The Drell-Yan q_T Spectrum and Its Uncertainty at N^{3}LL$'$

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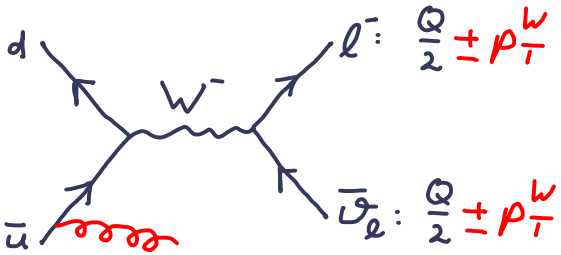
[to appear soon]

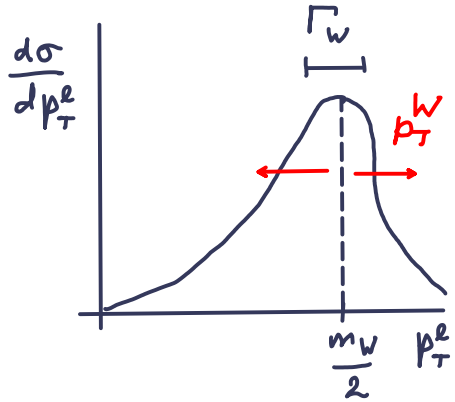
in collaboration with

G. Billis, M. Ebert, F. Tackmann

Motivation: Measuring m_W at ~~the LHC~~ any hadron collider

Want to measure m_W, but too much information about the neutrino is lost:





$\Rightarrow$ Need precise theory predictions for $\mathrm{d}\sigma/\mathrm{d}p_T^Z$ and $\mathrm{d}\sigma/\mathrm{d}p_T^W$ to model the p_T^W spectrum using precisely measured p_T^Z as input

$$\begin{aligned} m_W^{\text{ATLAS}} &= 80370 \pm 7_{\text{stat.}} \\ &\quad \pm 11_{\text{exp. syst.}} \\ &\quad \pm 14_{\text{theory}}\,\text{MeV} \\ &= 80370 \pm 19\,\text{MeV} \end{aligned}$$

[ATLAS, 1701.07240]

$$\begin{aligned} m_W^{\text{LHCb}} &= 80354 \pm 23_{\text{stat.}} \\ &\quad \pm 10_{\text{exp. syst.}} \\ &\quad \pm 17_{\text{theory}} \\ &\quad \pm 9_{\text{PDF}}\,\text{MeV} \\ &= 80354 \pm 32\,\text{MeV} \end{aligned}$$

[LHCb, 2109.01113]

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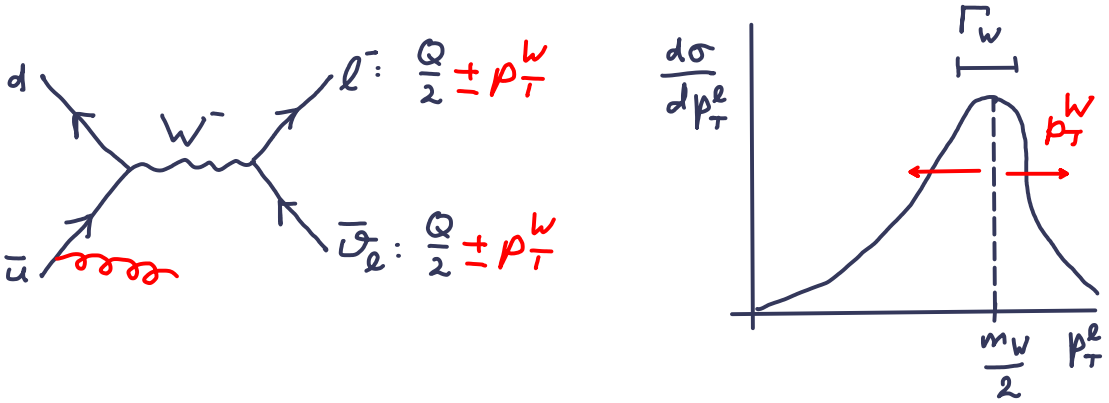

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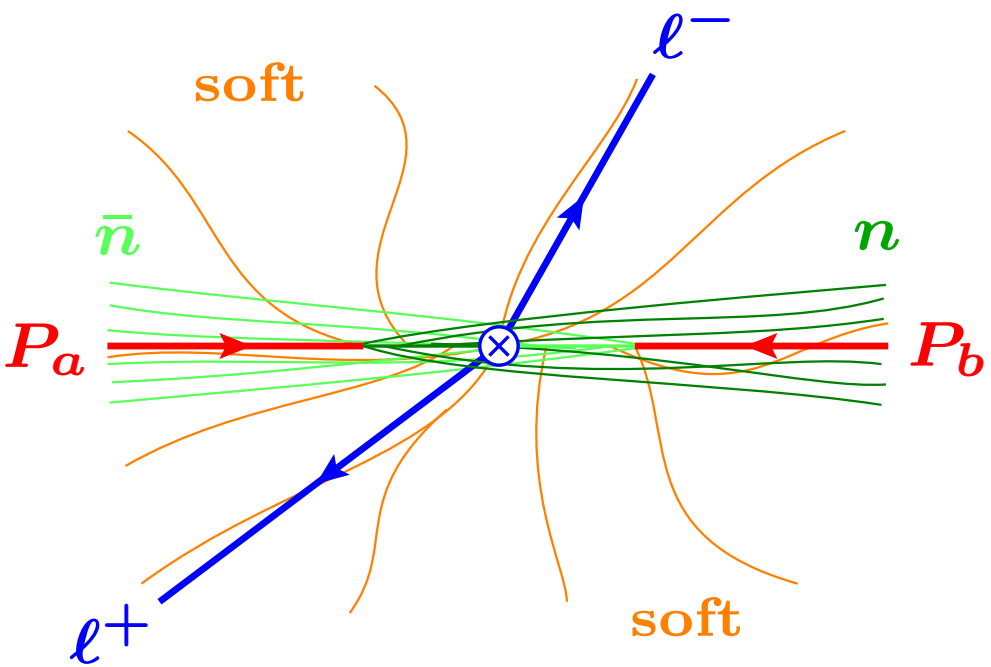
~~Challenges~~ Opportunities for theory

- Need sub-percent precision on $\mathrm{d}\sigma/\mathrm{d}p_T^Z$ and $\mathrm{d}\sigma/\mathrm{d}p_T^W$
  - Leave no stone unturned: QCD three-loop corrections, QED radiative corrections, quark mass effects, parametric and nonperturbative uncertainties
- Resum singular terms & large logarithms $\frac{\alpha_s^n}{q_T}\left(\ln\frac{q_T}{Q}\right)^{2n-1}$ to all orders in α_s

Perturbative ingredients: Factorized singular cross section at N^{3}LL′

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = \frac{\mathrm{d}\sigma^{\mathrm{res}}_{\mathrm{fact}}}{\mathrm{d}q_T} + \left[\frac{\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{full}}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{fact}}}{\mathrm{d}q_T}\right] \equiv \frac{\mathrm{d}\sigma^{\mathrm{res}}_{\mathrm{fact}}}{\mathrm{d}q_T} + \frac{\mathrm{d}\sigma^{\mathrm{nons}}_{\mathrm{fact}}}{\mathrm{d}q_T}$$

$$\frac{\mathrm{d}\sigma_{\mathrm{fact}}}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}q_T} = \sum_q H_{q\bar{q}}(Q,\mu)\; q_T \int_0^\infty \mathrm{d}b_T\, b_T\, J_0(q_T b_T)$$
$$\times\, f_q^{\mathrm{TMD}}(x_a, b_T, \mu, \zeta)\, f_{\bar{q}}^{\mathrm{TMD}}(x_b, b_T, \mu, \zeta) + (q \leftrightarrow \bar{q})$$



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Implemented in `SCETlib` C++ numerical library [Ebert, JKLM, Tackmann]:

- Three-loop hard function [Baikov et al. '09; Lee et al. '10; Gehrmann et al. '10, '20; Czakon et al. '21]
- Three-loop matching of TMD PDFs onto collinear PDFs
  [Li, Zhu, '16; Luo, Yang, Zhu, Zhu '19; Ebert, Mistlberger, Vita '20]
  - Prediction includes complete three-loop RG boundary conditions (N^{3}LL′)
  - Integral of spectrum is N^{3}LO-accurate
- Four-loop cusp, three-loop noncusp anomalous dimensions
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  [Resummation & use in subtraction: Ebert, JKLM, Stewart, Tackmann '20]
  [See talk by Alessandro Guida on Tuesday for impact on PDF fits!]

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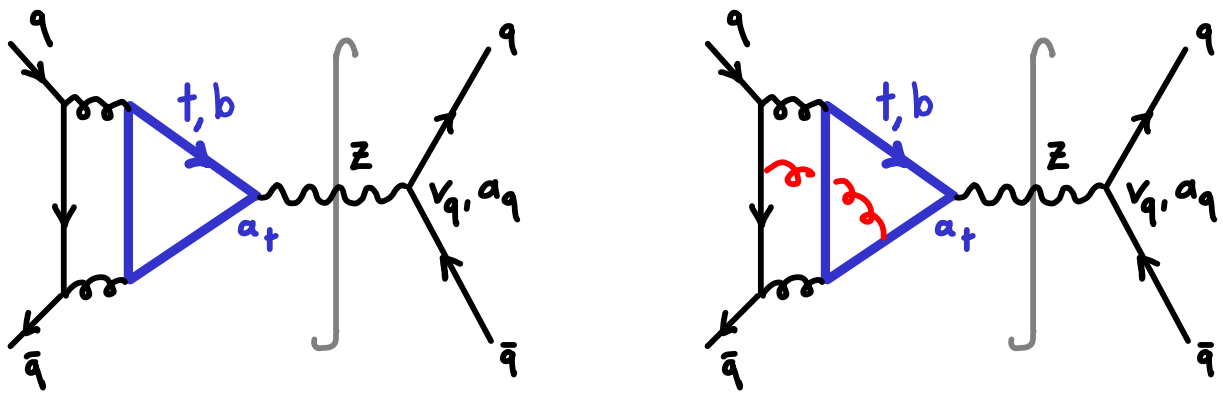

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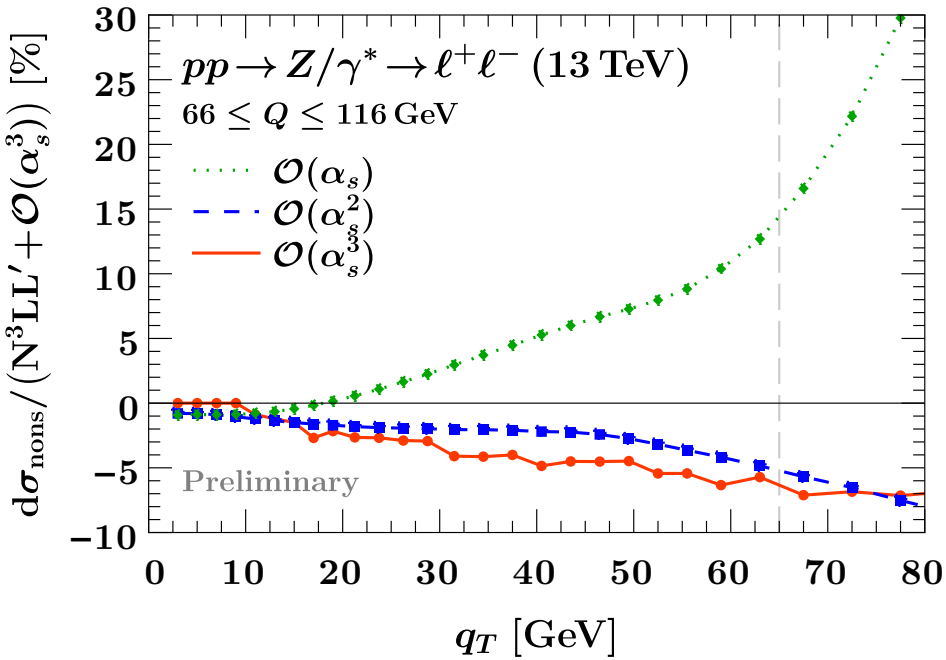
Perturbative ingredients: Fixed-order matching

