

Recent Lattice-QCD Calculations of Parton Distributions

QUANTUM 3

HUEY-WEN LIN

This work of HL is supported by the NSF under grant PHY 1653405 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

Level 3: 3,000 Bonus: 16

Level 8: 24,000 Bonus: 11

PLAY OPTIONS

NSF

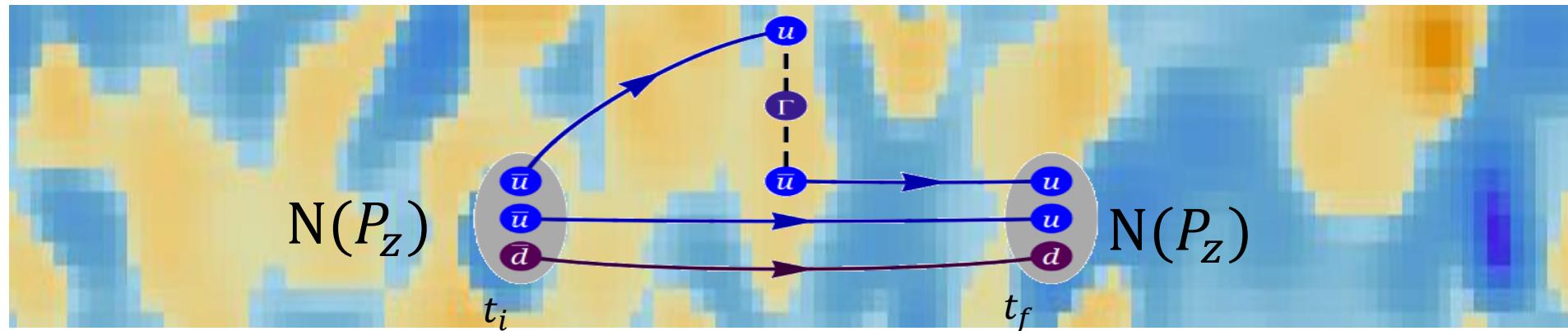
@LinQCD

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

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Lattice Parton Method

§ Large-momentum effective theory (LaMET)/Quasi-PDF (X. Ji, 2013; See 2004.03543 for review)



§ Compute quasi-distribution via

$$\tilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z) \Gamma \exp \left(-ig \int_0^z dz' A_z(z') \right) \psi(0) \right| P \right\rangle$$

§ Recover true distribution (take $P_z \rightarrow \infty$ limit)

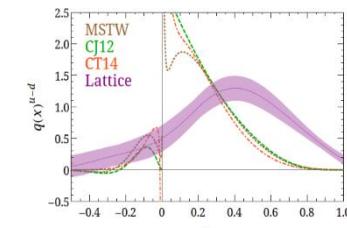
$$\tilde{q}(x, \mu, P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C \left(\frac{x}{y}, \frac{\mu}{P_z} \right) \mathbf{q}(y, \mu) + \mathcal{O} \left(\frac{M_N^2}{P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{(xP_z)^2}, \frac{\Lambda_{\text{QCD}}^2}{((1-x)P_z)^2} \right)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

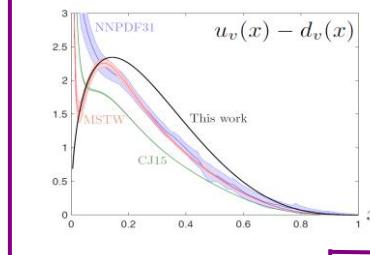
Lattice Parton Calculations

§ Rapid developments!

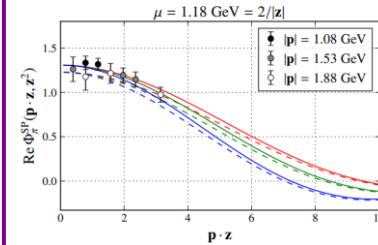
First unpol. PDF lattice calculation



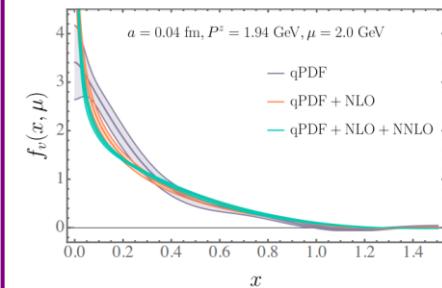
First lattice pseudo-PDFs



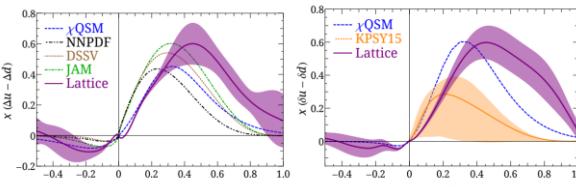
Euclidean correlation functions



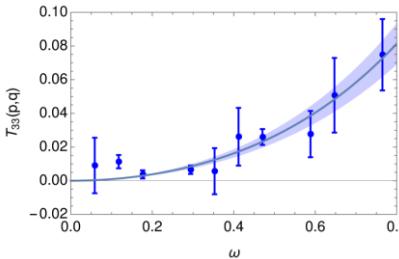
1st NNLO PDF



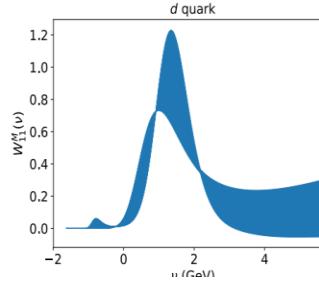
Pol. PDFs and mass corrections



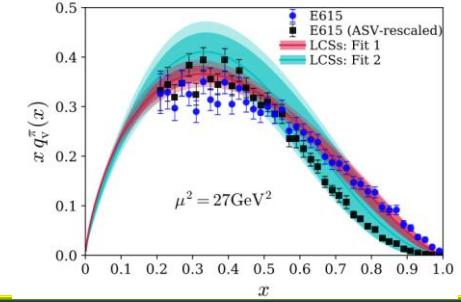
Compton amplitude



Hadronic tensor



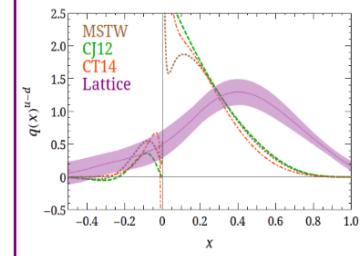
LCS



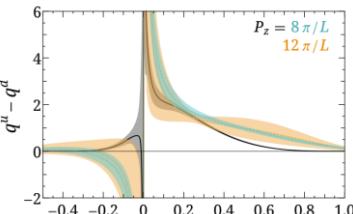
Lattice Parton Calculations

§ Physics quantity milestones

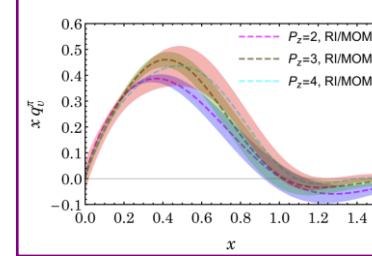
First unpol. lattice PDF



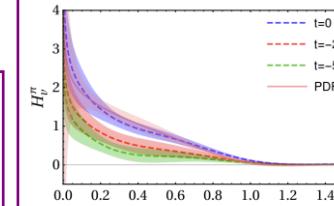
First PDFs at M_π^{phys}



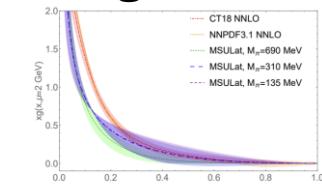
Pion v-PDF



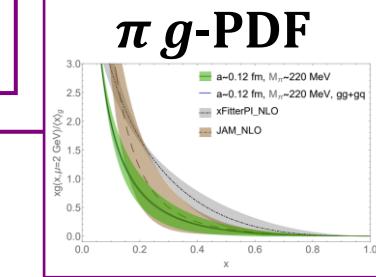
Pion GPD



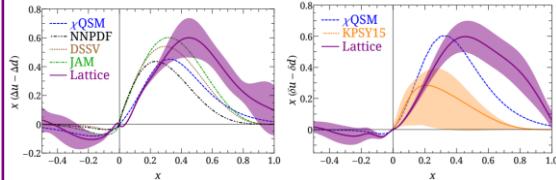
$N g$ -PDF



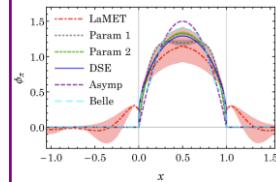
πg -PDF



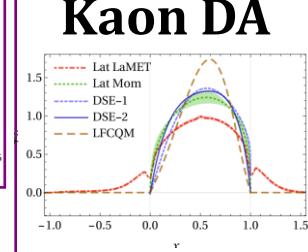
Pol. PDFs and mass corrections



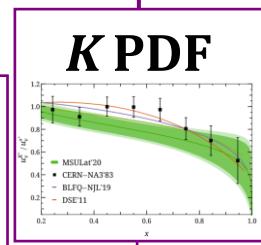
Pion DA



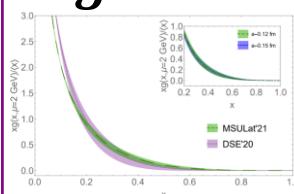
Kaon DA



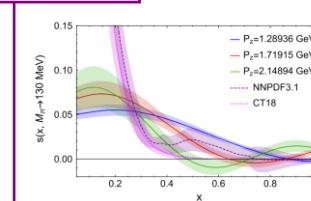
K PDF



Kaon g -PDF



s, c PDF

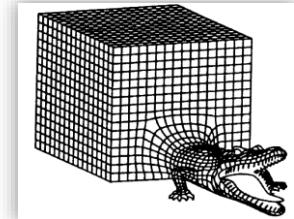
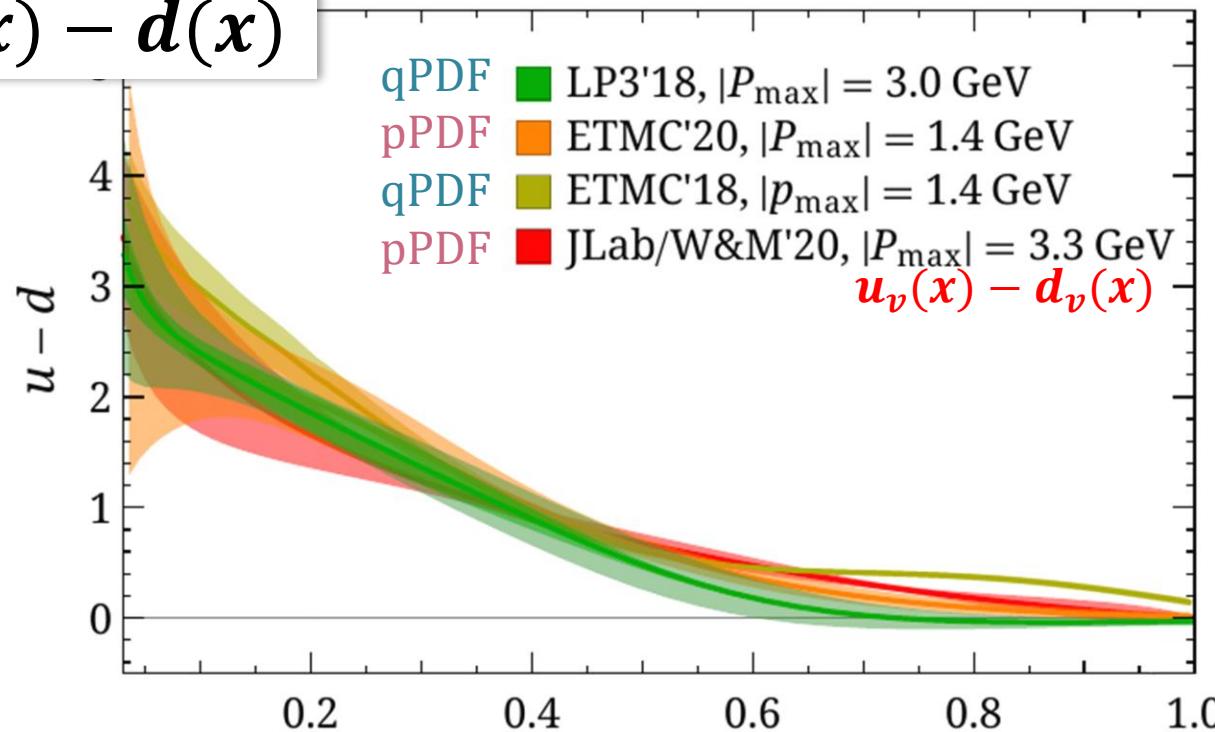


