



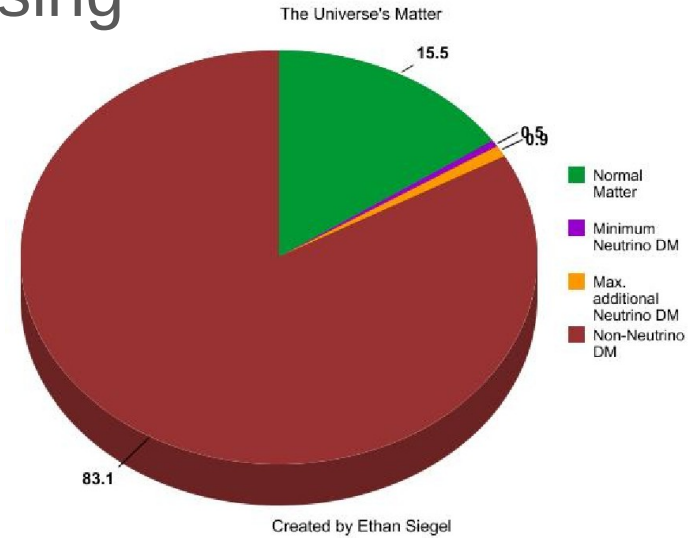
Unconventional thermal(-ish) dark matter

Anastasiia Filimonova

DIS2022
Santiago de Compostela

DM particles: intriguing and confusing

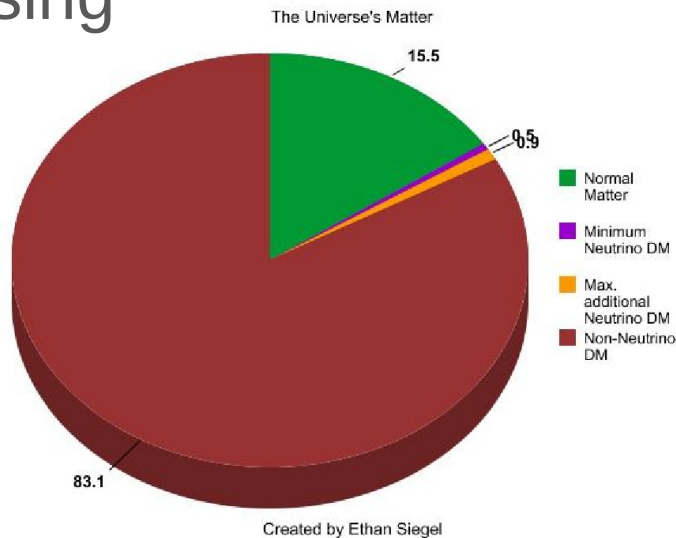
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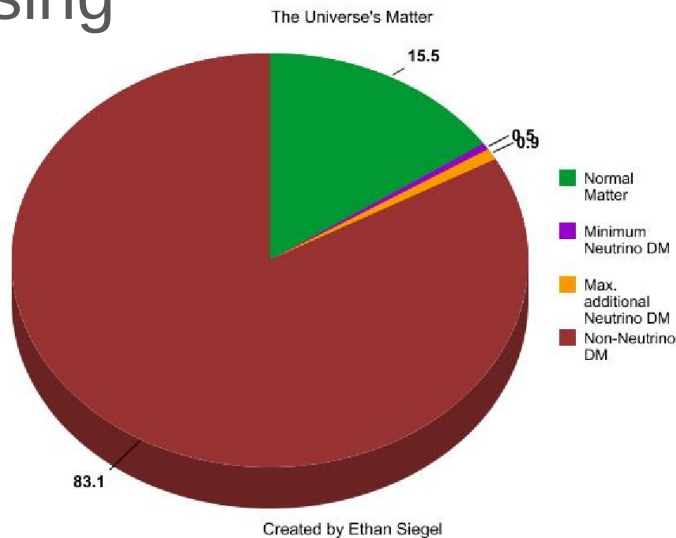
- Interact through **gravity**
- **Massive** (to cluster)
- If DM particles ever were relativistic – they should have **slowed down early** in the history of the Universe
- **Electrically neutral** (do not interact with photons)
- **Stable** on cosmological scales



DM particles: intriguing and confusing

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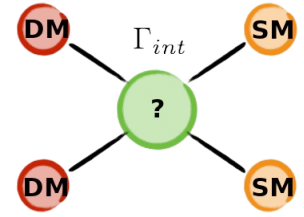
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What we don't know:

- Other interactions?
- Mass, spin....?
- Several species?

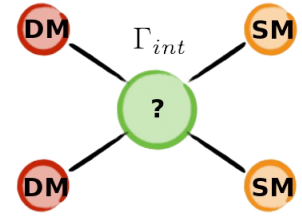
Why the electroweak scale?



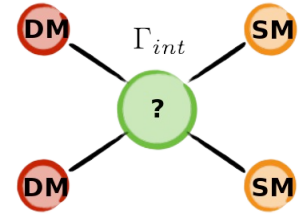
Why the electroweak scale?

Assumption:

thermal production



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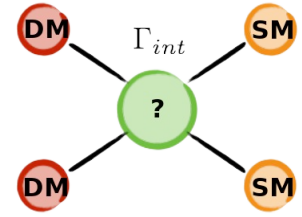


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$\Gamma_{int} > H$ In equilibrium with plasma

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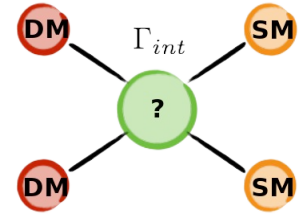
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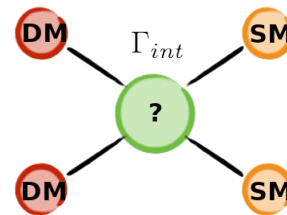
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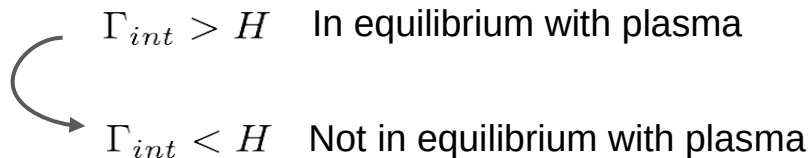
Relic abundance



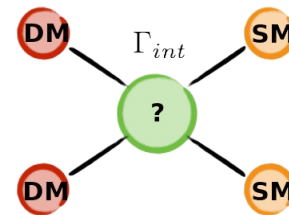
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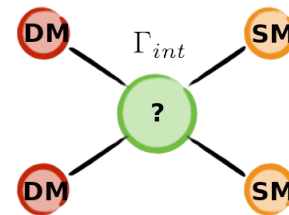
Relic abundance



Very simplified WIMPS

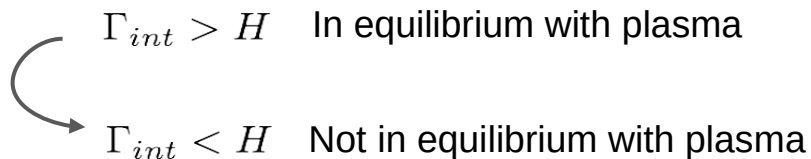
One new "heavy" particle

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Relic abundance

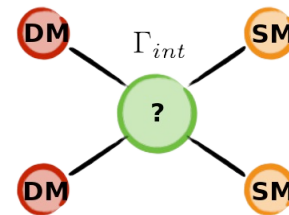
Very simplified WIMPS

One new "heavy" particle

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{m_{DM}^2} + n_{DM} \langle \sigma v \rangle \sim H$$

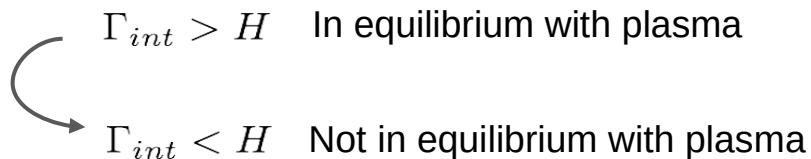
at decoupling

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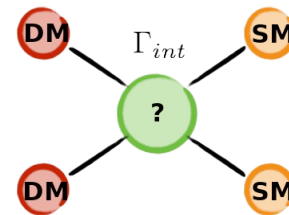
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at decoupling

$$\Omega_{DM} h^2 \sim \underbrace{0.12}_{\text{Planck}} \frac{10^{-26} \text{cm}^3 \text{s}^{-1} (\text{or } 10^{-9} [\text{GeV}]^{-2})}{\langle \sigma v \rangle}$$

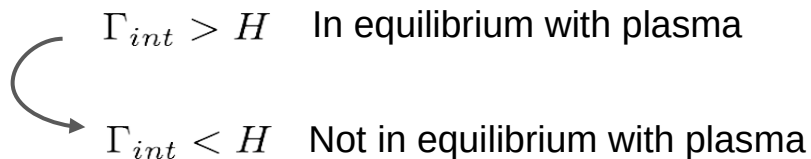
Planck

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→ Relic abundance

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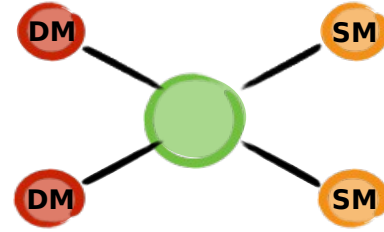
"Natural" choice:

$$\alpha \sim \alpha_{EW}$$

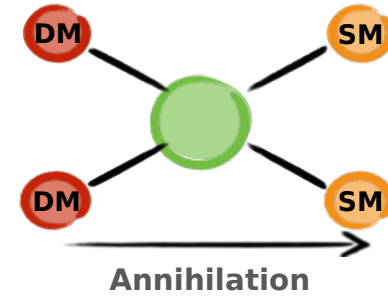
$$m_{DM} \sim O(100 \text{GeV})$$

And all these constraints...

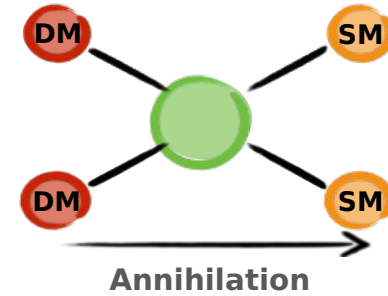
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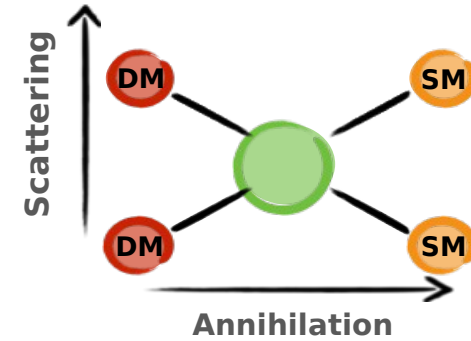


Annihilation (indirect detection)



Fermi satellite

And all these constraints...



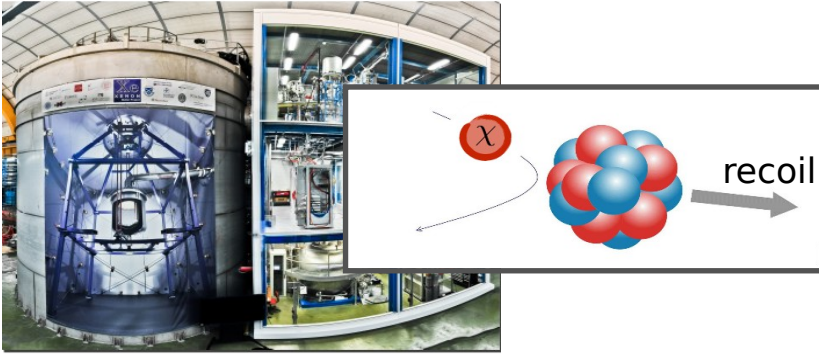
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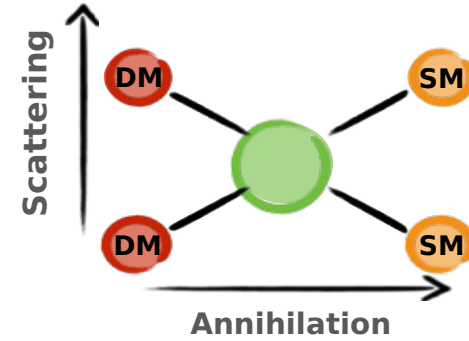
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And all these constraints...

