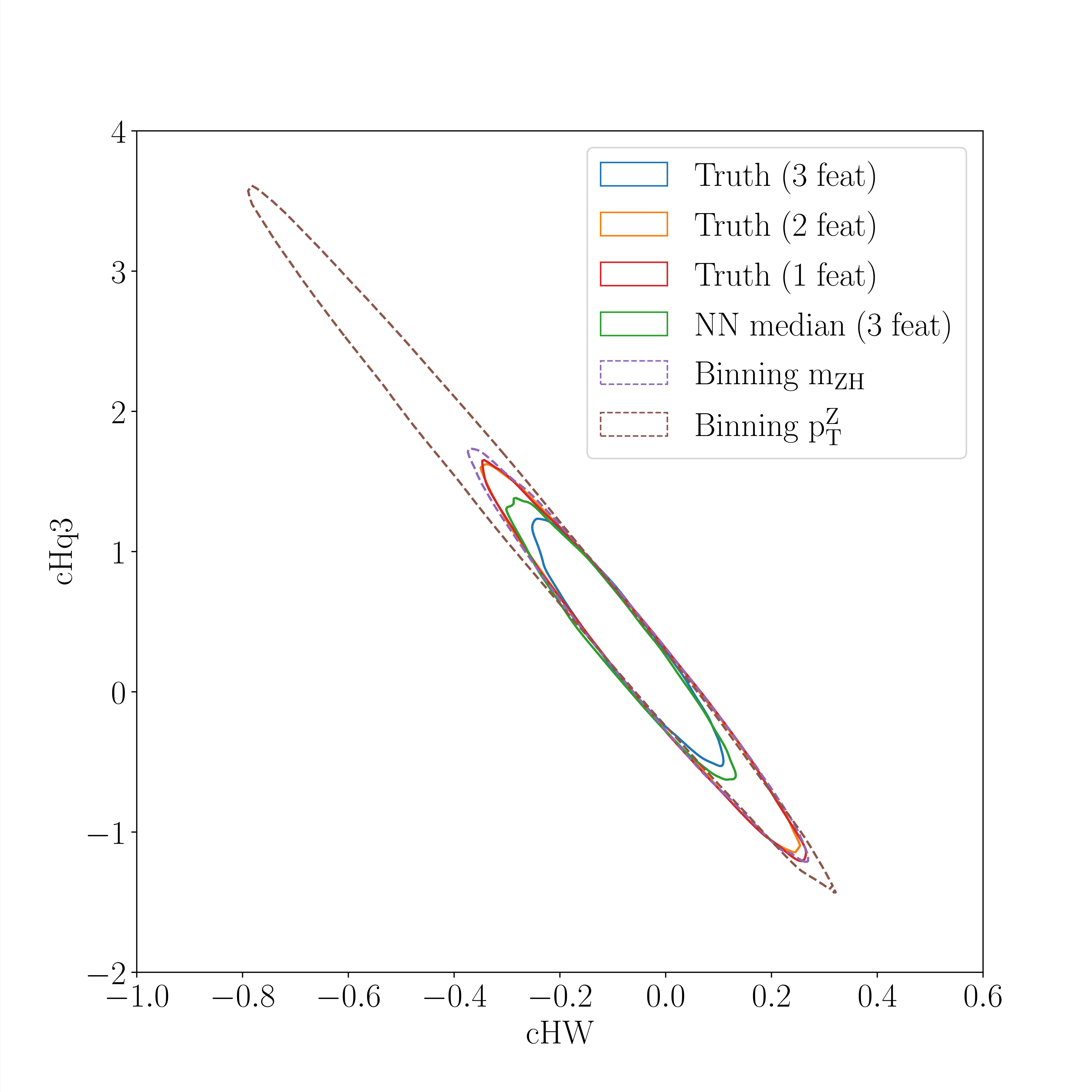

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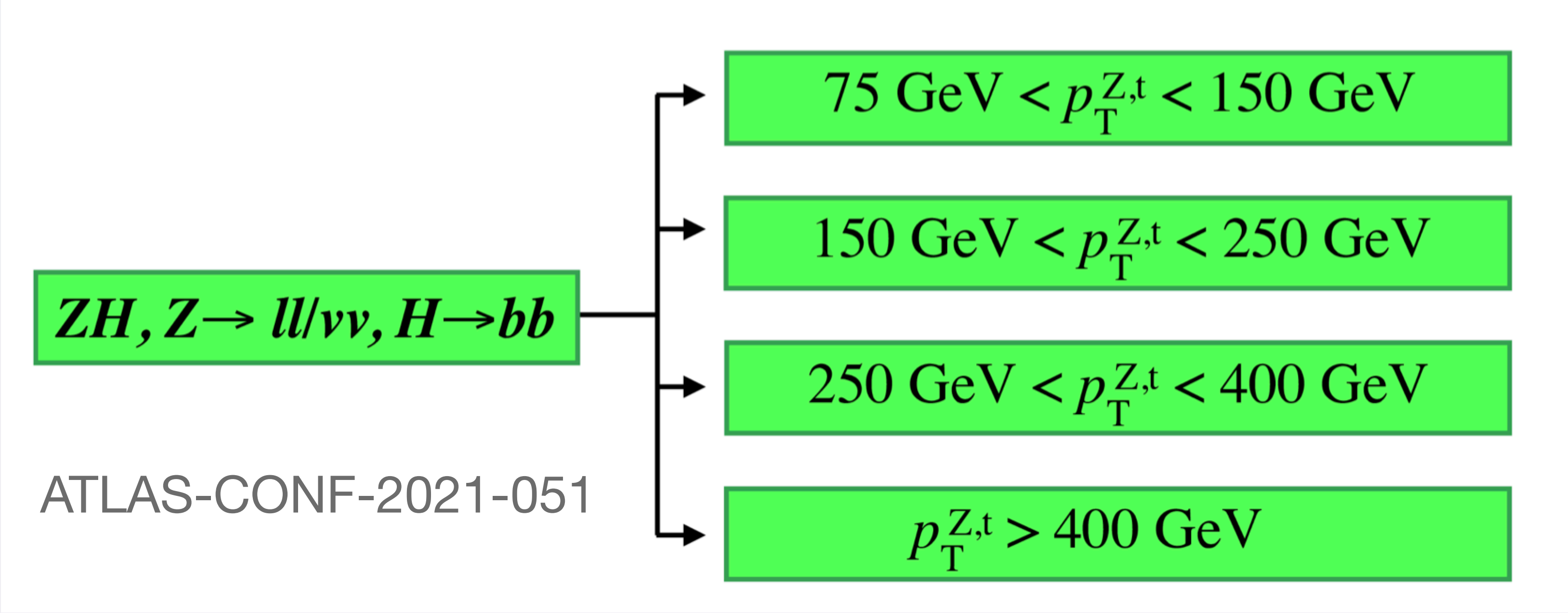

