

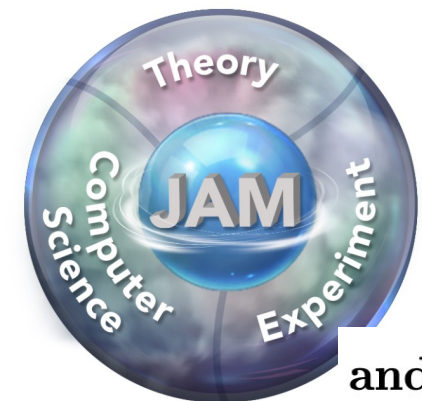
Complementarity of experimental and lattice QCD data on pion parton distributions

Patrick Barry¹, Colin Egerer¹, Joseph Karpie², Wally Melnitchouk¹, Chris Monahan^{1,3},
Kostas Orginos^{1,3}, Jianwei Qiu^{1,3}, David Richards¹, Nobuo Sato¹, Raza Sufian^{1,3}, Savvas
Zafeiropoulos⁴

¹**Jefferson Lab**, ²Columbia University, ³College of William & Mary, ⁴Aix Marseilles Univ.

DIS 2022, Santiago de Compostela, May 5th, 2022

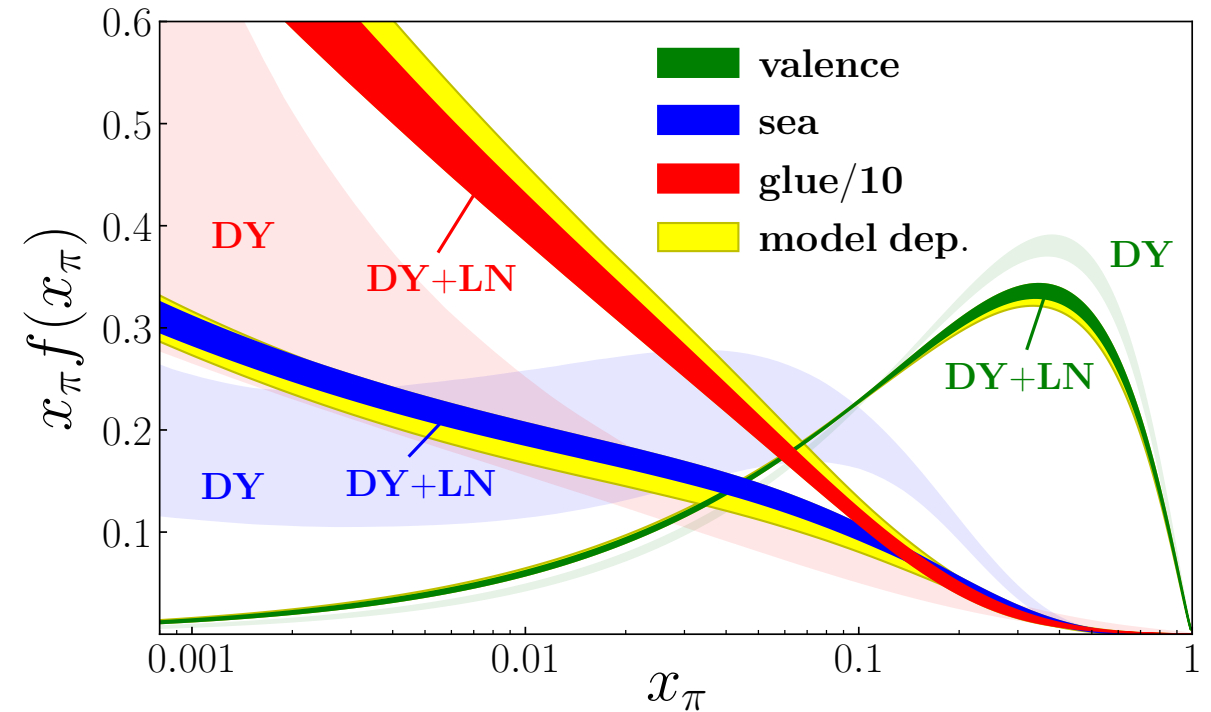
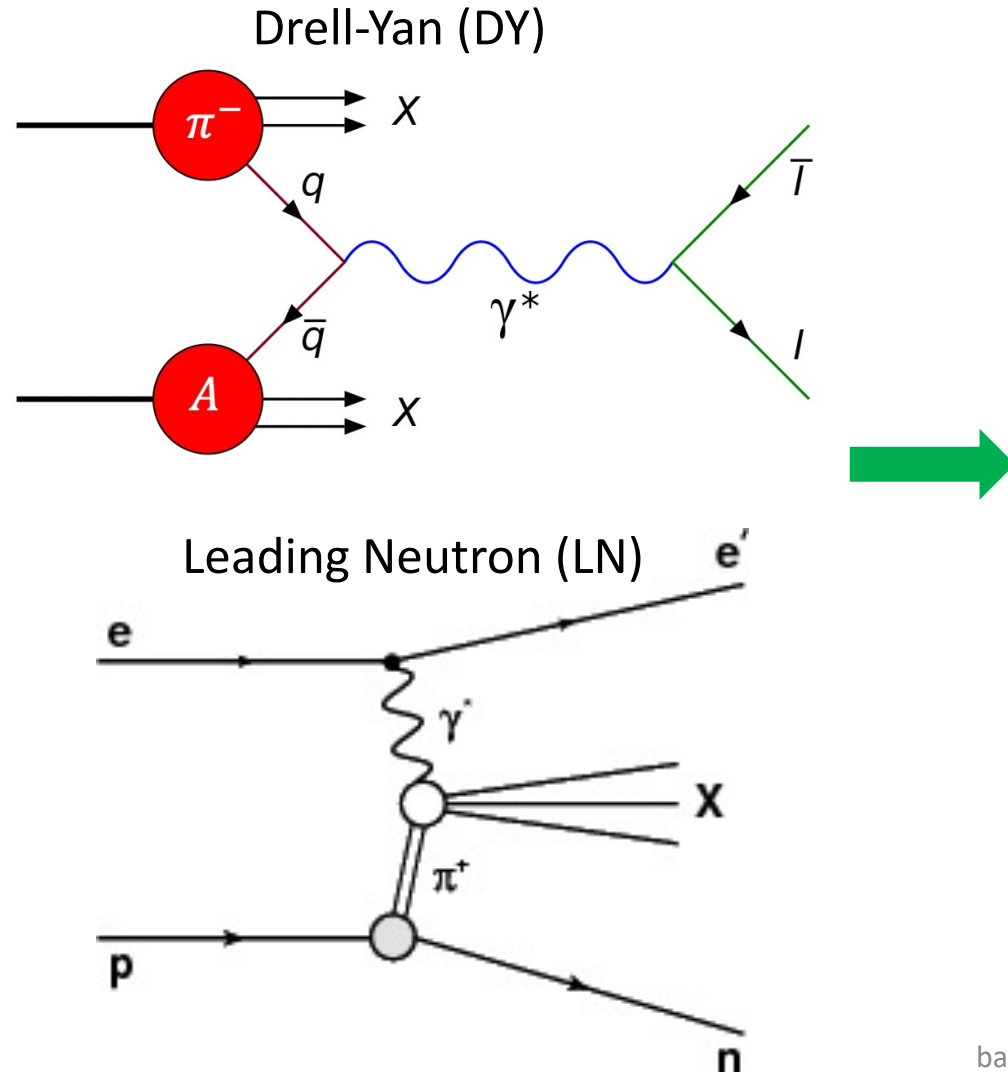
Based on a recent preprint: 2204.00543



