

Approximating missing higher-orders in transverse momentum distributions using resummations

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Abstract

We present a more reliable approach to approximate the unknown next-to-next-to-next-to-leading order (N3LO) transverse momentum distribution of colourless final states, namely the Higgs boson produced via gluon fusion and the lepton pair produced via Drell–Yan (DY) mechanism. The approximation we construct relies on the combination of various resummation formalisms – namely threshold, small-pt and high energy resummations – by exploiting the singularity structure of the large logarithms in Mellin space. We show that for the case of Higgs boson production, the approximate N3LO transverse momentum distribution amounts to a correction of a few percent with respect to the NNLO result with a reduction in the scale dependence.

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1 Introduction

In order to push forward precision and discovery physics at the LHC, the three pillars of QCD – namely fixed-order calculations, resummations, and PDFs – need to be determined at the highest accuracy possible (sub one percent). While significant, much work remains on the side of perturbative calculations, both in terms of fixed-order computations and resummations. Given that pushing the accuracy of the fixed-order computations is a gigantic task, it is crucial to estimate the missing higher-order contributions to the best of our abilities. Currently, the most commonly used way of estimate theoretical uncertainties associated with missing higher-order contributions is by varying the unphysical scales involved in the process according to a *scale variation prescription*. On one hand, the use of scale variation to estimate the missing higher-order uncertainties (MHOU) presents several advantages. First, due to the fact that the scale dependence of the strong coupling and PDFs are universal, scale variations can be used to estimate theoretical uncertainties for any perturbative processes. Second, the constraint imposed by the renormalization group invariance

ensures that as the order of the perturbative calculations increases, the scale dependence decreases. Third, the estimation of MHOU resulting from scale variations produces smooth functions of the kinematics, accounting for the correlations in the nearby regions of the phase space. On the other hand, the scale variation method has a number of caveats, chief among which is the fact that it does not allow for a probabilistic interpretation. In addition, there is the ambiguity in defining the central scale around which the variation should be performed and the ranges at which the scales are allowed to vary. But most importantly, scale variation misses uncertainties associated to new singularities appearing at higher-orders but not present at lower-orders. Various approaches [1–3] have recently merged in order to address the shortcomings related to the scale variation method. However, most of these approaches are only applicable to particular types of processes and observables. In Ref. [1], Cacciari and Houdeau proposed a new approach of estimating MHOU using a Bayesian model. In short, the method consists on adopting some assumptions on the progression of the perturbative expansion, then based on the knowledge of the first few orders, one can infer on the *hidden parameters* that are assumed to bound the structure of the perturbative coefficients, allowing for an inference on the unknown subsequent contributions. While this approach has proved to perform well for QCD observables at e^+e^- colliders [1], its reliability when it comes to proton-proton collider observables is subject to question. In Ref. [3], Bonvini built upon Cacciari-Houdeau’s work to construct more general, flexible, and robust models. The various models have been validated on various inclusive observables at the LHC. However, none of these models can as of yet be used for differential observables as the correlations between the different regions are not accounted for.

A possible way to estimate (or rather approximate) the impacts of unknown higher-orders in perturbation theory is by using the information provided by the resummed calculations. Given that the information on the various kinematic limits that appear in fixed-order calculations are contained to all-order in resummed expressions, it should be possible to consistently combine these various limits to approximate the subsequent unknown contributions. A similar approximation was done decades ago in the context of the Higgs boson production from gluon fusion at the total inclusive level [4]. In the following, we provide a proof of concept on how to combine the different resummation formulae to approximate the N3LO transverse momentum distributions for the Higgs boson productions in the infinite top mass limit. The eventual aim of the project is to extend the formalism to DY processes and use it to approximate MHOU in PDF fits. Throughout this manuscript, the counting refers to the order at the integrated level, i.e. N3LO refers to $\mathcal{O}(\alpha_s^5)$ in the expansion of the transverse momentum distributions.

2 Higgs boson production in HEFT

To the present day, the inclusive and transverse momentum distributions of the Higgs boson produced via gluon fusion are known to a fairly high precision. The total inclusive cross section with finite top-quark mass is known up to N3LO [5]. Recently, the N3LO* transverse momentum distributions for the Higgs produced with an associated jet in the final state have become available [6]. However, since these results are often obtained numerically, reading off the coefficients that are relevant for the comparison to the all-order computations is not practically feasible.

