

A general heavy-flavor mass scheme for charge-current DIS at NNLO and beyond

& implications for EW phenomenology

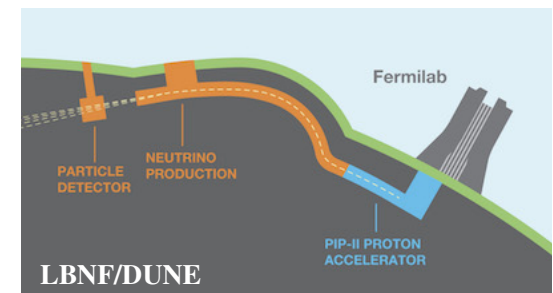
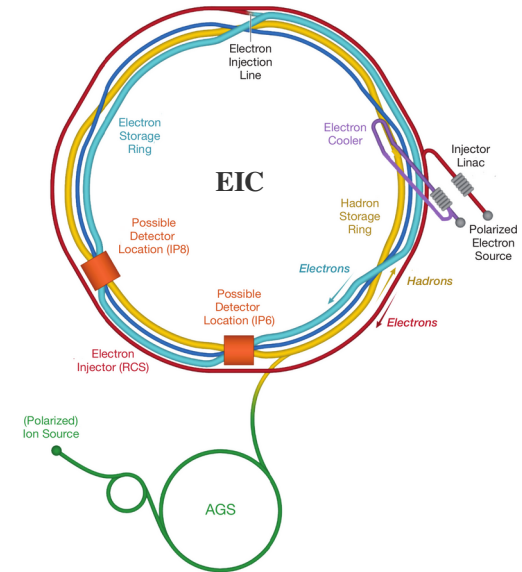
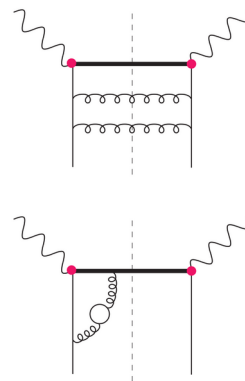
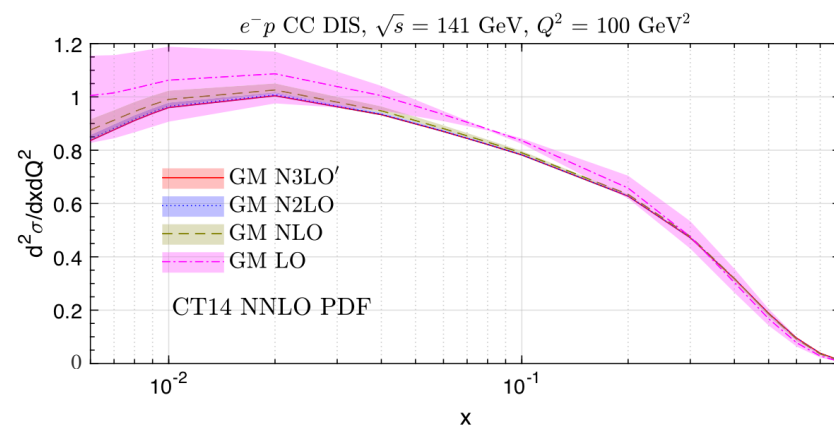
Tim Hobbs, Fermilab and IIT

4th May 2022

...with Jun Gao, Pavel Nadolsky, ChuanLe Sun and C.-P. Yuan

Based on: Phys. Rev. D**105** (2022) 1, L011503

[arXiv: 2107.00460]



 **Fermilab**

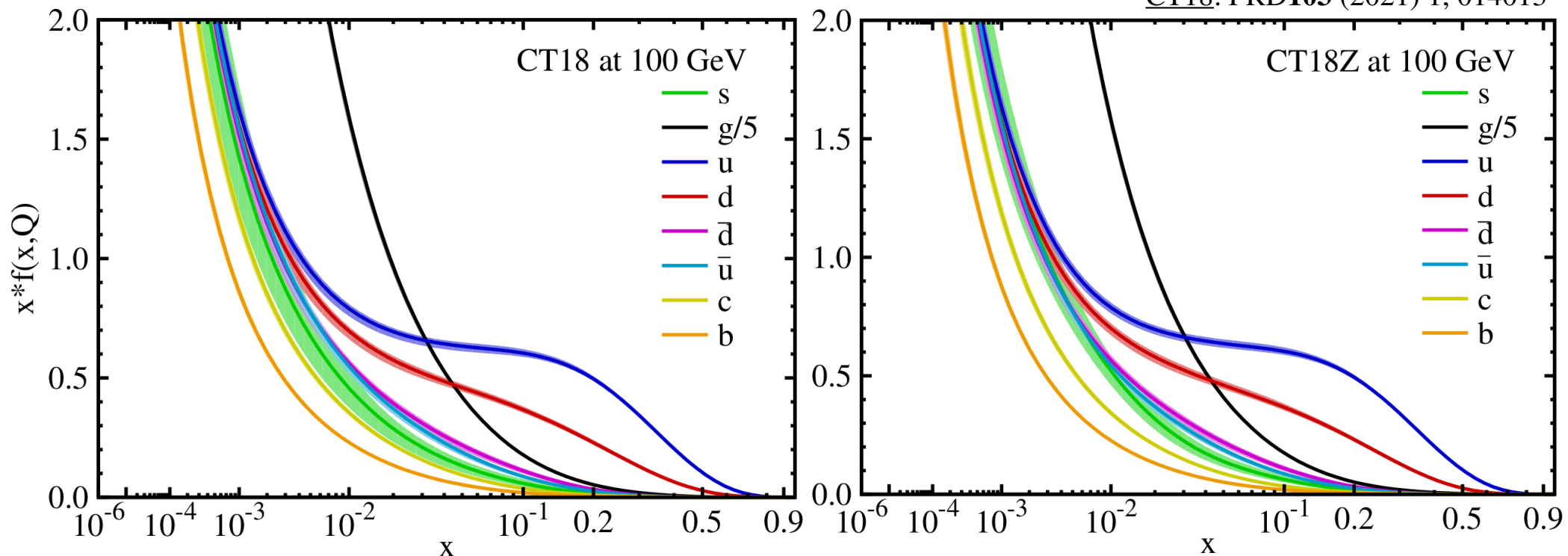
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- theory accuracy now/approaching (N)NNLO in α_s for typical processes