and HadStruc Collaborations

barryp@jlab.org

Experiments to probe pion structure



PHYSICAL REVIEW LETTERS **121**, 152001 (2018)

Featured in Physics

First Monte Carlo Global QCD Analysis of Pion Parton Distributions

P. C. Barry,¹ N. Sato,² W. Melnitchouk,³ and Chueng-Ryong Ji¹

Large momentum fraction behavior

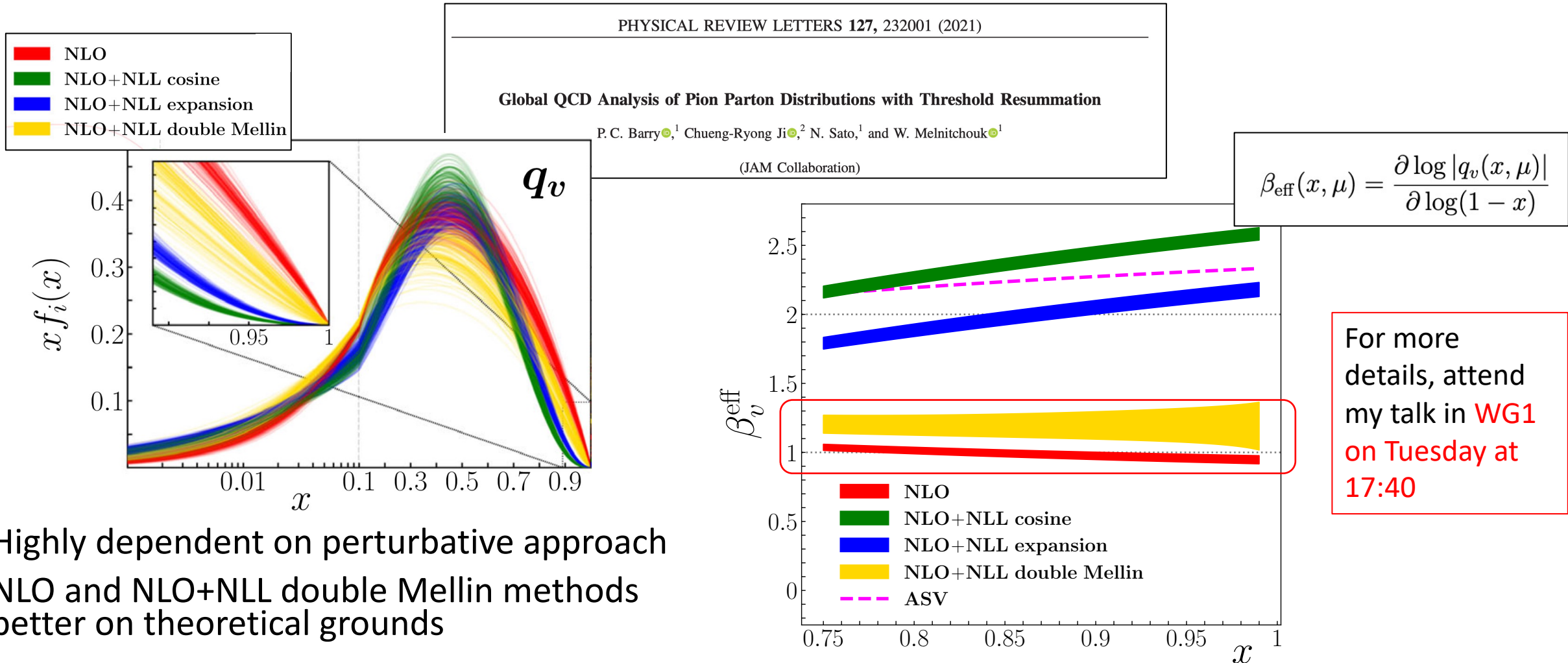
- Many theoretical papers have studied the behavior of the valence quark distribution as $x \rightarrow 1$ and
- Debate whether $q_v^\pi(x \rightarrow 1) \sim (1 - x)$ or $(1 - x)^2$

R. J. Holt and C. D. Roberts, [Rev. Mod. Phys. **82**, 2991 \(2010\)](#).
W. Melnitchouk, [Eur. Phys. J. A **17**, 223 \(2003\)](#).
G. R. Farrar and D. R. Jackson, [Phys. Rev. Lett. **43**, 246 \(1979\)](#).
E. L. Berger and S. J. Brodsky, [Phys. Rev. Lett. **42**, 940 \(1979\)](#).
M. B. Hecht, C. D. Roberts, and S. M. Schmidt, [Phys. Rev. C **63**, 025213 \(2001\)](#).
Z. F. Ezawa, [Nuovo Cimento A **23**, 271 \(1974\)](#).
P. V. Landshoff and J. C. Polkinghorne, [Nucl. Phys. **B53**, 473 \(1973\)](#).
J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, [Phys. Rev. D **8**, 287 \(1973\)](#).
T. Shigetani, K. Suzuki, and H. Toki, [Phys. Lett. B **308**, 383 \(1993\)](#).

A. Szczepaniak, C.-R. Ji, and S. R. Cotanch, [Phys. Rev. D **49**, 3466 \(1994\)](#).
R. M. Davidson and E. Ruiz Arriola, [Phys. Lett. B **348**, 163 \(1995\)](#).
S. Noguera and S. Scopetta, [J. High Energy Phys. **11** \(2015\) 102](#).
P. T. P. Hutaauruk, I. C. Cloët, and A. W. Thomas, [Phys. Rev. C **94**, 035201 \(2016\)](#).
T. J. Hobbs, [Phys. Rev. D **97**, 054028 \(2018\)](#).
K. D. Bednar, I. C. Cloët, and P. C. Tandy, [Phys. Rev. Lett. **124**, 042002 \(2020\)](#).

G. de Téramond, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, and A. Deur, [Phys. Rev. Lett. **120**, 182001 \(2018\)](#).
J. Lan, C. Mondal, S. Jia, X. Zhao, and J. P. Vary, [Phys. Rev. Lett. **122**, 172001 \(2019\)](#).
J. Lan, C. Mondal, S. Jia, X. Zhao, and J. P. Vary, [Phys. Rev. D **101**, 034024 \(2020\)](#).
L. Chang, K. Raya, and X. Wang, [Chin. Phys. C **44**, 114105 \(2020\)](#).
A. Kock, Y. Liu, and I. Zahed, [Phys. Rev. D **102**, 014039 \(2020\)](#).
Z. F. Cui, M. Ding, F. Gao, K. Raya, D. Binosi, L. Chang, C. D. Roberts, J. Rodríguez-Quintero, and S. M. Schmidt, [Eur. Phys. J. C **80**, 1064 \(2020\)](#).