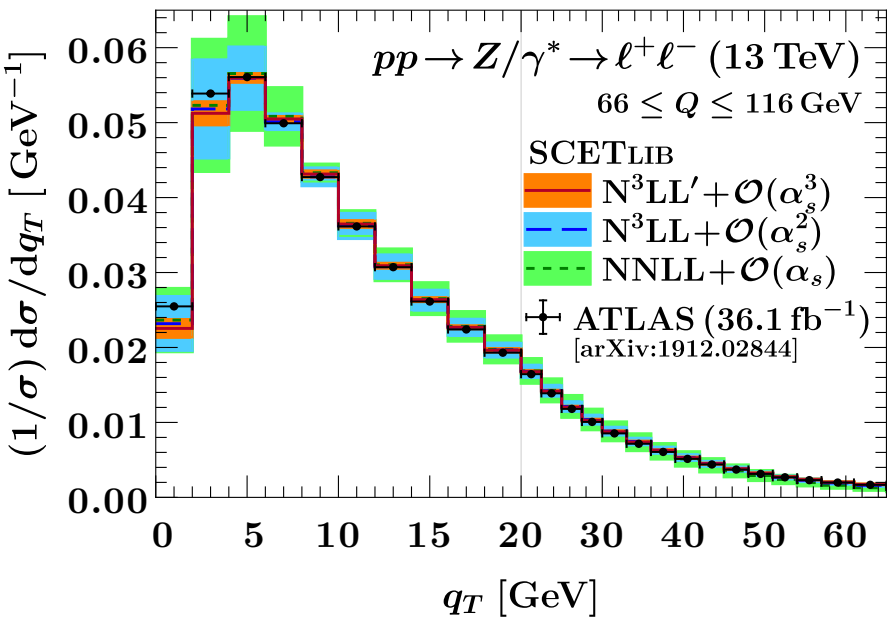
$$\frac{\mathrm{d}\sigma_{\mathrm{nons}}}{\mathrm{d}q_T} = \frac{\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{full}}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma^{\mathrm{FO}}_{\mathrm{sing}}}{\mathrm{d}q_T} = \frac{1}{q_T}\mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

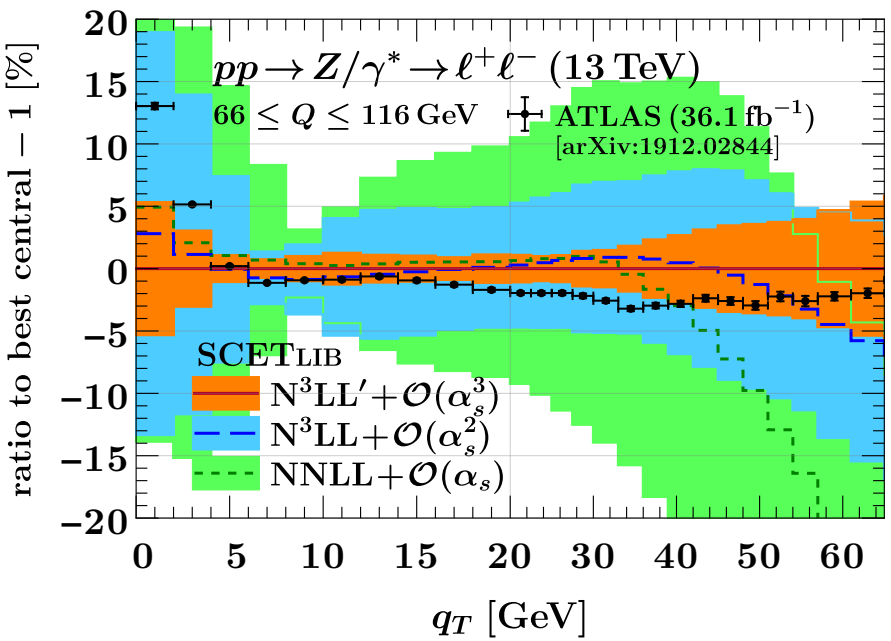


- In-house analytic implementation of all helicity structure functions at $\mathcal{O}(\alpha_s)$
- Fiducial Z+jet MC data at $\mathcal{O}(\alpha_s^2)$ from MCFM
  [Campbell, Ellis, et al. '99, '15]
- Very recently: Precise fiducial Z+jet MC data at $\mathcal{O}(\alpha_s^3)$ from NNLOjet
  [Chen et al., 2203.01565 – many thanks to the NNLOjet collaboration for providing the raw data.]

Results: Central prediction and perturbative convergence for $Z \to \ell^+\ell^-$

- Central results use `MSHT20nnlo` with $\alpha_s(m_Z) = 0.118$, $n_f = 5$
- NNLO (= three-loop!) PDF evolution formally sufficient at N^{3}LL′:
  - DGLAP kernels are a noncusp anomalous dimension
  - Scale dependence formally cancels within three-loop TMD PDF function
  - Separate question whether PDFs should have been extracted using three-loop $\hat{\sigma}_{ij}$ …

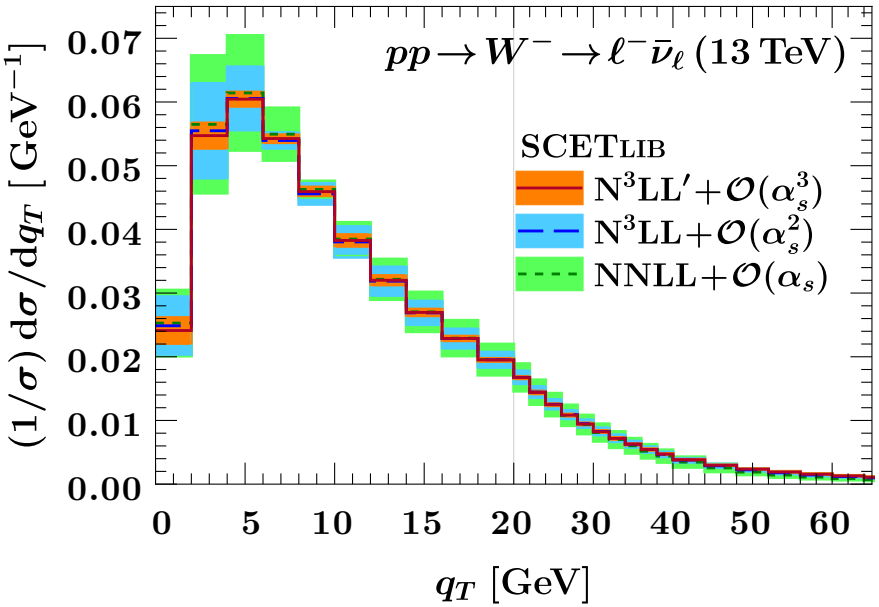


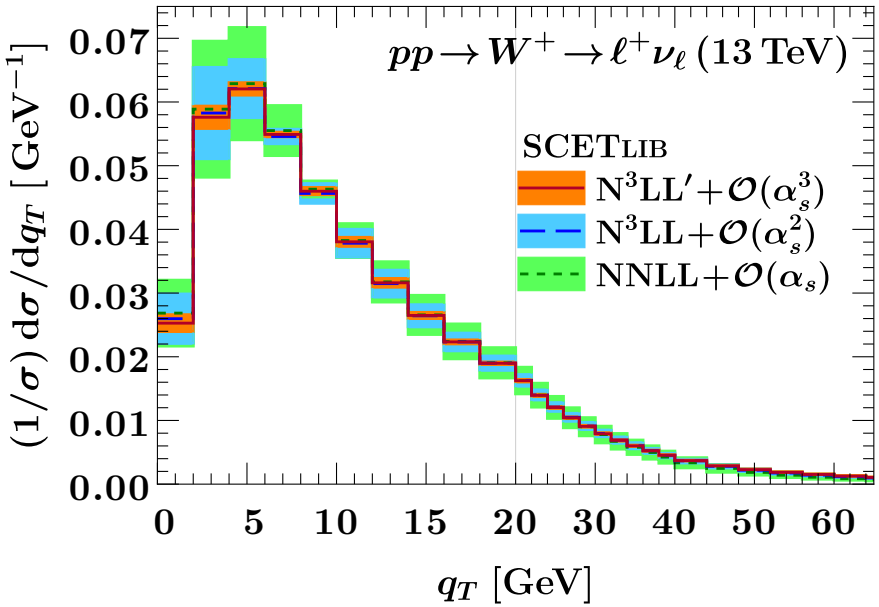


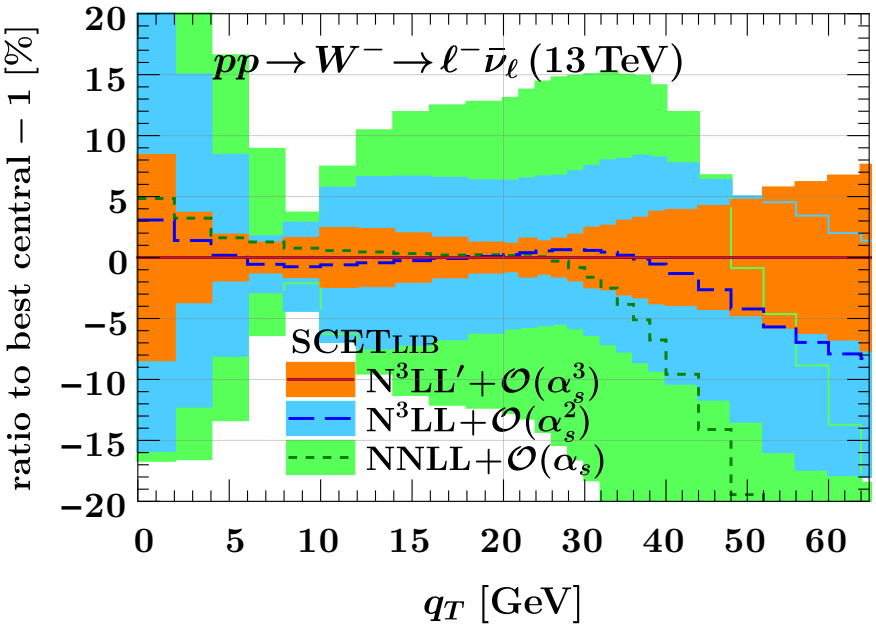
- Excellent perturbative convergence towards three-loop result
- Higher orders are covered by uncertainty estimate at lower orders

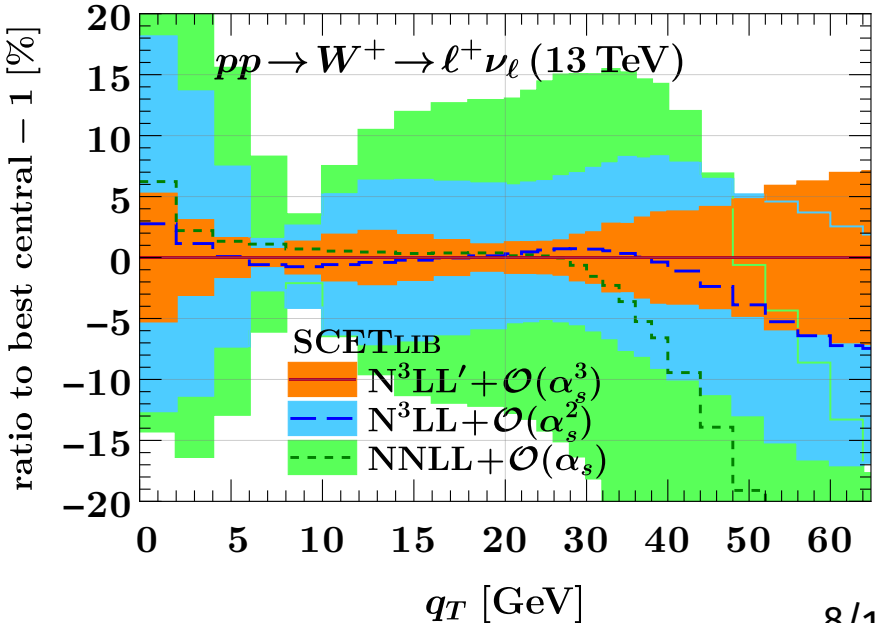
  [See backup for how they are estimated]