Scattering (direct detection)



Xenon experiment



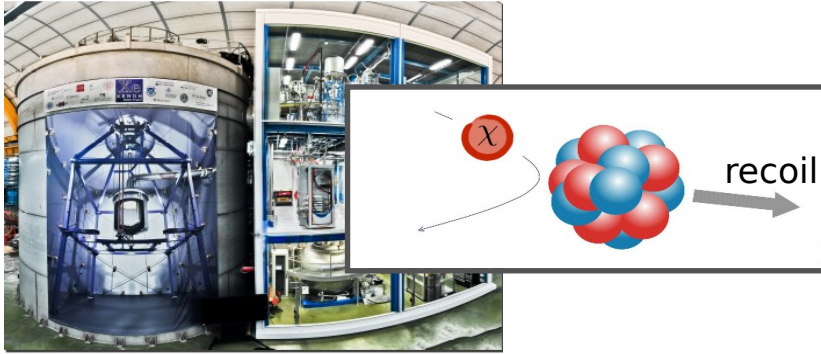
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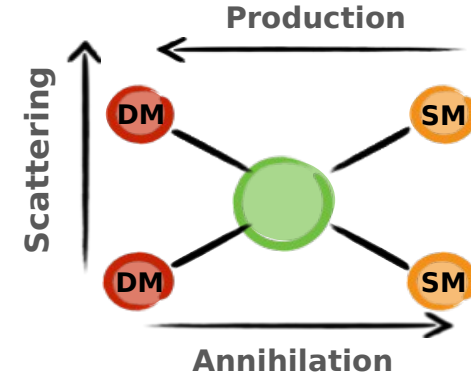
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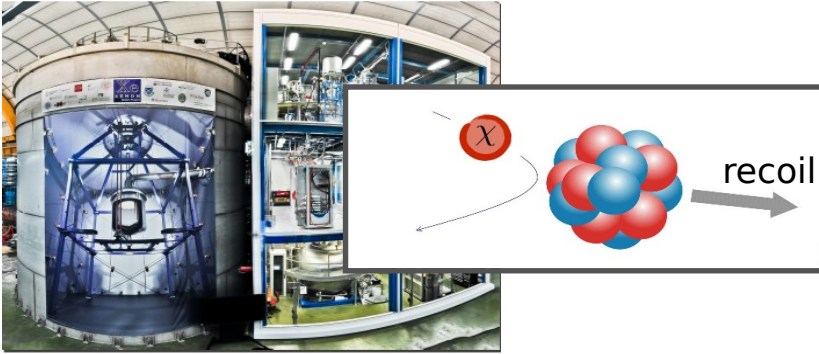
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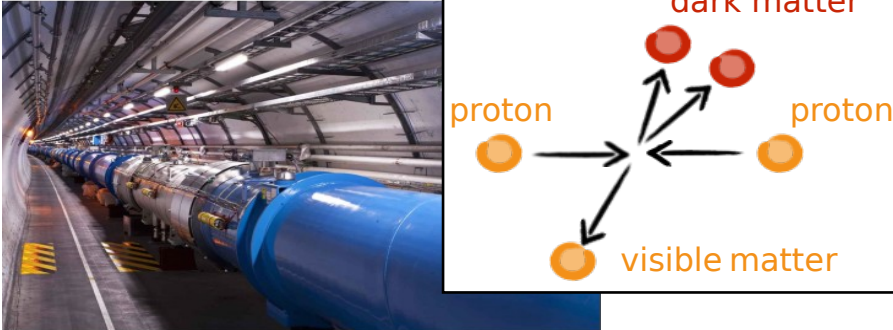
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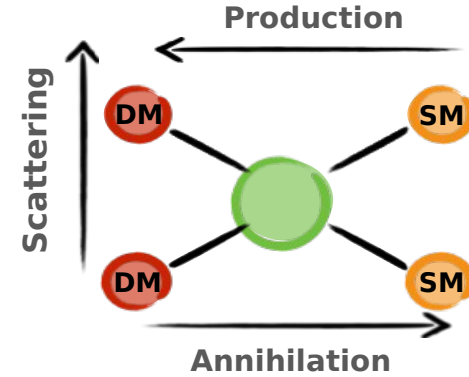


Xenon experiment

Production



Large Hadron Collider

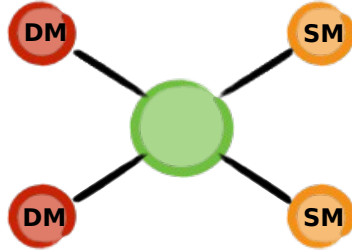


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Conventional (WIMP) candidates under tension



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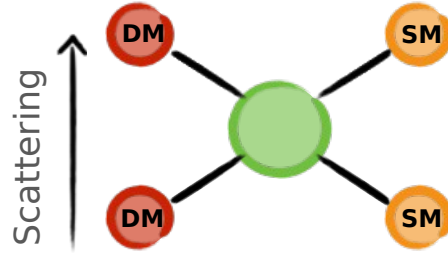
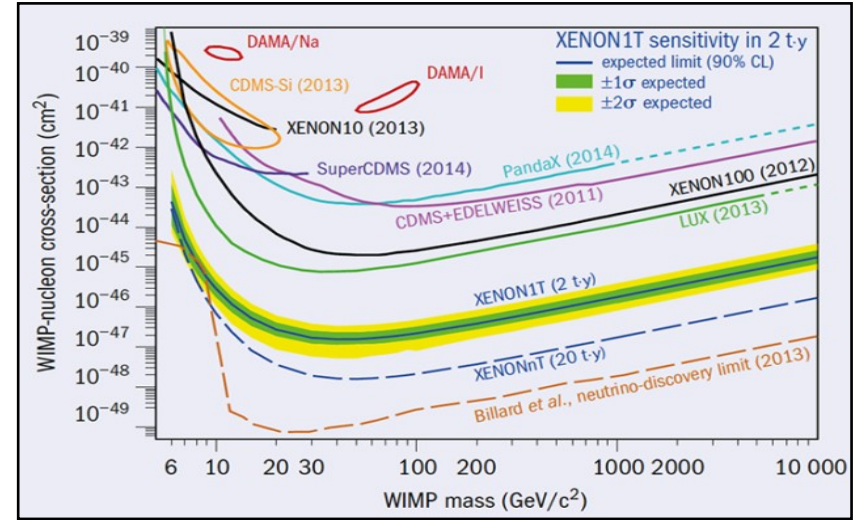
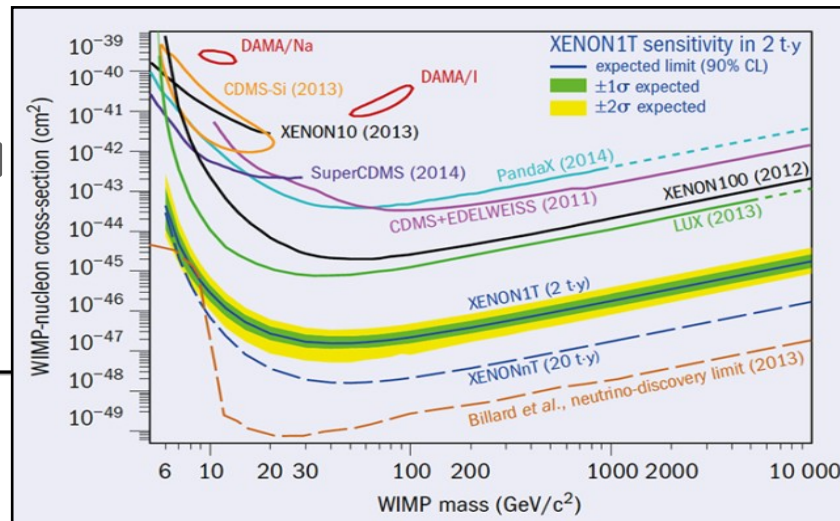
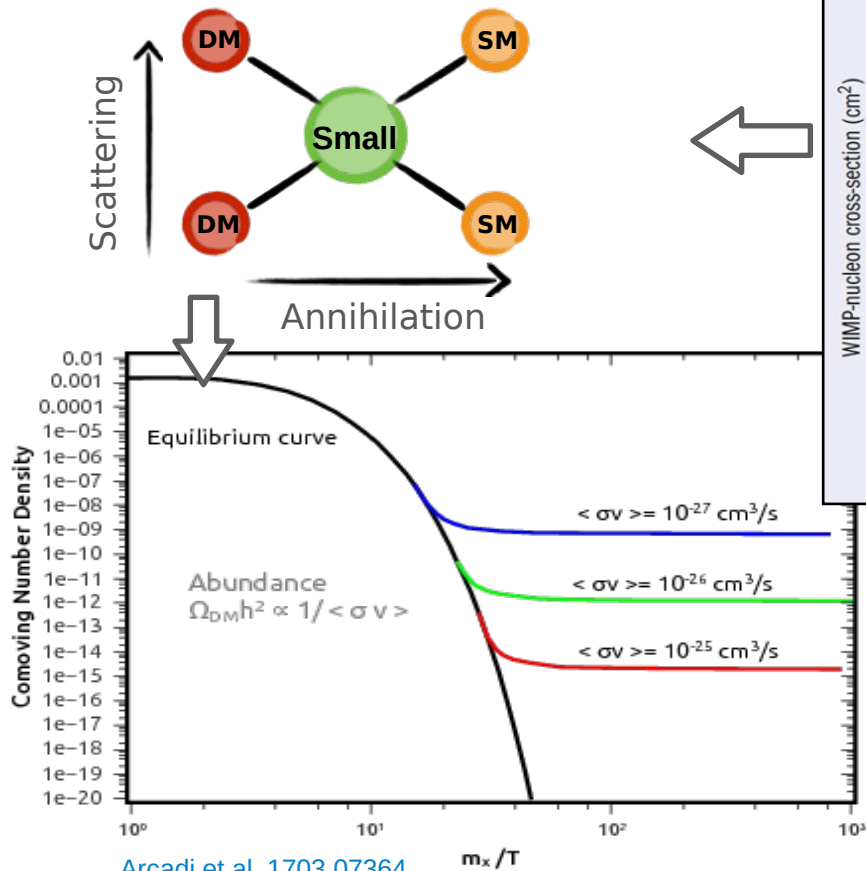