\rightarrow NLO EW corrections, especially for LHC data [WG3, K. Xie](#)

[see plenary, J. Gao; WG1, P. Nadolsky](#)

[CT18: PRD103 \(2021\) 1, 014013](#)



- fit a wide assortment of data from various underlying processes; scales

\rightarrow CC DIS (*e.g.*, HERA) important complement in CT; needs NNLO treatment

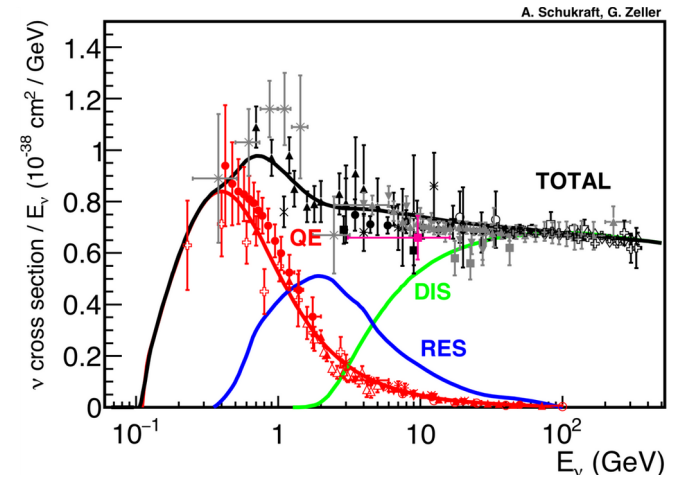
CC DIS: motivation

- charged-current DIS: central component in several next-generation QCD expt. programs

→ ν A, forward physics: DUNE, FASER ν

→ precision QCD: EIC, LHeC

example: DUNE target sensitivity to $\delta_{\text{CP}} \rightarrow$ control over CC DIS for $E_\nu \sim \text{few GeV}$

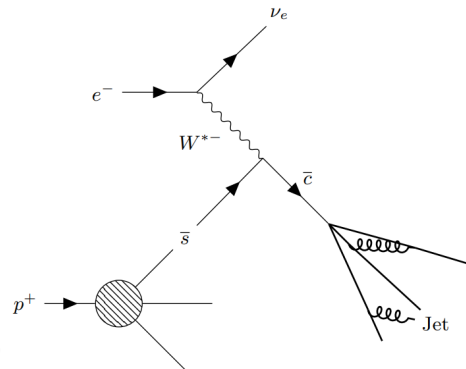
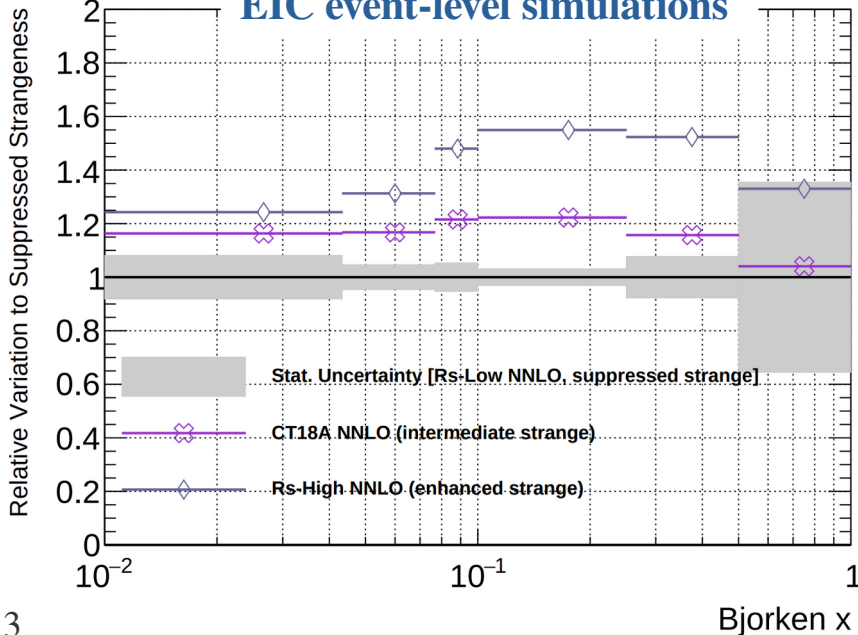


- W -exchange processes probe unique combinations of in-nucleon flavor currents; *e.g.*, nucleon $s(x, Q)$

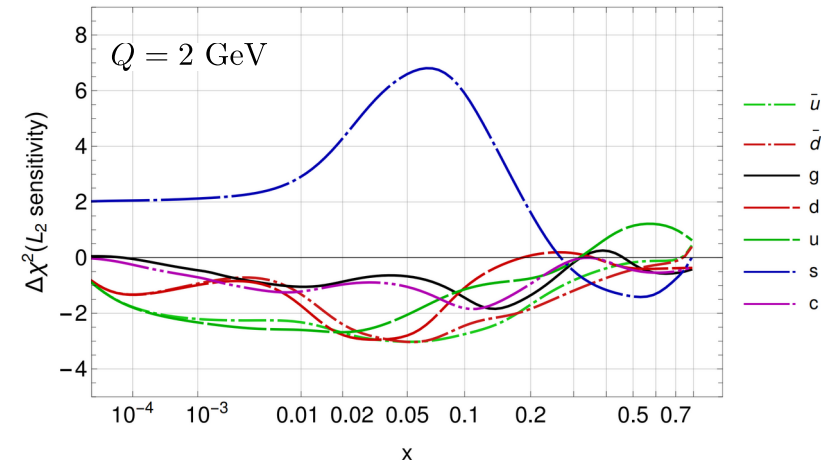
Phys. Rev. D **103** (2021) 7, 074023
Arratia, Furlotova, TJH, Olness, Sekula

