PB TMD fits at NLO with dynamical resolution scale

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Outline

- Recap of Parton Branching method
- Fixed and Dynamical soft-gluon resolution scale $z_M$
- Fits with fixed $z_M$ at NLO
- Fits with dynamical $z_M$ at NLO

Merged talk:
- $Z+b$ jet production in 4FL and 5FL
Recap of PB TMDs

TMD evolution in the PB formalism:

\[ \widetilde{A}_a(x, k_\perp, \mu^2) = \]
\[ A_a(x, k_\perp, \mu_0^2) \Delta_a(\mu^2) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2} \Theta(\mu^2 - \mu'^2) \Theta(\mu'^2 - \mu_0^2) \frac{\Delta_a(\mu^2)}{\Delta_a(\mu_0^2)} \int_z^{Z_M} dz \int \frac{d^2q_\perp}{\pi^2} \Gamma(\alpha_s q_\perp^2, z) \]

- **Splitting functions**: \( P_{ab}^R(z) \): The real emission parts of the DGLAP splitting function:
  - Probability that a branching will happen

- **Sudakov form factor**: \( \Delta_a = \exp(- \int_{\ln \mu_0^2}^{\ln \mu^2} d(\ln \mu'^2) \sum_b \int_0^{Z_M} dz \int \frac{d^2q_\perp}{\pi^2} \Gamma(\alpha_s q_\perp^2, z) P_{ba}^R(\alpha_s, z)) \)
  - The probability of an evolution without any resolvable branching

- **Resolution scale**: \( Z_M \):
  - Resolvable branching: \( z < Z_M \)
  - Non-resolvable branching: \( z > Z_M \)

At every step kinematics can be calculated!

[Hautmann et al., JHEP 01 (2018) 070, 1708.03279]
Recap of PB TMDs

Iterative form of the PB evolution equation:

\[
\tilde{A}_a(x, k_\perp, \mu^2) = \tilde{A}_a(x, k_\perp, \mu_0^2) \Delta_a(\mu^2) + \Sigma_b \int_{\ln \mu_0^2}^{\ln \mu^2} d \ln \mu_1^2 \times \frac{\Delta_a(\mu^2)}{\Delta_a(\mu_1^2)} \int_x^{z_M} dz \; P_{ab}^R(z, \alpha_s(q_\perp)) \Delta_b(\mu_1^2) \times \tilde{A}_b \left(\frac{x}{z}, k_\perp + (1 - z) \mu_1, \mu_0^2\right) + \ldots
\]

Solvable by MC iterative technique:

- generated \( \mu_1^2 \): if \( \mu_1^2 > \mu^2 \) stop, otherwise splitting,
- generated the next scale \( \mu_2^2 \): if \( \mu_2^2 > \mu^2 \) stop, otherwise splitting,
- ...
Angular Ordering:

Color coherence phenomena:

• Angular ordering of the soft gluon emissions
  \[ \theta_{i+1} > \theta_i \]
  \[ |q_{\perp,i}| = (1 - z_i) \ |E_i| \sin \theta_i \]

  Associating \(|E_i| \sin \theta_i\) with \(\mu'\)
  \[ q_{\perp,i}^2 = (1 - z_i)^2 \ \mu_i'^2 \]

• The argument of \(\alpha_s\) should be \(q_\perp^2\)
  \[ \alpha_s(q_\perp^2) = \alpha_s((1 - z)^2 \mu'^2) \]

• resolvable & non-resolvable \(\rightarrow\) condition on \(\min q_{\perp,i}^2 \rightarrow z_M\)
  \[ z_M = 1 - \left( \frac{q_0}{\mu'} \right) \]
Fixed and dynamical resolution scale

- **Fixed $z_M$:**
  - $\mu$ independent
  
  $z_M = 1 - \epsilon$

  where $\epsilon$ is small: $10^{-3}$, $10^{-4}$, $10^{-5}$,...