Physical Pion Mass Results

§ Summary of physical pion mass results

❖ Recent study increase boost momenta $P_z > 3 \text{ GeV}$

$u(x) - d(x)$



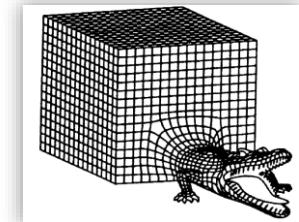
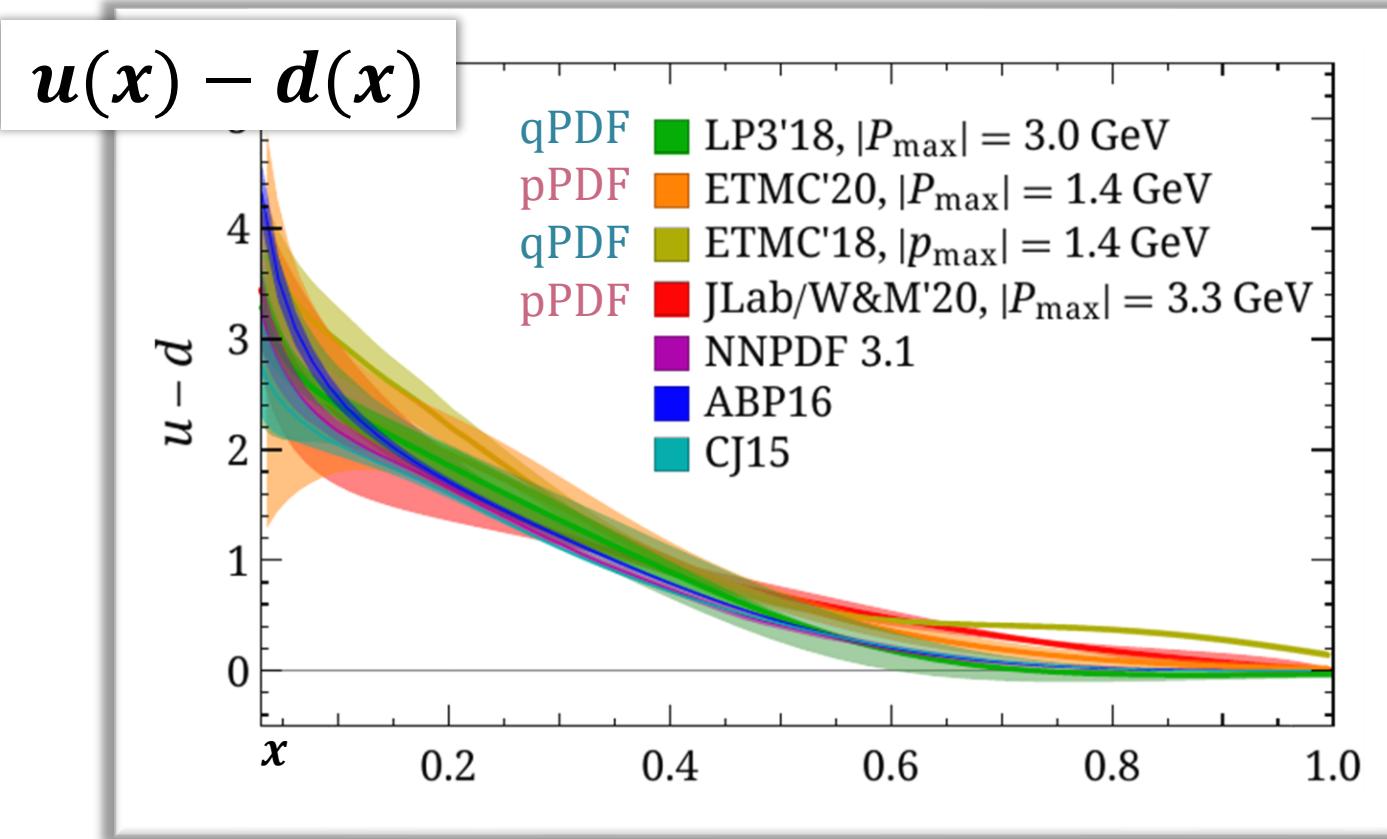
Finite volume,
Discretization,
...

2006.08636, PDFLattice2019 report

Physical Pion Mass Results

§ Summary of physical pion mass results

❖ Recent study increase boost momenta $P_z > 3$ GeV



Finite volume,
Discretization,
...

2006.08636, PDFLattice2019 report

First Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit

❖ Lattice details: clover/2+1+1 HISQ (MSULat)

$$a \approx \{0.06, 0.09, 0.12\} \text{ fm},$$

$$M_\pi \in \{135, 220, 310\}-\text{MeV pion},$$

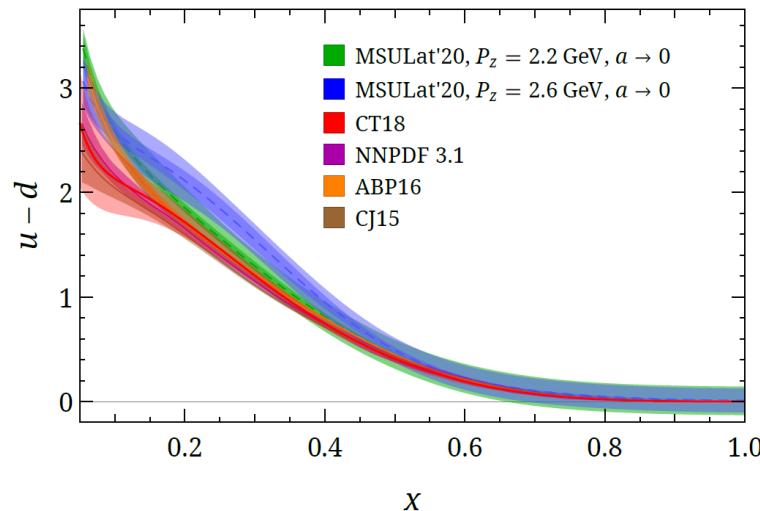
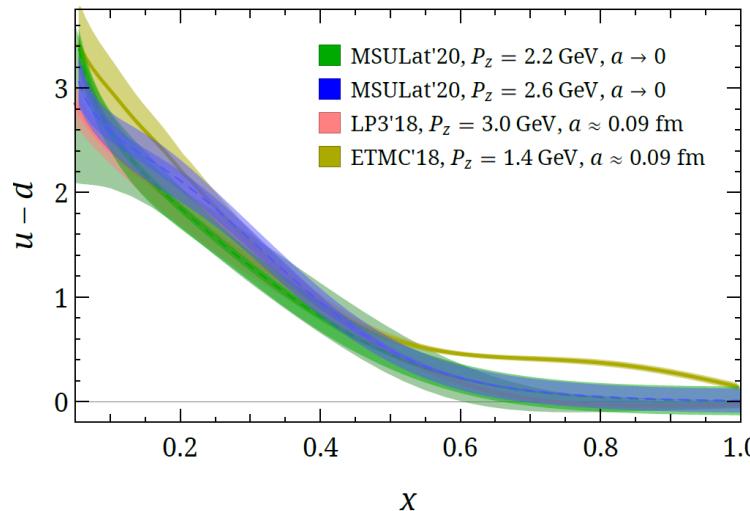
$$M_\pi L \in \{3.3, 5.5\}.$$

$$P_z \approx 2 \text{ GeV}$$



2011.14971, HL et al (MSULat)

❖ Naïve extrapolation to physical-continuum limit

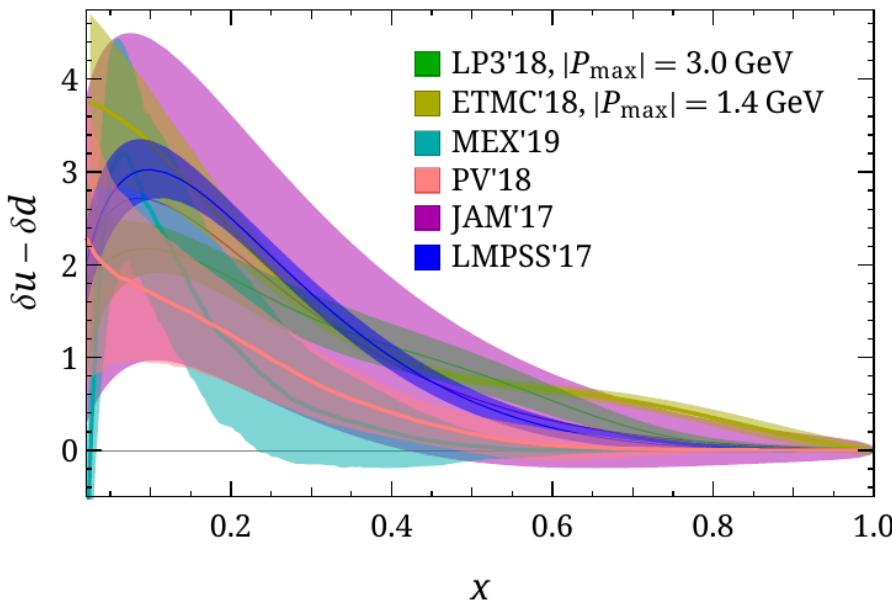
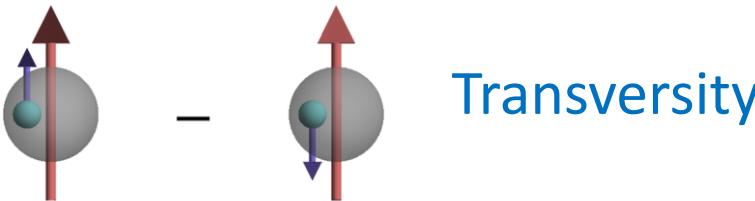


Transversity

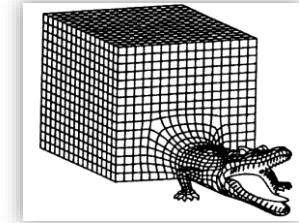
§ Summary of physical pion mass results

❖ Quasi-PDF method

$$\delta u(x) - \delta d(x)$$

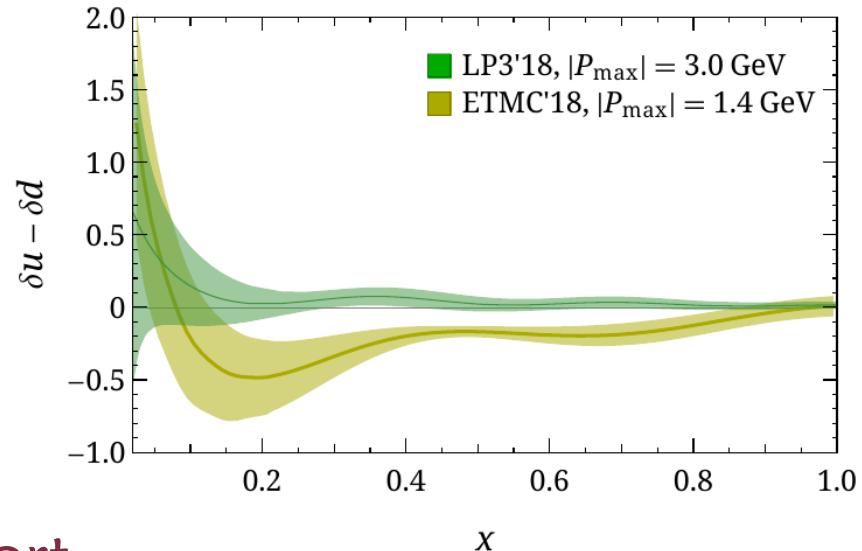


Transversity



Finite volume,
Discretization,
...