[arXiv: 2006.12520]

EIC event-level simulations



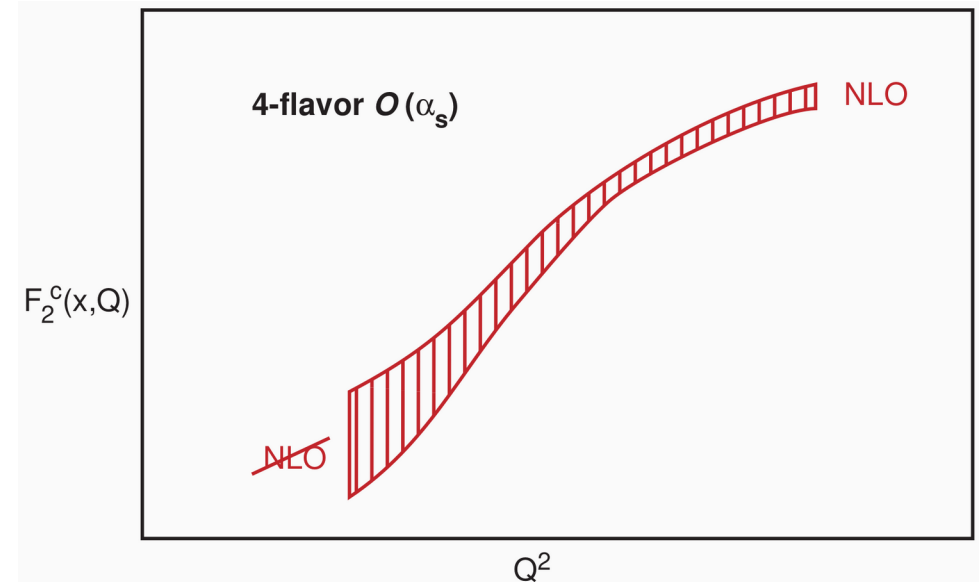
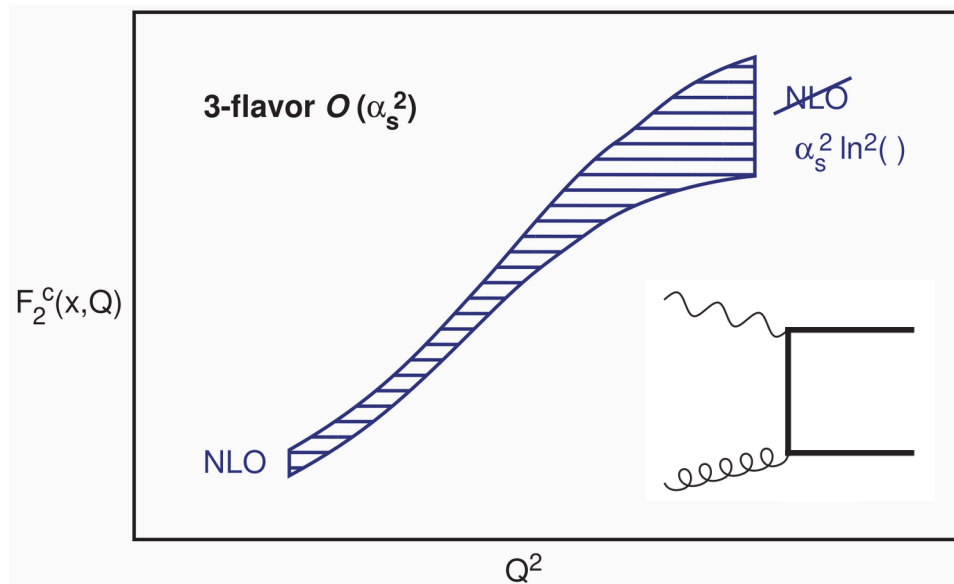
NuTeV $\mu^+\mu^- \nu$ -production, CT18 NNLO



- CC DIS data cover **wide range in Q ; higher pQCD accuracy needed** for perturbative stability, PDF extractions

evolution schemes as general problem in QCD

- higher order(s) in pQCD: improved accuracy in Wilson coeff., control over scale dependence
- at given fixed order, nontrivial relationship with chosen heavy-quark (HQ) scheme



- fixed flavor-number (FFN): $Q \gtrsim M_Q$; flavor-creation (FC) processes with $n_f = 3$
- zero-mass (ZM) variable flavor-number: $Q \gg M_Q$; flavor-excitation (FE) processes with $n_f = 4$

- 2 paradigms adapted to different regimes w.r.t. HQ mass scale; \exists **interpolation scheme?**

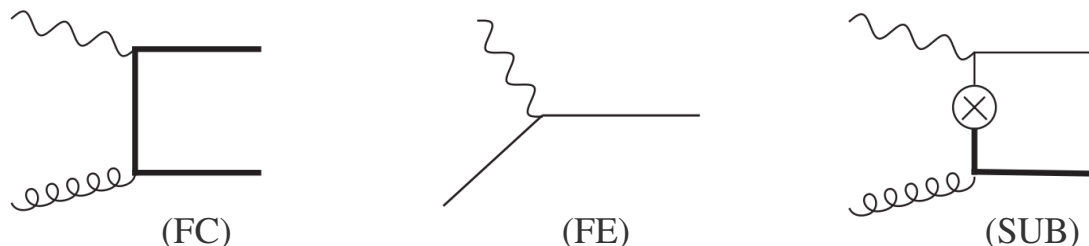
general-mass schemes: S-ACOT- χ

- variable flavor-number scheme to interpolate between ZM and FFN regimes: **ACOT**

Aivazis, Collins, Olness, Tung; PRD**50** (1994) 3085-3118

→ systematic approach to incorporating HQ mass dependence

- introduce subtraction term(s) to eliminate double counting between FC/FE contributions:



$$\left\{ \begin{array}{l} Q \gtrsim M_Q \implies (\text{SUB}) \approx (\text{FE}) \text{ such that } n_f = 3 \text{ FC dominates} \\ Q \gg M_Q \implies (\text{SUB}) \approx (\text{FC}) \text{ such that } n_f = 4 \text{ FE dominates} \end{array} \right.$$

$$\chi(x, Q, M_Q) = x \left(1 + \frac{1}{Q^2} \left[\sum_{\text{F.S.}} M_Q \right]^2 \right)$$