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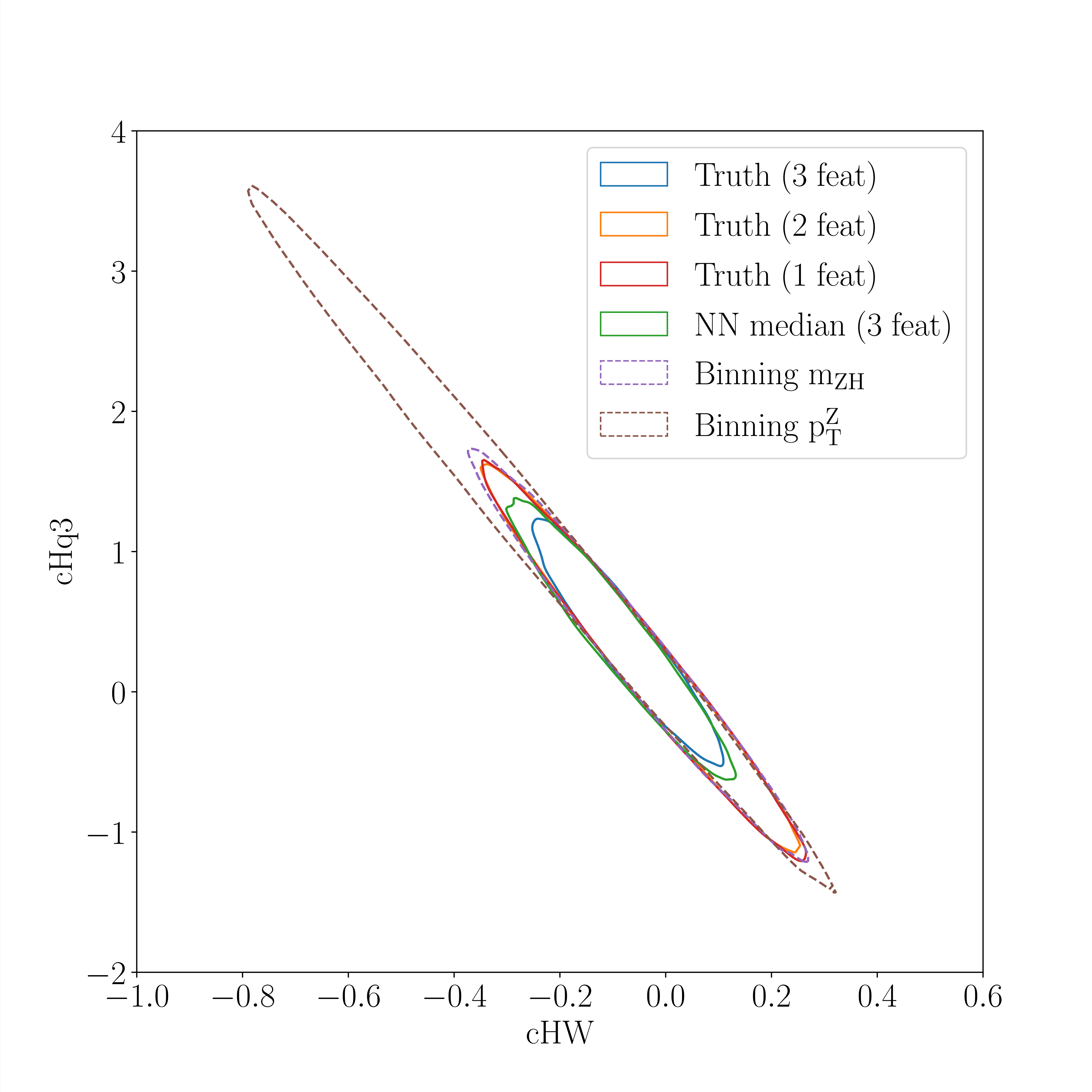