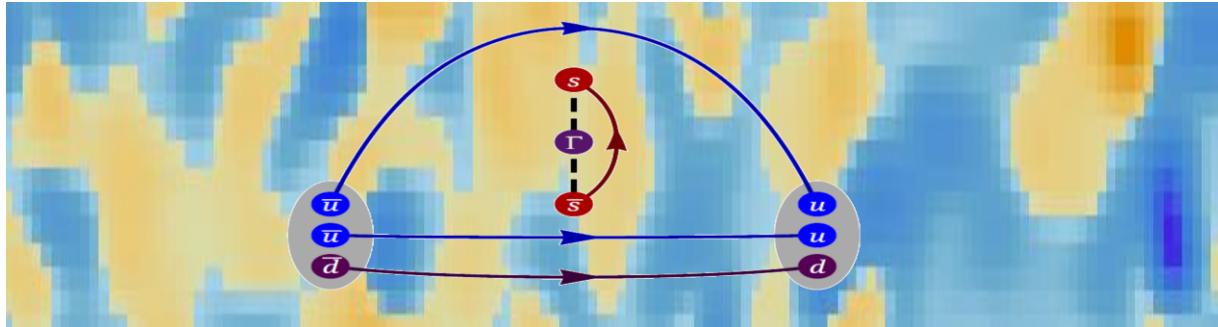
$$\delta \bar{d}(x) - \delta \bar{u}(x)$$



2006.08636, PDFLattice2019 report

First Lattice Strange PDF

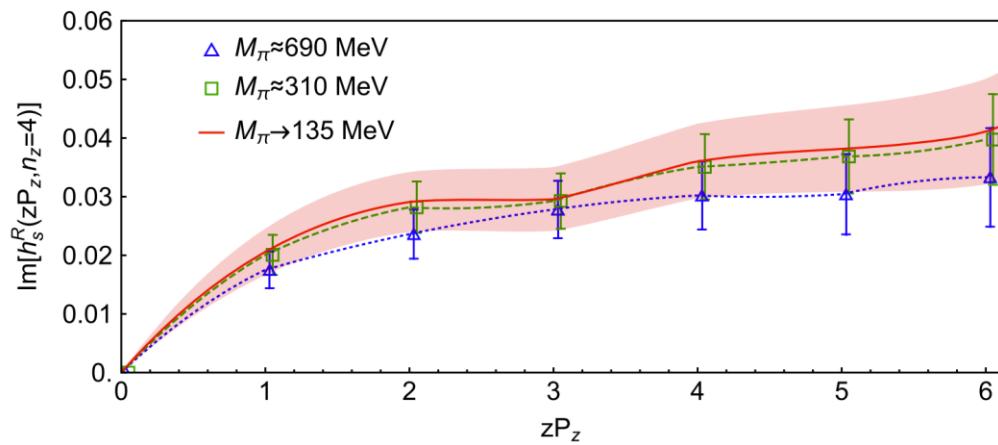
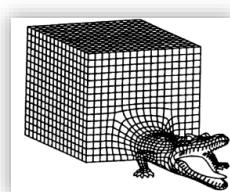
§ On the lattice, one needs to calculate the following



2005.12015, Zhang, Lin, Yoon

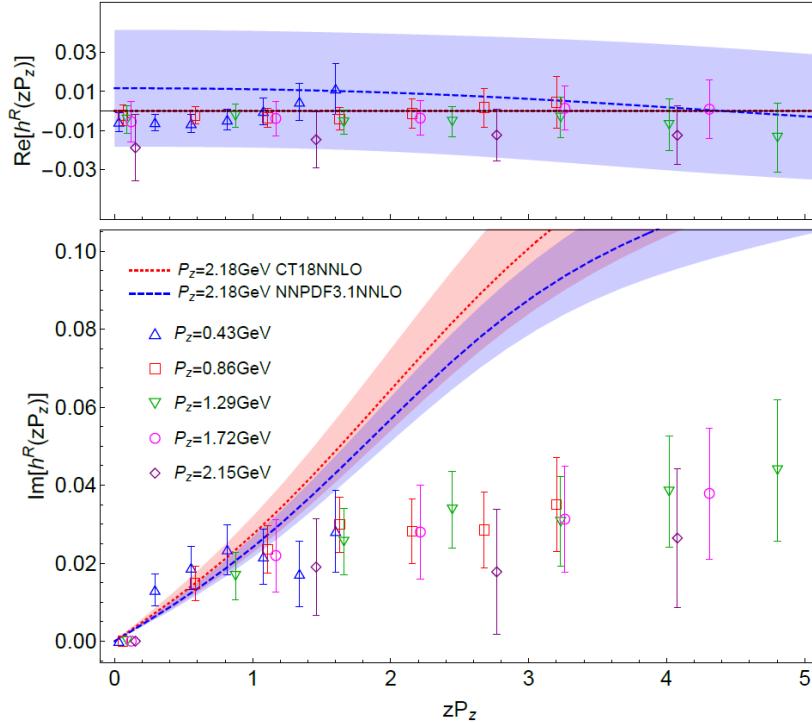
§ Results by MSULat/quasi-PDF method

- ❖ Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum
 - ❖ 7,184,000 strange loops
 - ❖ 344,832 nucleon correlators
- ❖ RI/MOM renormalization
- ❖ Extrapolated to
 $M_\pi \approx 140$ MeV



First Lattice Strange PDF

§ Lattice matrix elements



§ From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^1 \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + O\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

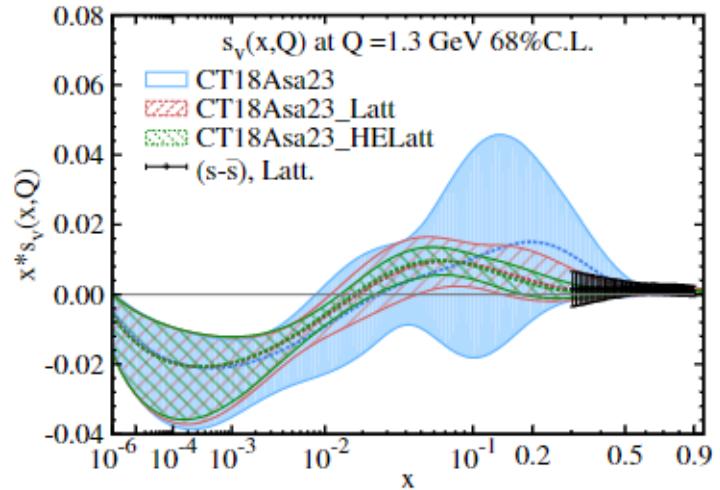
❖ Strange-antistrange symmetry

$$\text{Re}[h(z)] \propto \int dx (s(x) - \bar{s}(x)) \cos(xzP_z)$$

$$\text{Im}[h(z)] \propto \int dx (s(x) + \bar{s}(x)) \sin(xzP_z)$$

C T E Q

Preliminary Results

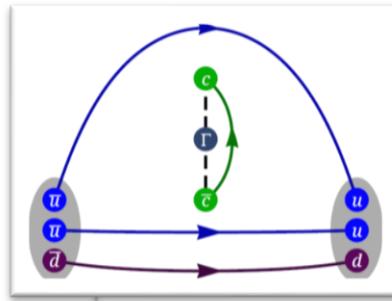


First Lattice Charm PDF

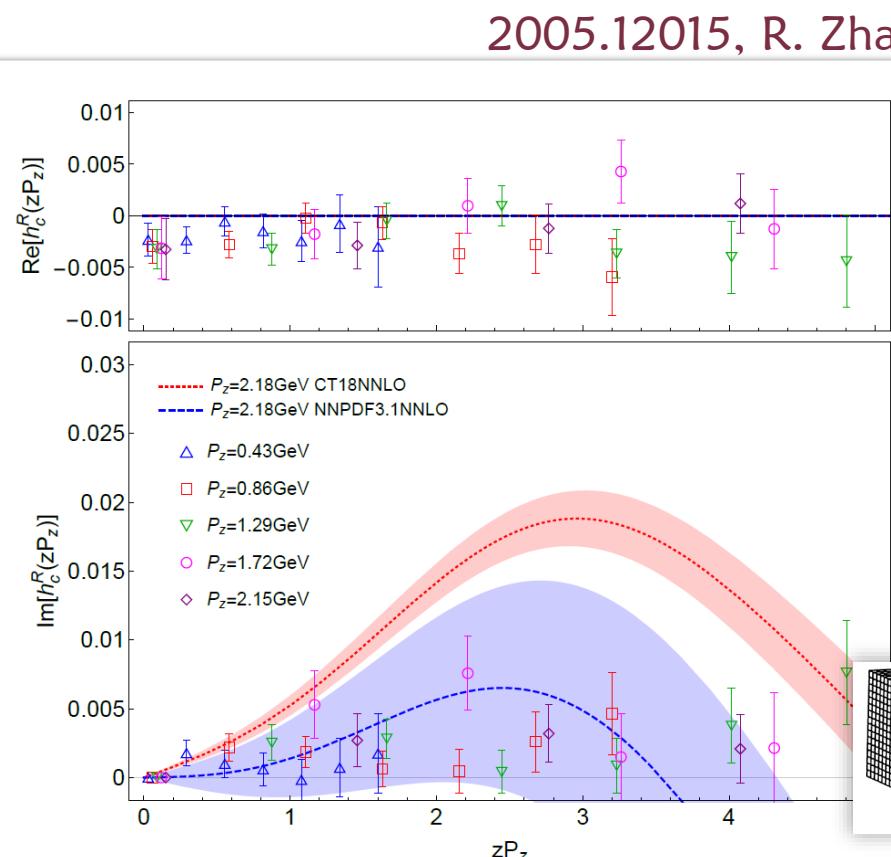
§ Large uncertainties in global PDFs

§ Results by MSULat/quasi-PDF method

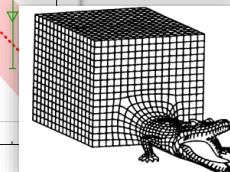
❖ Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Rui Zhang
(MSU)