JAM analysis with threshold resummation

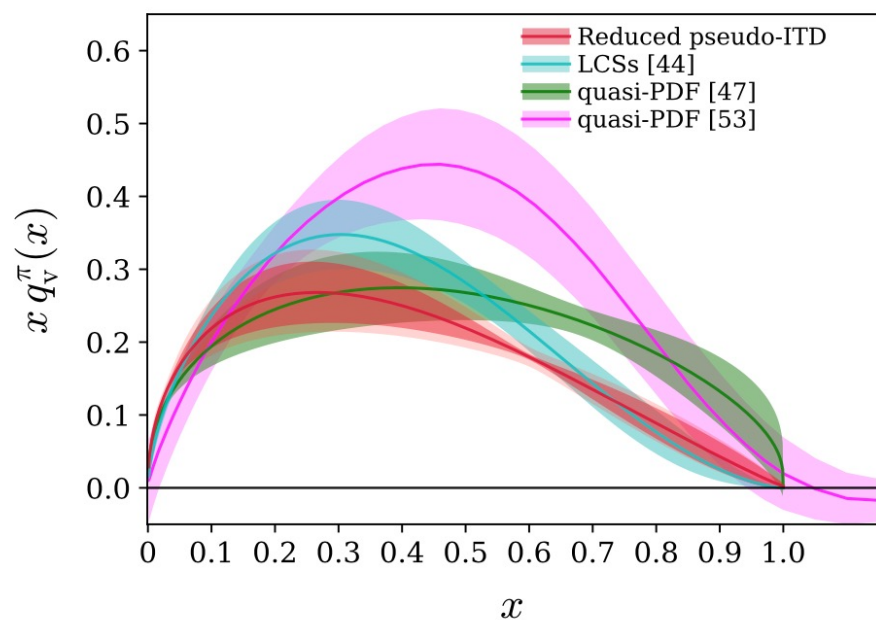


For more details, attend my talk in **WG1** on Tuesday at 17:40

- Highly dependent on perturbative approach
- NLO and NLO+NLL double Mellin methods better on theoretical grounds

Lattice QCD Activity

- Simulations on the lattice have been done to investigate this structure



Phys. Rev. D **100**, 114512 (2019).

Subset of pion lattice
QCD analyses

J.-H. Zhang, J.-W. Chen, L. Jin, H.-W. Lin, A. Schäfer, and Y. Zhao, *Phys. Rev. D* **100**, 034505 (2019), [arXiv:1804.01483 \[hep-lat\]](#).

Z.-Y. Fan, Y.-B. Yang, A. Anthony, H.-W. Lin, and K.-F. Liu, *Phys. Rev. Lett.* **121**, 242001 (2018), [arXiv:1808.02077 \[hep-lat\]](#).

R. S. Sufian, J. Karpie, C. Egerer, K. Orginos, J.-W. Qiu, and D. G. Richards, *Phys. Rev. D* **99**, 074507 (2019), [arXiv:1901.03921 \[hep-lat\]](#).

J.-W. Chen, H.-W. Lin, and J.-H. Zhang, *Nucl. Phys. B* **952**, 114940 (2020), [arXiv:1904.12376 \[hep-lat\]](#).

T. Izubuchi, L. Jin, C. Kallidonis, N. Karthik, S. Mukherjee, P. Petreczky, C. Shugert, and S. Syritsyn, *Phys. Rev. D* **100**, 034516 (2019), [arXiv:1905.06349 \[hep-lat\]](#).

B. Joó, J. Karpie, K. Orginos, A. V. Radyushkin, D. G. Richards, R. S. Sufian, and S. Zafeiropoulos, *Phys. Rev. D* **100**, 114512 (2019), [arXiv:1909.08517 \[hep-lat\]](#).

H.-W. Lin, J.-W. Chen, Z. Fan, J.-H. Zhang, and R. Zhang, *Phys. Rev. D* **103**, 014516 (2021), [arXiv:2003.14128 \[hep-lat\]](#).

R. S. Sufian, C. Egerer, J. Karpie, R. G. Edwards, B. Joó, Y.-Q. Ma, K. Orginos, J.-W. Qiu, and D. G. Richards, *Phys. Rev. D* **102**, 054508 (2020), [arXiv:2001.04960 \[hep-lat\]](#).

N. Karthik, *Phys. Rev. D* **103**, 074512 (2021), [arXiv:2101.02224 \[hep-lat\]](#).

Z. Fan and H.-W. Lin, *Phys. Lett. B* **823**, 136778 (2021), [arXiv:2104.06372 \[hep-lat\]](#).

How to relate PDFs with lattice observables?