Results: Predictions for $W^{\pm} \to \ell\nu$

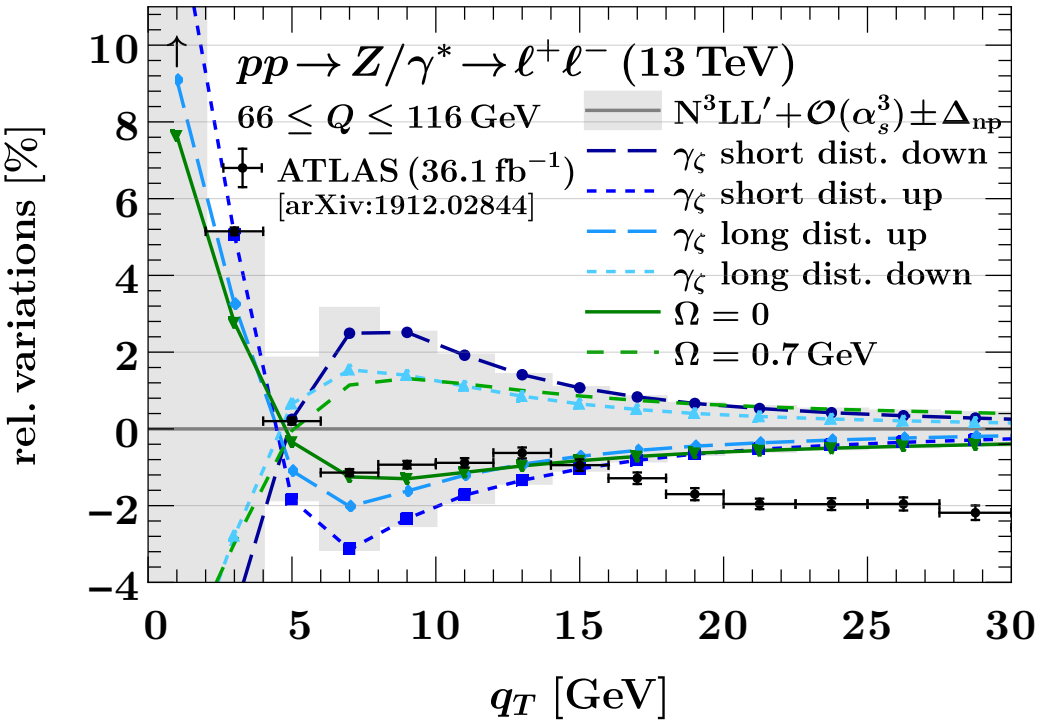






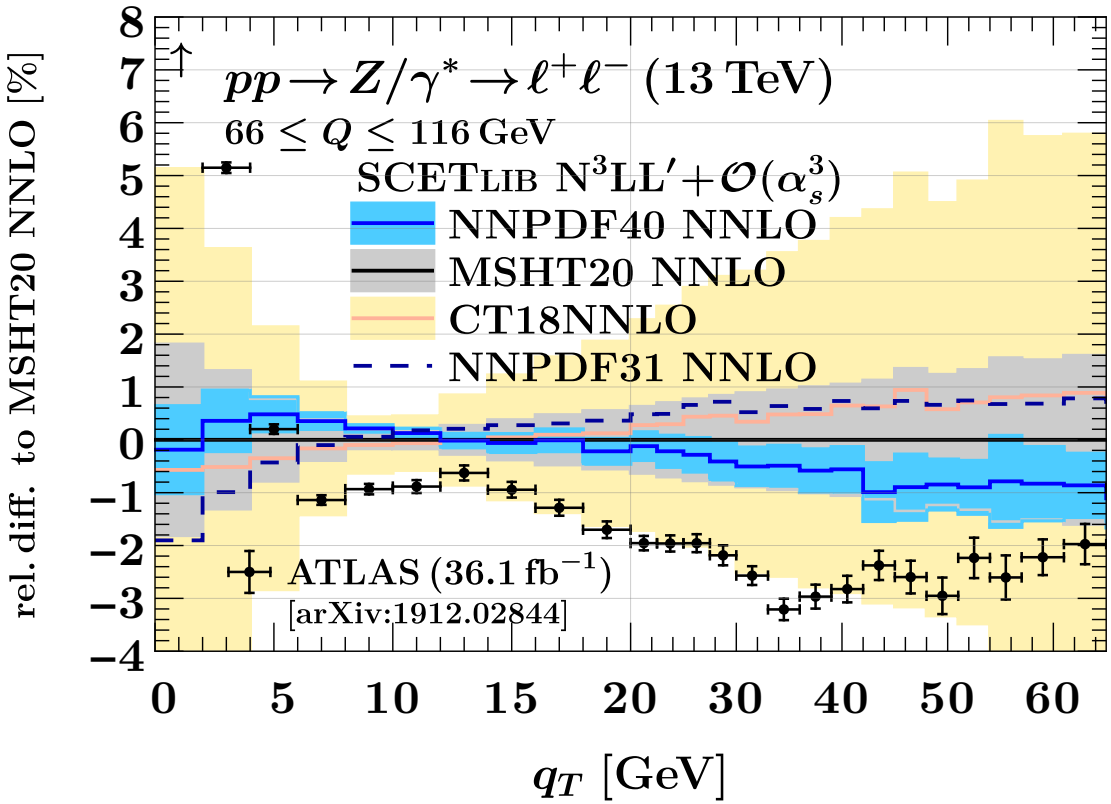


Results: Estimate of nonperturbative contributions



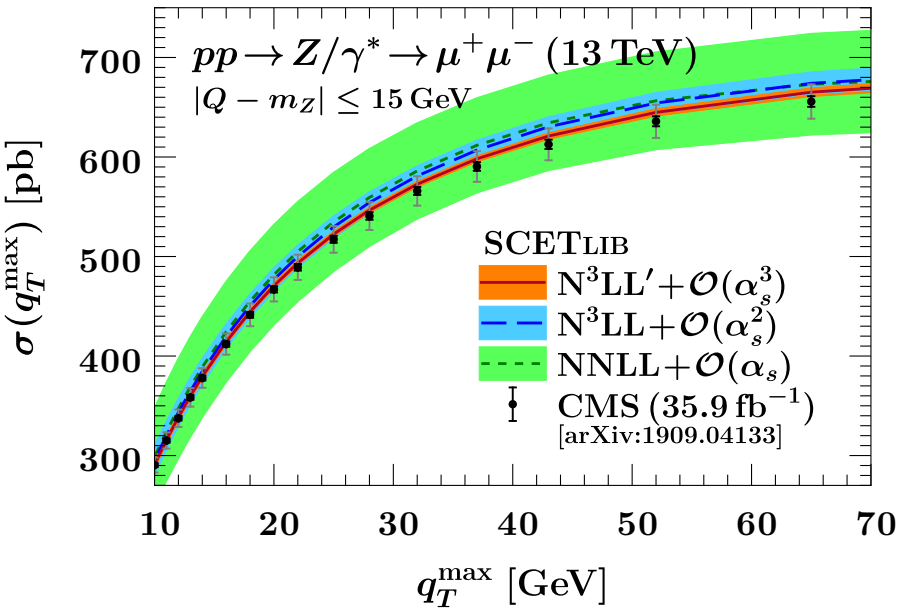
- Sources of nonperturbative corrections at small q_T:
  - Collins-Soper kernel (drives TMD evolution)
  - Intrinsic parton transverse momentum (TMD boundary conditions)
- Vary CS kernel model to cover spread of recent lattice results
  [See backup for details; see also talk by P. Shanahan on Monday!]
- Taken at face value, the lowest bins seem to prefer *weaker* NP effects
- Overshoot data at $q_T = 20 - 30\,\mathrm{GeV}$, way outside NP effect strength

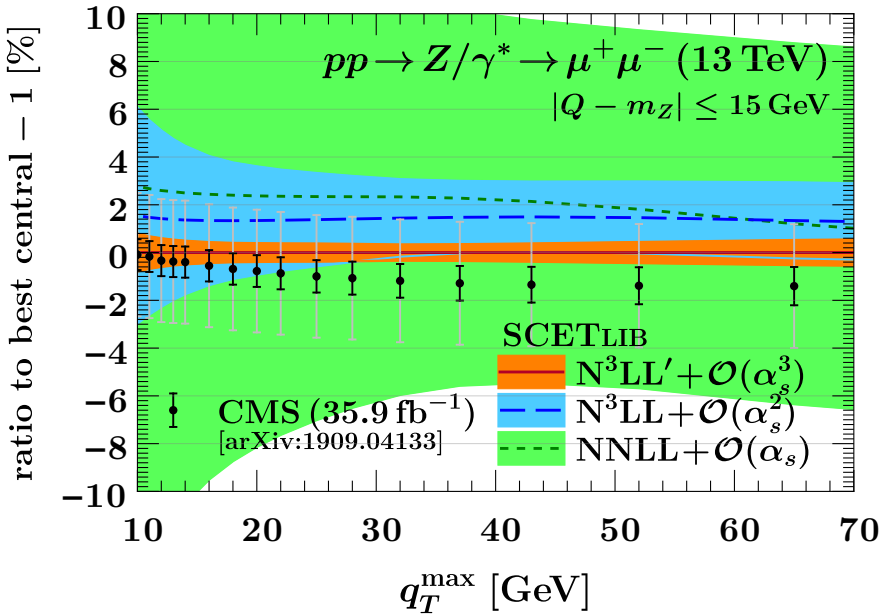
Results: Impact of PDFs on normalized Z spectrum

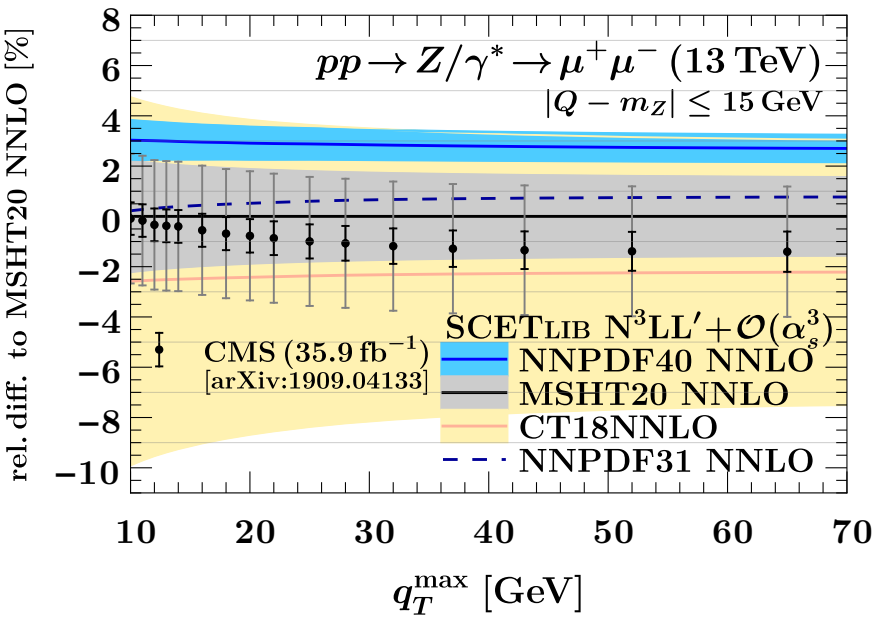


- Resummed 3-loop cross section is *analytic*, hold small $\alpha_s^{2,3}$ nonsingular fixed
  - Whole figure with complete PDF uncertainties at few 100 CPUh!
- PDF uncertainty largely cancels in normalized spectrum
- Cannot explain overshoot at $q_T = 20 - 30\,\mathrm{GeV}$