Gluon PDF in Nucleon

§ Gluon PDF using pseudo-PDF

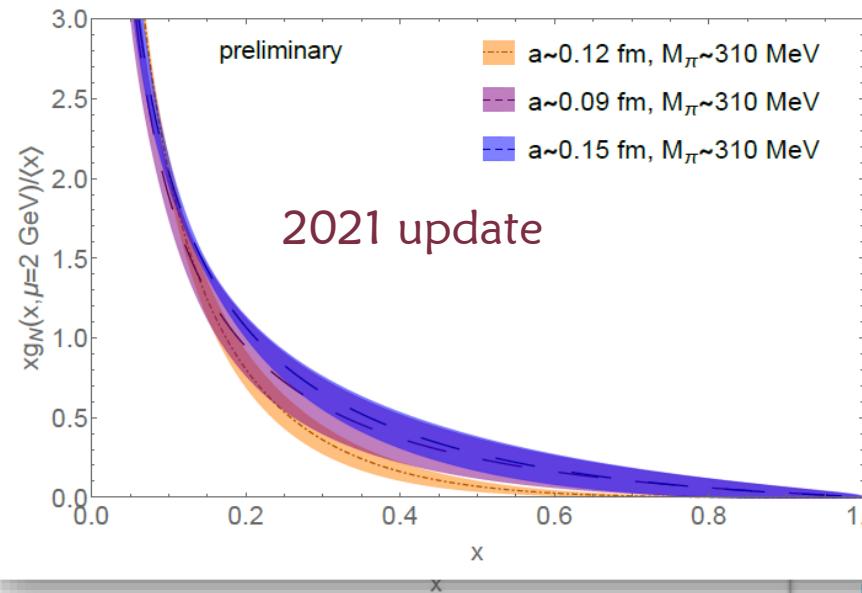
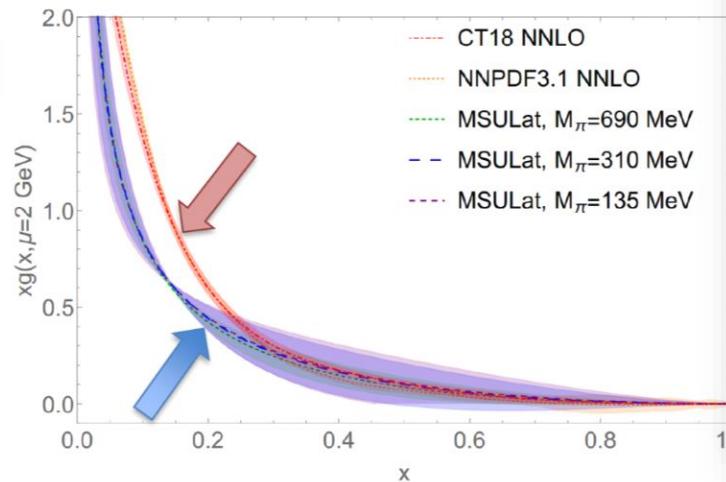
❖ Lattice details: clover/2+1+1 HISQ 0.12 fm,

310-MeV sea pion

❖ Study strange/light-quark

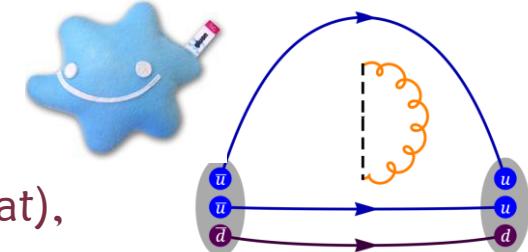
Z. Fan. et al (MSULat),
2007.16113

The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNI \square gluon unpolarized PDF at $\mu = 2 \text{ GeV}$ in the $\overline{\text{MS}}$ scheme.



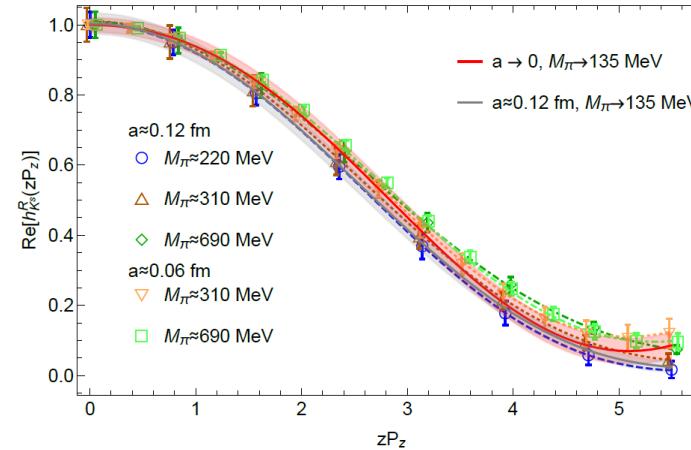
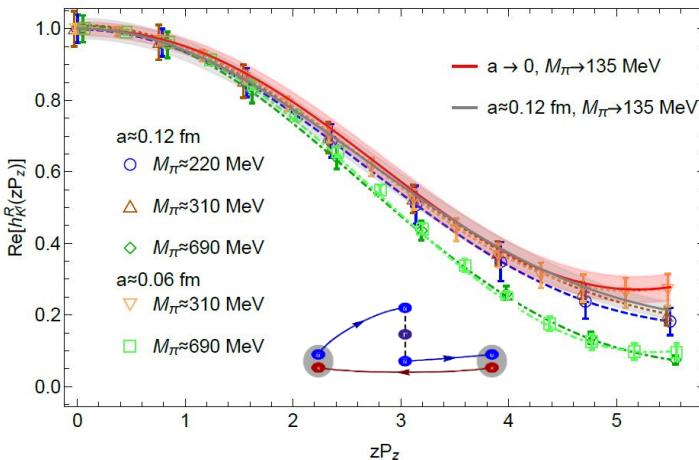
Slide by Zhouyou Fan @ DNP 2020

G: Zhouyou Fan

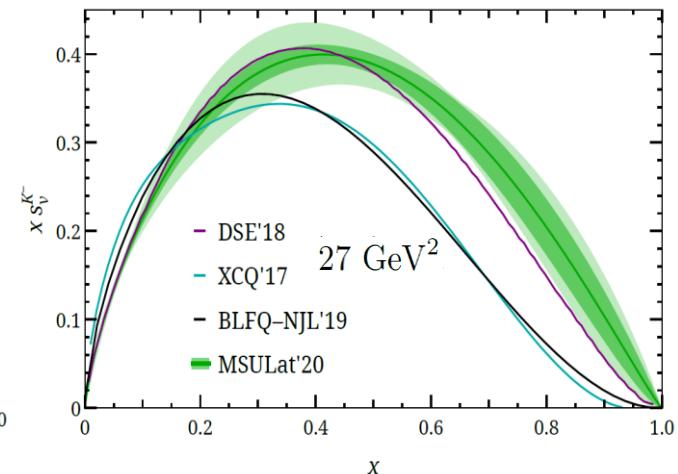
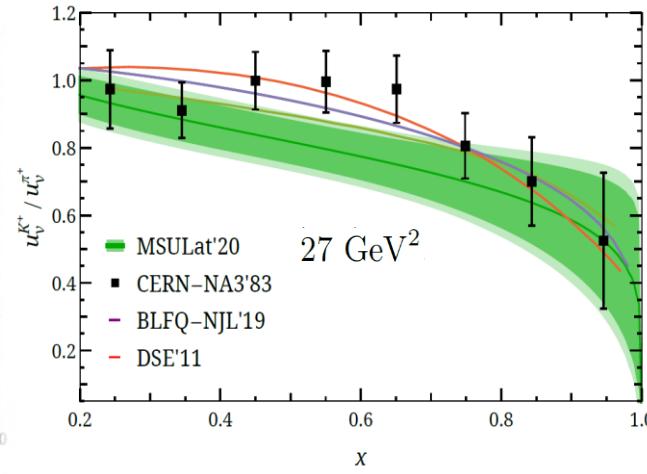
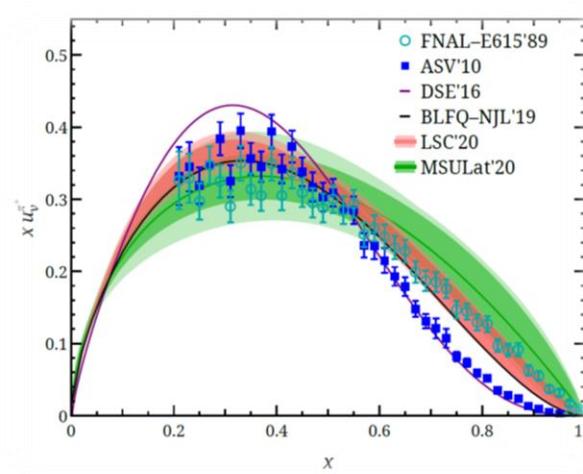


Meson PDFs

§ Valence-quark PDFs of Pion/Kaon using quasi-PDF in the continuum limit

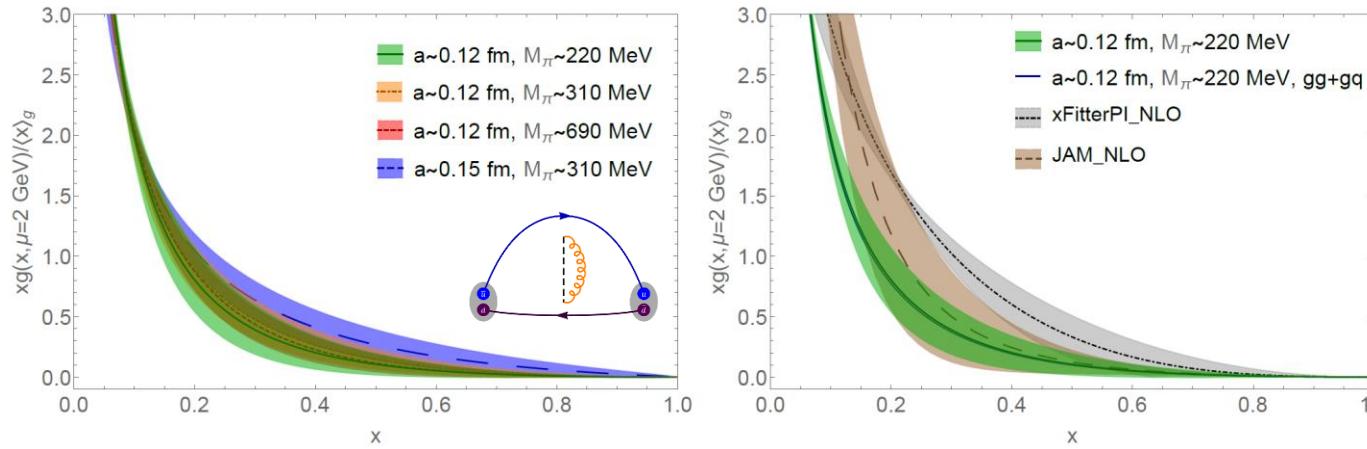


MSULat,
2003.14128



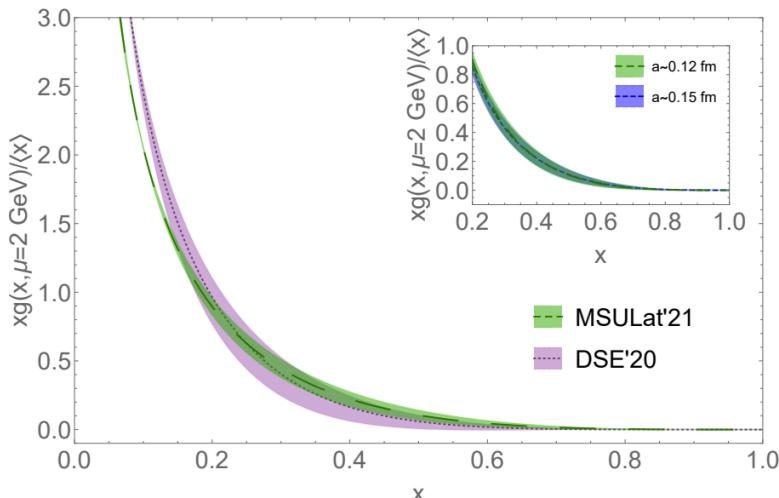
Meson Gluon PDFs

§ First pion and kaon gluon PDFs using pseudo-PDF



Zhouyou Fan
(MSU)