- Make use of short distance factorization and appropriate matching coefficients

$$\begin{aligned}\Sigma_{n/h}(\nu, z^2) &\equiv \langle h(p) | T\{\mathcal{O}_n(z)\} | h(p) \rangle \\ &= \sum_i f_{i/h}(x, \mu^2) \otimes C_{n/i}(x\nu, z^2, \mu^2) \\ &\quad + \mathcal{O}(z^2 \Lambda_{\text{QCD}}^2)\end{aligned}$$

- Structure just like experimental cross sections – good for global analysis

Reduced Ioffe time pseudo-distribution (Rp-ITD)

- Lorentz-invariant Ioffe time pseudo-distribution:

$$\mathcal{M}(\nu, z^2) = \frac{1}{2p^0} \langle p | \bar{\psi}(0) \gamma^0 \mathcal{W}(z; 0) \psi(z) | p \rangle$$

Quark and antiquark fields

Gauge link

“Ioffe time”

$$\nu = p \cdot z$$

$$z = (0, 0, 0, z_3)$$

Observable is the *reduced* Ioffe time pseudo-distribution (Rp-ITD)

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0, z^2)}$$

Ratio cancels UV divergences

Fitting the Data and Systematic Corrections

Valence quark
distribution in pion

Wilson coefficients
for matching

$$\text{Re } \mathfrak{M}(\nu, z^2) = \int_0^1 dx \, q_v(x, \mu_{\text{lat}}) \mathcal{C}^{\text{Rp-ITD}}(x\nu, z^2, \mu_{\text{lat}}) + z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi(L-z)} F_1(\nu) + \dots$$

Integration lower bound is 0

Systematic corrections to parametrize

- $z^2 B_1(\nu)$: power corrections
- $\frac{a}{|z|} P_1(\nu)$: lattice spacing errors
- $e^{-m_\pi(L-z)} F_1(\nu)$: finite volume corrections

Other potential
systematic
corrections the data
is not sensitive to

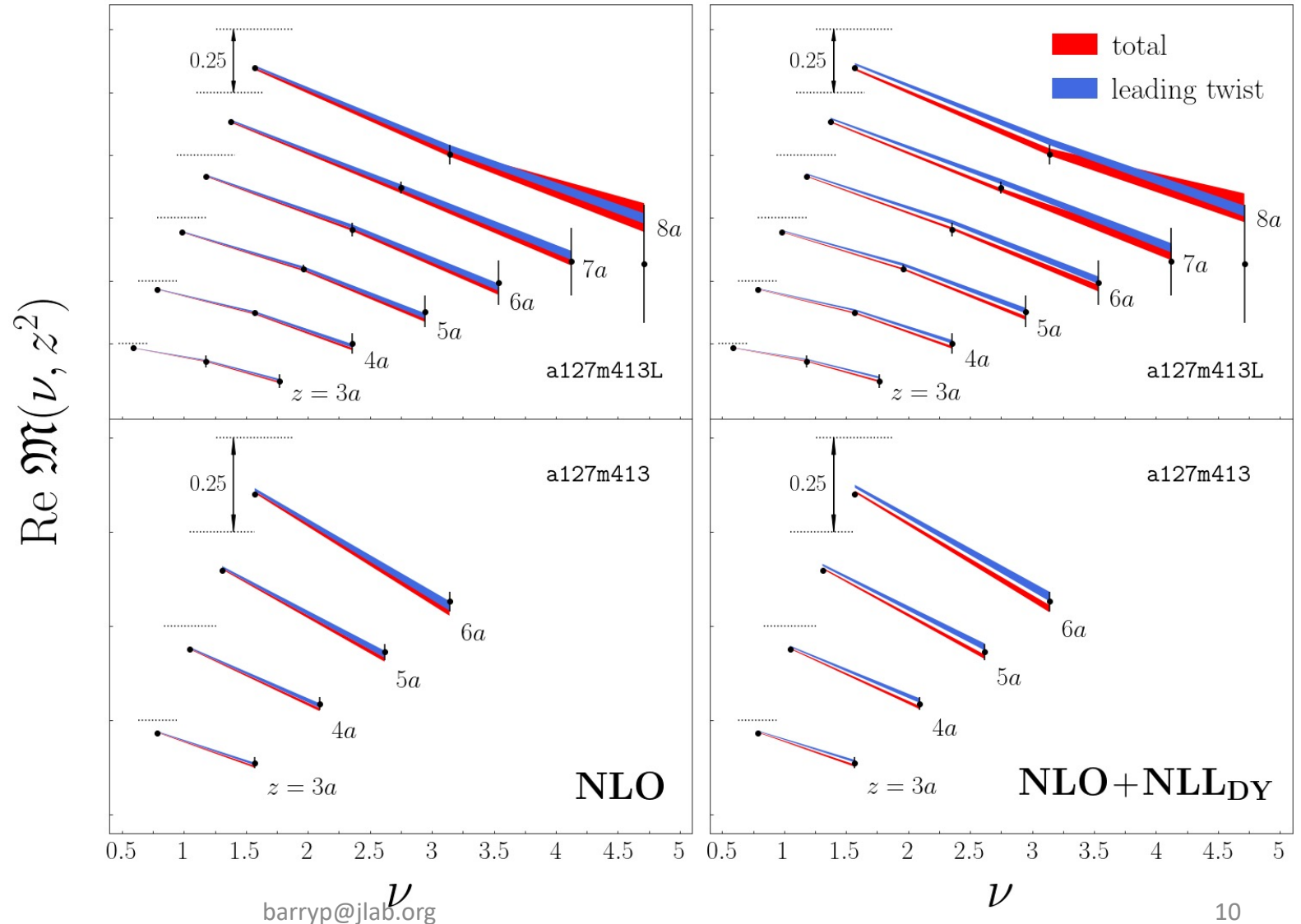
Goodness of fit

- Scenario A:
experimental data
alone
- Scenario B:
experimental + lattice,
no systematics
- Scenario C:
experimental + lattice,
with systematics