Cumulative unnormalized cross sections for N^{3}LO PDF fits

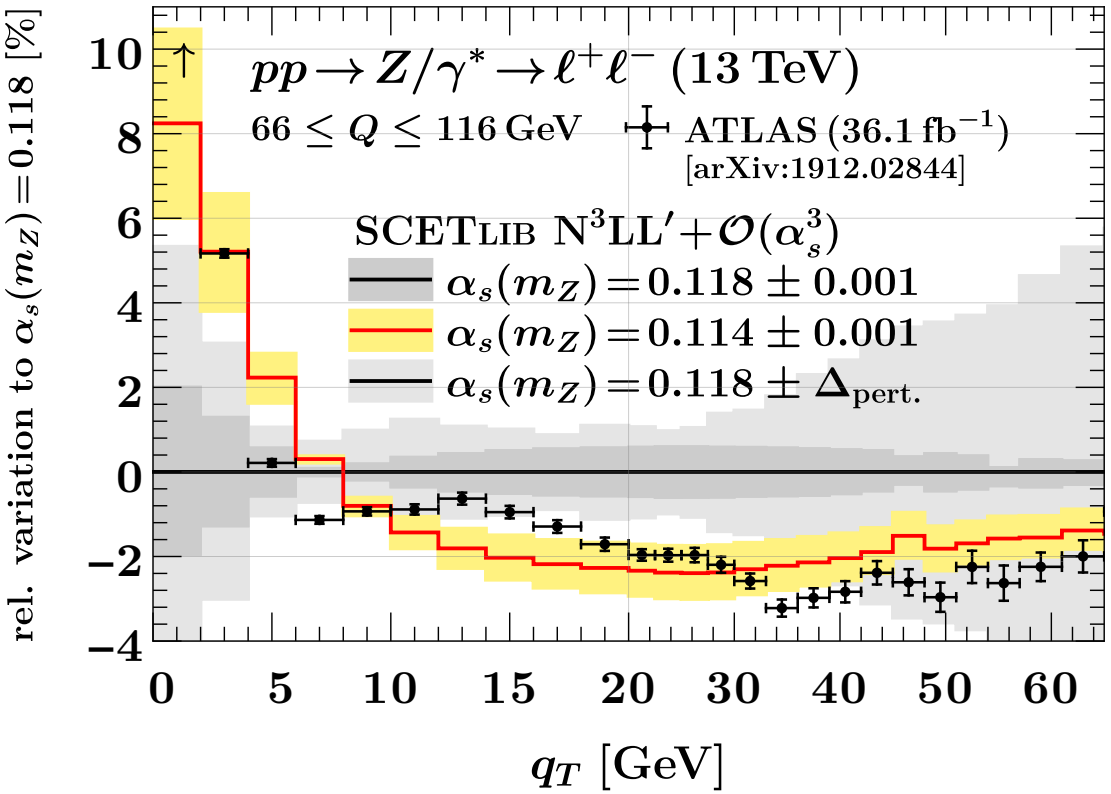






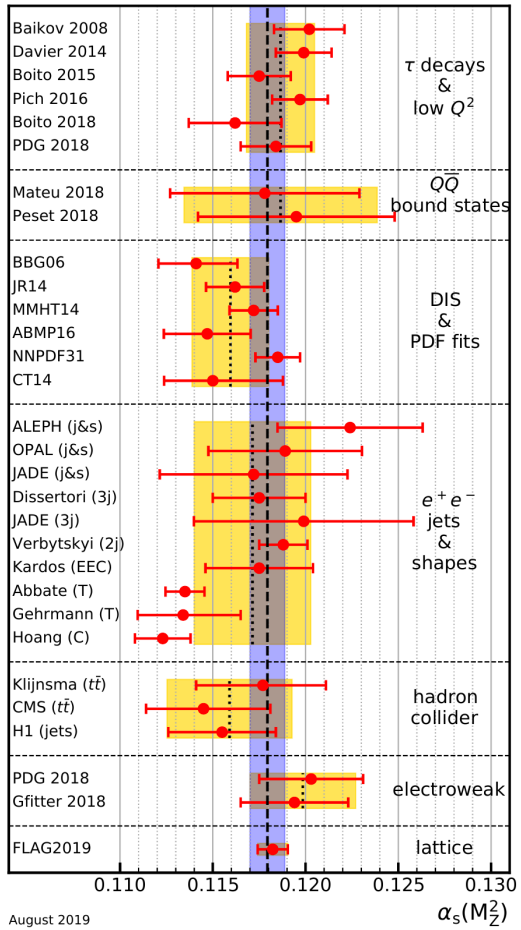
- Cumulative cross section distinguishes recent PDF sets
- Nonperturbative effects $\leq 0.1\%$ past $q_T^{\max} \sim 20\,\mathrm{GeV}$
- Great target for N^{3}LO PDF fits (limiting K-factors to σ_{nons}!)

Results: Impact of α_s on normalized Z spectrum



- Parametric uncertainty due to $\alpha_s(m_Z)$ on par with perturbative uncertainty
- Overshoot at $q_T = 20 - 30\,\text{GeV}$ is naturally explained by lower $\alpha_s(m_Z)$

This is not unprecedented …



- Lower values of $\alpha_s(m_Z)$ have previously been reported in fits to e^+e^- event shapes (thrust and C parameter)

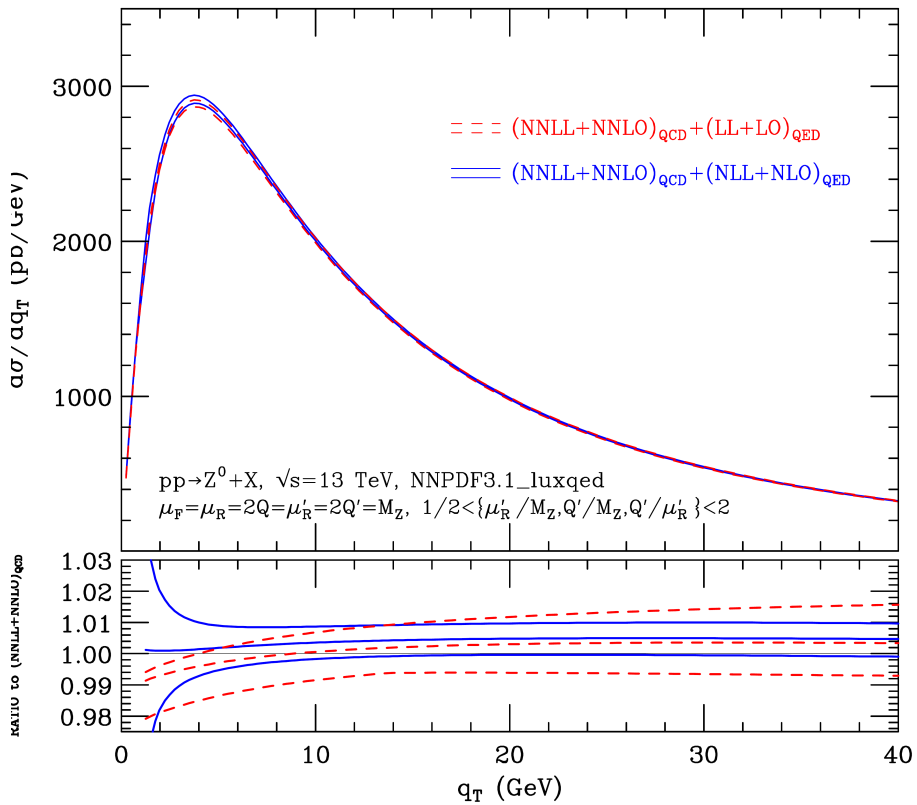
  DISCLAIMER: This was *not* an actual fit to $\{\alpha_s(m_Z), \Omega, \omega_\zeta^{(2)}\}$.

- Like $p_T^{Z/W}$, these are driven by all-order resummation …

. . . but many caveats remain

Systematics at the theory frontier:

- QED resummation effects for on-shell Z well understood
  [Bacchetta, Echevarria '18; Cieri, Ferrera, Sborlini '18; Billis, Tackmann, Talbert '19]
  - Expected to be $\sim 1\%$, but would bring the tail up *more*



[Cieri, Ferrera, Sborlini 1805.11948]

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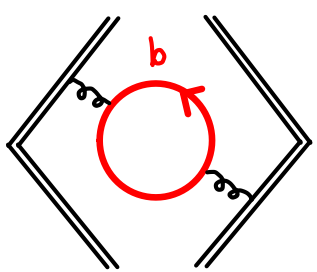
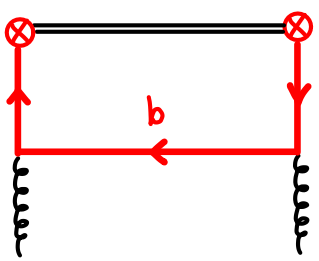
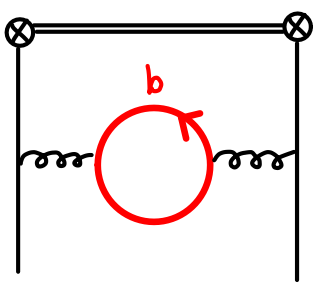
- In progress: Interface resummation with QED/weak corrections to full process with realistic lepton definitions (beyond mixed $\alpha\alpha_s$)
- Subleading power resummation & factorization for *nonsingular* cross section
  [Progress towards doing this at least for $\mathcal{O}(q_T/Q)$ azimuthal correlations]
  [Moos, Scimemi, Rodini, Vladimirov '21-'22; Ebert, Gao, Stewart '21 $\longrightarrow$ see talk by A. Vladimirov!]
- Full resummed treatment of mass effects/flavor thresholds
  - Expect impact on spectrum (and cumulative cross section) to be suppressed by $\# m_b^2/q_T^2$?







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- Presented third-order predictions for Z and $W^{\pm}$ q_T spectra at the LHC
  - Residual perturbative uncertainty at percent level in the peak
- Three-loop resummed SCETlib predictions are analytic & fast also with cuts
  - Assessing PDF and α_s uncertainties possible *directly at three loops*
  - Cumulative cross sections up to $q_T^{\max} \approx 40\,\mathrm{GeV}$
    extremely promising targets for N^{3}LO PDF fits (or reweighting)
- Even small changes $\alpha_s(m_Z) \pm 0.001$ strongly impact the peak shape
  - Effect for $q_T \leq 20\,\mathrm{GeV}$ as important for TMD fits as collinear PDF uncertainty
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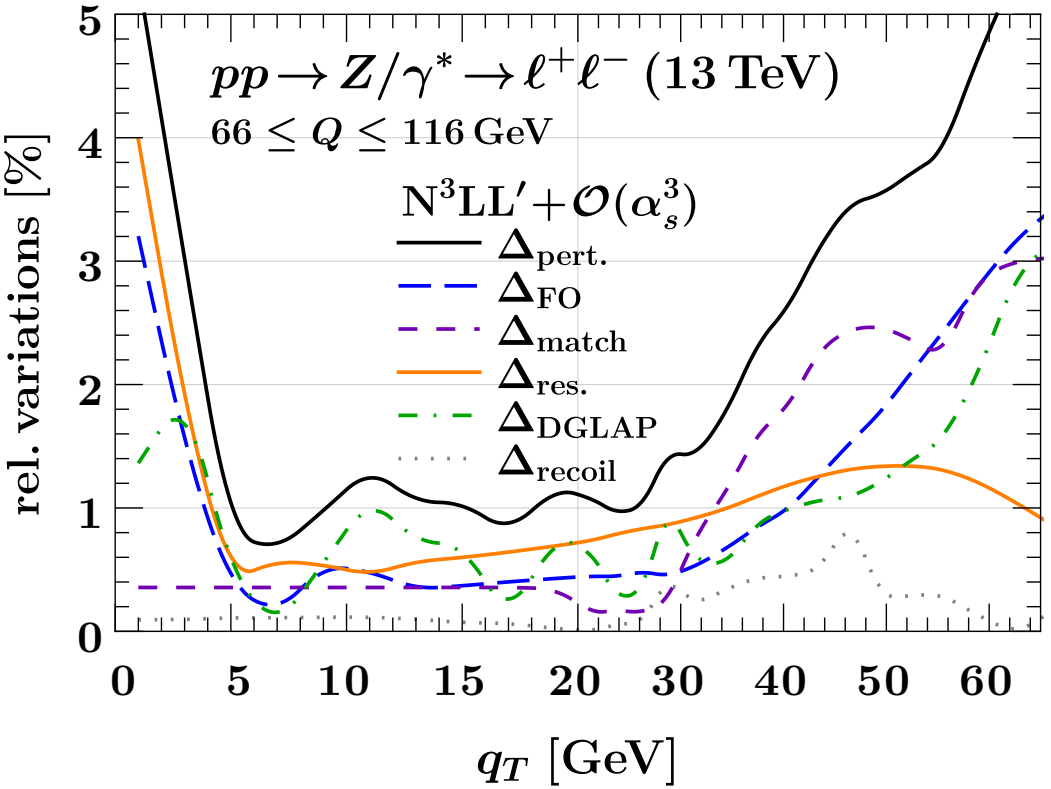
Thank you for your attention!

Backup

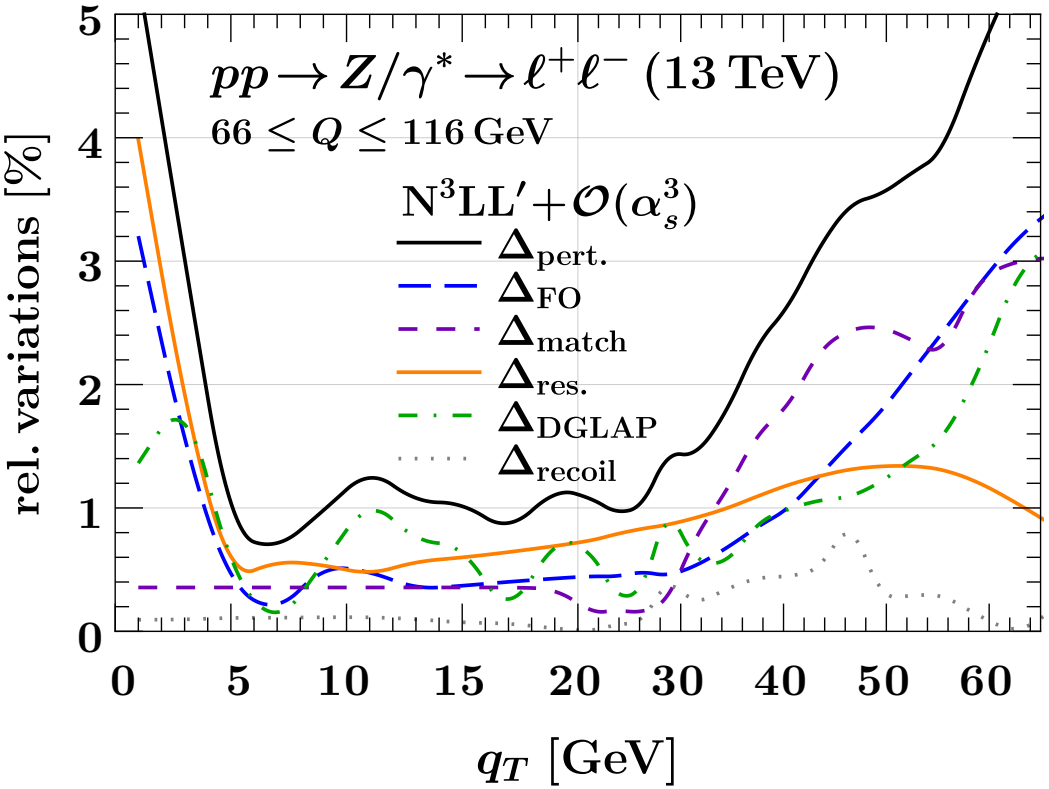
Breakdown of perturbative uncertainties



$$\Delta_{\mathrm{pert.}} = \Delta_{\mathrm{FO}} \oplus \Delta_{\mathrm{match}} \oplus \Delta_{\mathrm{res}} \oplus \Delta_{\mathrm{DGLAP}} \oplus \Delta_{\mathrm{recoil}}$$