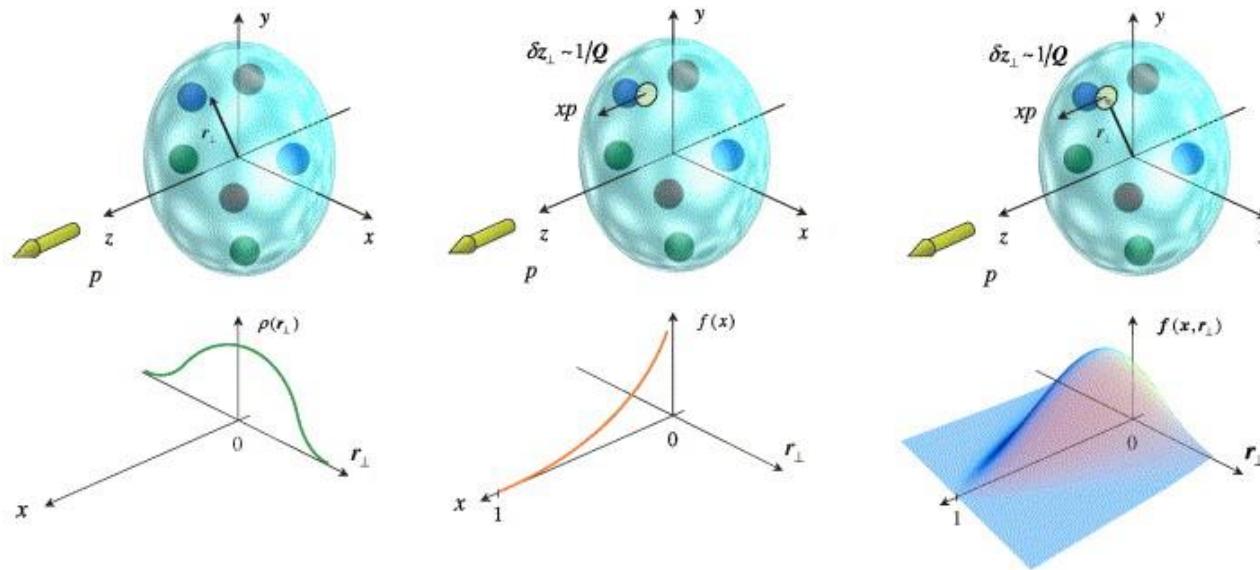
2007.16113, 2104.06372, Fan et al (MSULat)



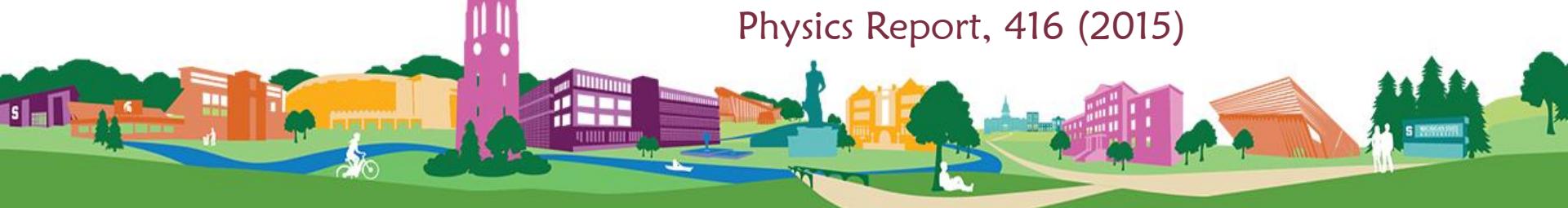
Alejandro
Salas-Chavira
(MSU)

2112.03124 , Salas-Chavira et al (MSULat)

Bjorken- x Dependent GPDs



Picture from A. Belitskya and A Radyushkin,
Physics Report, 416 (2015)



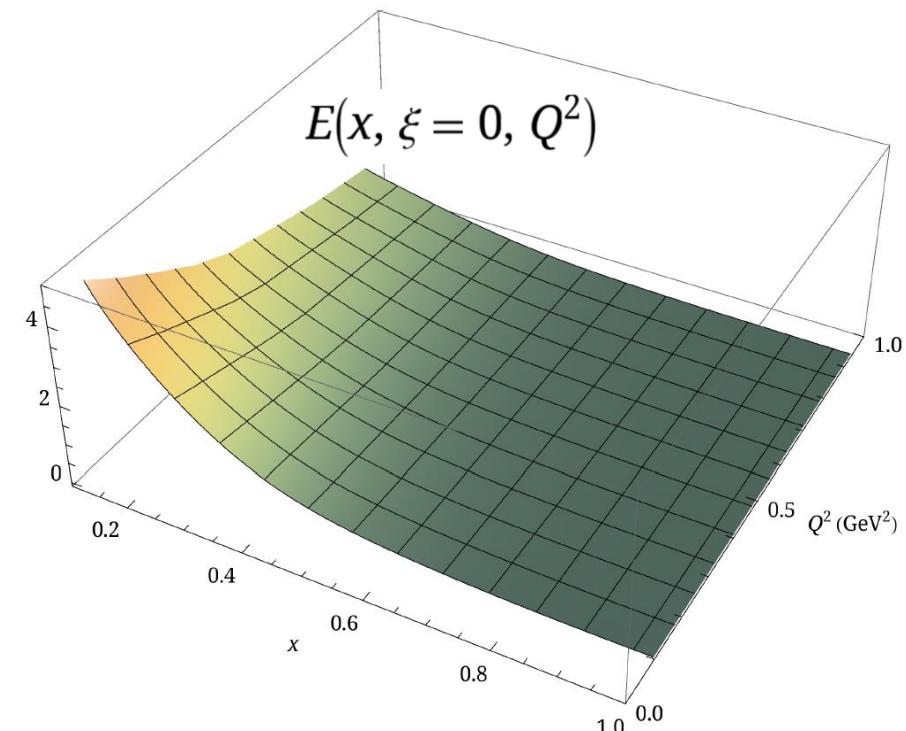
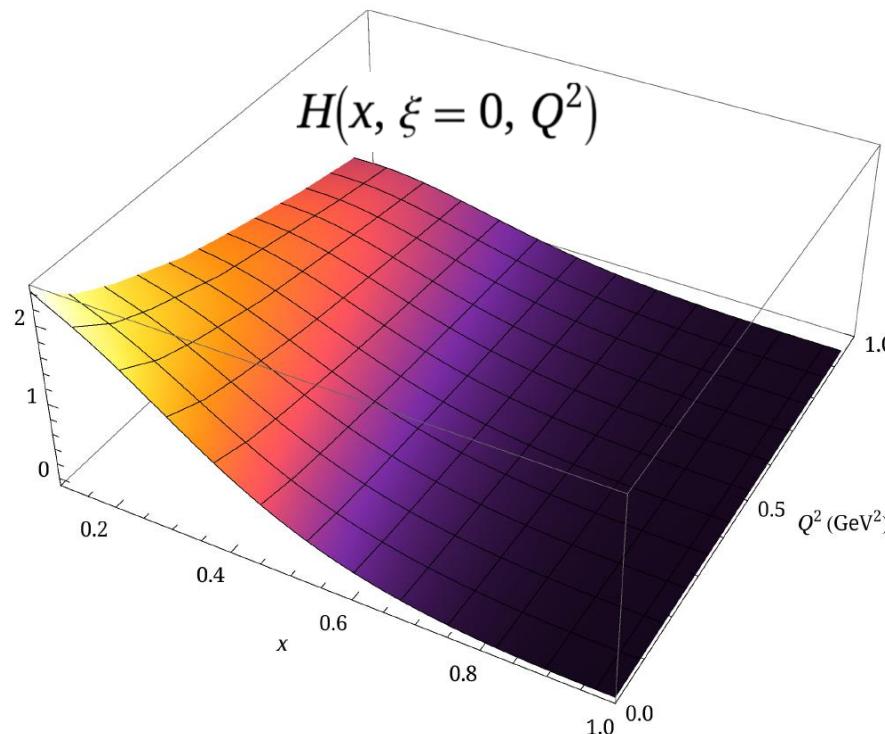
Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

❖ MSULat: clover/2+1+1 HISQ

0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

❖ $\xi = 0$ isovector nucleon quasi-GPD results



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

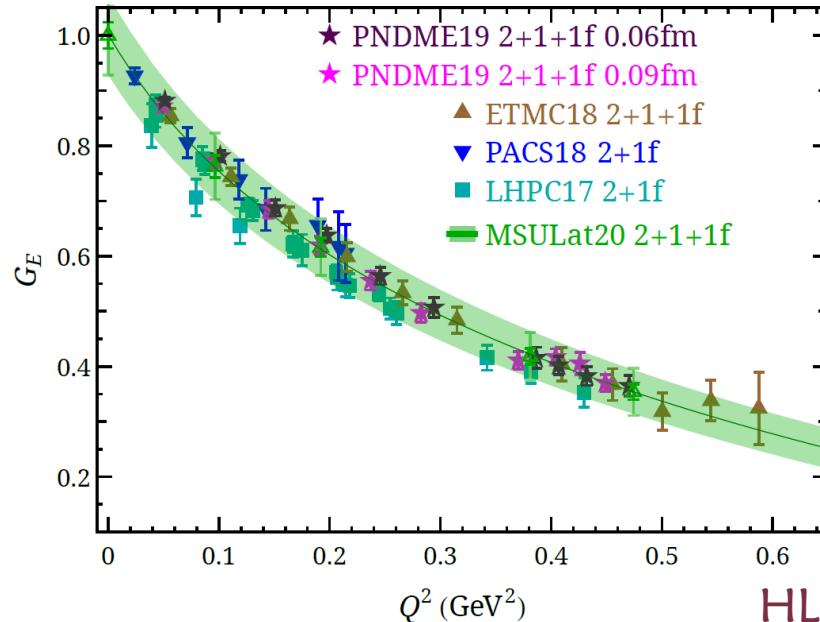
❖ Lattice details: clover/2+1+1 HISQ (MSULat)