| Process | Experiment | N_{dat} | Scenario A | | Scenario B | | Scenario C | |
|---------------|----------------|------------------|----------------|--------------------|----------------|--------------------|----------------|--------------------|
| | | | NLO | +NLL _{DY} | NLO | +NLL _{DY} | NLO | +NLL _{DY} |
| | | | $\bar{\chi}^2$ | | $\bar{\chi}^2$ | | $\bar{\chi}^2$ | |
| DY | E615 | 61 | 0.84 | 0.82 | 0.83 | 0.82 | 0.84 | 0.82 |
| | NA10 (194 GeV) | 36 | 0.53 | 0.53 | 0.52 | 0.54 | 0.51 | 0.53 |
| | NA10 (286 GeV) | 20 | 0.80 | 0.81 | 0.78 | 0.79 | 0.74 | 0.81 |
| LN | H1 | 58 | 0.36 | 0.35 | 0.39 | 0.39 | 0.38 | 0.37 |
| | ZEUS | 50 | 1.56 | 1.48 | 1.62 | 1.69 | 1.59 | 1.62 |
| Rp-ITD | a127m413L | 18 | – | – | 1.04 | 1.06 | 1.05 | 1.04 |
| | a127m413 | 8 | – | – | 1.98 | 2.63 | 1.00 | 1.18 |
| Total | | 251 | 0.82 | 0.80 | 0.89 | 0.92 | 0.85 | 0.86 |