Image: XENON Collaboration



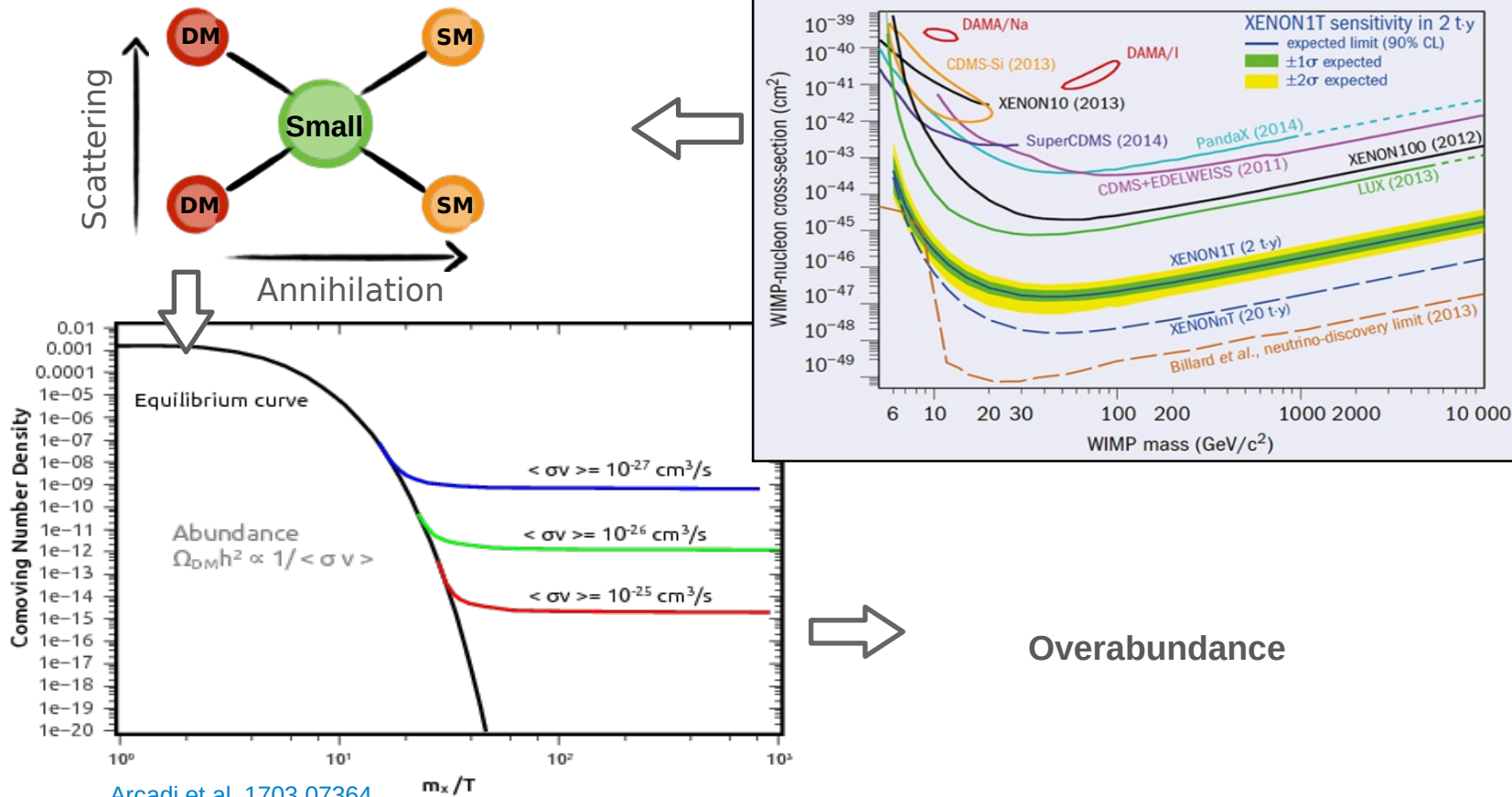
Conventional (WIMP) candidates under tension

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Alternatives?

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Light: $m < \text{few GeV}$

Direct detection

e.g. Essig et al. [1509.01598], [1907.07682]

Colliders + beyond

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Telescopes

e.g. Antares: [1912.05296],
CTA: Rinchiuso et al. [2008.00692],
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Non-thermal

Freeze-in

e.g. Hall et al. [0911.1120], Bélanger et al. [1811.05478]

Early kinetic decoupling, **coscattering**

e.g. Binder et al. [1706.07433], D'Agnolo et al. [1705.08450]

Different mass-coupling relation?

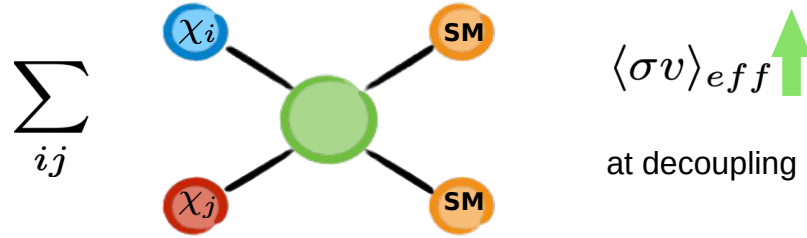
WIMPlless miracle, coannihilation, forbidden dark matter

e.g. Feng & Kumar [0905.3039], Griest & Seckel 1991, D'Agnolo & Ruderman [1505.07107]

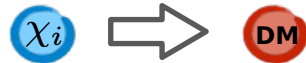
From coannihilation to cospattering (inelastic scattering)

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Coannihilation
Griest & Seckel 1991



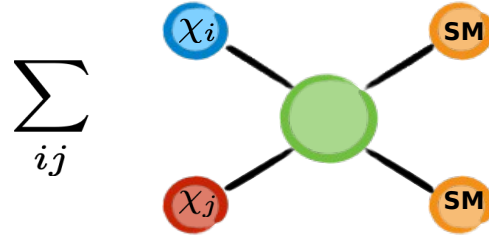
But eventually




No Direct detection signal

From coannihilation to cospattering (inelastic scattering)

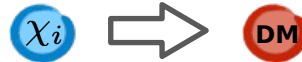
Coannihilation
Griest & Seckel 1991



$\langle\sigma v\rangle_{eff}$ 
at decoupling

$$n_{dark}\langle\sigma v\rangle_{eff} \sim H$$

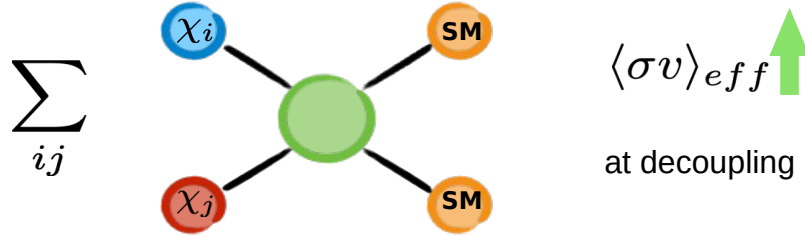
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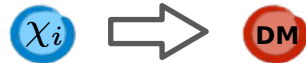
Coannihilation
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Boltzmann suppressed

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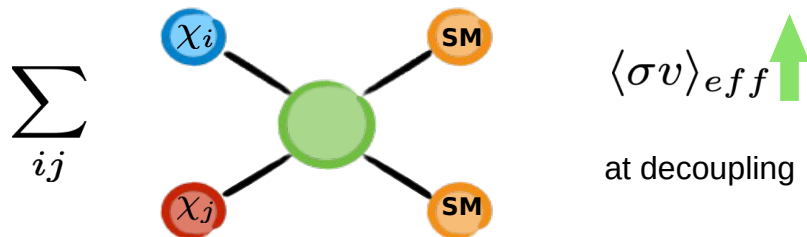


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Cospattering

(inelastic scattering)

Not always thermal

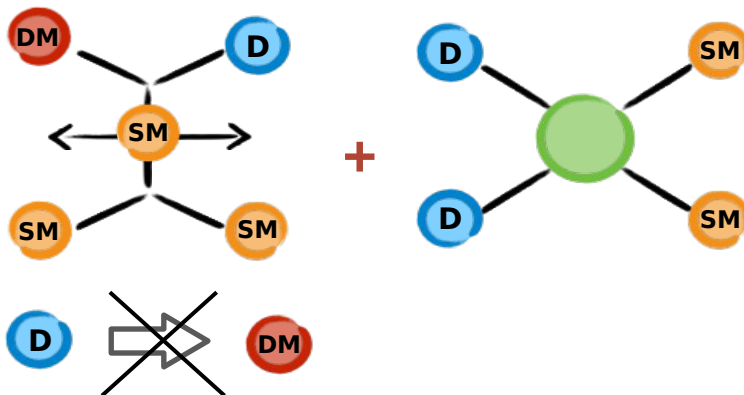
(early kinetic decoupling)

D'Agnolo et al. [1705.08450]

AF & S. Westhoff [1812.04628]

AF, S Junius, LL Honorez & S.

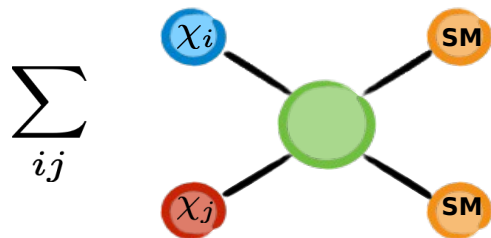
Westhof [2201.08409]



From coannihilation to cospattering (inelastic scattering)

Coannihilation

Griest & Seckel 1991



$$\langle \sigma v \rangle_{eff} \uparrow$$

at decoupling

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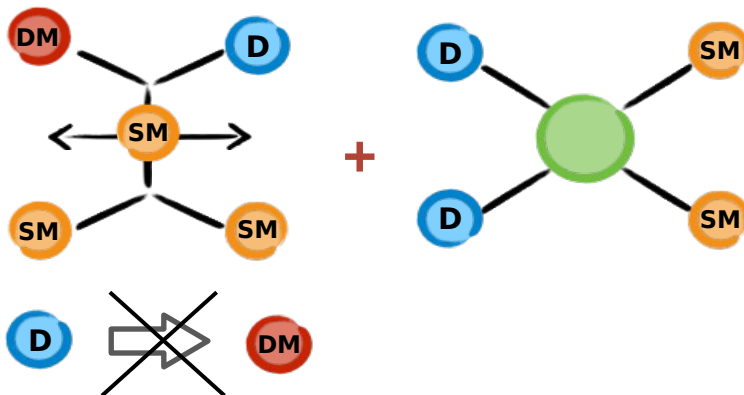
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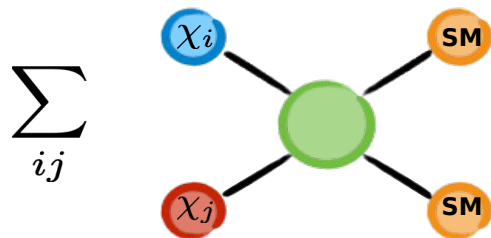



$$n_{SM} \langle \sigma v \rangle_{DM \rightarrow M} \sim H$$

From coannihilation to cospattering (inelastic scattering)

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$\langle \sigma v \rangle_{eff}$ 

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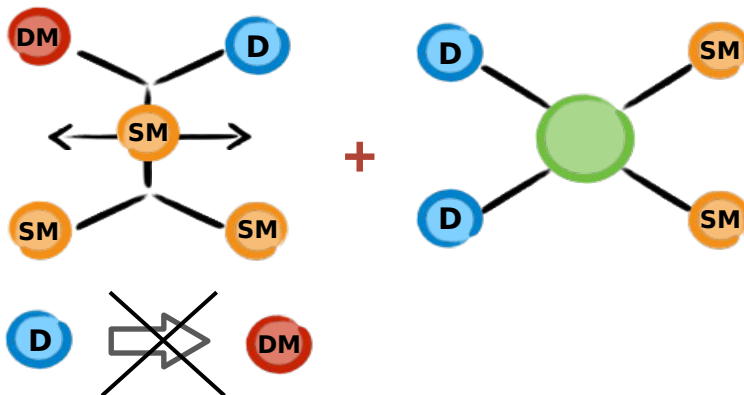
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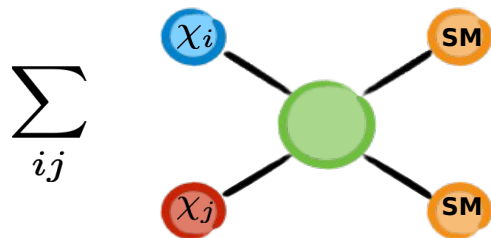
$$\textcircled{n_{SM}} \langle \sigma v \rangle_{DM \rightarrow M} \sim H$$

Very high

From coannihilation to cospattering (inelastic scattering)

Coannihilation

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$\langle \sigma v \rangle_{eff} \uparrow$

at decoupling

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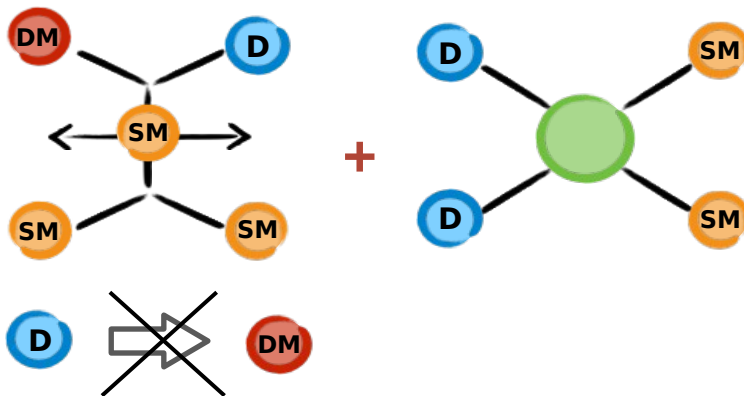
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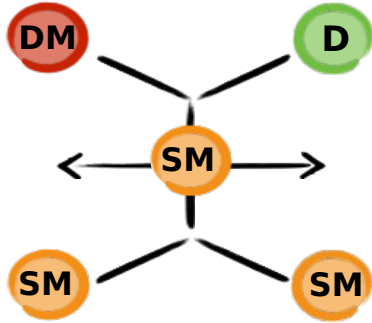
Very high



Correct relic abundance
for much smaller couplings

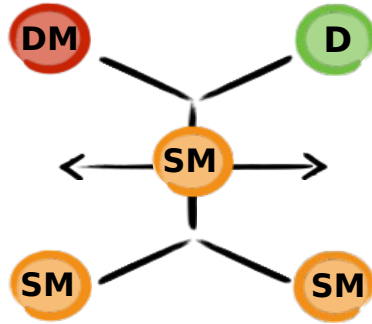
So just continue searching?