In the following sections, we present a formalism for the construction of the Higgs transverse momentum distribution beyond NNLO by combining the information on the singularity structure at small- N and large- N which can be predicted by the high-energy and threshold resummation formulae, respectively. Threshold resummation embodies to all orders in α_s logarithms of the form $\ln N$ that drives the transverse momentum spectra in the limit $N \rightarrow \infty$, while the $(N-1)^n$ behaviour for $N \rightarrow 1$ is fully determined to all orders by the small- x resummation. Therefore, for a partonic cross section known up to N^n LO (i.e. $\mathcal{O}(\alpha_s^{n+2})$), the approximate expression is constructed as a combination of fixed-order calculations and expansion from resummations:

$$\left[\frac{d\hat{\sigma}_{ab}}{d\xi_p} \right]^{N^{n+1}\text{LO}}(N, \xi_p) = \left[\frac{d\hat{\sigma}_{ab}}{d\xi_p} \right]^{N^n\text{LO}}(N, \xi_p) + \frac{d\hat{\sigma}_{ab}^{(n+1)}}{d\xi_p}(N, \xi_p) \quad \text{with} \quad \frac{d\hat{\sigma}_{ab}^{(n)}}{d\xi_p} = \frac{d\hat{\sigma}_{ab}^{\text{he},(n)}}{d\xi_p} + \frac{d\hat{\sigma}_{ab}^{\text{th},(n)}}{d\xi_p}, \quad (1)$$

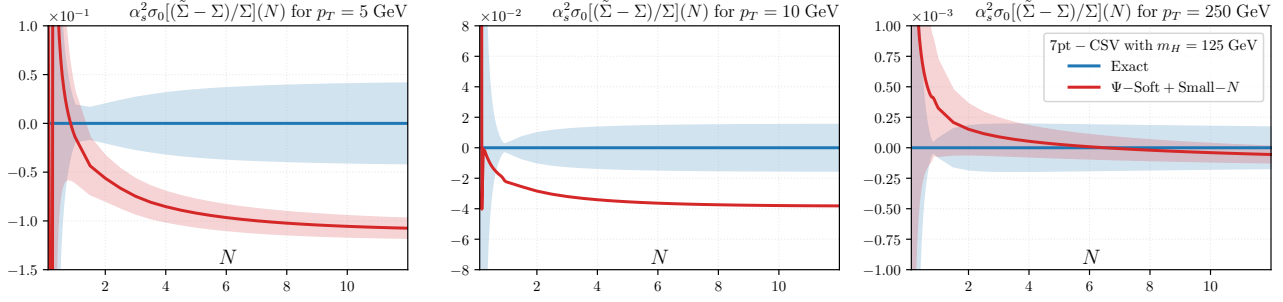


Figure 1: Plots comparing the relative difference between the exact and approximate solutions at NNLO for the gg -channel at different values of p_T . The error bands have been computed using the 7-point scale variation method. The codes used to generate the predictions both in momentum and Mellin space are publicly available [8, 9].

where $\xi_p \equiv p_T^2/M^2$ is a dimensionless variable with M the invariant mass of the produced Higgs boson. Notice that no matching function is introduced when combining the two resummed cross sections. This means that the second part of Eq. (1) is only valid if the small- N behaviour controlled by the high-energy contribution is not spoiled by the threshold component and vice-versa.

In order to construct our approximate expression, let us first describe the large- N approximation of the partonic cross section in Eq. (1). The analytical expression of the threshold resummation for transverse momentum distributions with colour singlet in the final state has been derived in [7]. By expanding the expression as a series in α_s one can extract the terms relevant for the approximation. However, the way in which the logarithms of N appear in the expansion does not correspond to the Mellin transform of the x -space expressions which spoils the finite- N behaviour. In addition, the resulting expressions display unphysical singularities. Indeed, as opposed to exhibiting poles at small- N , the expressions contain a logarithmic branch cut at $N = 0$. The correct singularity structure in the small- N region of the resummed expression can be restored by resorting to the Ψ -soft prescription in which the logarithms of N are simply replaced by the digamma functions as has been done for inclusive Higgs production in Ref. [4]. Doing so yields the following resummed expression:

$$\frac{d\hat{\sigma}_{ab}^{\text{th}}}{d\xi_p}\left(N, \xi_p, \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right) = \tilde{C}_{ab}(N, \xi_p) \sum_{n=1}^{\infty} \alpha_s^n \sum_{k=0}^{2n} \tilde{g}_{n,k}\left(N, \xi_p, \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right) (\psi_0(N) + \gamma_E)^k \quad (2)$$

where \tilde{C} collects all the non-logarithmic dependence and $\tilde{g}_{n,k}$ are numerical coefficients. We should emphasize that in the above equation the born-level cross section which contains $\mathcal{O}(\alpha_s^2)$ contribution is included in the definition of the coefficient \tilde{C} .