- “simplified” ACOT (S-ACOT): neglect full HQ mass dependence in FE graphs
- S-ACOT- χ : smooth HQ thresholds, include approx. HQ mass dependence: $C_i(x) \rightarrow C_i(\chi)$

- formulation necessitates careful tracking of diagrams to organize calculation correctly

template calculation: NC DIS at NNLO

Guzzi, Nadolsky, Lai, Yuan Phys. Rev. D **86**, 053005 (2012)

[arXiv: 1108.5112]

- at structure-function level, factorization allows separation of coeff. functions, PDFs:

$$F(x, Q) = \sum_{i=1}^{N_f^s} e_i^2 \sum_{a=0}^{N_f} [C_{i,a} \otimes \Phi_{a/p}] (x, Q) \quad (F = F_{2,L})$$

- compute S-ACOT- χ coeff. functions: expand in α_s each term in auxiliary partonic struct. func.:

$$F_{i,b}(\hat{x}, Q) = \sum_{a=0}^{N_f} [C_{i,a} \otimes \Phi_{a/b}] (\hat{x}, Q)$$

→ matching terms order-by-order,

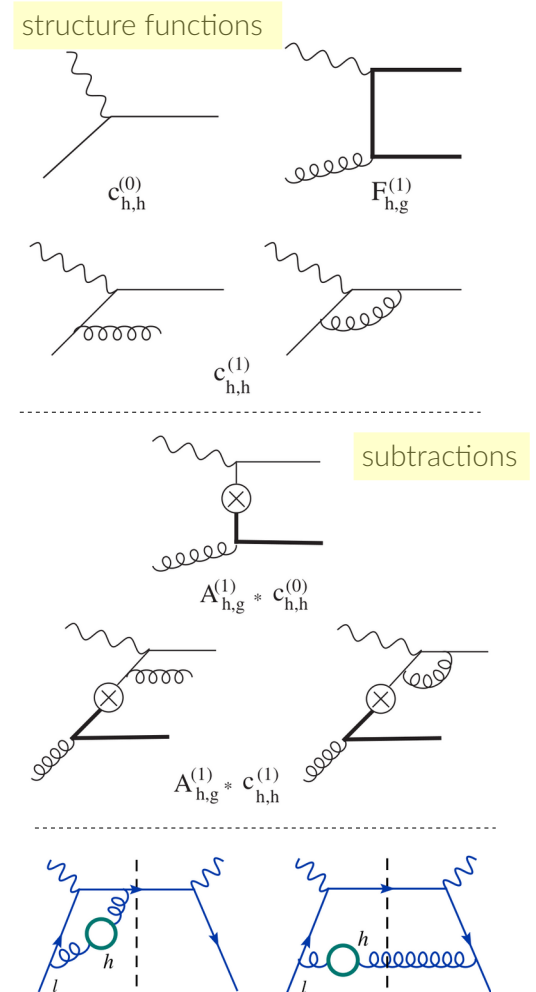
$$\begin{cases} C_{i,b}^{(0)}(\hat{x}) = F_{i,b}^{(0)}(\hat{x}) \\ C_{i,b}^{(1)}(\hat{x}) = F_{i,b}^{(1)}(\hat{x}) - [C_{i,a}^{(0)} \otimes A_{ab}^{(1)}](\hat{x}) \\ C_{i,b}^{(2)}(\hat{x}) = F_{i,b}^{(2)}(\hat{x}) - [C_{i,a}^{(0)} \otimes A_{ab}^{(2)}](\hat{x}) - [C_{i,a}^{(1)} \otimes A_{ab}^{(1)}](\hat{x}) \end{cases}$$

- organize into heavy-, light-quark pieces: $F = \sum_{l=1}^{N_l} F_l + F_h$

$$C_{h,g}^{(2)} = \hat{F}_{h,g}^{(2)} - A_{hg}^{(2)} - c_{h,h}^{(1)} \otimes A_{hg}^{(1)}$$

→ S-ACOT- χ : massless FE, χ -rescaled

→ light-quark SFs: additional flavor non-sing. (NS) *disconnected* graphs:



CC DIS at NNLO and beyond (i)

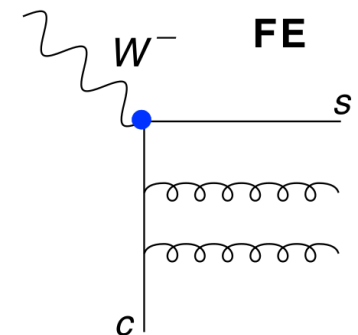
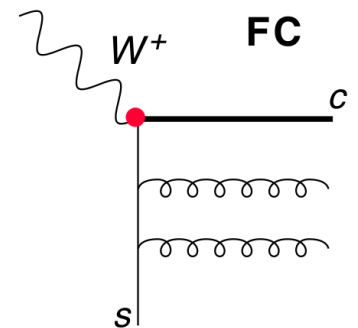
- conceptually, organization of CC DIS calculation resembles NC
- gauge-boson coupling introduces charge/flavor-changing vertices

$$F_l \iff W \bar{q}_l q_l$$

$$F_h \iff W \bar{q}_h q_l \text{ or } W \bar{q}_l q_h$$

→ appearance of heavy flavor occurs at different orders w.r.t. NC

- HQ contributions begin only at NNLO for F_l
- FE and FC diagrams involving HQ start from LO for F_h