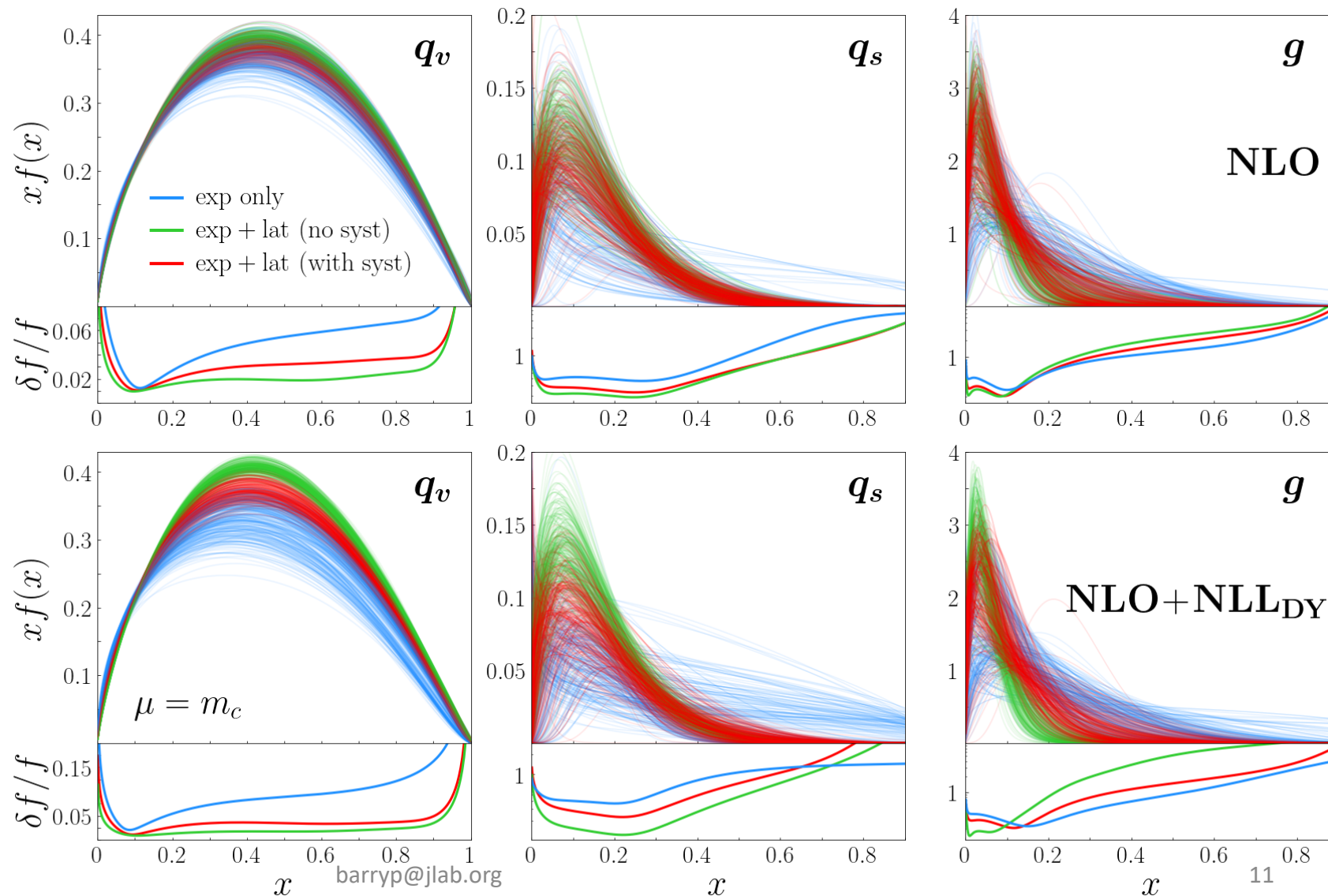
Agreement with the data

- Results from the full fit and isolating the leading twist term
- Difference between bands is the systematic correction



Resulting PDFs

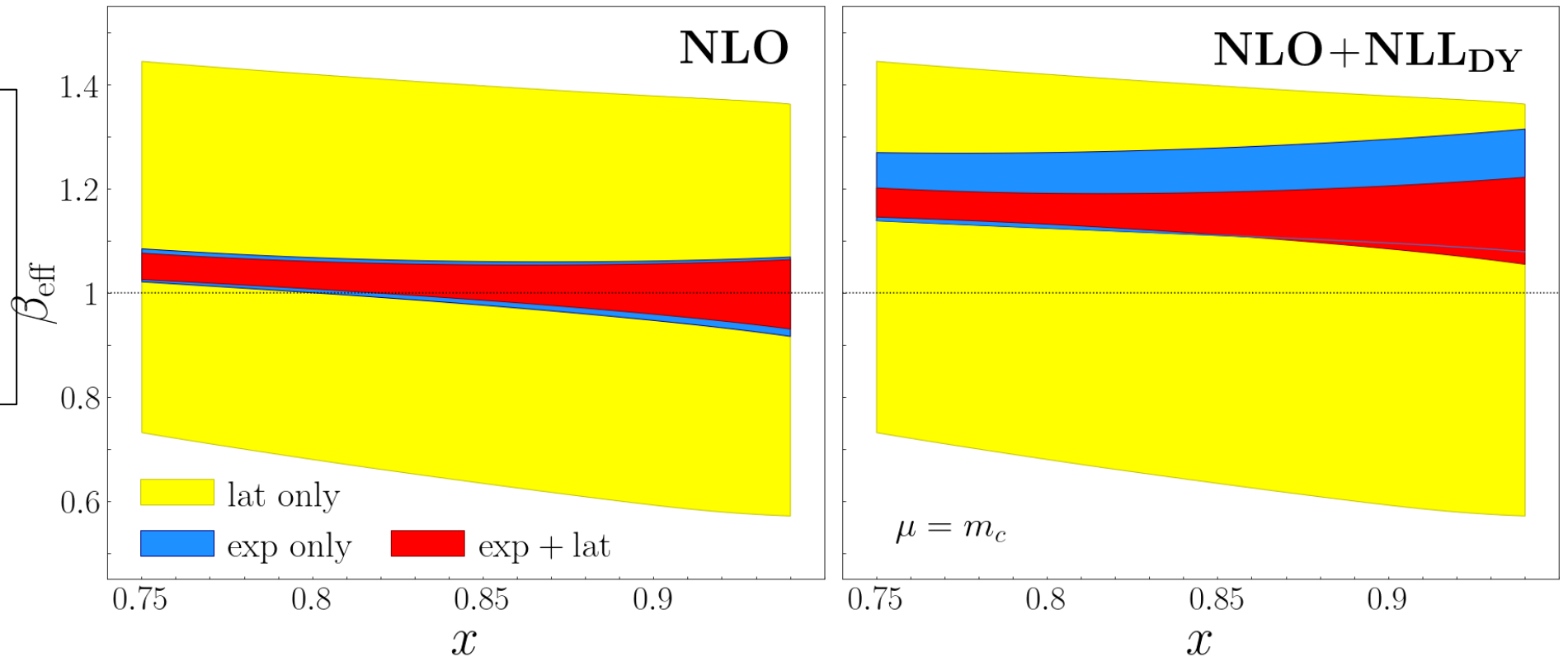
- PDFs and relative uncertainties
- Including lattice reduces uncertainties
- NLO+NLL_{DY} changes a lot – unstable under new data



Effective β from $(1 - x)^{\beta_{\text{eff}}}$

$$\beta_{\text{eff}}(x, \mu) = \frac{\partial \log |q_v(x, \mu)|}{\partial \log(1 - x)}$$