The leading logarithmic (LLx) high-energy resummation for the transverse momentum distribution of the Higgs boson in HEFT has been derived in Ref. [10]. The computations were performed by keeping the initial-state gluons off their mass-shell, $p_i^2 = |p_{T,i}|^2$ from which the *impact parameter* that defines the cross section is derived. For the gg -channel, for instance, the expanded resummed expression up to NNLO is written as:

$$\frac{d\hat{\sigma}_{gg}^{\text{he}}}{d\xi_p}(N, \xi_p) = \sigma_{H,gg}^{\text{Born}} \left[\left(\frac{\alpha_s}{\pi}\right) \frac{C_A}{N} \frac{2}{\xi_p} + \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{2C_A}{N}\right)^2 \frac{\ln \xi_p}{\xi_p} \right] + \mathcal{O}(\alpha_s^3). \quad (3)$$

In view of combining the high-energy approximation and the threshold approximation, one has to make sure that the small- N contributions vanish at moderately large- N . From Eq. (3) one can see that not only the distribution always vanish in the large- N limit but also the vanishing point is located at the vicinity of $N \sim 1$ where effects from the threshold limit start to contribute.

In Fig. 1 we compare the approximation to the exact NNLO result. By focusing first on the small- N region ($N < 1$), it is apparent that the high-energy approximation reproduces the fixed-order computations fairly

well. Not only do the uncertainty bands of the two results overlap, but in all cases the uncertainty bands of the exact results are contained in the approximation. Moving to the region where $N > 1$, one notices that small discrepancies persist between the exact and the approximate results.

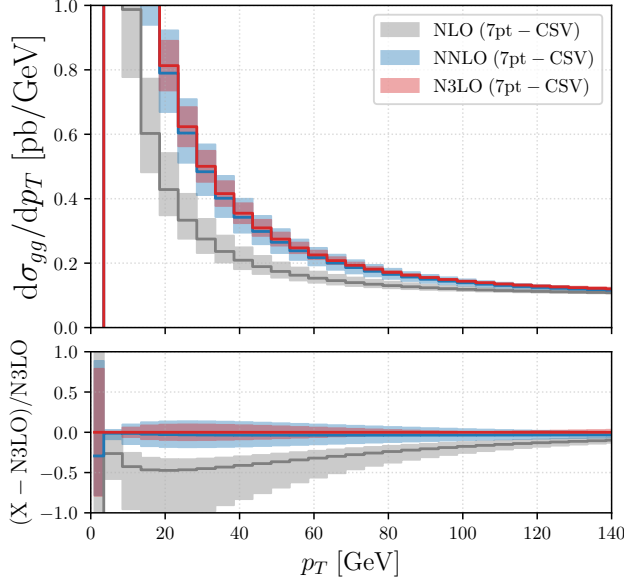


Figure 2: Approximate N3LO higgs transverse momentum distribution in HEFT approximation at $\sqrt{s} = 14$ TeV.

These discrepancies however reduces as the value of the transverse momentum increases. The approximation would improve slightly if the contributions from the small- p_T resummation were included. Nevertheless, omitting the small- p_T contributions in the HEFT is justified by the fact that it coincides with the high-energy contributions at small- p_T and large- N .

In Fig. 2 we show the approximate N3LO Higgs transverse momentum distribution at the hadronic level. The inverse Mellin transform is computed using the contour deformation defined by the *Minimal Prescription* as described in [11]. For comparisons, both the exact NLO and NNLO are also included. As in the previous figures, the uncertainty bands have been computed using the 7-point scale variation. One can see that the approximate N3LO transverse momentum distribution amounts to a correction of a few percent with respect to the NNLO result. As expected, the uncertainty band from the N3LO approximation is smaller compared to the one from NNLO with the former fully contained in the latter.

3 Drell–Yan processes

The results presented here are for the Higgs production in the infinite top mass limit provides a relatively simple case study as a proof of concept for the methodology. However, the eventual aim is to provide a set of parton distribution functions (PDFs) accounting for MHOU approximated using resummations. For this reason it is important to apply the methodology also to DY processes as it provides valuable information about the proton structure.

The way in which the approximate N3LO expression is constructed for DY case is very similar to the Higgs described above with the main difference that in the DY case the contributions from the small- p_T resummation have to be taken into account. While the analytical expressions of the small- p_T and threshold resummation formulae are available in the literature [7, 11, 12], only the large- b (or equivalently small- p_T) version is known for the high-energy limit in Ref. [13], which extends the formalism for the differential cross-section for Higgs production as described in Ref. [10]. Since the resummation formalism in the small- p_T is known for DY processes, we are instead interested in the limit of large transverse momentum. This requires redoing the calculation for DY transverse momentum presented in Ref. [13]. It is finally worth noting that the derivation of the full expression is a complicated task given the non-trivial relation between $\ln N$ and $\ln p_T$.

4 Conclusions

We explored the idea of using all-order computations to approximate contributions from missing higher-orders. The combination of the various resummation formalisms were carried out in Mellin space where one can fully study the singularity structure of the resummed expressions. Such an approximation seem to yield reasonable predictions as attested by the partonic and hadronic results. However, further work is required in order to apply the approximation also to DY processes in particular for the goal of using the approximation in PDF fits.

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