- Fixed-order uncertainty, keeps resummed logarithms unchanged
- Estimated by standard variations of overall $\mu_R = \mu_{\mathrm{FO}}$
- All scales (except μ_f) are chosen $\propto \mu_{\mathrm{FO}}$, so e.g. μ_H/μ_S unchanged
- Frozen out at $b_T \lesssim 1/\Lambda_{\mathrm{QCD}}$ by μ_X^* prescription $\Rightarrow$ disentangled from NP

Breakdown of perturbative uncertainties



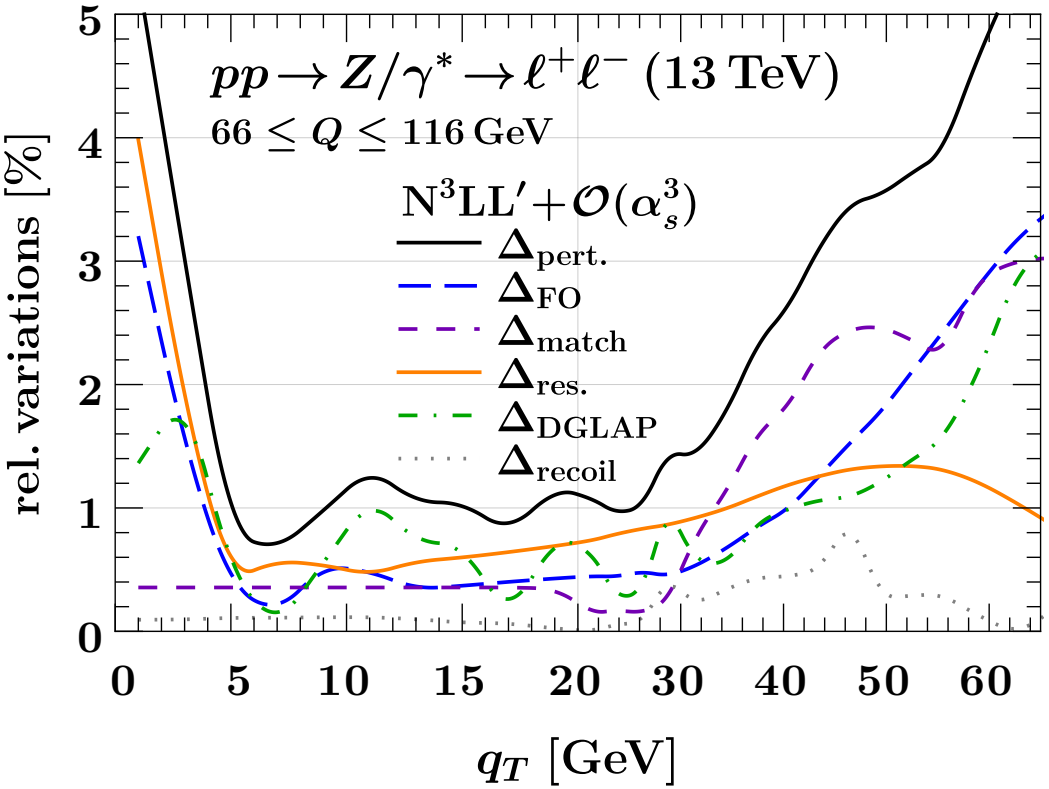
$$\Delta_{\mathrm{pert.}} = \Delta_{\mathrm{FO}} \oplus \Delta_{\mathrm{match}} \oplus \Delta_{\mathrm{res}} \oplus \Delta_{\mathrm{DGLAP}} \oplus \Delta_{\mathrm{recoil}}$$

- Uncertainty from matching scheme between resummed peak and fixed-order tail
- Estimated by varying the $x = q_T/Q$ transition points in hybrid profile as

$$\{x_1, x_2, x_3\} = \{0.3, 0.6, 0.9\} \pm \{0.1, 0.15, 0.2\}$$

- Checked that *inclusive* integrated cross section is recovered within Δ_{match}

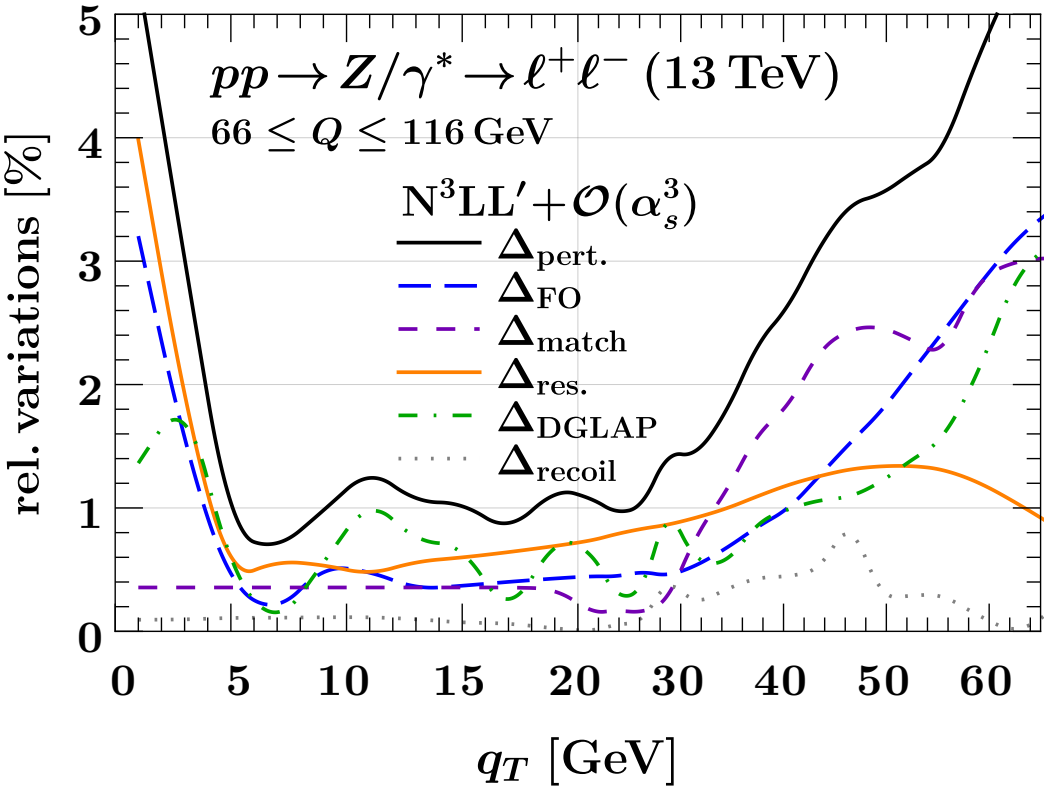
Breakdown of perturbative uncertainties



$$\Delta_{\mathrm{pert.}} = \Delta_{\mathrm{FO}} \oplus \Delta_{\mathrm{match}} \oplus \Delta_{\mathrm{res}} \oplus \Delta_{\mathrm{DGLAP}} \oplus \Delta_{\mathrm{recoil}}$$

- Probes higher-order resummed logarithms
- Estimated by envelope of 36 different combinations of independently varying $\{\mu_B, \mu_S, \nu_B, \dots\}$ in $\sigma^{(0)} = H\, B \otimes B \otimes S$
- Also frozen out at $b_T \lesssim 1/\Lambda_{\mathrm{QCD}}$ by μ_X^* prescription $\Rightarrow$ disentangled from NP

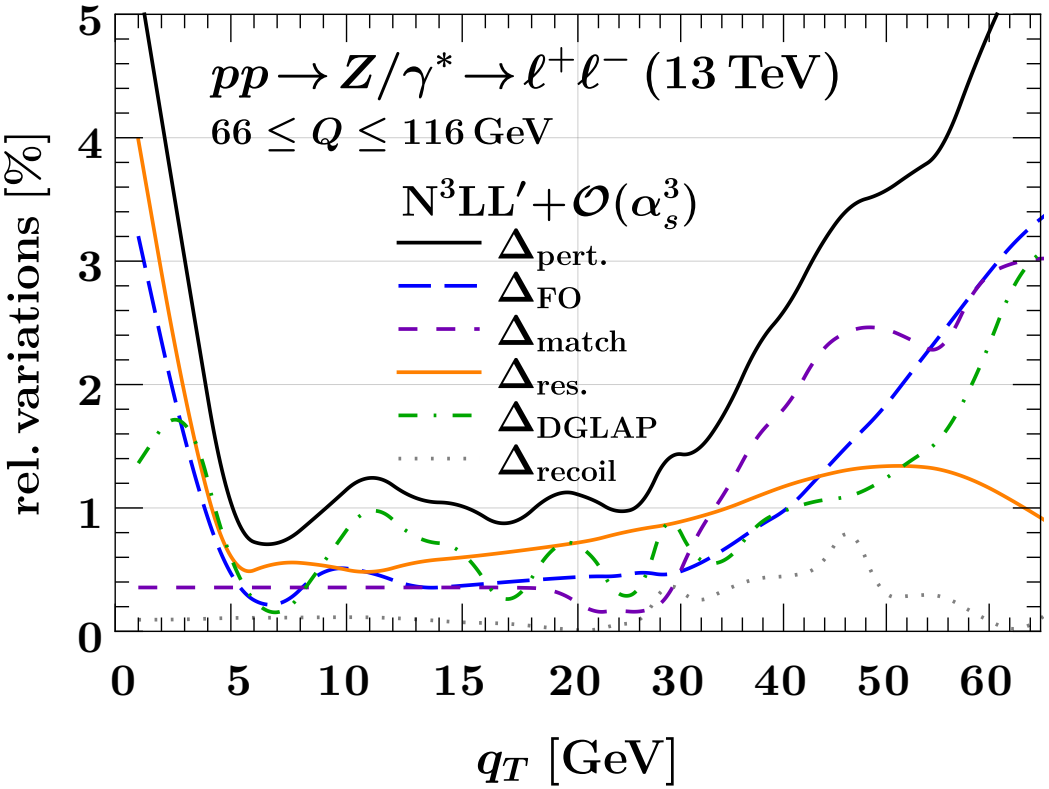
Breakdown of perturbative uncertainties



$$\Delta_{\mathrm{pert.}} = \Delta_{\mathrm{FO}} \oplus \Delta_{\mathrm{match}} \oplus \Delta_{\mathrm{res}} \oplus \Delta_{\mathrm{DGLAP}} \oplus \Delta_{\mathrm{recoil}}$$

- Estimate of missing higher orders (four loops) in DGLAP running
- Estimated both in peak and tail by joint variations of $\mu_f(b_T, q_T, Q)$ and $\mu_F(Q)$
- Oscillatory due to b_T-space features at uncancelled m_b threshold

Breakdown of perturbative uncertainties

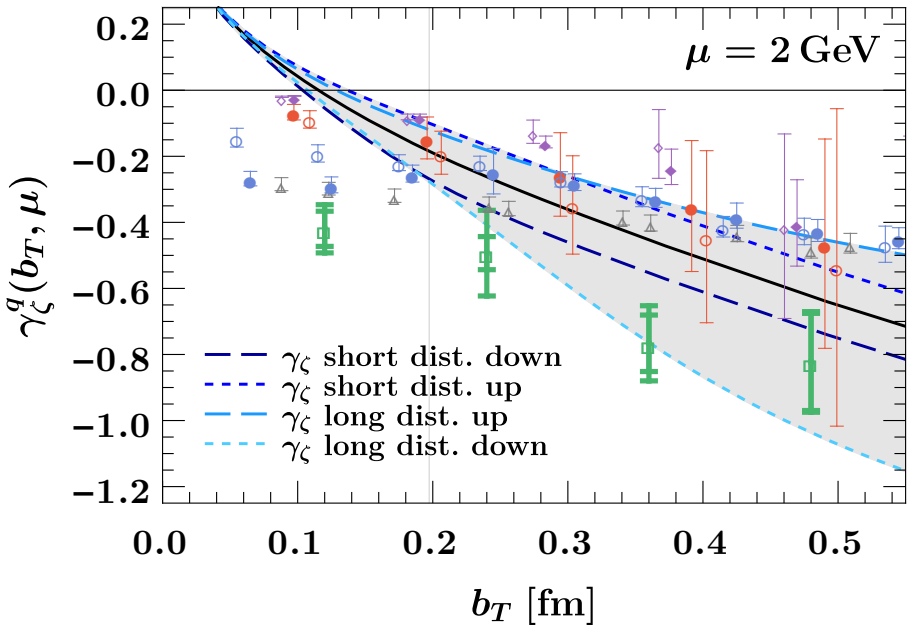


$$\Delta_{\mathrm{pert.}} = \Delta_{\mathrm{FO}} \oplus \Delta_{\mathrm{match}} \oplus \Delta_{\mathrm{res}} \oplus \Delta_{\mathrm{DGLAP}} \oplus \Delta_{\mathrm{recoil}}$$