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- Partner decays are slow during decoupling.
- **But they are also long-lived at colliders!**

So just continue searching?

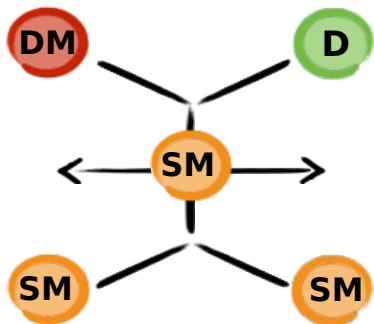


- Partner decays are slow during decoupling.
- But they are also long-lived at colliders!

- Both dark states present at decoupling — compressed spectrum.

$$\begin{array}{l} \text{D} \\ \text{DM} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \quad \frac{\Delta m}{m} \simeq 10\%$$

So just continue searching?

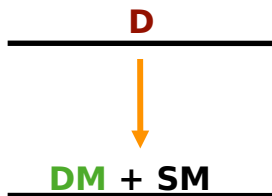


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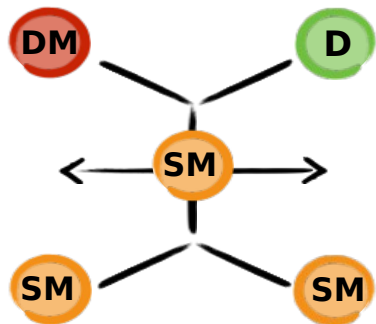
- Both dark states present at decoupling — compressed spectrum.

Two horizontal black lines represent energy levels. The top line is labeled with a red 'D' to its left. The bottom line is labeled with a green 'DM' to its left. To the right of the lines is the equation $\frac{\Delta m}{m} \simeq 10\%$.

we search for



So just continue searching?

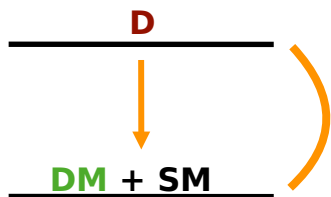


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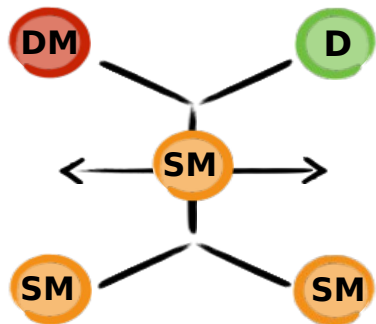
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we search for



Defines p_T of SM

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- Partner decays are slow during decoupling.
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- Both dark states present at decoupling — compressed spectrum.



Defines p_T of SM



Particles are soft

Two horizontal lines, one above the other. The top line is labeled 'D' in red. The bottom line is labeled 'DM' in green.

$$\frac{\Delta m}{m} \simeq 10\%$$

A large, hollow, downward-pointing arrow.

LLPs @ colliders: heavy vs. light

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~Electroweak scale

pp-experiments: LHC

Soft products



High backgrounds



New experimental developments

(e.g. cross-triggers)

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MeV-GeV scale

Also at ee-experiments: Belle II

Very clean signatures

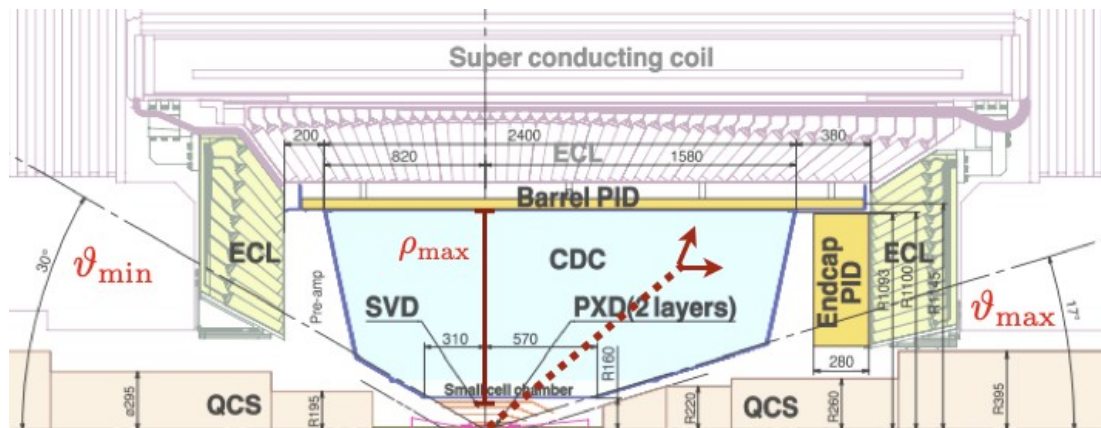
Bonus:

B-mesons are produced almost at rest

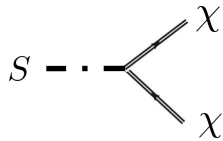
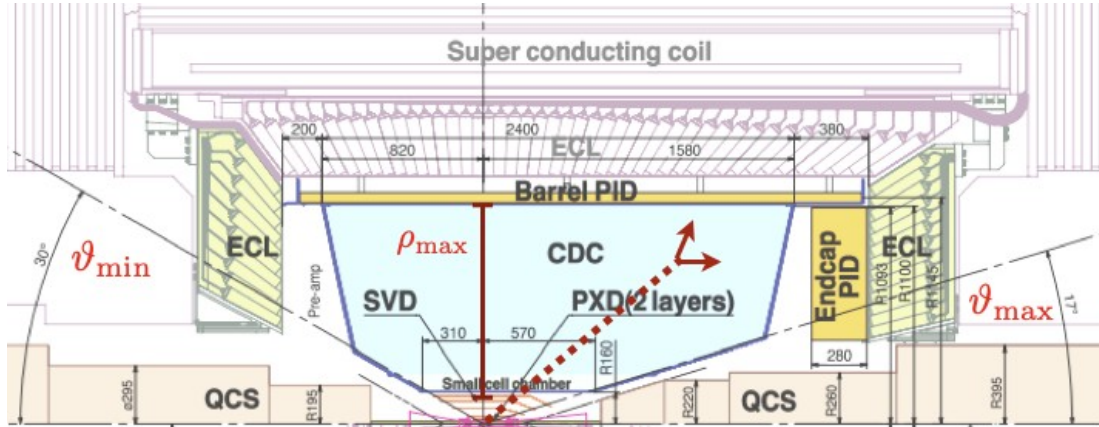


Large lifetimes can be probed

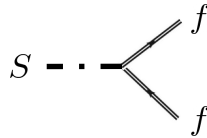
LLP and missing energy searches at Belle II



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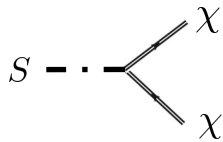
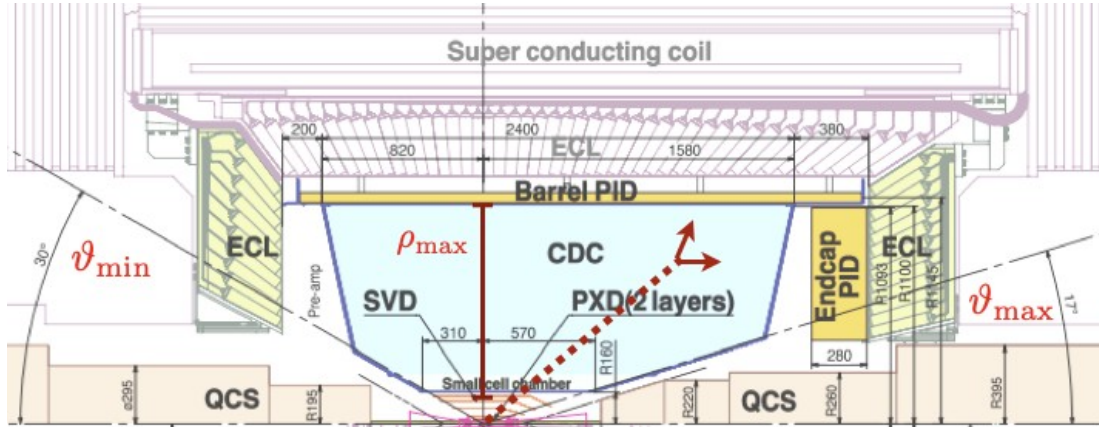


Missing energy

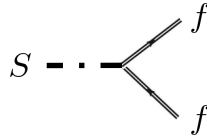


Displaced searches

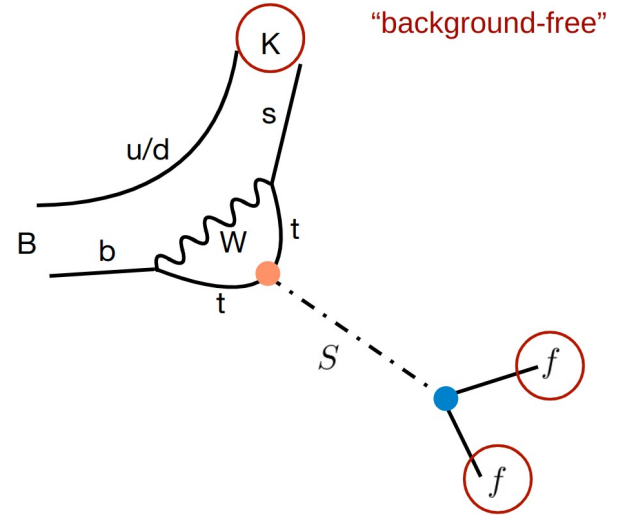
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Missing energy



Displaced searches

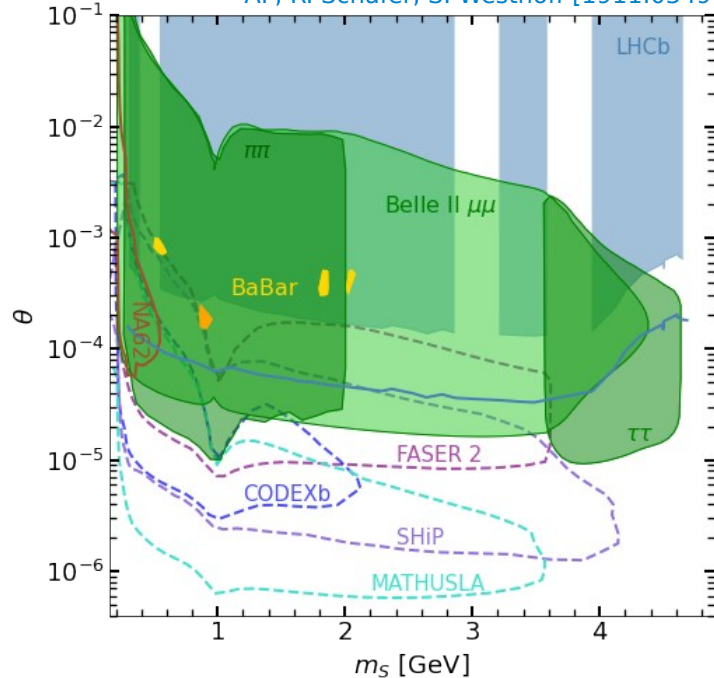


LLP and missing energy searches at Belle II are very promising!