0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV

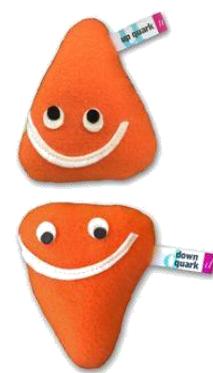
❖ $\xi = 0$ isovector nucleon quasi-GPD results

$$\int_{-1}^{+1} dx x^{n-1} = \sum_{i=0, \text{even}}^{n-1} (-2\xi)^i A_{ni}^q(t) + (-2\xi)^n C_{n0}^q(t) \Big|_{n \text{ even}}$$

$n = 1$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001



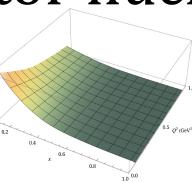
Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

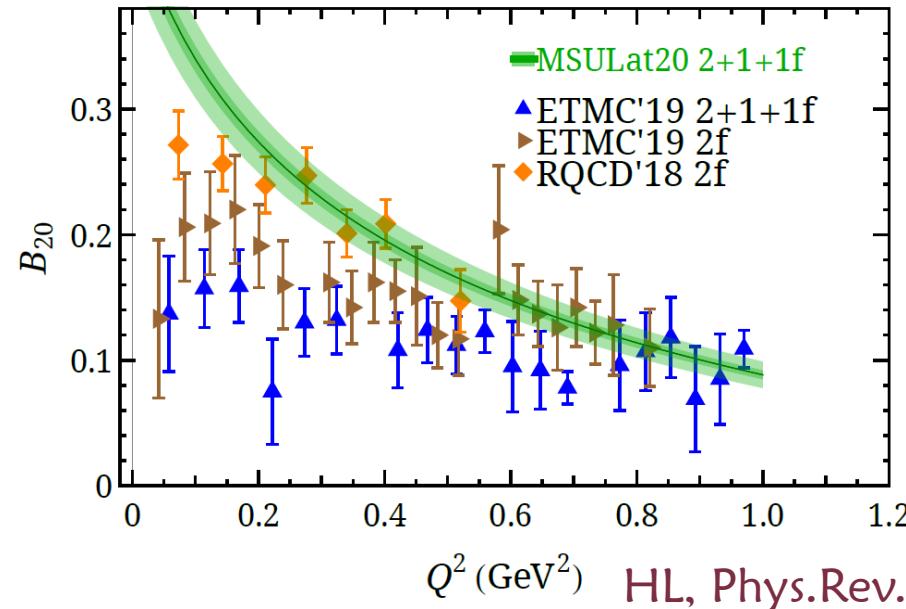
❖ Lattice details: clover/2+1+1 HISQ (MSULat)

0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV

❖ $\xi = 0$ isovector nucleon quasi-GPD results

$$\int_{-1}^{+1} dx x^{n-1} = \sum_{i=0, \text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t) \Big|_{n \text{ even}}$$


$n = 2$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001



Nucleon Tomography

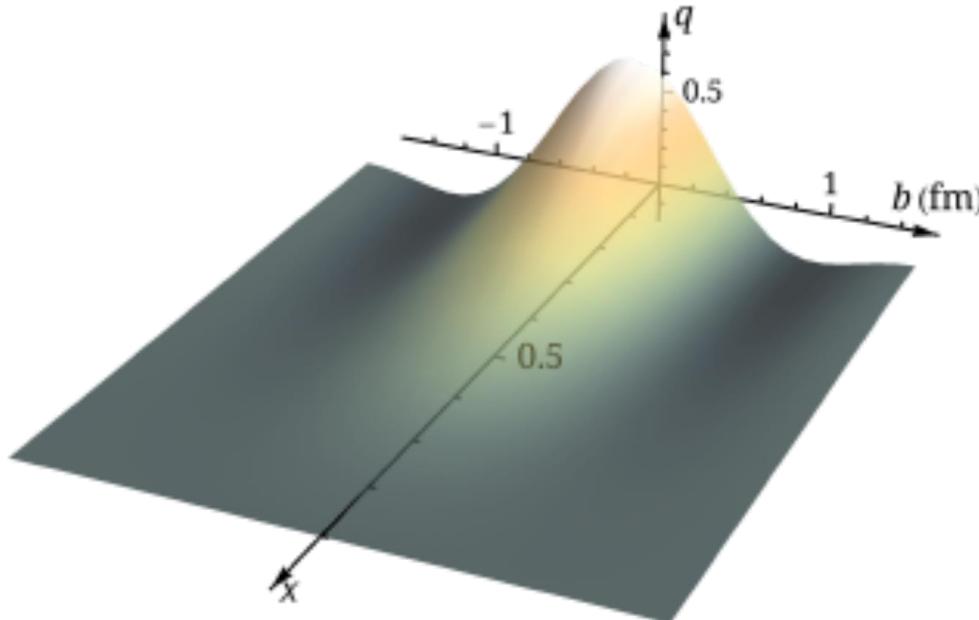
§ Nucleon GPD using quasi-PDFs at physical pion mass

❖ Lattice details: clover/2+1+1 HISQ (MSULat)

0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV

❖ $\xi = 0$ isovector nucleon quasi-GPD results

$$q(x, b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Tomography

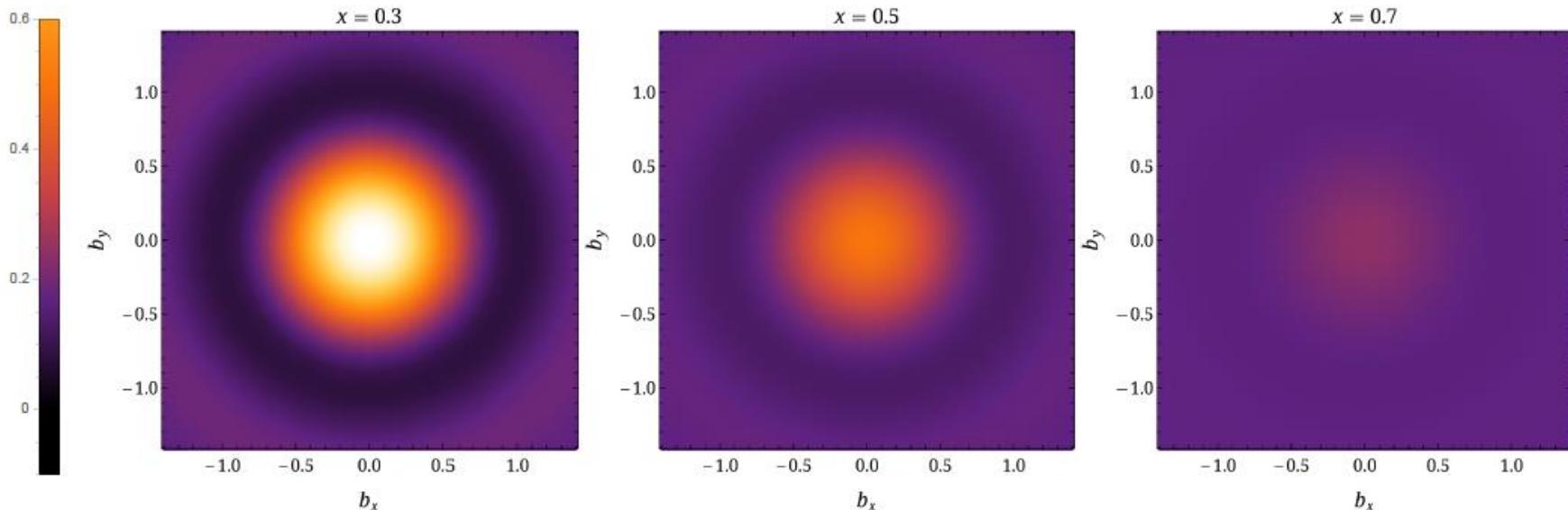
§ Nucleon GPD using quasi-PDFs at physical pion mass

❖ Lattice details: clover/2+1+1 HISQ (MSULat)

0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV

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$$q(x, b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q} \cdot \vec{b}}$$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Polarized GPDs

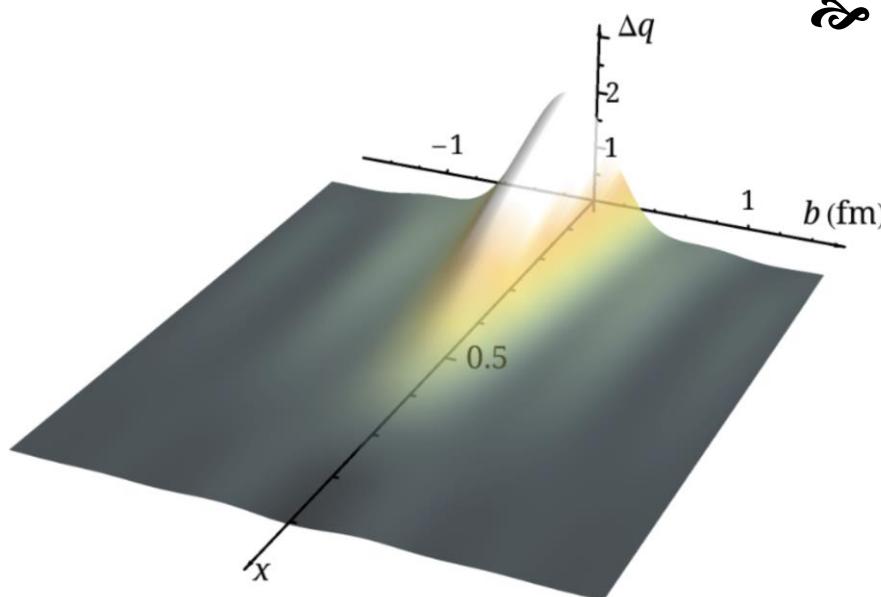
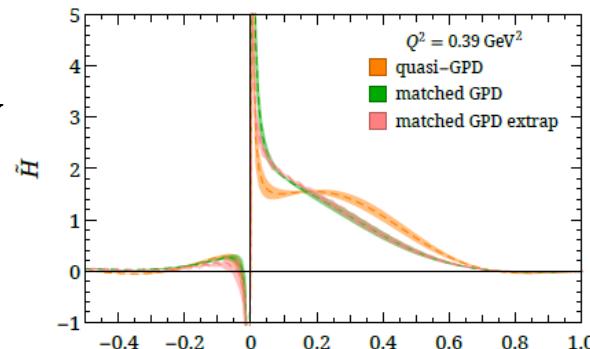
§ Helicity GPD (\tilde{H}) using quasi-PDFs at physical pion mass

❖ MSULat: clover/2+1+1 HISQ

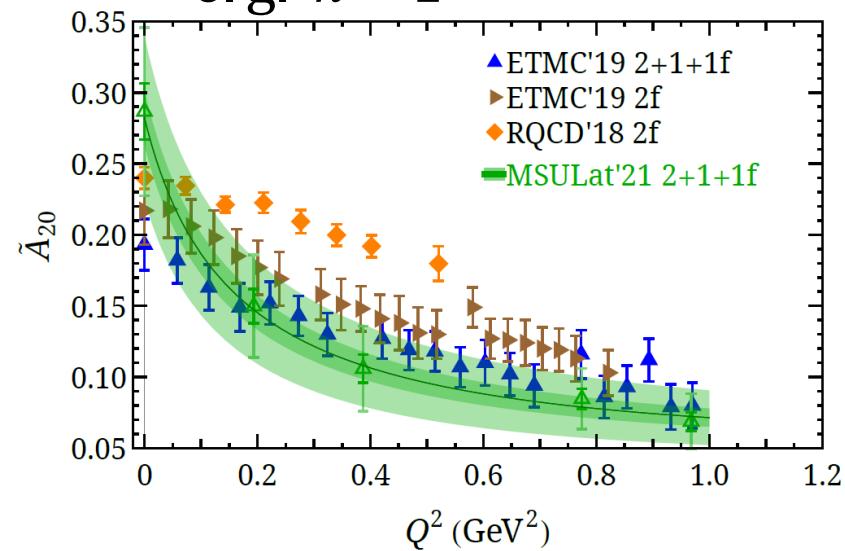
0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