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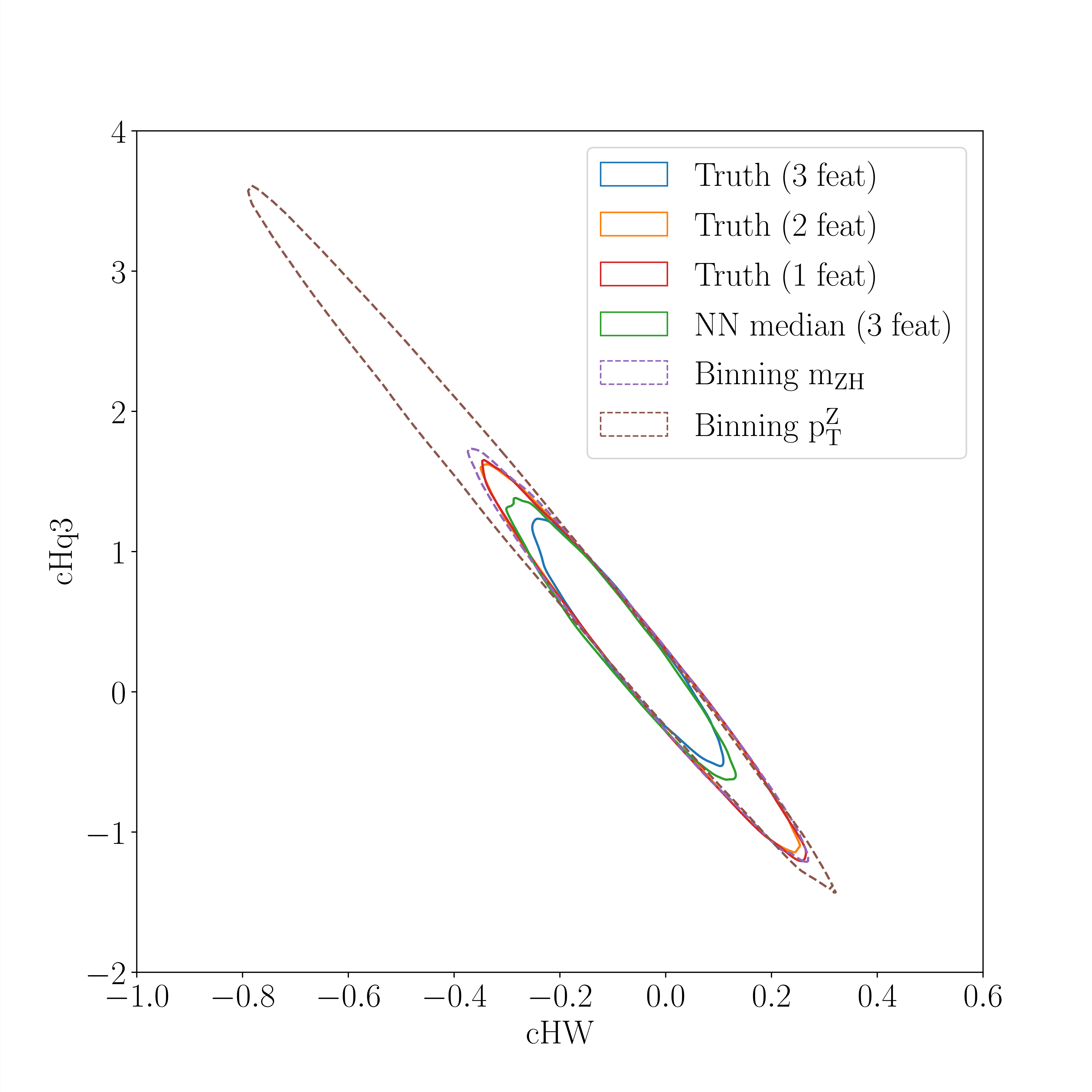