Dark scalar:

Displaced stashes can compete with proposed long-baseline experiments

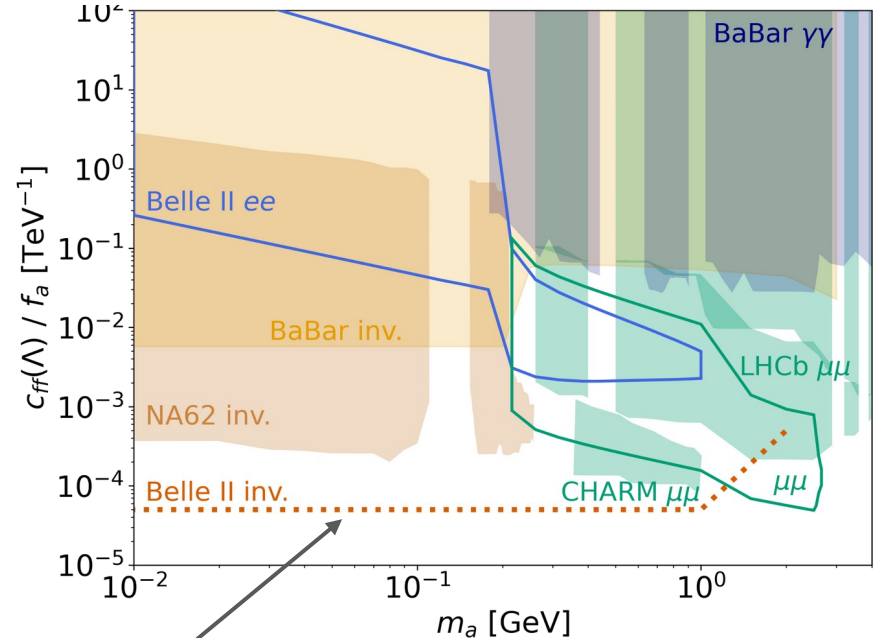
AF, R. Schäfer, S. Westhoff [1911.03490]



ALPs (fermion coupling only):

Complementarity of invisible and displaced searches.

T. Ferber, AF, R. Schäfer, S. Westhoff [2201.06580]



A new search strategy for two-body invisible decays

Alternative dark matter candidates?

Light: $m < \text{few GeV}$

Direct detection

e.g. Essig et al. [1509.01598], [1907.07682]

Colliders + beyond

e.g. Belle II physics Book [1808.10567],
Physics Beyond Colliders at CERN [1901.09966]

Heavy: $m > \text{few TeV}$

Telescopes

e.g. Antares: [1912.05296],
CTA: Rinchuso et al. [2008.00692],
Hess: Rinchuso et al. [1908.04317],
IceCube: Kachelriess et al. [1805.04500]

Non-thermal

Freeze-in

e.g. Hall et al. [0911.1120], Bélanger et al. [1811.05478]

Early kinetic decoupling, cospinning

e.g. Binder et al. [1706.07433], D'Agnolo et al. [1705.08450]

Different mass-coupling relation?

WIMPlless miracle, coannihilation, forbidden dark matter

e.g. Feng & Kumar [0905.3039], Griest & Seckel 1991, D'Agnolo & Ruderman [1505.07107]

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BSM (heavy) DM + SM mediator

+ let's keep DM thermal (we like it)

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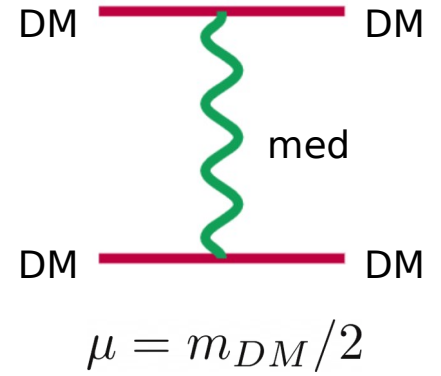
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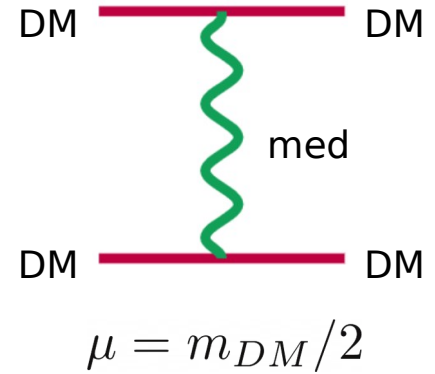
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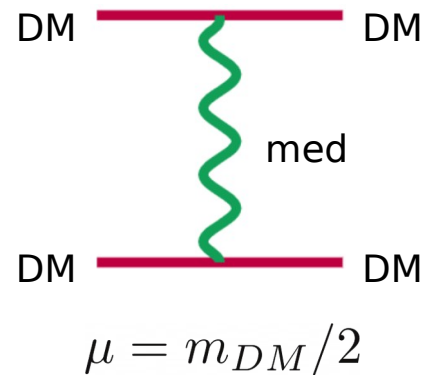


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for bound states

for scattering states

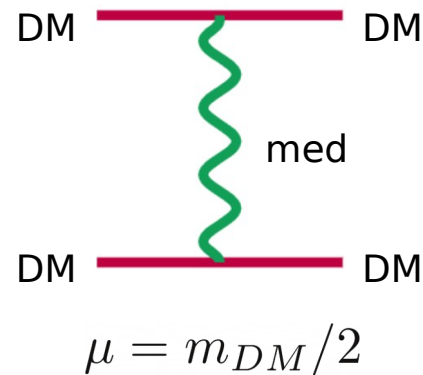


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Typically, viable scenarios have dark matter at multi-GeV or TeV scale

Long-range interactions and their implications

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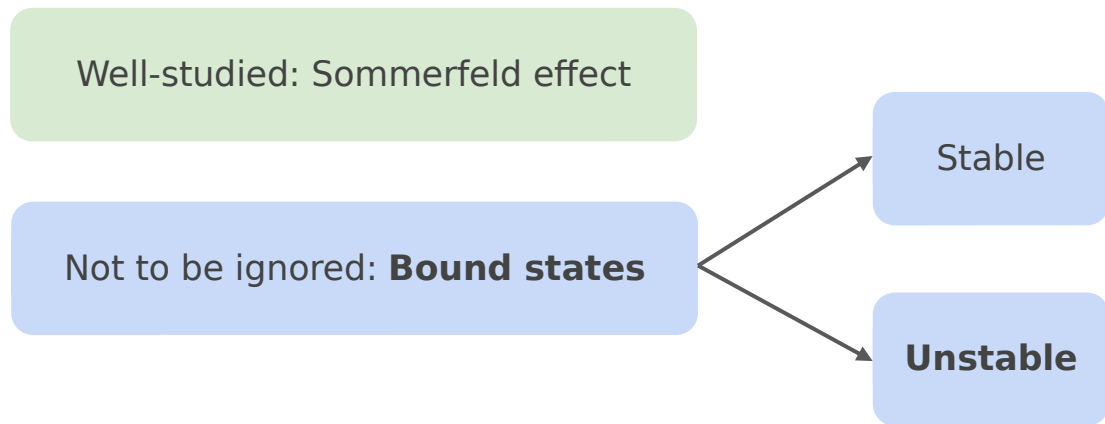
Well-studied: Sommerfeld effect