- S-ACOT- χ patterns of subtractions, HQ mass dependence in CC NLO Wilson coeffs.:

$$\left\{ \begin{array}{l} C_{l,l}^{(1)} = c_{l,l}^{(1)}(z), \quad C_{l,g}^{(1)} = c_{l,g}^{(1)}(z), \quad C_{h,h}^{(1)} = c_{l,l}^{(1)}(\chi) \\ C_{h,l}^{(1)} = H_l^{(1)}(z) - C_{h,l}^{(0)} \otimes A_{ll}^{(1)} \\ C_{h,g}^{(1)} = H_g^{(1)}(z) - C_{h,l}^{(0)} \otimes A_{lg}^{(1)} - C_{h,h}^{(0)} \otimes A_{hg}^{(1)} \end{array} \right.$$

- HQ contributions explicitly appear in light-quark SF at NNLO; subtracted NS coeff.:

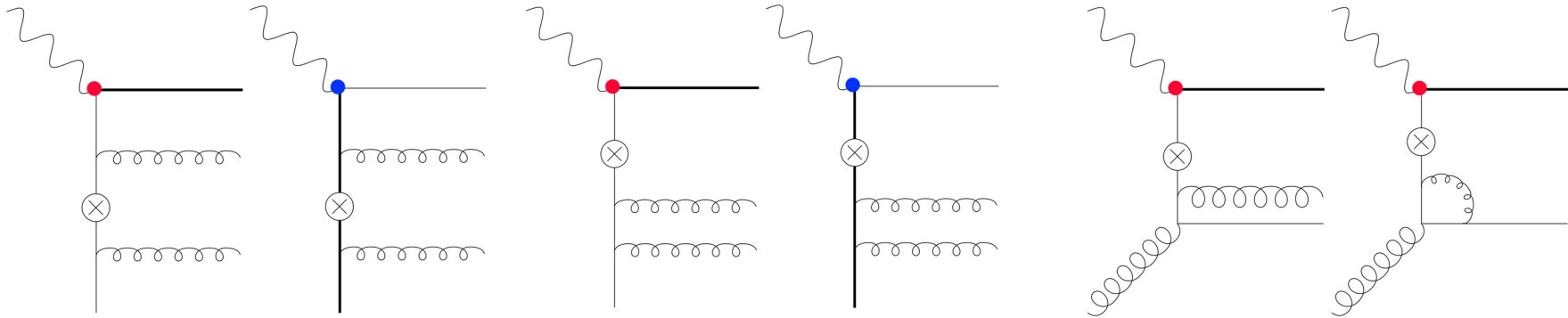
$$C_{l,g}^{(2)} = c_{l,g}^{(2)}(z), \quad C_{l,h}^{(2)} = c_{l,h}^{(2)}(\chi)$$

$$C_{l,l}^{(2)} = c_{l,l}^{(2)}(z) + \tilde{C}_{l,l}^{(NS,2)}(z)$$

- careful ordering of diagrams by flavor structure, topology is crucial

CC DIS at NNLO and beyond (ii)

→ representative CC NNLO subtraction diagrams – identifiable with coeff. expressions



$$C_{h,h}^{(2)} = c_{h,h}^{(2)}(\chi)$$

$$C_{h,l}^{(2)} = H_l^{(2)}(z) - \Delta C_{h,l}^{(2)}$$

$$C_{h,g}^{(2)} = H_g^{(2)}(z) - \Delta C_{h,g}^{(2)}$$

constructed from 2-loop operator matrix elements

▪ finally, ZM N³LO Wilson coeffs. available!

→ evaluate approximate N³LO (*i.e.*, N³LO')

▪ S-ACOT- χ patterns of subtractions, HQ mass dependence in CC NLO Wilson coeffs.:

$$\left\{ \begin{array}{l} C_{l,l}^{(1)} = c_{l,l}^{(1)}(z), \quad C_{l,g}^{(1)} = c_{l,g}^{(1)}(z), \quad C_{h,h}^{(1)} = c_{l,l}^{(1)}(\chi) \\ C_{h,l}^{(1)} = H_l^{(1)}(z) - C_{h,l}^{(0)} \otimes A_{ll}^{(1)} \\ C_{h,g}^{(1)} = H_g^{(1)}(z) - C_{h,l}^{(0)} \otimes A_{lg}^{(1)} - C_{h,h}^{(0)} \otimes A_{hg}^{(1)} \end{array} \right.$$