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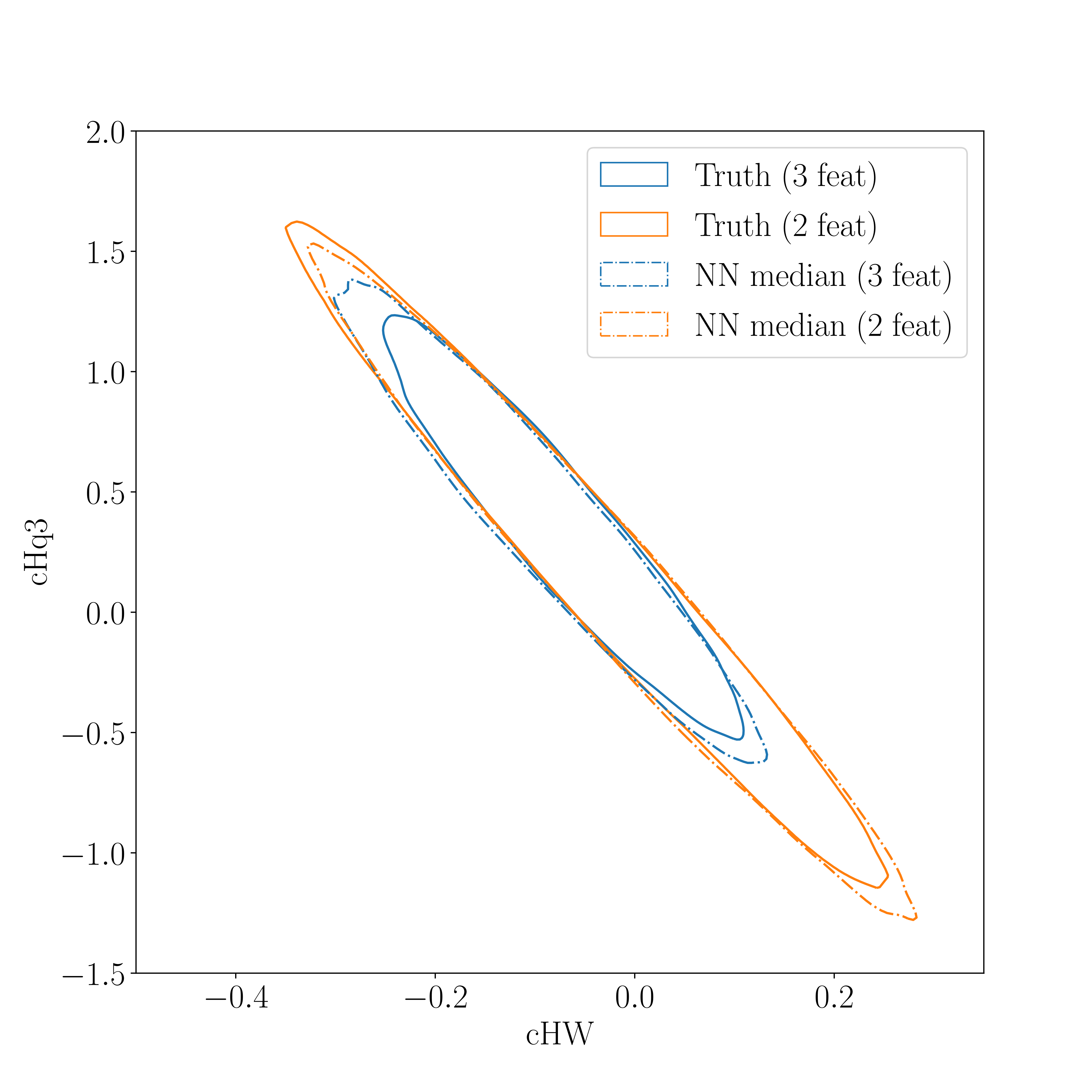
- NN reproduces the truth (**within uncertainties?**)