Long-range interactions and their implications

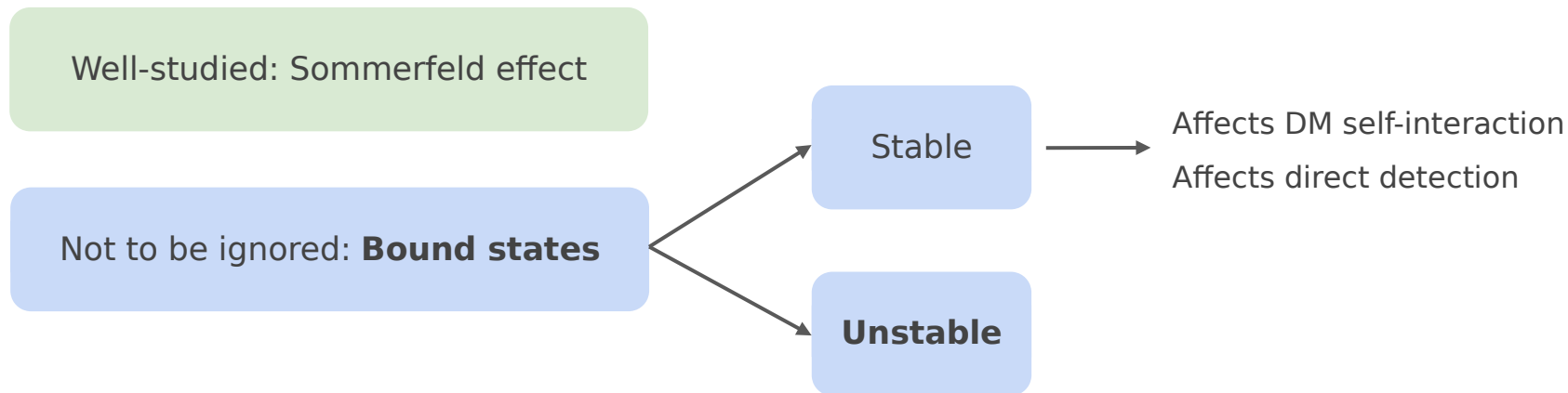
Well-studied: Sommerfeld effect

Not to be ignored: **Bound states**

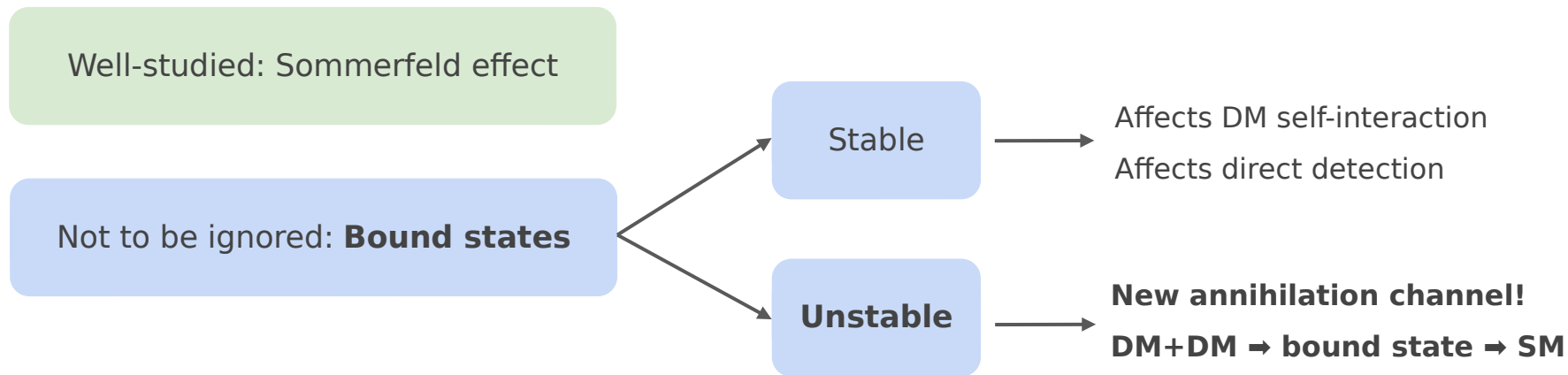
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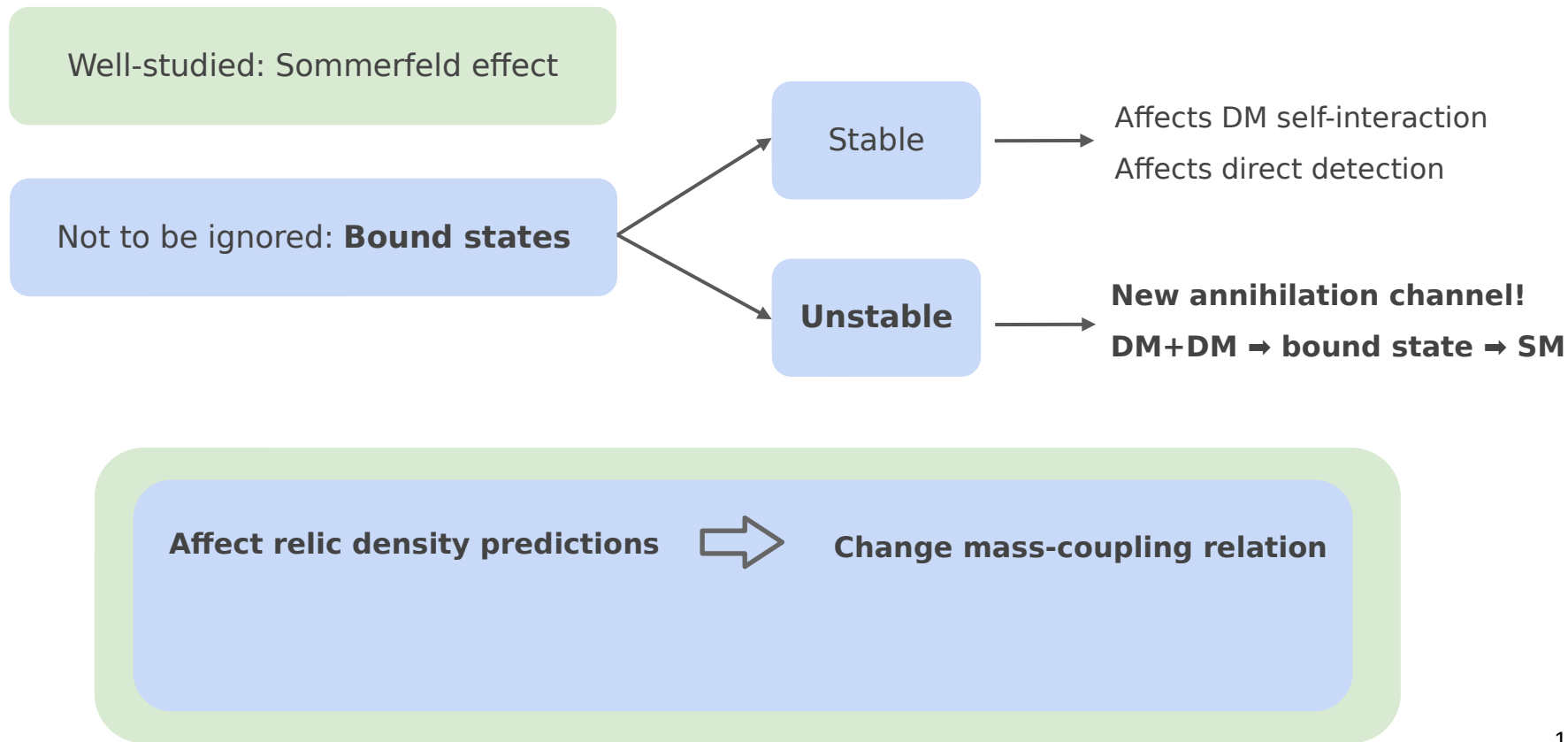
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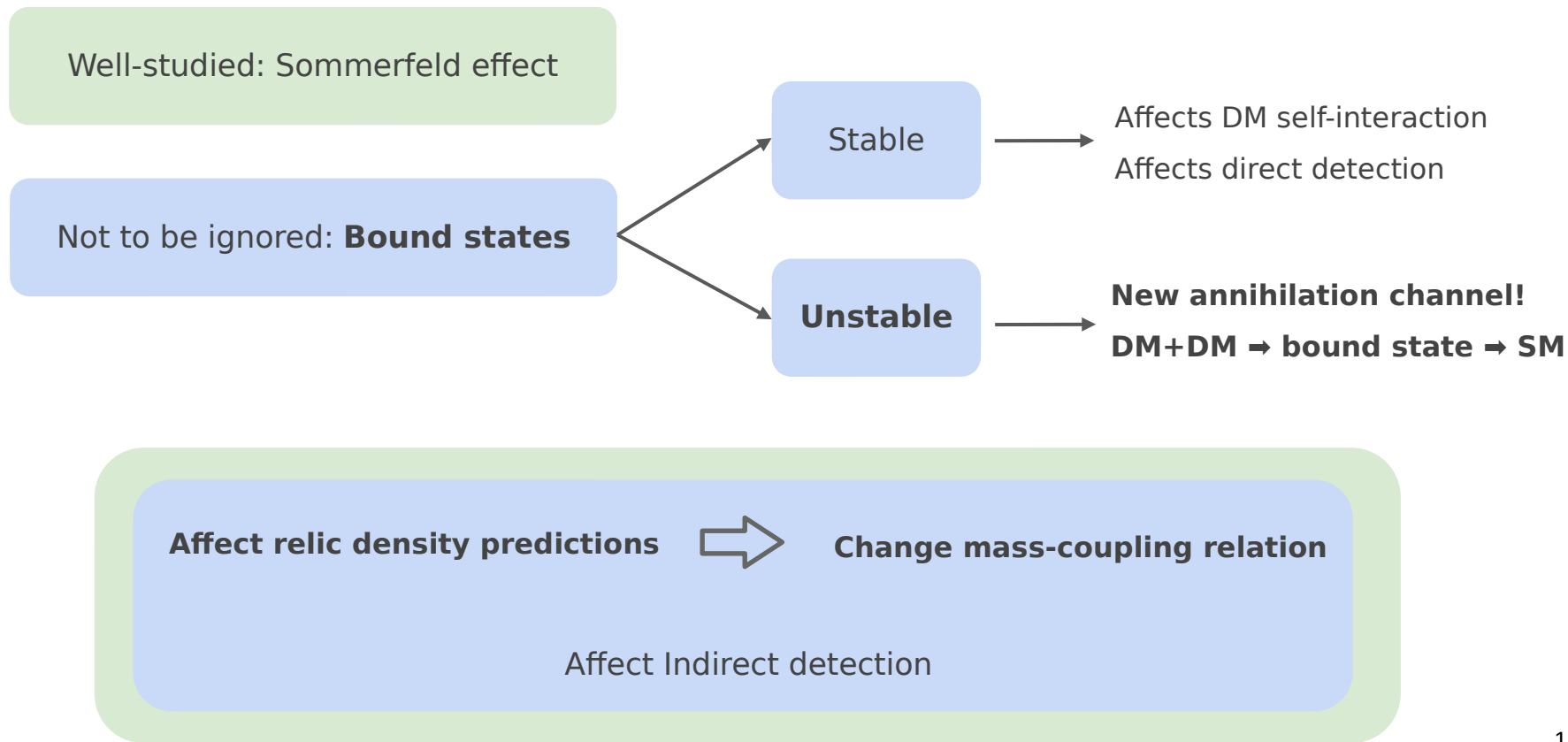
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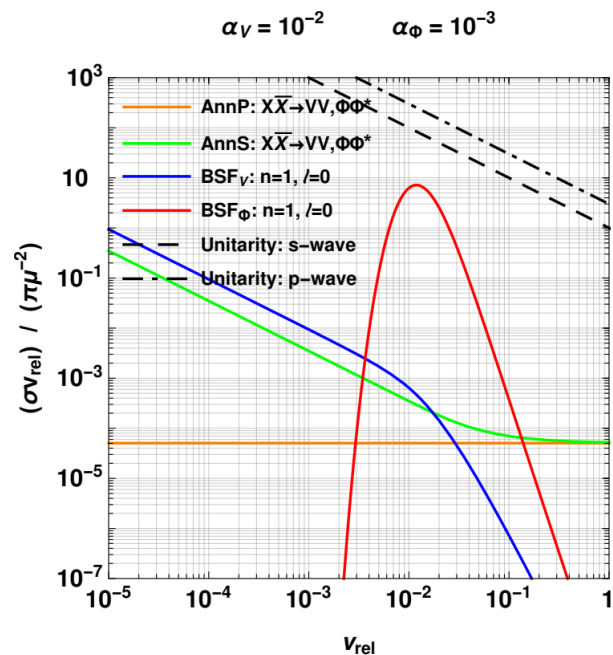
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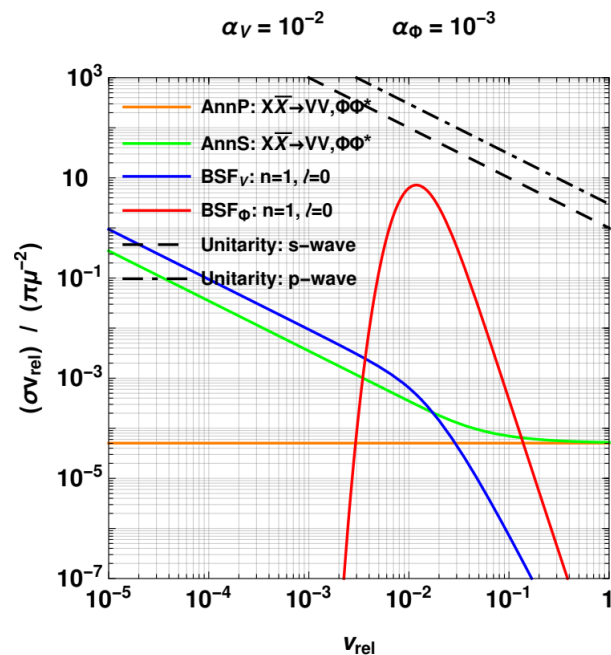


Dramatic example: scalar mediator



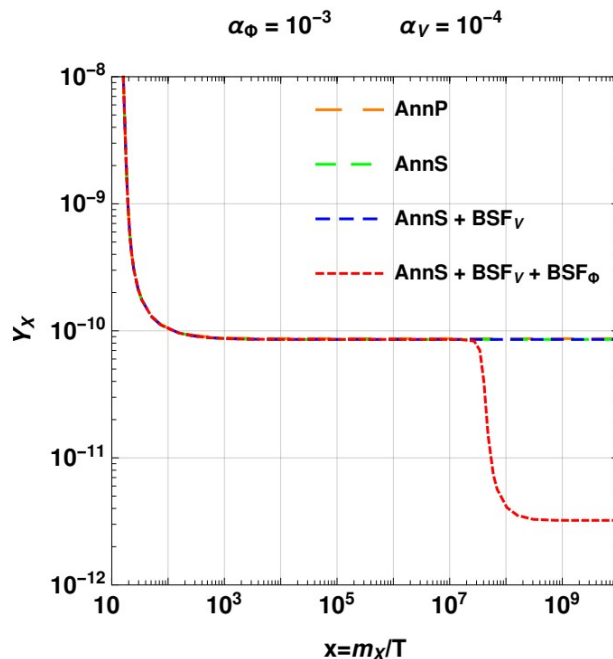
Huge enhancement from BSF_Φ

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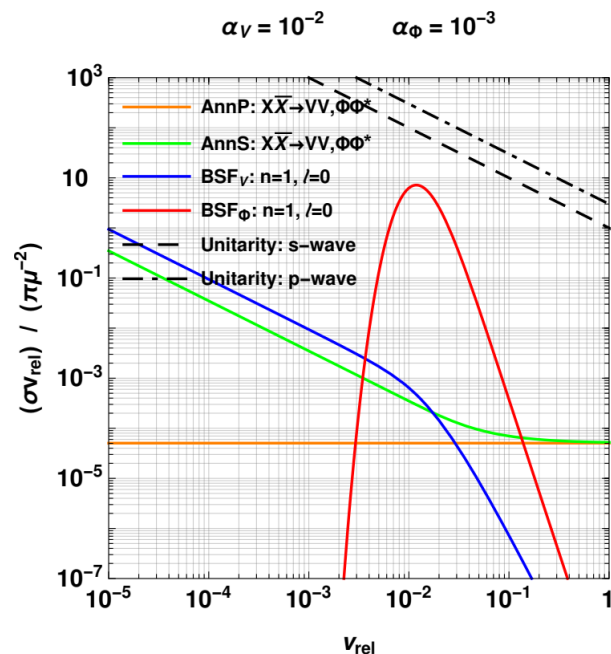


Huge enhancement from BSF_Φ

Relic density predictions change by orders of magnitude!