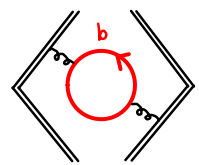
- RPI-I transformation of n_a^μ, n_b^μ in $W_{\mathrm{LP}}^{\mu\nu} \sim g_\perp^{\mu\nu}(n_a, n_b)$
- Induces $O(q_T^2/Q^2)$ change in spectrum due to fiducial cuts on $L_{\mu\nu}$
  [Ebert, JKLM, Stewart, Tackmann '20]
- Equivalent to changing "recoil prescription"/choice of Z rest frame by $\mathcal{O}(q_T/Q)$

Nonperturbative model for the Collins-Soper kernel

$$\frac{1}{2}\gamma^q_{\nu,\mathrm{NP}}(b_T) = \gamma^q_{\zeta\,\mathrm{NP}}(b_T) = c^i_\zeta \tanh\!\left(\frac{\omega^2_{\zeta,i}}{|c_\zeta|} b_T^2\right) = \mathrm{sgn}(c^i_\nu)\,\omega^2_{\zeta,i} b_T^2 + \mathcal{O}(b_T^4)$$



- Vary either ω_ζ (“short distance”) or c_ζ (“long distance”) to cover lattice results
  [Collection of lattice data from Shanahan, Wagman, Zhao, 2107.11930 → see talk by P. Shanahan]
- Pick central value of $\mathrm{sgn}(c^i_\nu)\,\omega^2_{\zeta,i}(1 \pm 2)$ to serve as bias correction for known leading (NNLL) bottom quark mass effect in γ^q_ζ:



$$\Delta\gamma^q_\zeta(b_T, m_b, \mu) = \frac{\alpha_s^2}{\pi^2} C_F T_F (m_b b_T)^2 \left(\ln\frac{b_T^2 m_b^2}{4e^{-2\gamma_E}} - 1\right) \approx -(0.25\,\mathrm{GeV})^2 b_T^2$$

NOTE Compatible with [Scimemi, Vladimirov ’19; Bacchetta et al. ’19], but aim for a-priori prediction

- Most general structure of leading NP correction $b_T^2 \boldsymbol{\Lambda_i^{(2)}(x)}$ is complicated
- However, can show that for a given process and fiducial volume, only *a single average coefficient* $\boldsymbol{\overline{\Lambda}}$ remains after the integral over hard phase space Φ_B:
  [Ebert, JKLM, Stewart, Sun '22]

$$\tilde{\sigma}(b_T) = \tilde{\sigma}^{(0)}(b_T)\Big\{1 + b_T^2\Big(2\boldsymbol{\overline{\Lambda}^{(2)}} + \gamma_{\zeta,q}^{(2)} L_{Q^2}\Big)\Big\} + \mathcal{O}\big[(\Lambda_{\rm QCD} b_T)^4\big]$$

$$\boldsymbol{\overline{\Lambda}^{(2)}} = \frac{\int \mathrm{d}\Phi_B\, A(\Phi_B) \sum_{i,j} \sigma_{ij}^B(Q)\, f_i^{(0)}(x_a, \mu_0)\, f_j^{(0)}(x_b, \mu_0)\big[\boldsymbol{\Lambda_i^{(2)}(x_a)} + \boldsymbol{\Lambda_j^{(2)}(x_b)}\big]}{2\int \mathrm{d}\Phi_B\, A(\Phi_B) \sum_{i,j} \sigma_{ij}^B(Q)\, f_i^{(0)}(x_a, \mu_0)\, f_j^{(0)}(x_b, \mu_0)}$$

- Idea: Promote $\boldsymbol{\overline{\Lambda}^{(2)}}$ to a single-parameter Gaussian model

$$f_i^{\rm NP}(x, b_T) = \exp(-\Omega^2 b_T^2) \quad \text{with} \quad \boldsymbol{\overline{\Lambda}^{(2)}} = -\Omega^2$$

- Take central $\Omega = 0.5\,\mathrm{GeV}$ and vary it as $\Omega = \{0, 0.7\}\,\mathrm{GeV}$
- For $q_T \gg \Lambda_{\rm QCD}$, this captures the most general form of the leading NP correction to the rapidity-integrated q_T spectrum

RG evolution, profile scales, and Landau pole prescription

- Use exact analytic solutions of virtuality and rapidity RG equation, combined with fast numerically exact solution of β function [Ebert '21]
  - Eliminates source of truncation error at fraction of cost of full Runge-Kutta
- Choose RG boundary scales as *hybrid profile scales* $\mu_X(b_T, q_T, Q)$:
  [Lustermans, JKLM, Tackmann, Waalewijn '19]

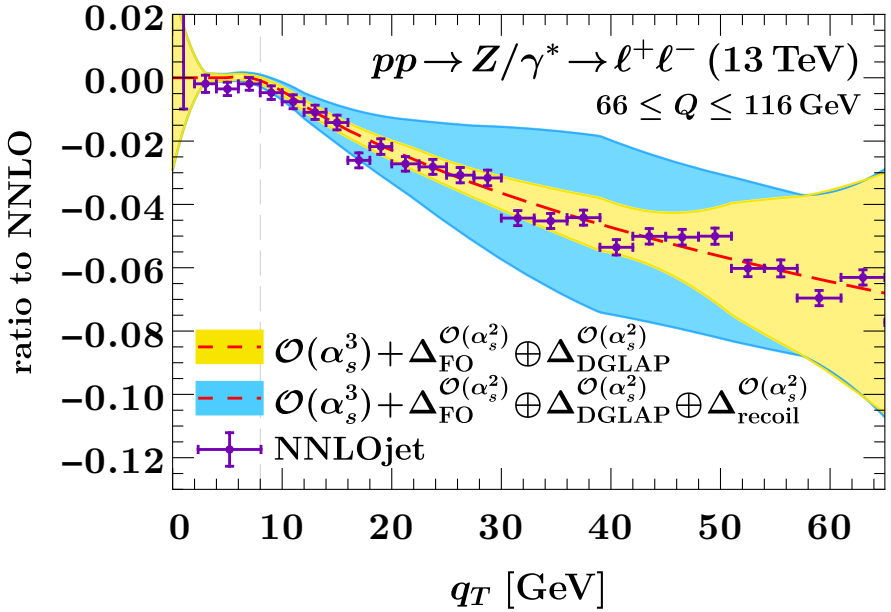
$$\mu_X(b_T, q_T \ll Q) = \frac{b_0}{b_T} \quad \text{but} \quad \mu_X(b_T, q_T \to Q) \to \mu_{\rm FO} = Q$$

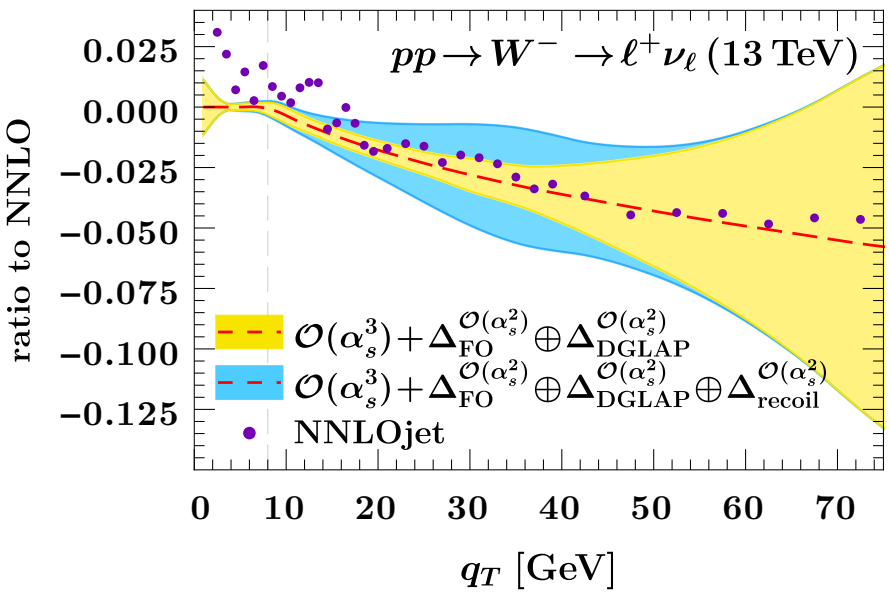
- Apply "local" b^* prescription starting at $\mathcal{O}(b_T^4)$ to virtuality scales *only*:

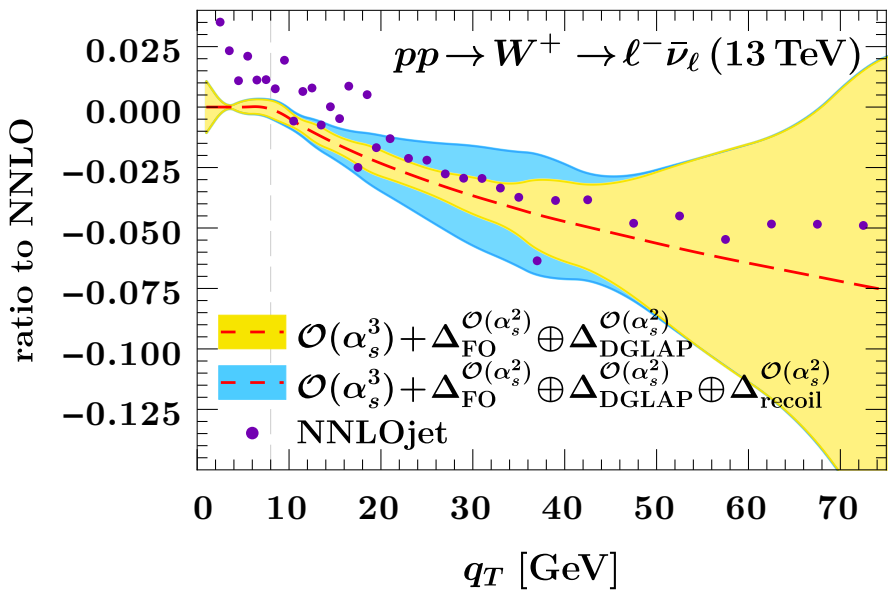
$$\mu_X \to \mu_X^* = \left[(\mu_X^{\rm min})^4 + \left(\frac{b_0}{b_T}\right)^4\right]^{1/4} = \frac{b_0}{b_T}\left\{1 + \mathcal{O}\left[(\mu_i^{\rm min} b_T)^4\right]\right\}$$

  - Avoids contaminating nonperturbative corrections at quadratic order
    [Conflict with b_T-space renormalon structure: Scimemi, Vladimirov '18]
    [Translation back to momentum space: Ebert, JKLM, Stewart, Sun '22]
  - For PDFs inside beam functions, use $\mu_f^{\rm min} = \min\{Q_0, m_c\}$

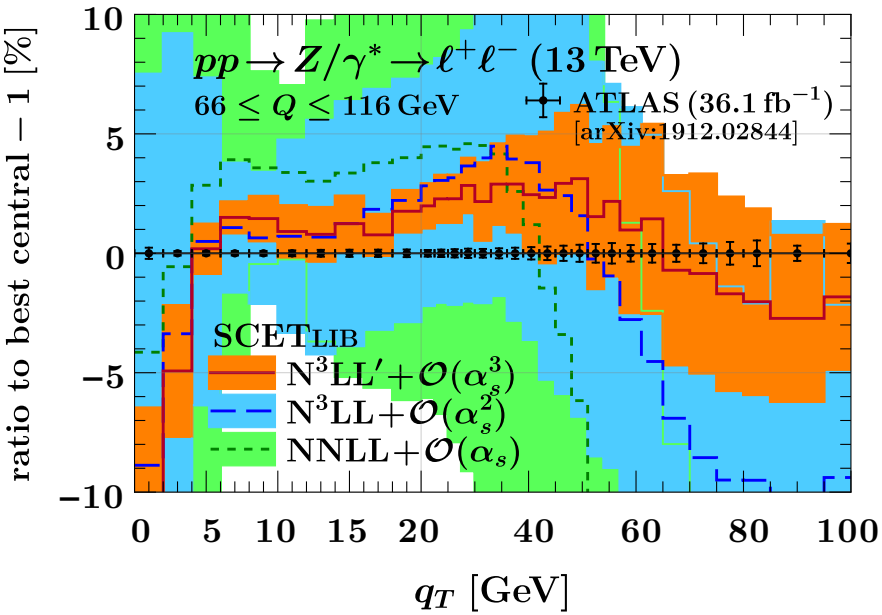
$\mathcal{O}(\alpha_s^3)$ nonsingular interpolations