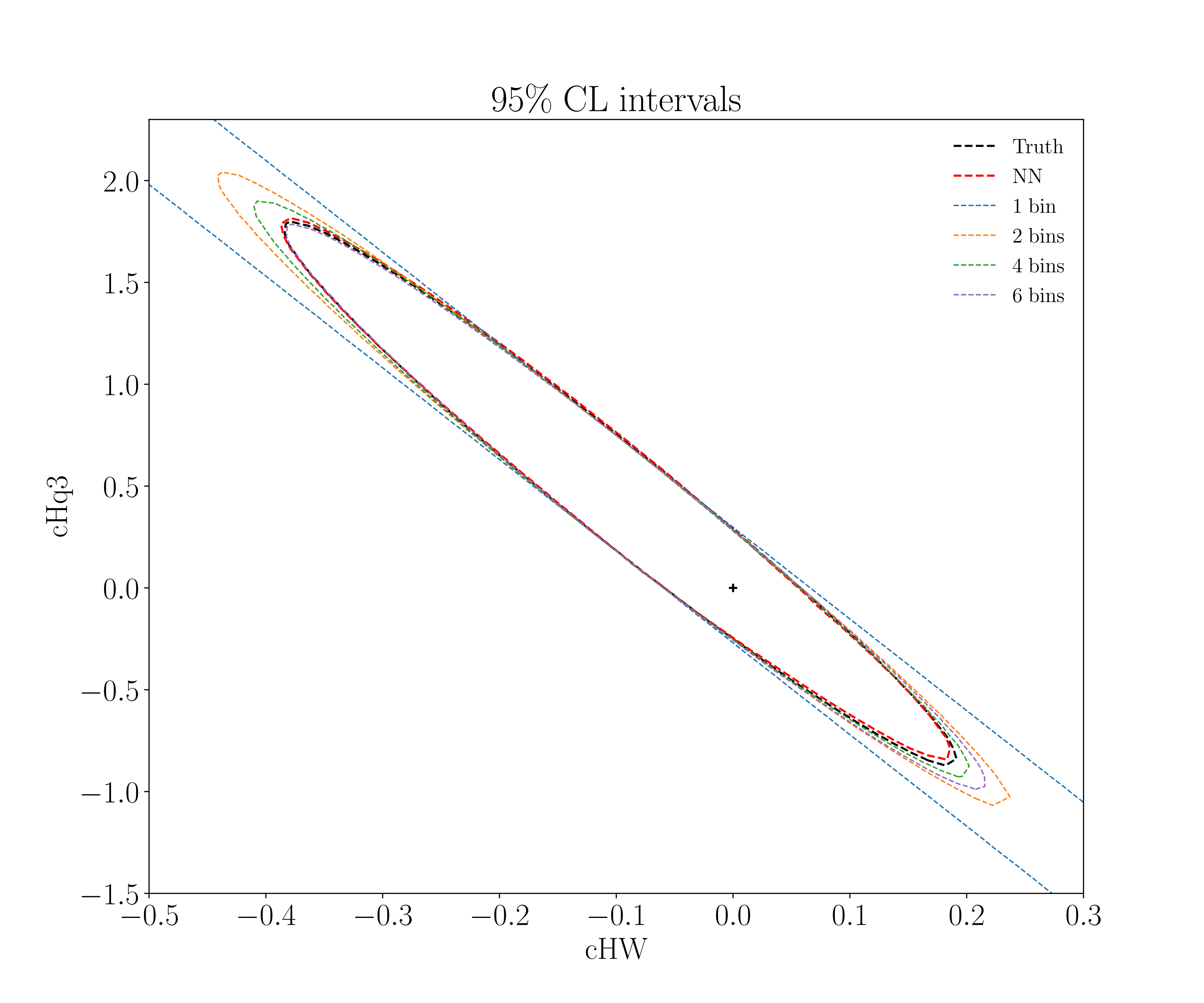


Conclusion and Outlook

- The SMEFT allows for a **model independent** framework to search for New Physics, taking **correlations** from different sectors into account