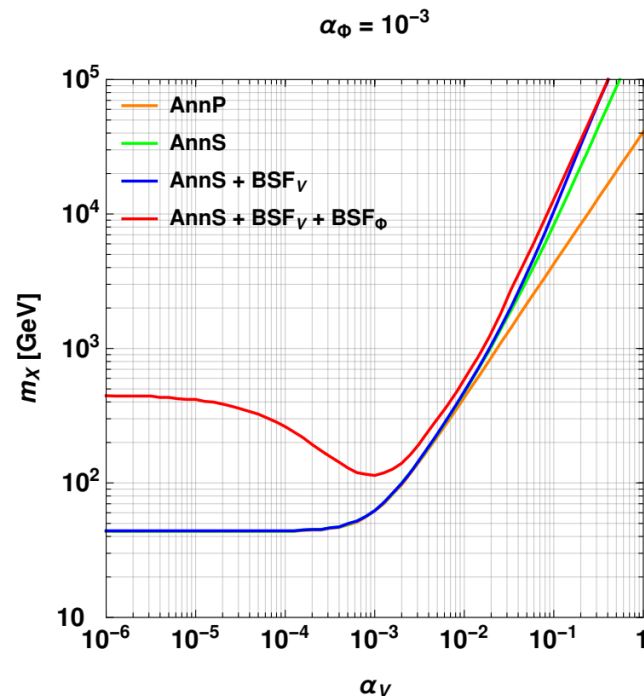
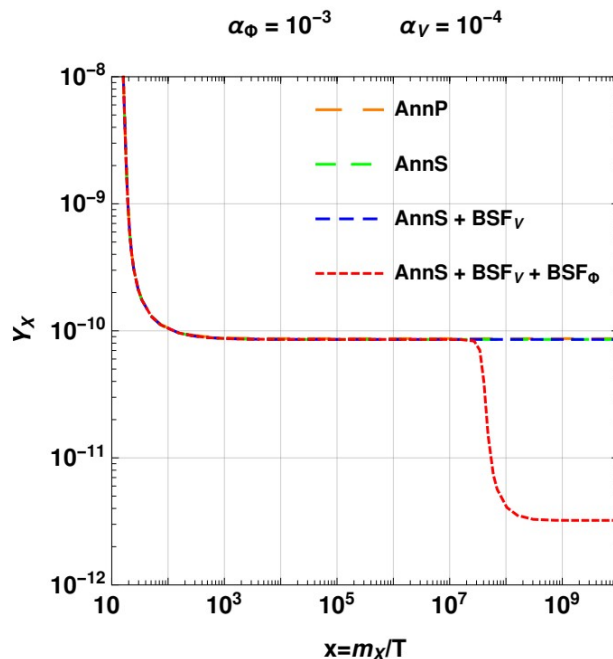


Dramatic example: scalar mediator



Huge enhancement from BSF_Φ

Relic density predictions change by orders of magnitude!



Changes all pheno predictions (DD, ID, colliders..!)

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Need a general formalism to account for bound states

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$$\frac{dY_j}{dx} = -\frac{\lambda}{x^2} \sum_i \langle \sigma_{ji}^{\text{ann}} v_{\text{rel}} \rangle (Y_j Y_i - Y_j^{\text{eq}} Y_i^{\text{eq}}) - \frac{\lambda}{x^2} \sum_i \sum_B \langle \sigma_{ji \rightarrow B}^{\text{BSF}} v_{\text{rel}} \rangle \left(Y_j Y_i - \frac{Y_B}{Y_B^{\text{eq}}} Y_j^{\text{eq}} Y_i^{\text{eq}} \right) - \Lambda x \sum_i \langle \Gamma_{j \rightarrow i} \rangle \left(Y_j - \frac{Y_i}{Y_i^{\text{eq}}} Y_j^{\text{eq}} \right)$$

$$\frac{dY_B}{dx} = -\Lambda x \left[\langle \Gamma_B^{\text{dec}} \rangle (Y_B - Y_B^{\text{eq}}) + \sum_{i,j} \langle \Gamma_{B \rightarrow ij}^{\text{ion}} \rangle \left(Y_B - \frac{Y_i Y_j}{Y_i^{\text{eq}} Y_j^{\text{eq}}} Y_B^{\text{eq}} + \sum_{B' \neq B} \langle \Gamma_{B \rightarrow B'}^{\text{trans}} \rangle \left(Y_B - \frac{Y_{B'}}{Y_{B'}^{\text{eq}}} Y_B^{\text{eq}} \right) \right) \right]$$

$$\text{with } \lambda \equiv \sqrt{\frac{\pi}{45}} m_{\text{Pl}} m g_*^{1/2} \quad \text{and} \quad \Lambda \equiv \frac{\lambda}{s x^3} = \sqrt{\frac{45}{4\pi^3}} \frac{m_{\text{Pl}}}{m^2} \frac{g_*^{1/2}}{g_{*S}}$$

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However, in most models, this system can be simplified to **one effective Boltzmann equation**

[Binder, AF, Petraki, White \[2112.00042\]](#), see also [Garny Heisig \[2112.01499\]](#)

Conclusions

- Strong direct detection constraints point us to **less conventional** dark matter scenarios
- Viable scenarios are **very different** at different energy scales
- Well-known models have features that dramatically affect dark matter phenomenology, and therefore **change the search strategies** (e.g. existence of long-lived particles or bound states)
- At MeV-GeV scale, displaced and invisible searches at **electron colliders are very promising**
- Displaced and invisible searches are **complementary**. They both require attention when exploring the parameter space of a feebly-interacting model
- **Bound states are a whole new avenue** in dark matter community. We are still developing the framework to understand them but they will definitely change many of our predictions

Backup

Dark scalar model

$$\mathcal{L} = -\frac{1}{2}m_\phi^2\phi^2 - \mu|H|^2\phi - y_\chi\bar{\chi}\chi\phi - \frac{1}{2}m_\chi\bar{\chi}\chi$$

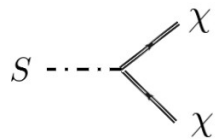
Can play role of DM candidate

Search regions

$$\Gamma_S = c_\theta^2 \Gamma_{\chi\bar{\chi}} + s_\theta^2 \Gamma_{\text{SM}}$$

$$m_S > 2m_\chi$$

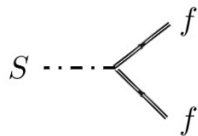
Invisible decays dominate



Missing energy

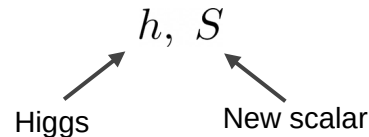
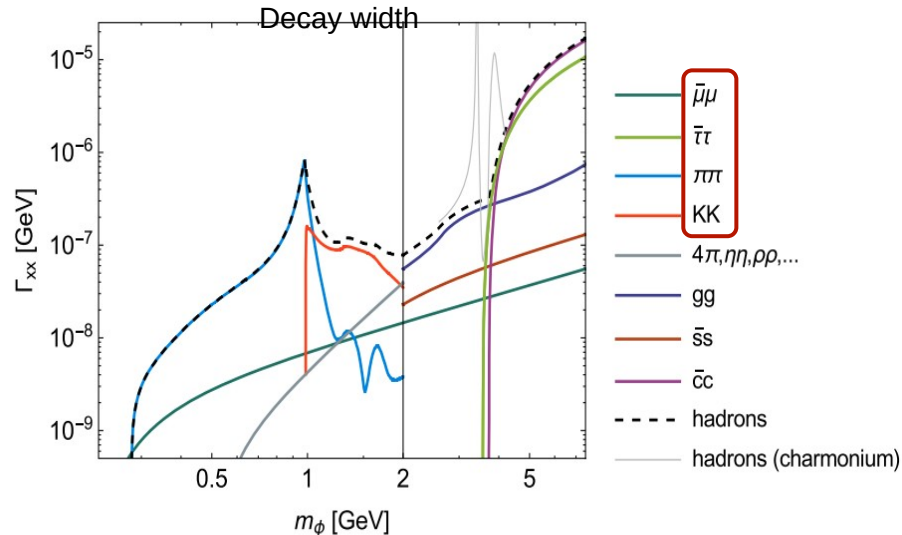
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Visible decays only