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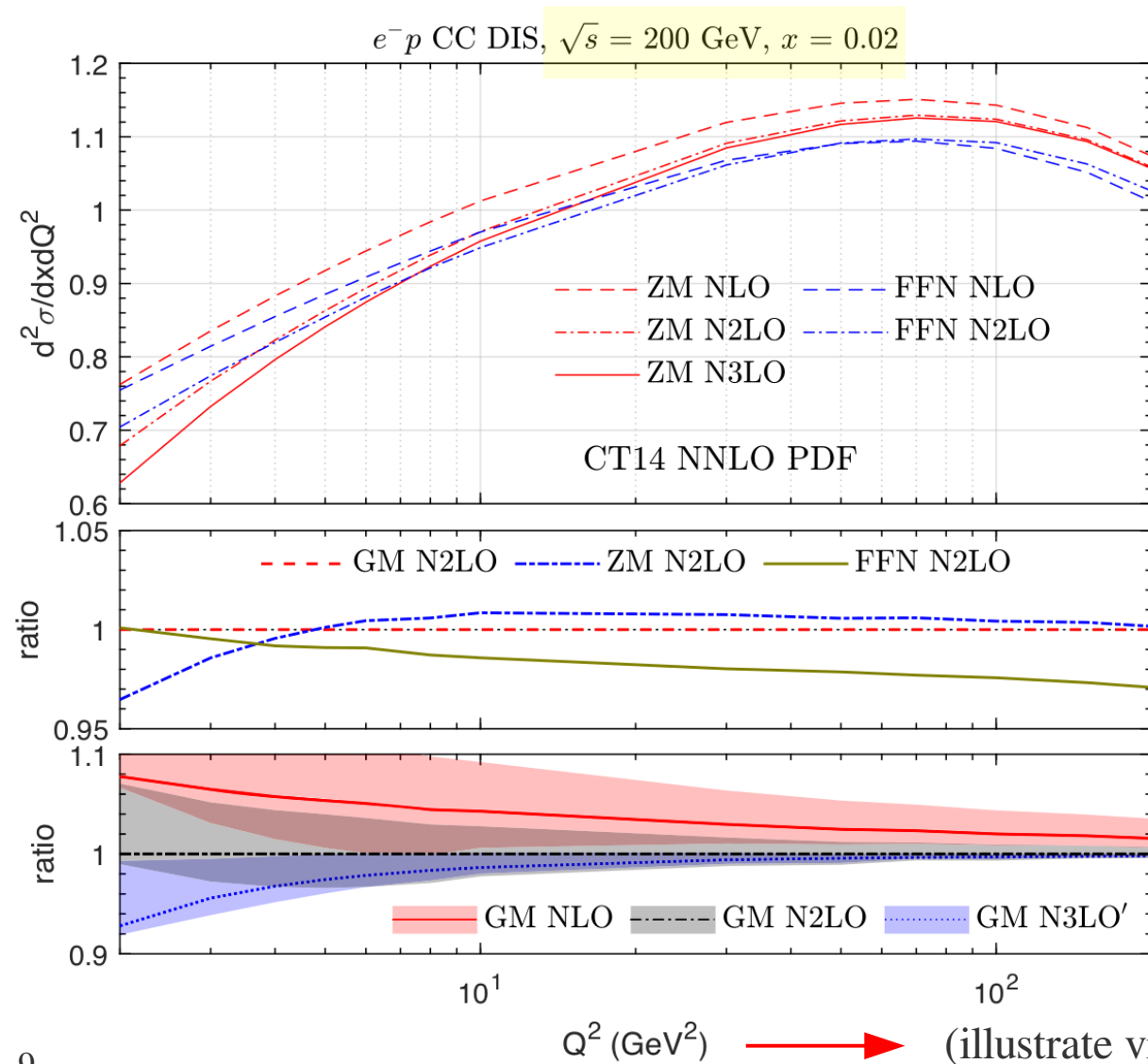
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▪ careful ordering of diagrams by flavor structure, topology is crucial

illustration for arbitrary DIS cross sections

- before expt.-specific predictions, compute generic DIS reduced cross section(s)

→ compare calculations of highest available order: FFN vs. ZM vs. GM schemes



NLO: ZM vs FFN, $\lesssim 6\%$
 NNLO: ZM vs FFN, $\lesssim 3\%$

GM interpolation between
 FFN at low Q , ZM at high Q

GM N³LO': GM NNLO
 + ZM N³LO
 [NNLO PDFs]

scale variations: shift μ_R, μ_F
 by $\times 2$

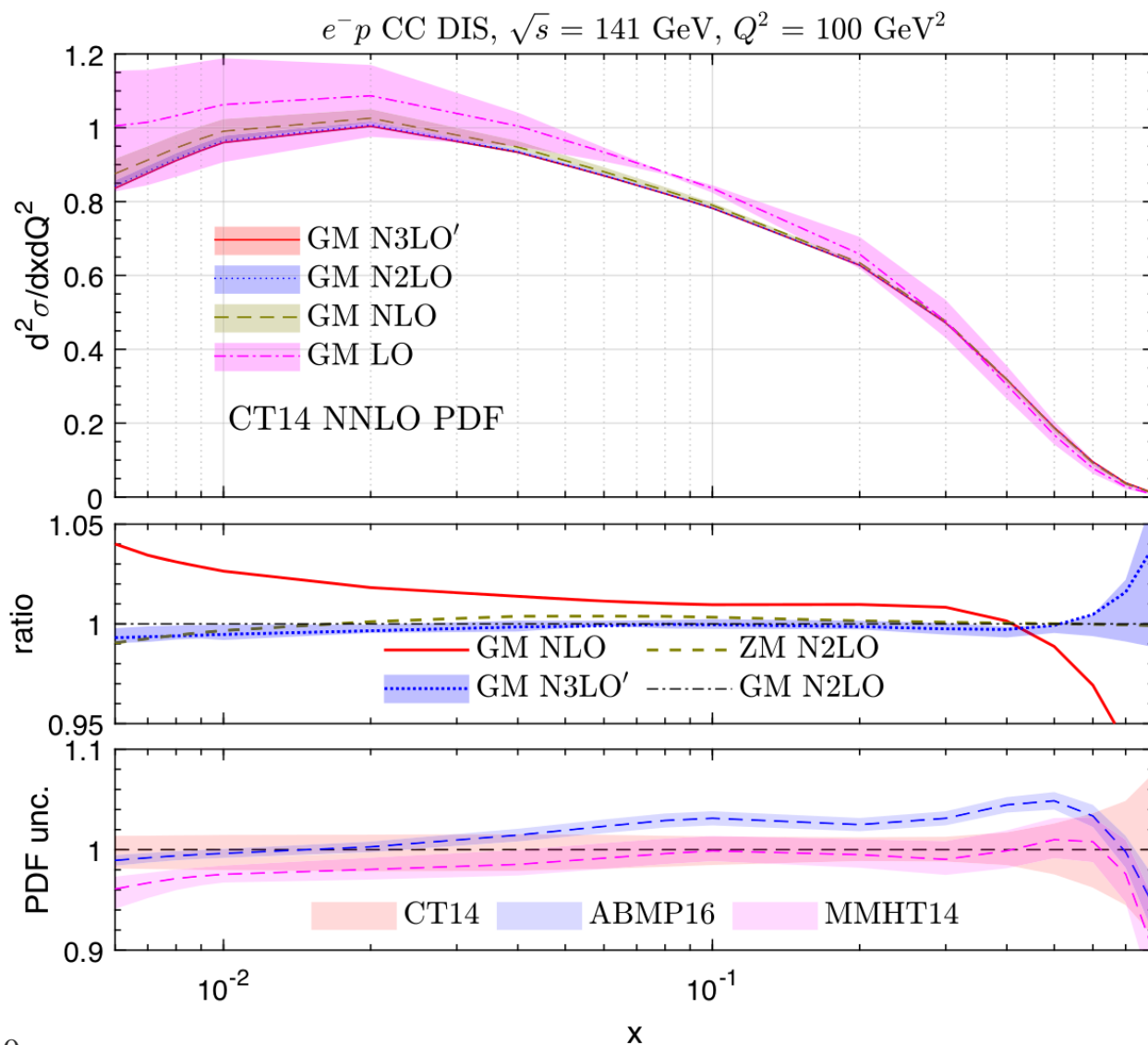
→ improved convergence with order!