- **Dynamical Resolution scale in Angular Ordering:**
  
  $z_M = 1 - \left(\frac{q_0}{\mu'}\right)$

  where $q_0$ is smallest emitted transverse momentum for resolvable partons

  - Sudakov form factor $\Delta_a$: non-resolvable region
  - Splitting functions $P_{ab}^R$: resolvable region

The Condition on $q_0$ of

$$z_M = 1 - \left( \frac{q_0}{\mu'} \right)$$

- Scale of strong coupling:
  $$\alpha_s(q^2_\perp) = \alpha_s((1 - z)^2 \mu'^2)$$

- Lowest scale in $\alpha_s$ corresponds to minimal $q_\perp$

- $q_{\perp,\text{min}} = q_0 \quad \& \quad q_0 > \Lambda_{QCD} \Rightarrow \text{we stay in the weak coupling region!}$

$\Lambda_{QCD} \approx 0.2 \text{ GeV}$
PB TMD fits at NLO with fixed $z_{\text{max}}$

The Past PB TMD fits at NLO calculation using angular ordering: fixed $z_M$
"NLO DIS Matrix Element (ME) and NLO evolution kernel"

- Associating the evolution scale with some physical interpretation:
  - Set 1
    \[ \alpha_s(\mu'^2) \]
  - Set 2
    \[ \alpha_s(q^2_{\perp}) = \alpha_s((1 - z)^2\mu'^2) \]


- Data set: HERA 1+ 2 inclusive DIS data

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<th>d.o.f</th>
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- Measurement of the inclusive DIS cross section obtained at HERA compared to predictions using Set 1 and Set 2
PB TMD fits at NLO with dynamical $z_{\text{max}}$

New study

From fixed resolution scale to dynamical resolution scale
PB TMD fits at NLO with dynamical $z_{\text{max}}$: $z_M = 1 - \left( \frac{q_0}{\mu^2} \right)$

New fits with dynamical $z_{\text{max}}$ at LO and NLO with HERA 1 + 2 Data set: Using \texttt{Fitter} arXiv:1709.01151v1

✓ Performing different fits, each time by varying $Q_{\text{min}}^2$ and on top of that with different $q_0$ values.

- At LO, for small $Q_{\text{min}}^2$ and $0.9 \text{ GeV} < q_0 < 1.2 \text{ GeV} \rightarrow 2.2 < \frac{\chi^2}{\text{dof}} < 3$
- AT NLO, for small $Q_{\text{min}}^2$ and all values of $q_0$, we have better fits with good $\frac{\chi^2}{\text{dof}}$!
The difference between LO and NLO

- Does the difference between LO and NLO come from the kernels? or ME?!..

- The difference is dominated by the kernel not ME!..

4 states for this purpose:
1. Fitting with NLO kernel & NLO ME
2. Fitting with NLO kernel & LO ME
3. Fitting with LO kernel & LO ME
4. Fitting with LO kernel & NLO ME

For $q_0=1.0 \text{ GeV}$
The difference between LO and NLO

• Which part of the kernel is responsible?

\[ P_{ab}(z, \mu^2) \text{? or } \alpha_s ? \]

4 states for this purpose:
1. Fitting with NLO \( P_{ab} \) & NLO \( \alpha_s \)
2. Fitting with NLO \( P_{ab} \) & LO \( \alpha_s \)
3. Fitting with LO \( P_{ab} \) & LO \( \alpha_s \)
4. Fitting with LO \( P_{ab} \) & NLO \( \alpha_s \)

The difference is dominated by the splitting functions not \( \alpha_s \)!
Which part of the splitting functions is responsible for the difference between LO and NLO?

- For high values of $q_0$ (e.g., [1.0 Gev, 1.2 Gev]) or low values of $z_M = 1 - \left(\frac{q_0}{\mu'}\right)$, LO and NLO have different behavior.

  The first piece for checking is $\frac{1}{z}$

- In the NLO, all the splitting functions have pieces with $(1/z)$ term:
  \[ P_{ab}(z, \mu^2) \sim P_{qq}(1/z, \mu^2), P_{qg}(1/z, \mu^2), P_{gg}(1/z, \mu^2), P_{gq}(1/z, \mu^2) \]

- In the LO, just the splitting functions with “gluon” in the final state have $(1/z)$ piece:
  \[ P_{gg}(z, \mu^2) = \frac{1}{1-z} + \frac{1}{z} - 2 + z(1-z), \]
  \[ P_{gq}(z, \mu^2) = \frac{1+(1-z)^2}{z} \]

- And the splitting functions with “quark” in the final state don’t have $(1/z)$ piece:
  \[ P_{qq}(z, \mu^2) = \frac{2}{1-z} - 1 - z, \]
  \[ P_{qg}(z, \mu^2) = z^2 + (1-z)^2 \]

- Is the lack of $(1/z)$ piece in LO splitting function with quark in the final state responsible for this difference?

  Let’s check it!
Does the difference come from 1/z piece of NLO splitting function?