Displaced searches

One new coupling

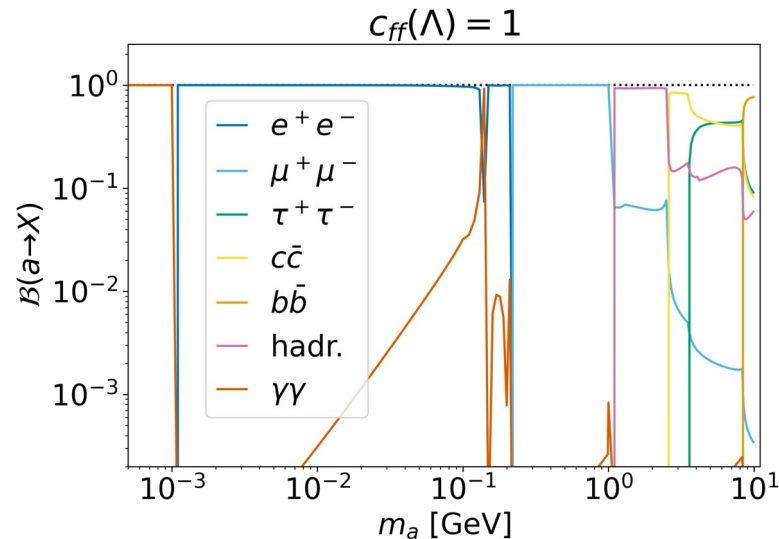


ALP model

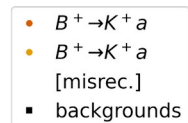
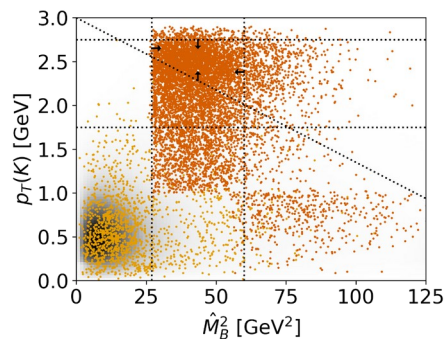
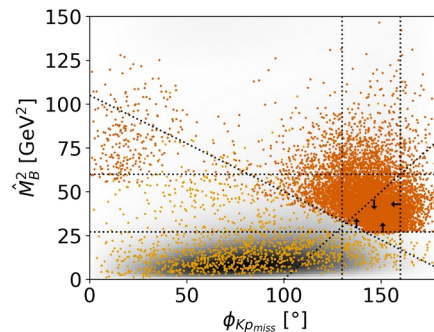
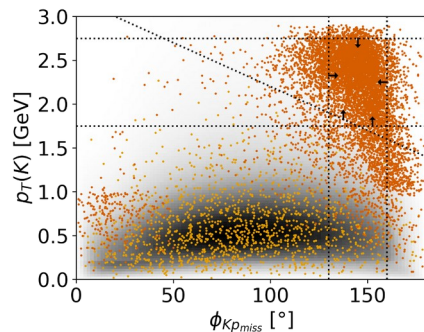
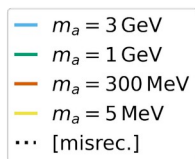
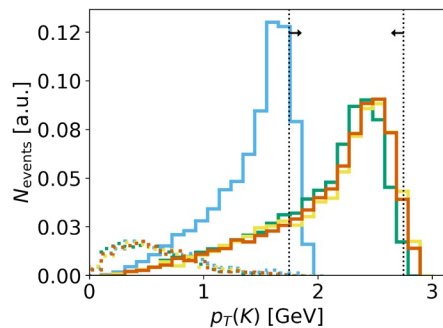
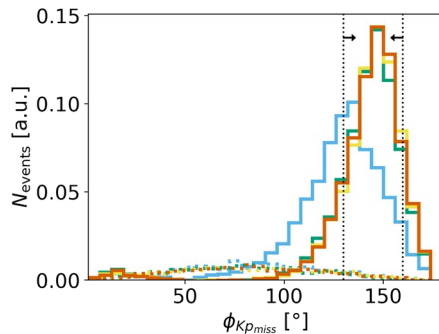
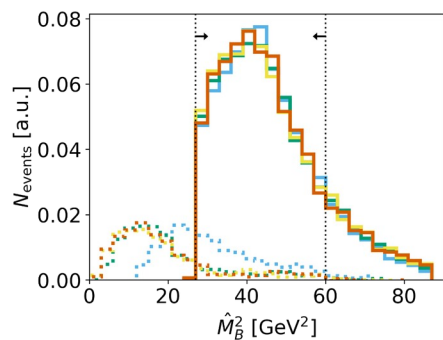
$$\mathcal{L}_{\text{eff}}(\mu > \mu_w) = \sum_f \frac{c_{ff}(\mu)}{2} \frac{\partial^\mu a}{f_a} (\bar{f} \gamma_\mu \gamma_5 f) + c_{WW}(\mu) \frac{a}{f_a} \frac{\alpha_2}{4\pi} W_{\mu\nu}^A \widetilde{W}^{\mu\nu, A}$$

$$\mathcal{B}(B^+ \rightarrow K^+ a) = 0.03 \left(\frac{c_{ff}(\Lambda)}{f_a [\text{TeV}]} + 0.0032 \frac{c_{WW}(\Lambda)}{f_a [\text{TeV}]} \right)^2 \frac{f_0^2(m_a^2)}{f_0^2(0)} \frac{\lambda^{1/2}(m_B^2, m_K^2, m_a^2)}{m_B^2 - m_K^2}$$

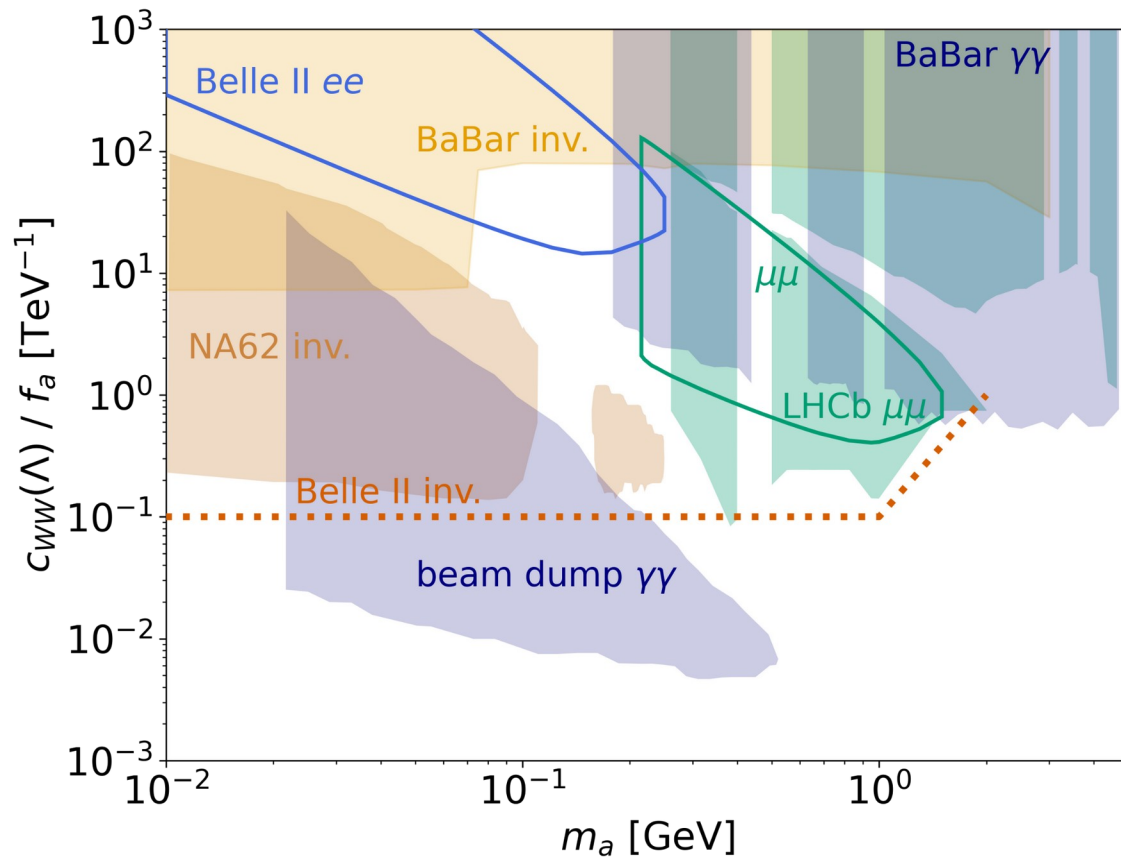
$$\Gamma_{a \rightarrow \ell \bar{\ell}} = 2\pi m_a \frac{|C_{\ell\ell}^{\text{eff}}(m_a)|^2 m_\ell^2}{\Lambda^2} \sqrt{1 - \frac{4m_\ell^2}{m_a^2}}$$



ALP invisible searches: kinematics and selection criteria



ALP model: projections for photon coupling only



Coupled system of Boltzmann equations

$$\frac{dY_j}{dx} = -\frac{\lambda}{x^2} \sum_i \langle \sigma_{ji}^{\text{ann}} v_{\text{rel}} \rangle (Y_j Y_i - Y_j^{\text{eq}} Y_i^{\text{eq}}) - \frac{\lambda}{x^2} \sum_i \sum_{\mathcal{B}} \langle \sigma_{ji \rightarrow \mathcal{B}}^{\text{BSF}} v_{\text{rel}} \rangle \left(Y_j Y_i - \frac{Y_{\mathcal{B}}}{Y_{\mathcal{B}}^{\text{eq}}} Y_j^{\text{eq}} Y_i^{\text{eq}} \right) - \Lambda x \sum_i \langle \Gamma_{j \rightarrow i} \rangle \left(Y_j - \frac{Y_i}{Y_i^{\text{eq}}} Y_j^{\text{eq}} \right)$$

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However:

- Assume fast transitions $\Gamma_{i \leftrightarrow j} \rightarrow Y_i / Y_i^{\text{eq}} = w$
- At high temperatures, ionisations are efficient;
at low temperatures - decays (directly or via transitions) are fast $\rightarrow \frac{dY_{\mathcal{B}}}{dx} \simeq 0$

This approximation was first proposed by Ellis, Luo Olive [1503.07142]

One effective Boltzmann equation

Some definitions:

Total DM yield

$$Y \equiv \sum_j Y_j$$

Total rates for a given bound state

$$\langle \Gamma_{\mathcal{B}}^{\text{ion}} \rangle \equiv \sum_{i,j} \langle \Gamma_{\mathcal{B} \rightarrow ij}^{\text{ion}} \rangle$$

$$\langle \Gamma_{\mathcal{B}}^{\text{trans}} \rangle \equiv \sum_{\mathcal{B}' \neq \mathcal{B}} \langle \Gamma_{\mathcal{B} \rightarrow \mathcal{B}'}^{\text{trans}} \rangle$$

$$\langle \Gamma_{\mathcal{B}}^{\text{tot}} \rangle \equiv \langle \Gamma_{\mathcal{B}}^{\text{dec}} \rangle + \langle \Gamma_{\mathcal{B}}^{\text{ion}} \rangle + \langle \Gamma_{\mathcal{B}}^{\text{trans}} \rangle$$

Matrix notations

$$\mathbb{F}_{\mathcal{B}\mathcal{B}'}^{\text{dec}} \equiv \delta_{\mathcal{B}\mathcal{B}'} \langle \Gamma_{\mathcal{B}}^{\text{dec}} \rangle$$

$$\mathbb{F}_{\mathcal{B}\mathcal{B}'}^{\text{ion}} \equiv \delta_{\mathcal{B}\mathcal{B}'} \langle \Gamma_{\mathcal{B}}^{\text{ion}} \rangle$$

$$\mathbb{F}_{\mathcal{B}\mathcal{B}'}^{\text{trans}} \equiv \delta_{\mathcal{B}\mathcal{B}'} \langle \Gamma_{\mathcal{B}}^{\text{trans}} \rangle$$

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$$\mathbb{T}_{\mathcal{B}\mathcal{B}'} \equiv \langle \Gamma_{\mathcal{B} \rightarrow \mathcal{B}'}^{\text{trans}} \rangle$$

$$\frac{dY}{dx} = - \frac{\lambda}{x^2} \langle \sigma^{\text{eff}} v_{\text{rel}} \rangle [Y^2 - (Y^{\text{eq}})^2]$$

$$\langle \sigma^{\text{eff}} v_{\text{rel}} \rangle \equiv \sum_{i,j} \frac{g_{i,\text{eff}} g_{j,\text{eff}}}{g_{\text{eff}}^2} \left(\langle \sigma_{ij}^{\text{ann}} v_{\text{rel}} \rangle + \sum_{\mathcal{B}} r_{\mathcal{B}} \langle \sigma_{ij \rightarrow \mathcal{B}}^{\text{BSF}} v_{\text{rel}} \rangle \right)$$

$$r_{\mathcal{B}} \equiv \sum_{\mathcal{B}'} \langle \Gamma_{\mathcal{B}'}^{\text{dec}} \rangle (\mathbb{F}^{\text{tot}} - \mathbb{T})_{\mathcal{B}'\mathcal{B}}^{-1}$$

Bound state
efficiency factor

$$0 \leq r_{\mathcal{B}} \leq 1$$

Saha ionisation equilibrium for **metastable** bound states

(Algebraic) eq. for bound states + detailed balance + definition of efficiency factor:

$$\frac{n_{\mathcal{B}}}{n_{\mathcal{B}}^{\text{eq}}} = \left(\frac{n_{\text{free}}}{n_{\text{free}}^{\text{eq}}} \right)^2 - \left[\left(\frac{n_{\text{free}}}{n_{\text{free}}^{\text{eq}}} \right)^2 - 1 \right] r_{\mathcal{B}}$$

$$\mu_{\mathcal{B}}/T = 2\mu_{\text{free}}/T + \ln \left[1 - (1 - e^{-2\mu_{\text{free}}/T}) r_{\mathcal{B}} \right]$$

$$r_{\mathcal{B}} \ll 1$$

$$\mu_{\mathcal{B}} = 2 \mu_{\text{free}}$$

(familiar expression)

$$r_{\mathcal{B}} \rightarrow 1$$

$$\mu_{\mathcal{B}} = 0$$