→ (illustrate virtuality dependence)

implications for CC DIS at EIC

- EIC will undertake various precision QCD measurements; EIC Yellow Report [arXiv: 2103.05419](https://arxiv.org/abs/2103.05419)

→ Inclusive Reactions Study (YR7.1.1): CC – including positron beam – access to d, s PDFs



- consider high-energy EIC collisions

→ reconstruction challenges:
CC events restricted to high Q^2

- strong perturbative convergence

→ for N³LO', scale variations
generally contained to $\lesssim 0.5 - 1\%$

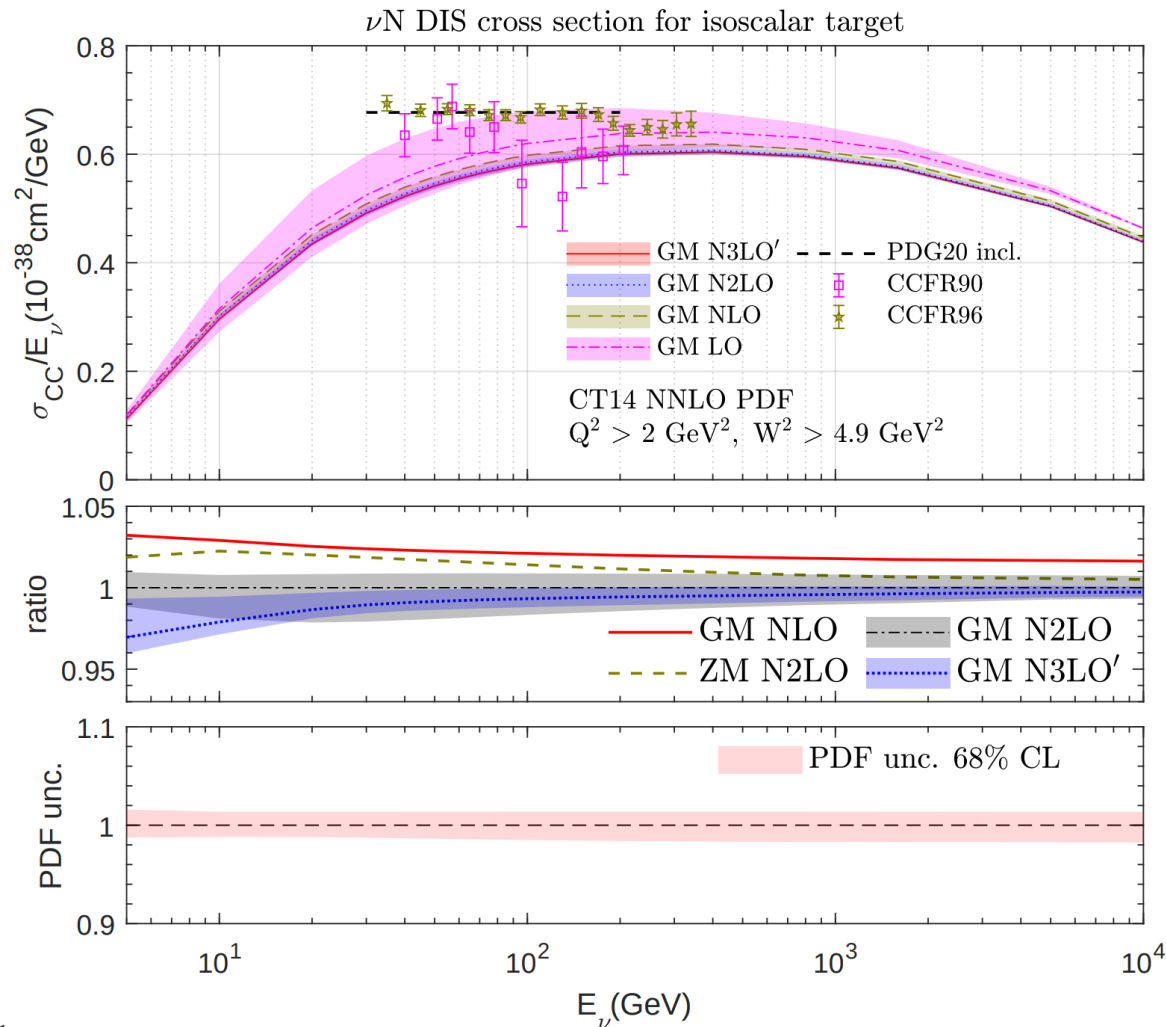
- significantly smaller than PDF-driven uncertainties, which can be as large as $\approx 2\%$

vital ingredient in EIC PDF program

precision QCD will also be necessary for νA

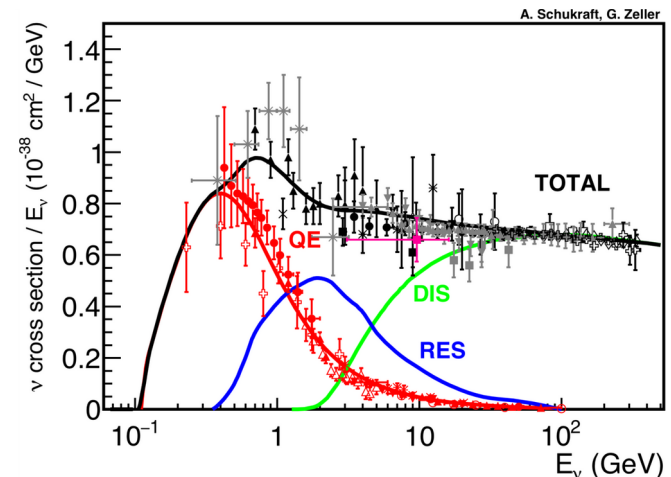
- forthcoming neutrino-nuclear experiments cover wide range of energies, E_ν
- even at DUNE, events coming from DIS represent $\gtrsim 40\%$