Calculations
from QCD do
not predict
 $\beta_{\text{eff}} = 2$



Conclusions

- Including both lattice and experimental data sheds light on the pion PDF itself as well as systematics associated with the lattice
- Consistency between experimental and lattice data
- $1 \lesssim \beta_v^{\text{eff}} \lesssim 1.2$
- Performed analysis with current-current lattice QCD data – too noisy to have a sizable impact
- Extensions to non-perturbative objects that are not well constrained by experiments could be aided by lattice calculations

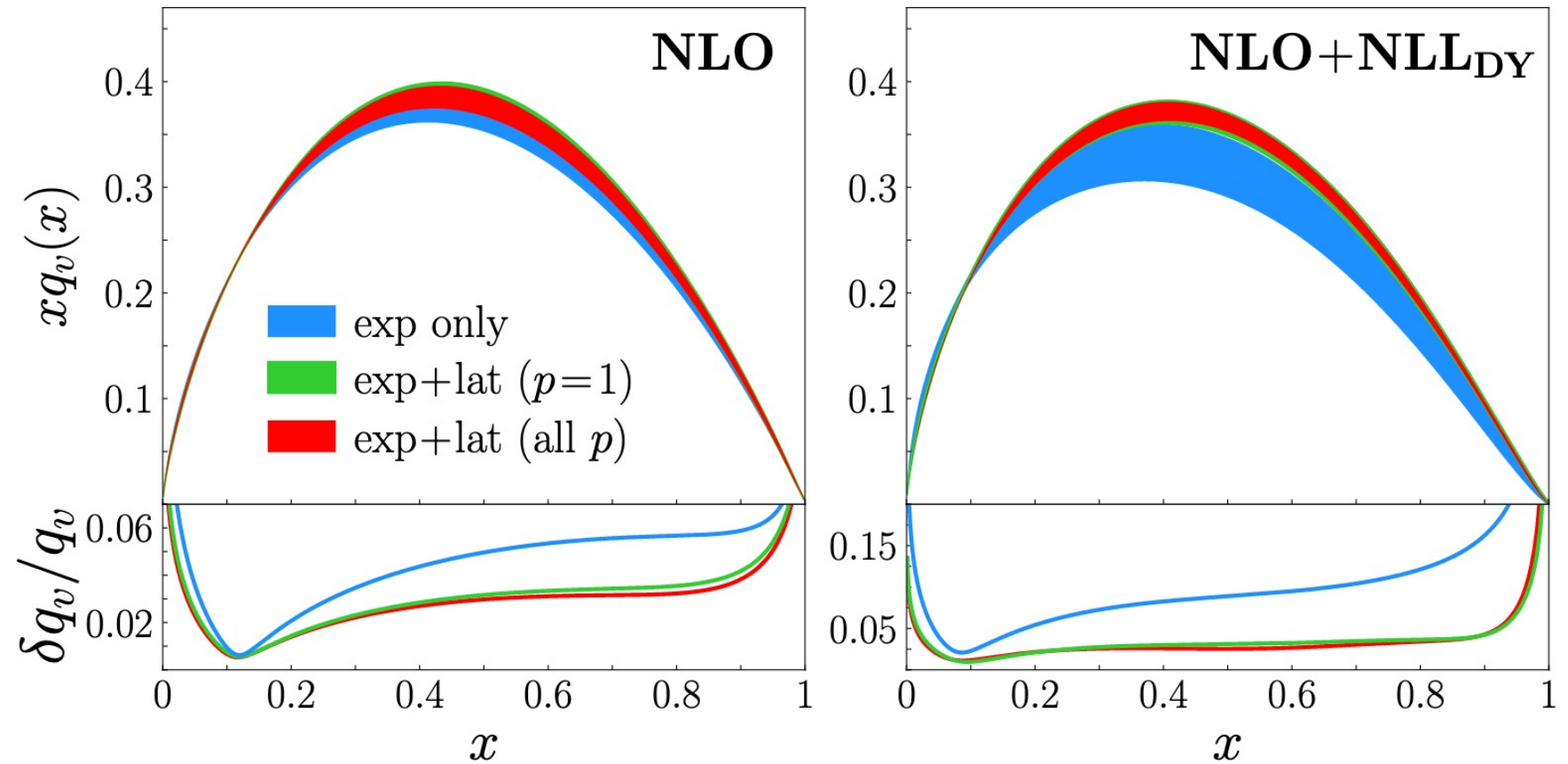
Backup Slides

Fitting only the $p = 1$ points

- Most precise points, but not large range in \log time
- Through analysis containing *only* lattice data, would not be sufficient to get a large x description of PDF

Resulting low-momentum PDFs

- These momentum points do entire job!



Quantifying individual systematic corrections on the lattice

- Breaking down by the 3 systematics

$$z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu) + e^{-m_\pi(L-z)} F_1(\nu)$$

- Dominance of power or spacing corrections depends on z
- Finite volume corrections don't matter