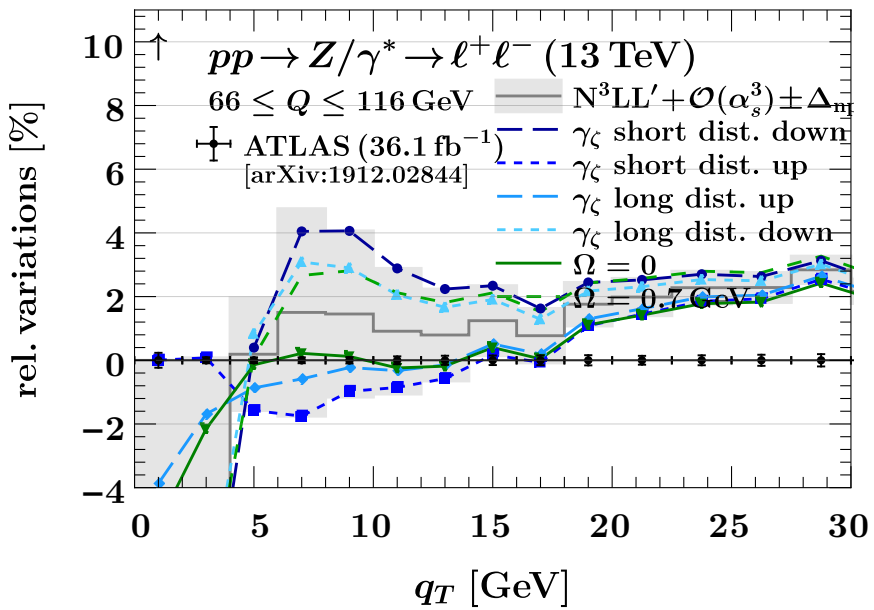




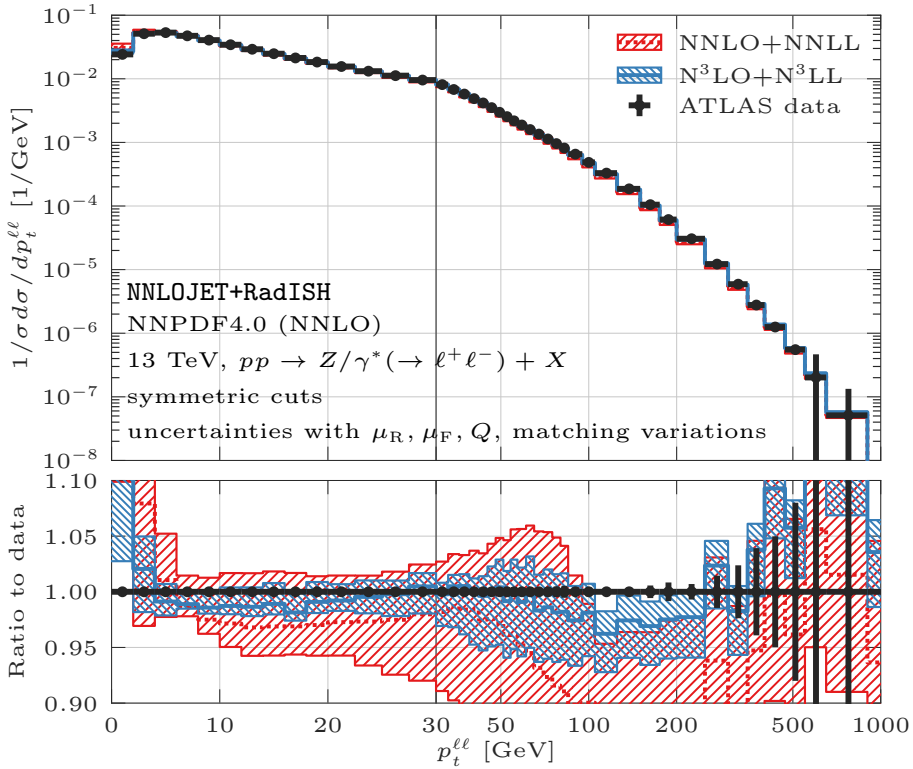


Comparison with RadISH (using identical NNLOjet fixed-order matching)





[Chen et al., 2203.01565]



- Can recover the data for $q_T \le 4\,\mathrm{GeV}$ with NP model $\approx$ *off*
- To recover the RadISH result at $\le 4\,\mathrm{GeV}$, would need large positive $\gamma_\zeta^{(2)}$ or $\bar{\Lambda}^{(2)}$
- In either case, cannot recover $\ge 20\,\mathrm{GeV}$ due to $\Lambda_{\rm QCD}^2/q_T^2$ scaling imposed by TMD factorization & OPE

Comparison with RadISH (using identical NNLOjet fixed-order matching)

- Common ingredient: Sudakov evolution kernels from $\mu_0 \sim Q$ to $\mu \sim 1/b_T, q_T$

  e.g.: $$K_\Gamma(\mu_0, \mu) = \int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \, \Gamma[\alpha_s(\mu')] \ln \frac{\mu'}{\mu_0}$$

- Implementation of Sudakov kernels in SCETlib is exactly equal to numerical solution of β function + numerical μ' integral
  - $\beta(\alpha_s)$ and $\Gamma(\alpha_s)$ truncated after α_s^4, no additional approximations or assumptions
  - Exact RGE closure $U(\mu_0, \mu)\, U(\mu, \mu_0) = 1$
  - Exact path independence in (μ, ν) or (μ, ζ) plane
- …but much faster, thanks to closed-form results in [Ebert, 2110.11360] in terms of a single polynomial root-finding problem

Comparison with RadISH (using identical NNLOjet fixed-order matching)

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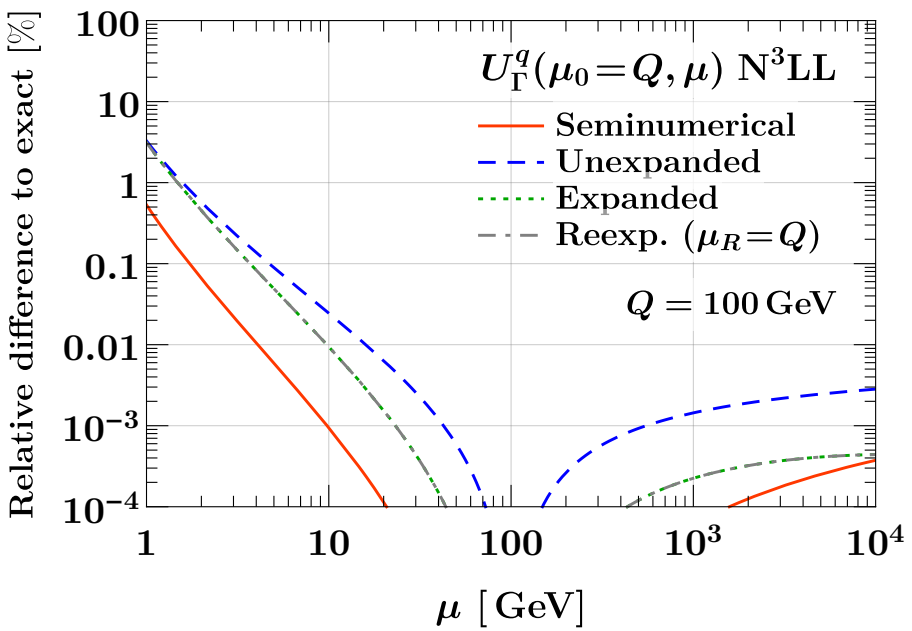
  e.g.: $$K_\Gamma(\mu_0, \mu) = \int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \, \Gamma[\alpha_s(\mu')] \ln \frac{\mu'}{\mu_0}$$

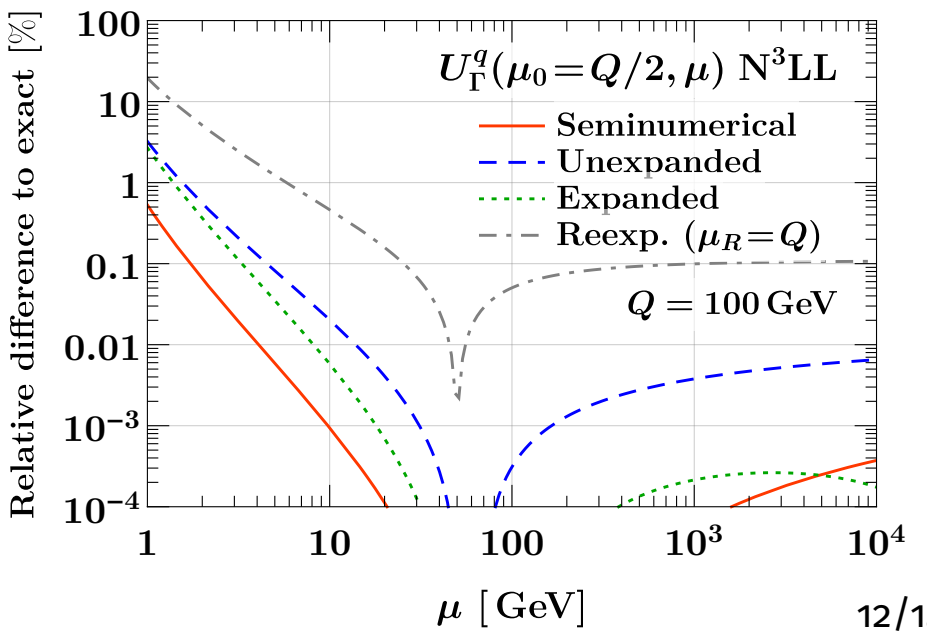
- Common to expand $K_\Gamma(\mu_0, \mu)$ in terms of $\alpha_s(\mu_0)$ throughout instead

  $\Rightarrow$ simpler analytic solution with $g^{(1)}$ a function of an $\mathcal{O}(1)$ argument:

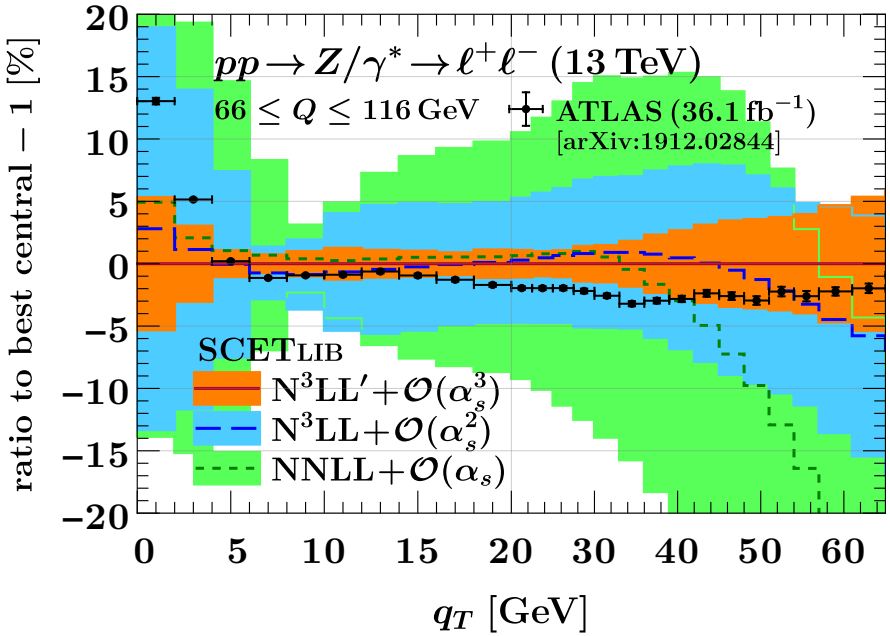
  $$K_\Gamma^{\text{exp.}}(\mu_0, \mu) = L g^{(1)}\big(\alpha_s(\mu_0) L\big) + \text{NLL}\,, \qquad L = \ln \frac{\mu_0}{\mu}$$