(important dependence on SF extrapolations, correlations in tunes with low-energy model parameters)



→ small DIS cross section variations influence DUNE sensitivity

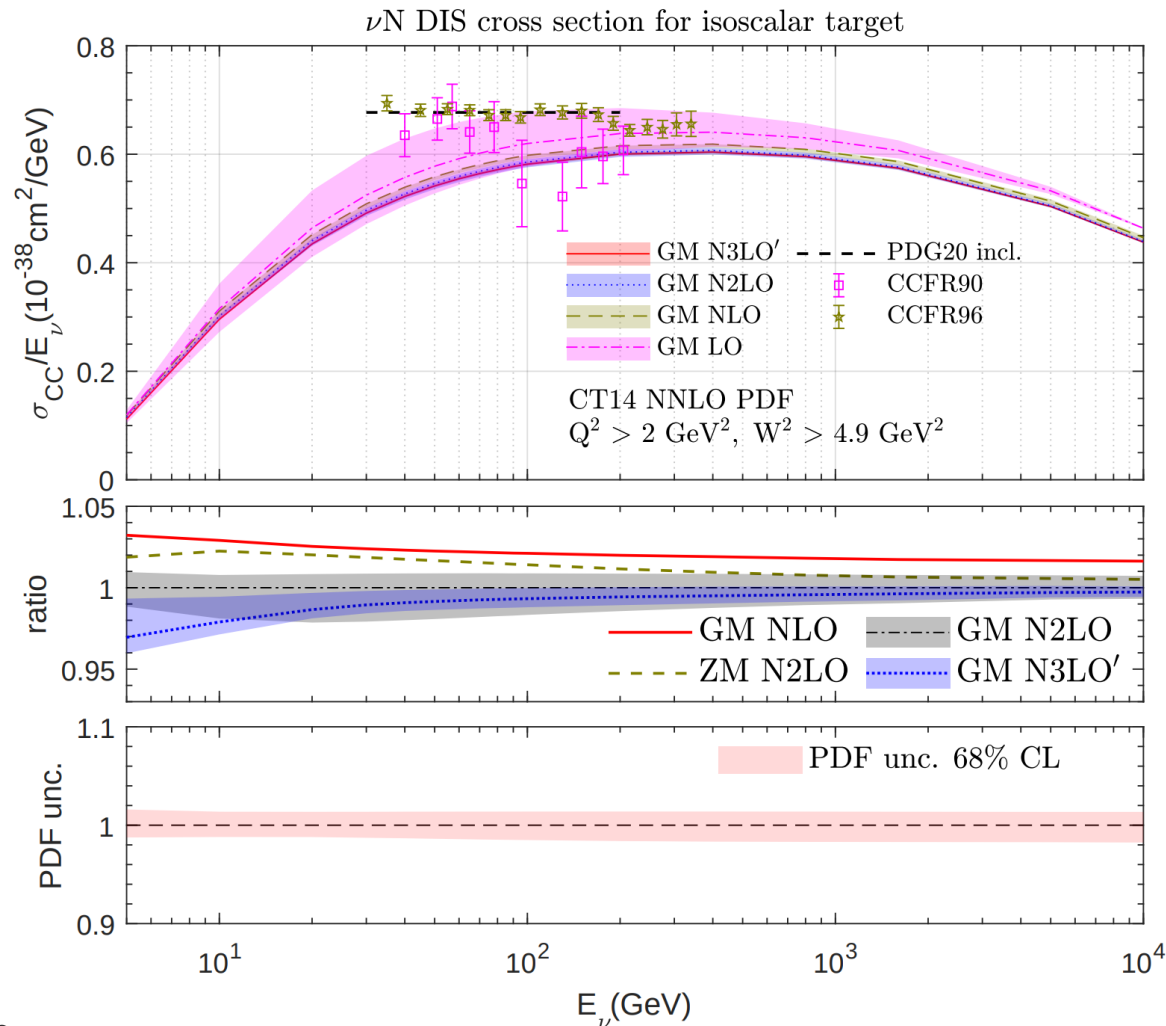
→ at higher energies, significant impact on sensitivity of forward-physics program at FASER ν (~ 100 s GeV); neutrino telescopes ($> \text{TeV}$)



precision QCD will also be necessary for νA

- ν cross sections generally diminished by LO \rightarrow (N)NNLO, by 6% for most E_ν
- as before, NNLO and N³LO' corrections greatly reduce scale variations

$\left\{ \begin{array}{l} E_\nu > 100 \text{ GeV: negligible} \\ \sim 1-3\% \text{ elsewhere} \end{array} \right.$



- in contrast, PDF uncertainties are $\sim 1-2\%$

\rightarrow strong pQCD theory for FASER ν program

- future analyses will witness an interplay between pQCD and **nuclear effects**

\rightarrow assessed nuclear correction using nCTEQ15: $\sim 0.5\%$ effect

conclusions: next steps, PDF implications

- have extended general-mass HQ scheme to CC DIS at NNLO; approximate N³LO
 - incorporates full HQ threshold dependence; interpolation between FFN, ZM approaches
 - dramatic reductions to dependence on perturbative QCD scale choices
 - consistency across broad ranges of $x, Q^2; E_\nu$
 - perturbative uncertainties at EIC reduced to sub-percent level for target kinematics
 - substantially boosts precision of inclusive measurements program; PDF sensitivity
 - (N)NNLO accuracy reduces ν DIS scale uncertainties to $\sim 1\text{-}3\%$; less at high energies
 - critical to achieving precision objectives in ν A programs at DUNE
-
- interfaces with PDF global analyses (and perhaps generators) will be valuable
 - higher pQCD accuracy suggests need for parallel enhancements in, *e.g.*, nuclear modeling, EW corrections, few-GeV nonperturbative theory