❖ $\xi = 0$ isovector nucleon (quasi-)GPD results

HL (MSULat), Phys.Lett.B 824 (2022) 136821



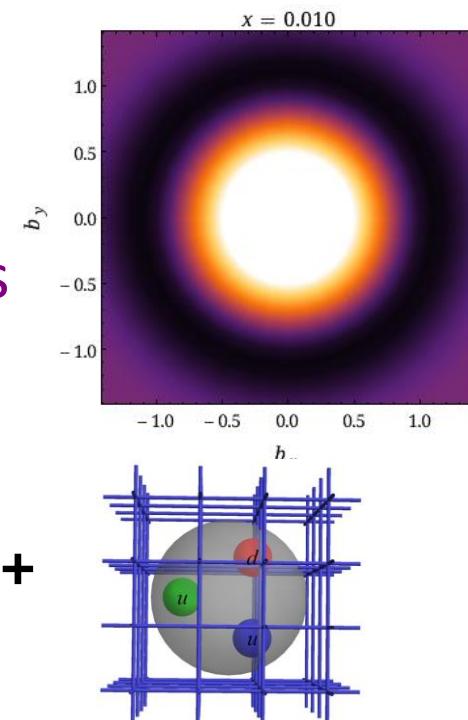
❖ Take the integral to form moments;
e.g. $n = 2$



Summary and Outlook

- § Exciting era using LQCD to study parton distributions
- § Overcoming longstanding limitations of moment method
 - ❖ Bjorken- x dependence of parton distributions are widely studied
 - ❖ More study of systematics planned for the near future
- § Precision and progress are limited on resources
 - ❖ Challenges = new opportunities quantities

§ In the future



The work of HL is sponsored by NSF CAREER Award under grant PHY 1653405 & RCSA Cottrell Scholar Award
Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources

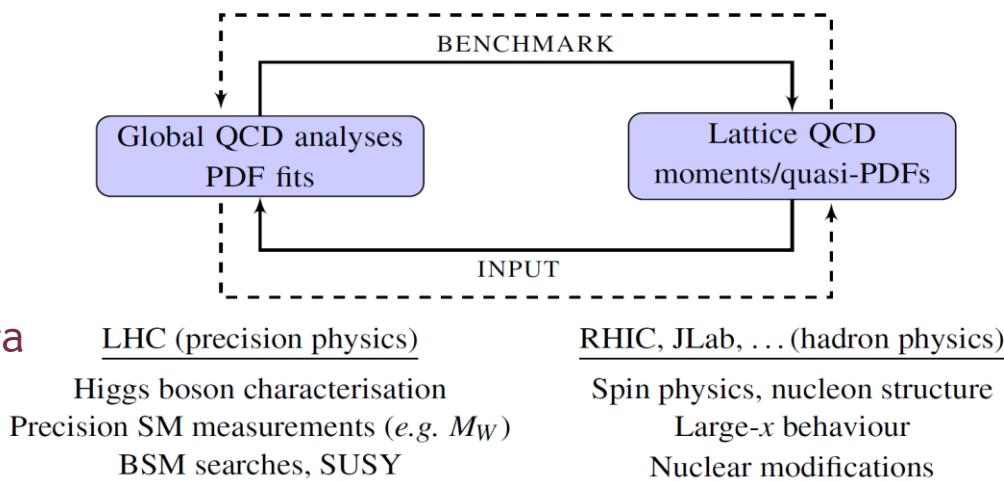
Backup Slides



How Can Lattice Help?

THE PDFLATTICE2017 WORKSHOP

Plot by
E. Nocera

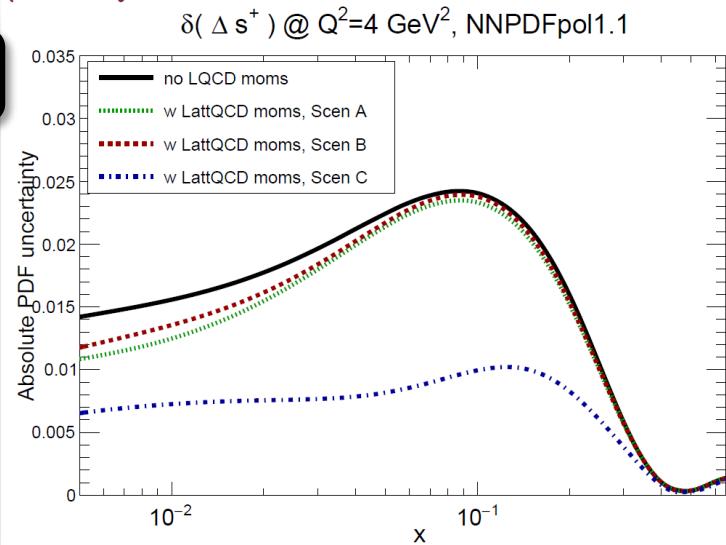


Whitepaper , Progress in Par. and Nuc. Phy. 100, 106 (2018)

A: 70% B: 50% C: 20%

§ Is there one quantity for which LQCD can achieve a precision at which it can make a significant difference?

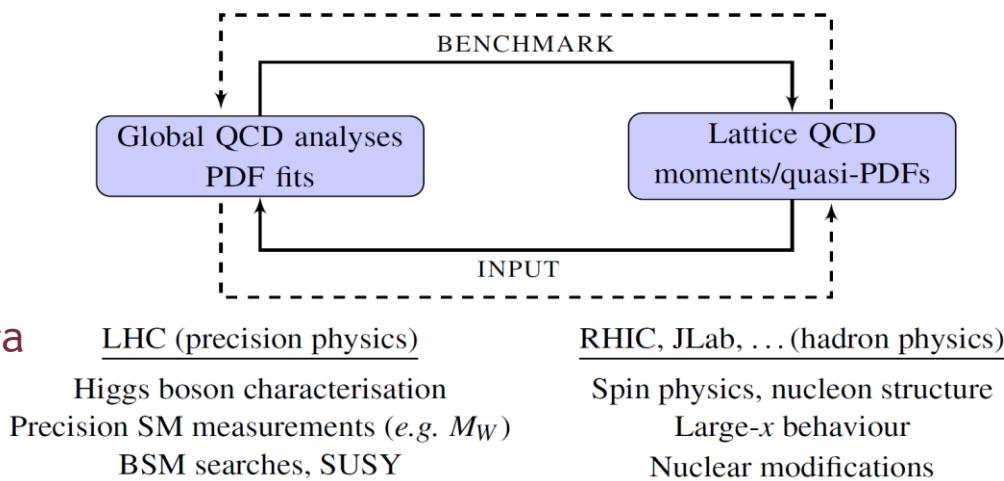
Example study



How Can Lattice Help?

THE PDFLATTICE2017 WORKSHOP

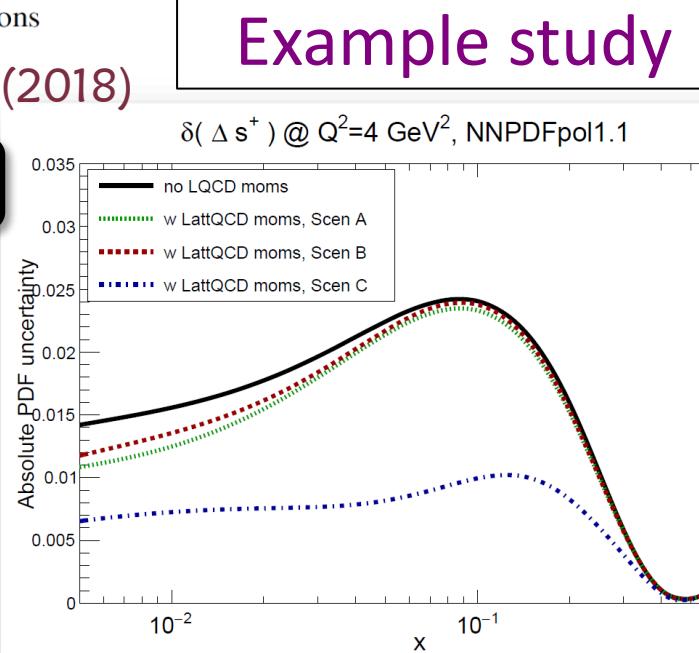
Plot by
E. Nocera



Whitepaper , Progress in Par. and Nuc. Phy. 100, 106 (2018)

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Polarized PDFs

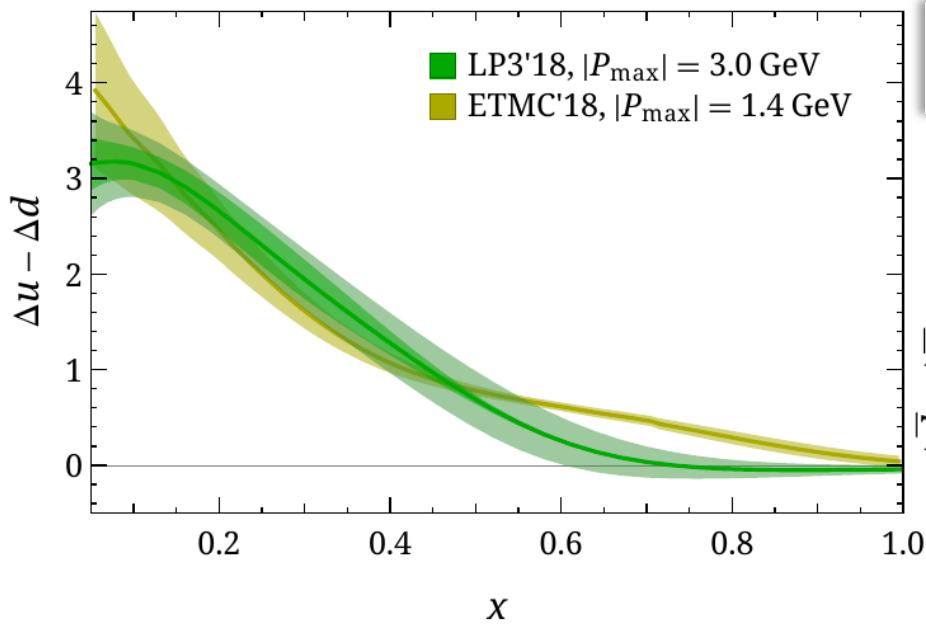
§ Summary of physical pion mass results

❖ Quasi-PDF method

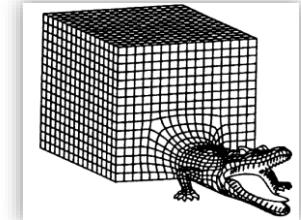
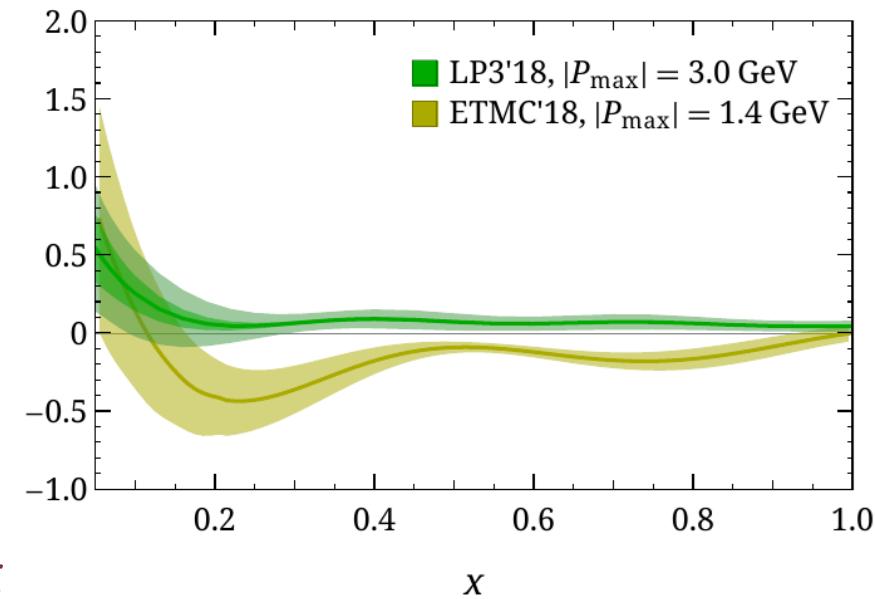


Helicity
long. polarized

$$\Delta u(x) - \Delta d(x)$$



$$\Delta \bar{u}(x) - \Delta \bar{d}(x)$$



Finite volume,
Discretization,
...