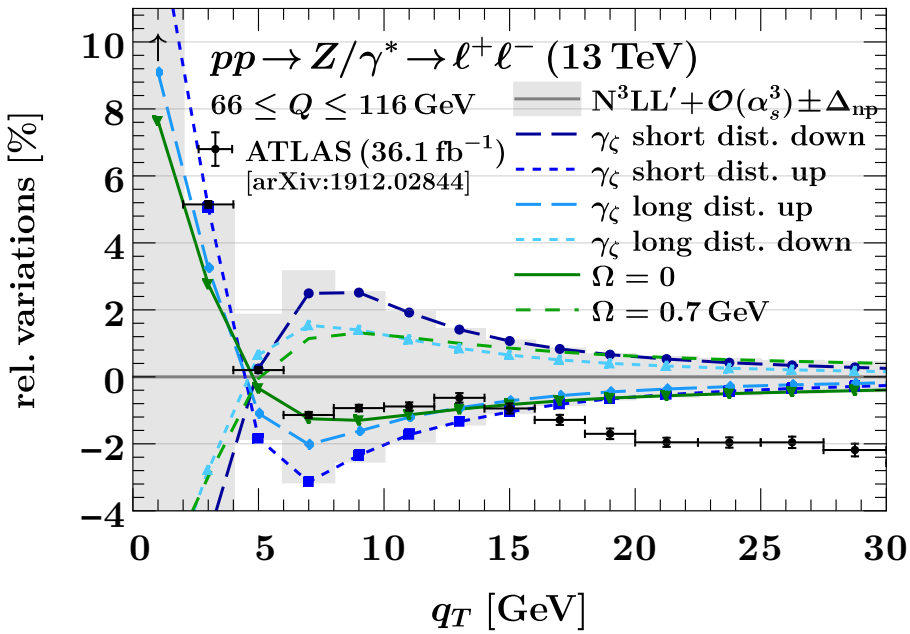
- However, reexpanding in terms of $\alpha_s(\mu_R)$, $\mu_R \neq \mu_0$ (read: μ_0 = resummation scale) leads to large truncation errors [Billis, Tackmann, Talbert, 1907.02971]

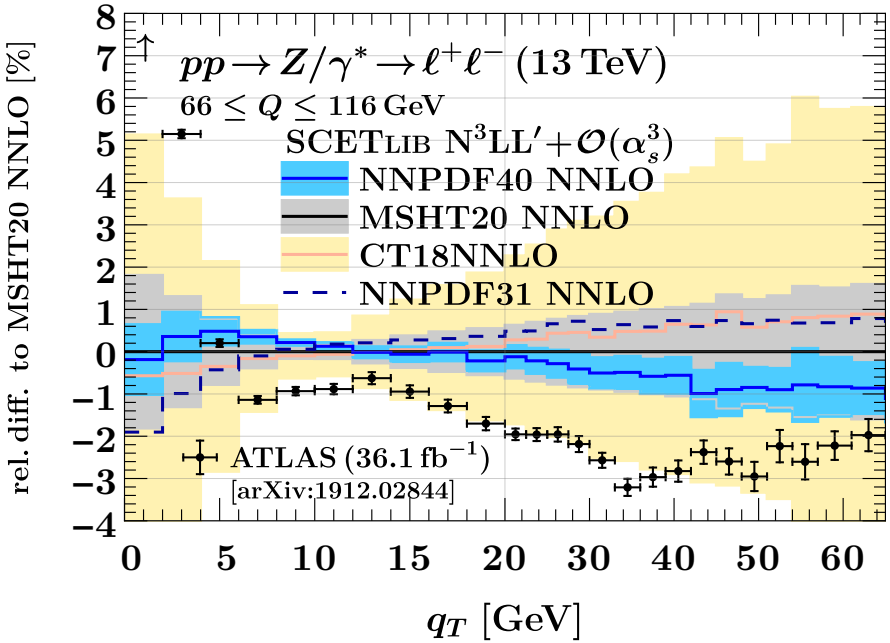


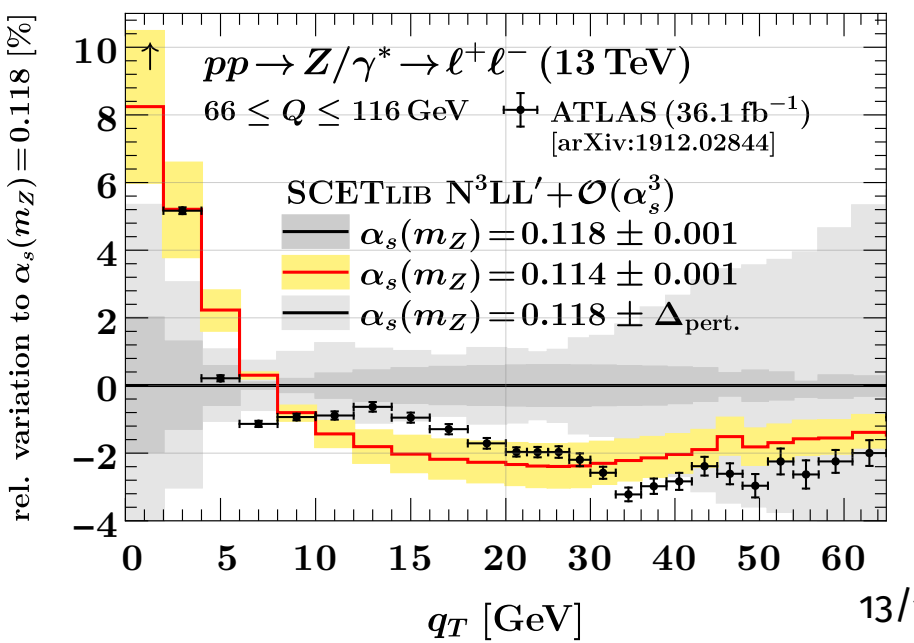
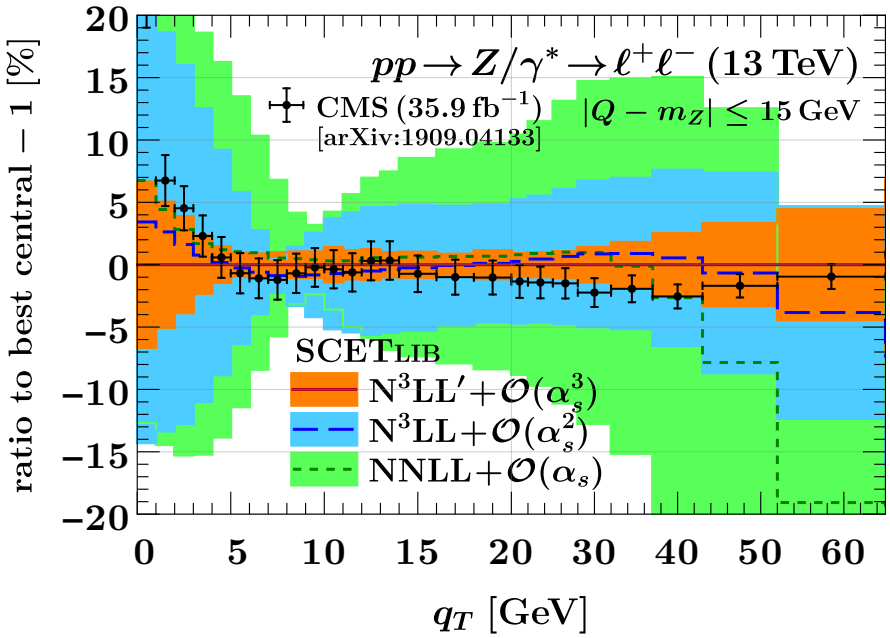
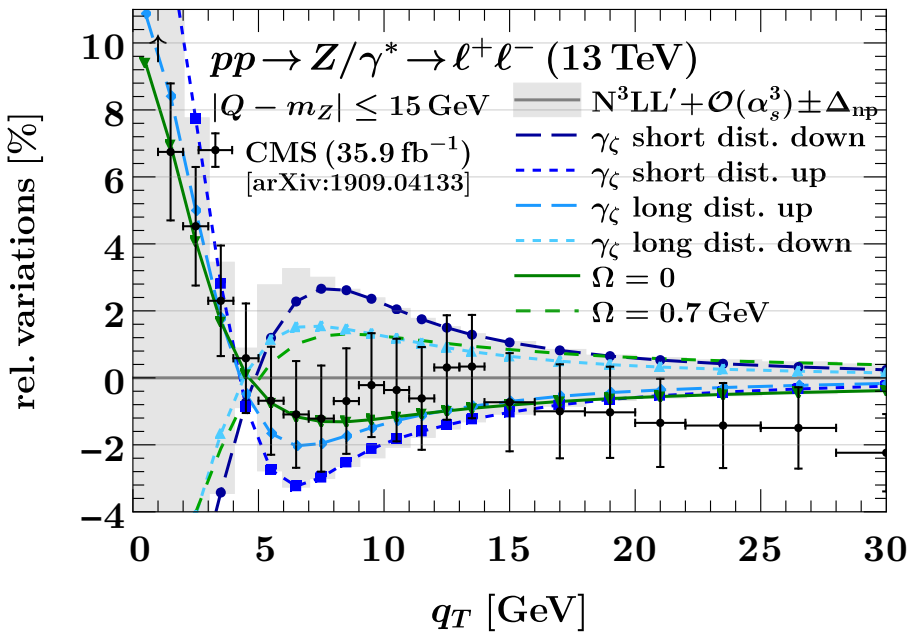
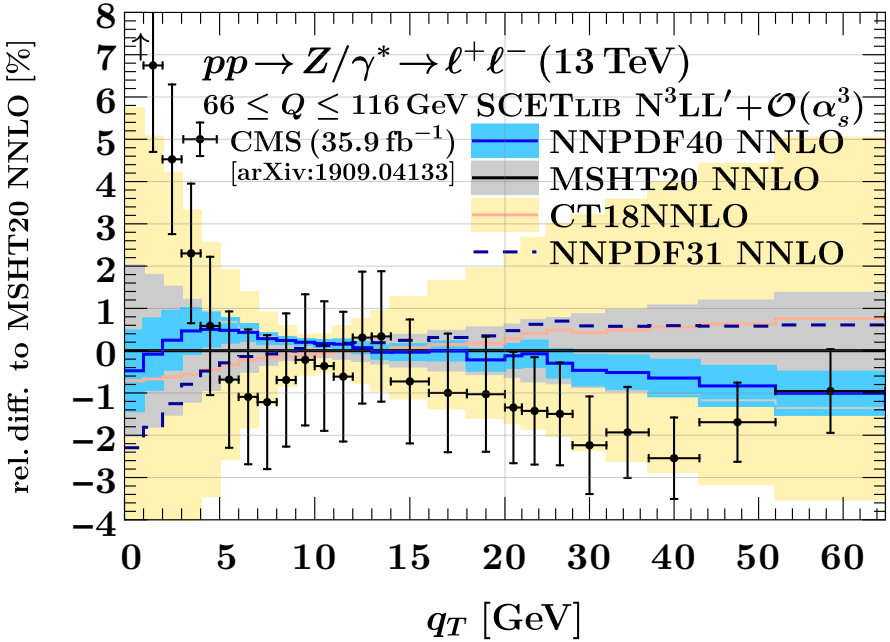
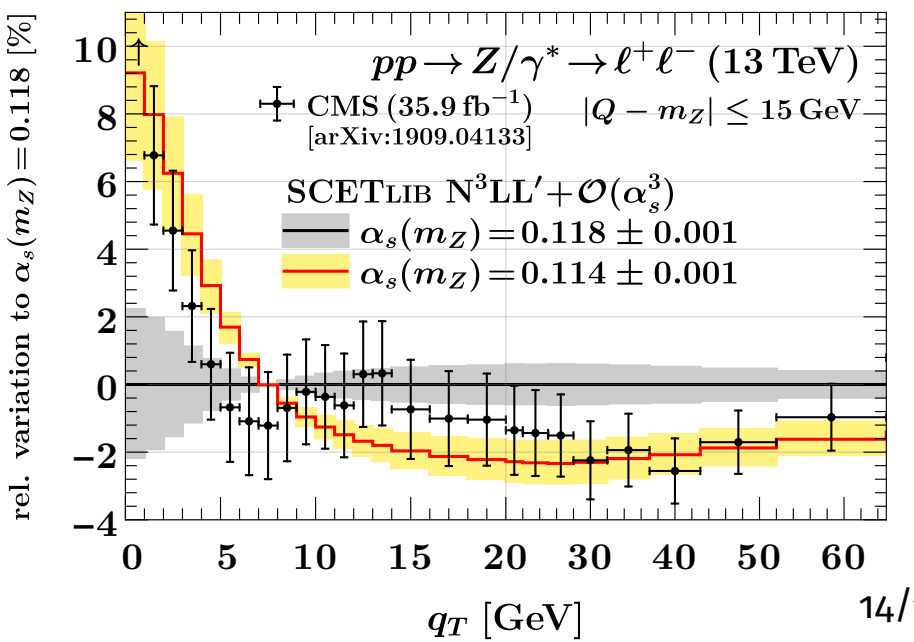


ATLAS normalized spectrum (Born leptons)









CMS normalized spectrum (dressed leptons)