For better understanding: “We added to the LO splitting functions ($P_{qq}, P_{qg}$) the 1/z pieces of NLO”

✓ $P_{qq} (z, \mu^2) = \frac{2}{1-z} - 1 - z + \left( \frac{1}{z} \right)$ pieces of $P_{qq}$ NLO

✓ $P_{qg} (z, \mu^2) = z^2 + (1 - z)^2 + \left( \frac{1}{z} \right)$ pieces of $P_{qg}$ NLO

✓ In NLO we have an extra (1/z) pieces in the quark channels compared with LO which is responsible for this difference!

✓ With this piece we are describing data well! Amount of $\chi^2_{dof}$ is reasonably good!

** For PB-TMD fit with dynamical zmax we obtain a reasonably good $\chi^2_{dof}$ at NLO! **
How does dynamical zmax affect the fitted TMD (iTMD)?

Set 2: fixed zmax & $\alpha_s(q_T^2) = \alpha_s((1 - z)^2 \mu'^2)$

The dynamical zmax fit implies an effect not only in the $k_T$ dependence but also in the $x$ dependence!
The predictions in dynamical $z_{\text{max}}$ frame

Predictions with ME generated by MCatNLO combined with obtained TMDs.
The merged talk: $Z+b$ jet production in 4FL and 5FL

5 FLNS
- Full coupled evolution with all flavours & $\alpha(M_{Z_{nf=5}})=0.118$
- HERAPDF parametrization form
- Using full HERAI+II inclusive DIS data
- $\chi^2/\text{dof}=1.21$


4 FLNS
- The same functional form and data as 5FL-parameters re-fitted
- $M_b \rightarrow \infty$ & $\alpha(M_{Z_{nf=4}})=0.1128$
- $\chi^2/\text{dof}=1.25$

[arXiv:2106.09791]

Matrix elements from MC@NLO (HERWIG6 subtraction)
- 5FLVNS: $Z + $ one parton process
- 4FLVNS: $Z + bb$ process

PDFs: TMDs (4FL & 5FL)
- 5FLVNS: $b$-quark is treated as a light quark
- 4FLVNS: no $b$-quark in the parton density

Parton shower following TMDs for intial state
- 5FL & 4FL PB-TMDs included in the Cascade3

Differential cross section for $Z + b\bar{b}$ as a function of $p_t(Z)$ as measured by CMS collaboration.

The full prediction + the result of using only the LHE files are shown.
Z+bb as a function of $\Delta \phi$ ($b\bar{b}$)

Differential cross section for Z +b$b\bar{b}$ as a function of $\Delta \phi$ ($b\bar{b}$) as measured by CMS collaboration

**5 FLNS**

CMS, 8 TeV, DeltaPhi bb, at least two b jets

- **Data**
- $Z+\bar{t}$ (5FL NLO)
- $Z+\bar{t}$ (5FL-PB-NLO-2018-set2 (scale))
- $Z+\bar{t}$ (5FL NLO LHE (scale))

**4 FLNS**

CMS, 8 TeV, DeltaPhi bb, at least two b jets

- **Data**
- $Z+bb$ (4FL NLO)
- $Z+bb$ (4FL-PB-NLO-2020-set2 (scale))
- $Z+bb$ (4FL NLO LHE (scale))

**4FL** : both b partons are already produced at the ME level

**5FL** : $bb\bar{b}$ must be simulated in the parton shower.
Breakdown of the different contributions to quantify their roles

4FL: weakly depends on PB-TMD/parton shower
5FL: significant contribution from TMD parton shower
Summary

- PB TMD fits at NLO with dynamical zmax for the first time!
- For PB-TMD fit to HERA data with dynamical zmax, we obtain a reasonably good $\frac{\chi^2}{dof}$ at NLO!
- The difference between LO and NLO fits is mostly due to (1/z) pieces in quark channel in NLO splitting functions!
- The dynamical zmax impacts both the k$_T$ dependence and the x dependence of the fitted parton distribution!
- The next step: Using the PB TMD with dynamical zmax in phenomenology of LHC and lower energy colliders!
- The 4FL and 5FL PB-TMD distributions used to calculate Z + bb production
  - Good agreement with measurements obtained by the CMS collaboration
  - The evolution of the PB-TMD parton densities as well as in the PB-TMD parton shower is checked.

Thank you …
BACK UP …
Back up…
PB TMD fits at NLO with dynamical $z_{\text{max}}$: 

NLO fits with dynamical $z_{\text{max}}$

$Q_{\text{min}}^2 = 2, 3.5, ..., 100$ GeV$^2$