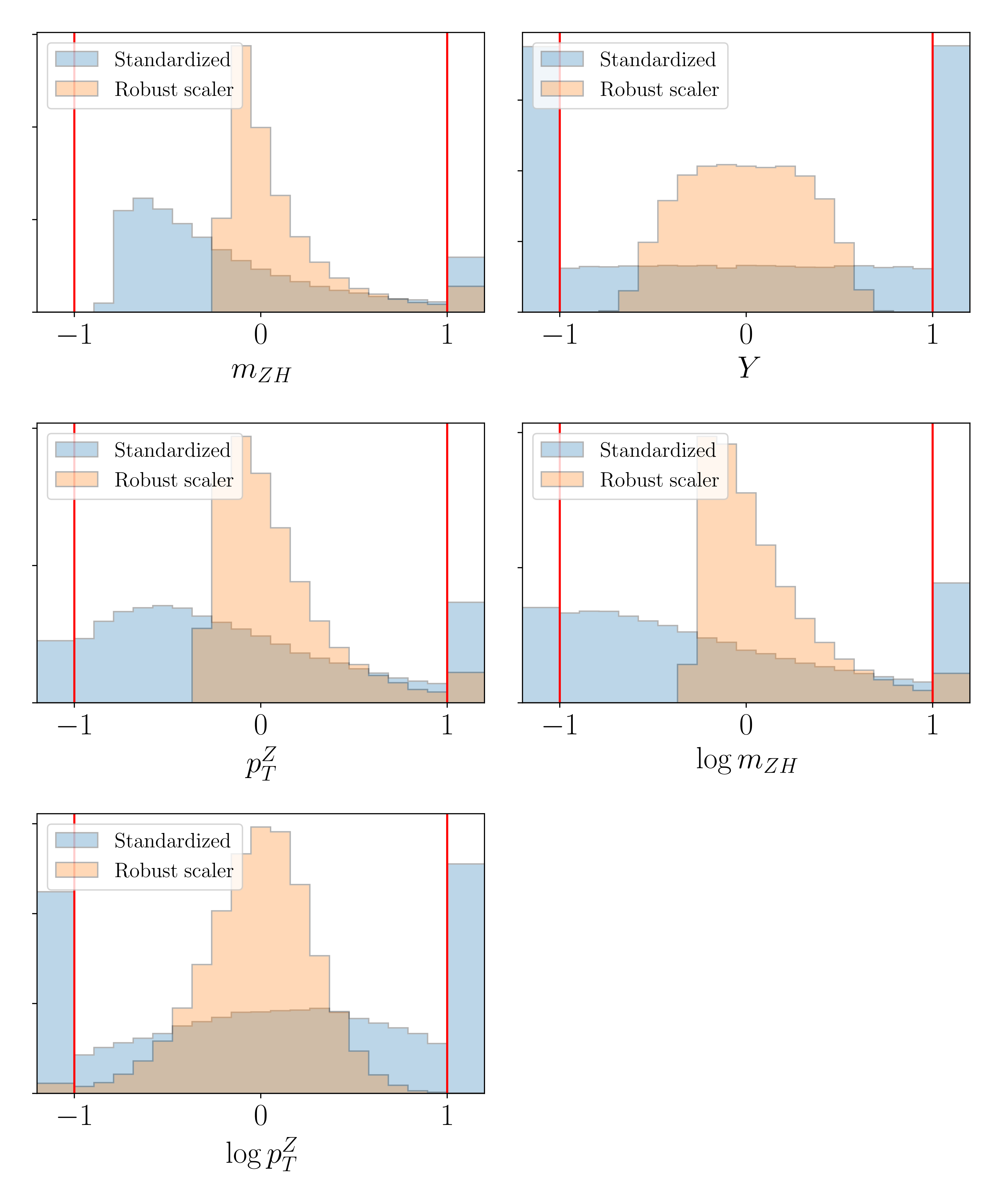
- Traditional SMEFT analyses can and should be **complemented** with **unbinned** measurements to optimise the constraining power on the Wilson coefficients

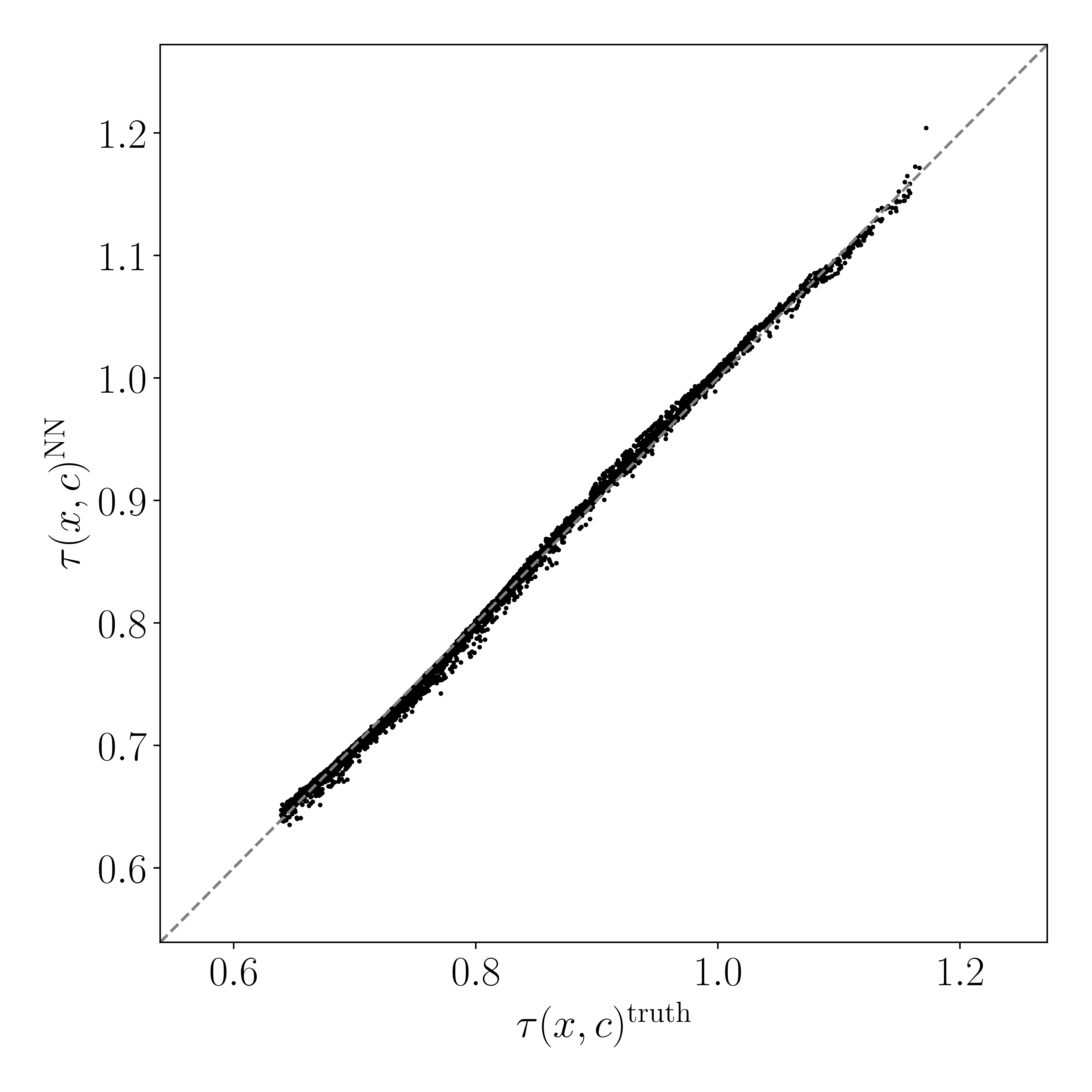
- Likelihood ratio parameterisations with NN are a **promising** way forward, and should be implemented in true global EFT fits

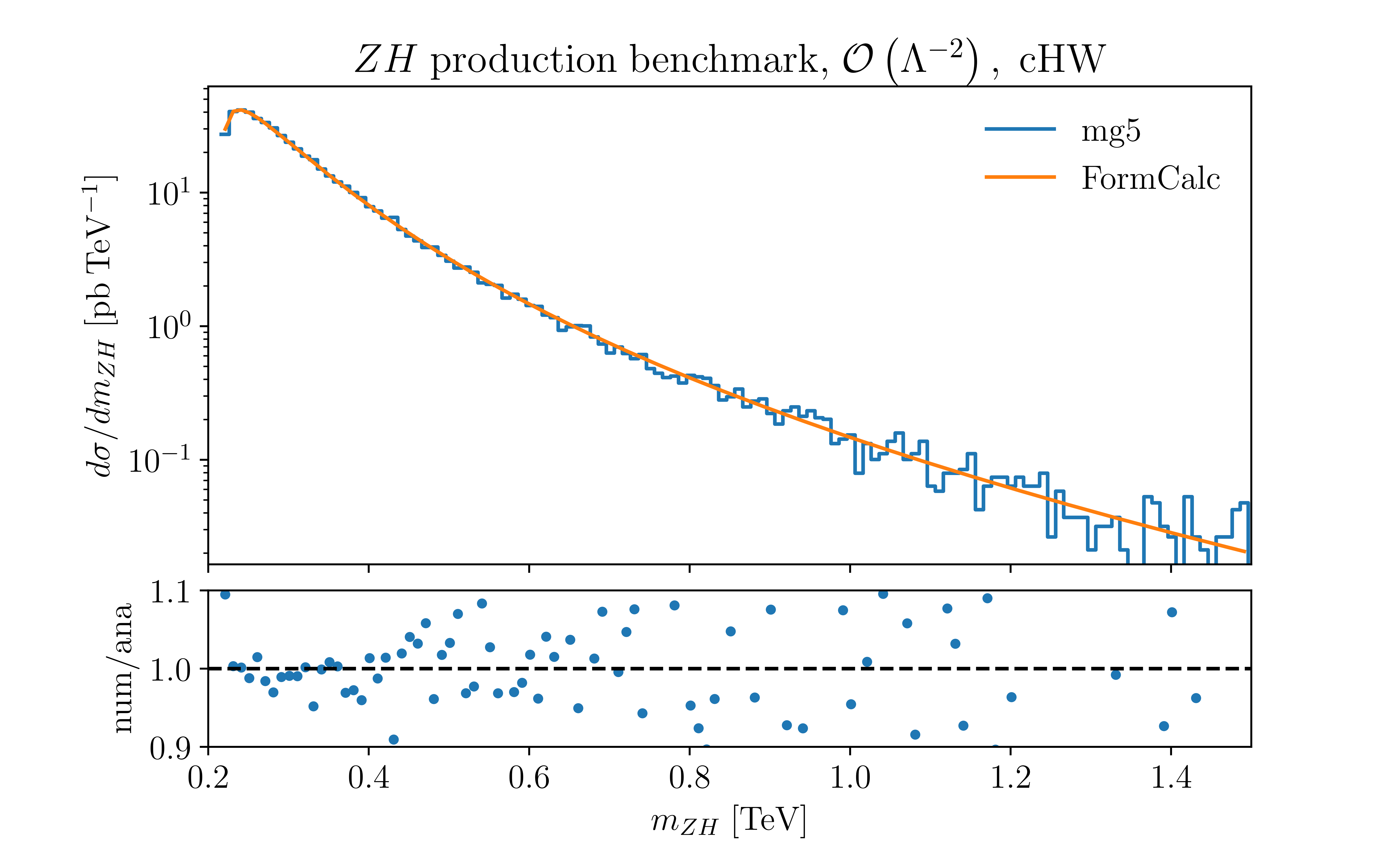


Thank you!
Questions?

Backup


Standardized
Robust scaler
−1
0
1
m_{ZH}
Y
p_T^Z
$\log m_{ZH}$
$\log p_T^Z$


1.2
1.1
1.0
0.9
0.8
0.7
0.6
$\tau(x,c)^{\mathrm{NN}}$
0.6
0.8
1.0
1.2
$\tau(x,c)^{\mathrm{truth}}$



ZH production benchmark, $\mathcal{O}\left(\Lambda^{-2}\right)$, cHW
mg5
FormCalc
$d\sigma/dm_{ZH}$ [pb TeV^{-1}]
10^{1}
10^{0}
10^{-1}
num/ana
1.1
1.0
0.9
0.2
0.4
0.6
0.8
1.0
1.2
1.4
m_{ZH} [TeV]