2006.08636, PDFLattice2019 report

Polarized PDFs

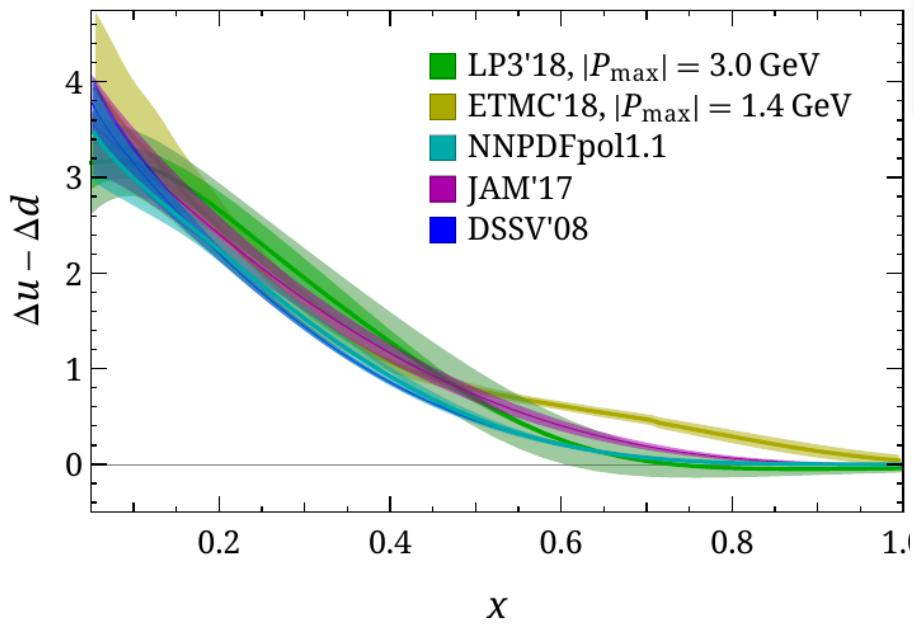
§ Summary of physical pion mass results

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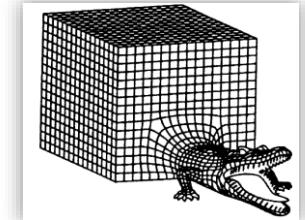
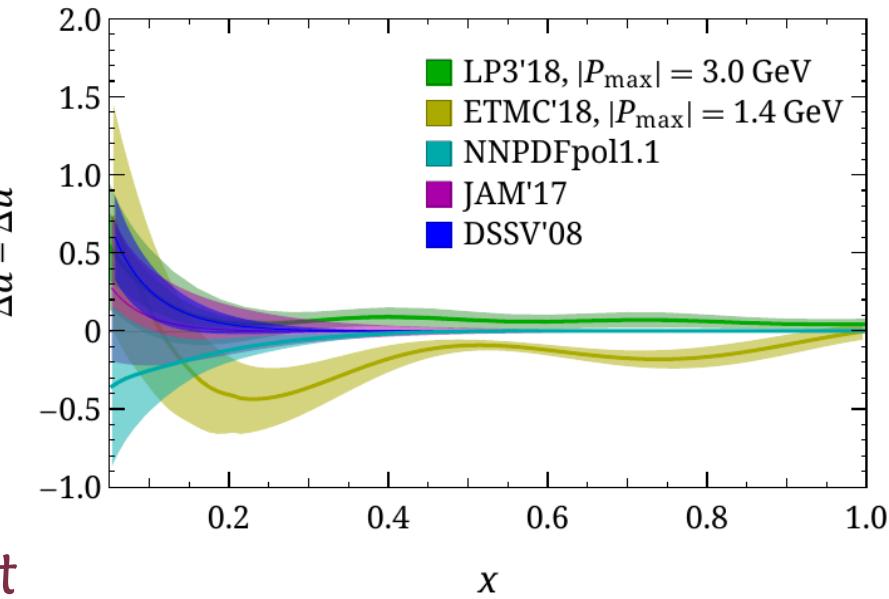


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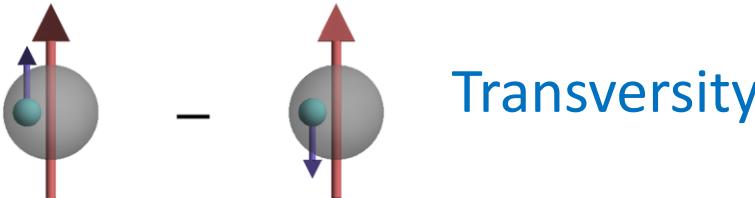
2006.08636, PDFLattice2019 report

Transversity

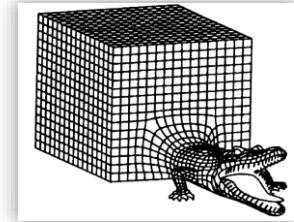
§ Summary of physical pion mass results

❖ Quasi-PDF method

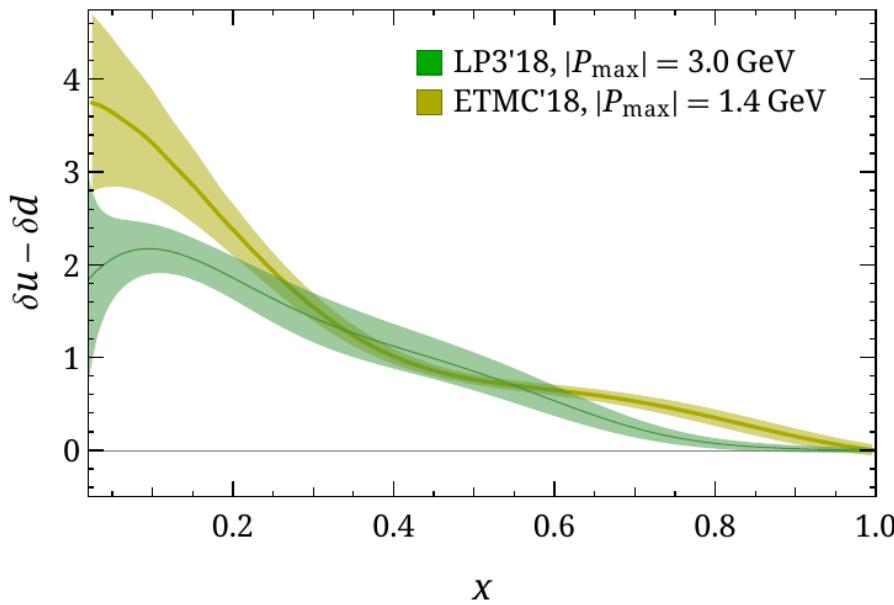
$$\delta u(x) - \delta d(x)$$



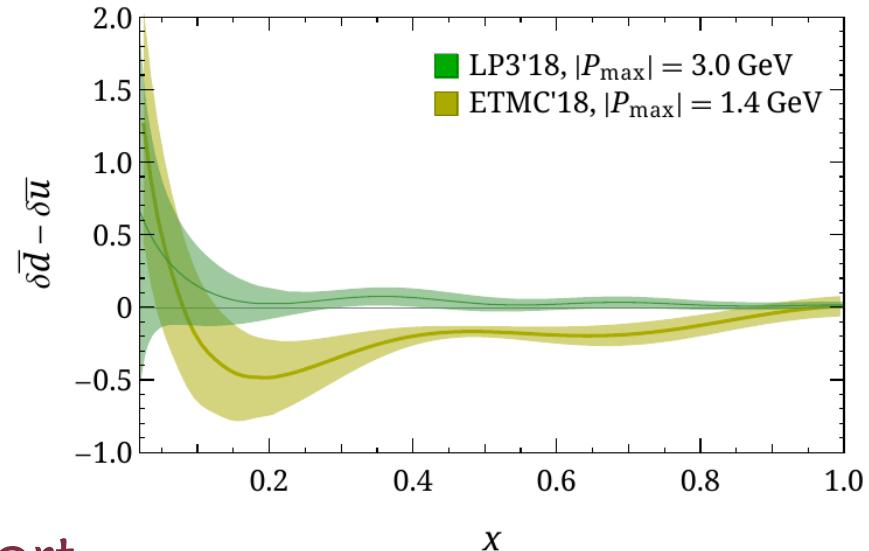
Transversity



Finite volume,
Discretization,
...



$$\delta \bar{d}(x) - \delta \bar{u}(x)$$



2006.08636, PDFLattice2019 report

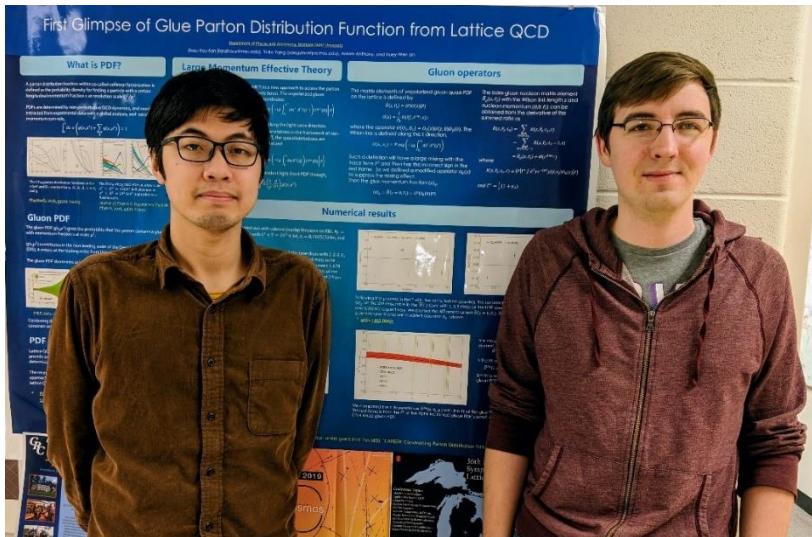
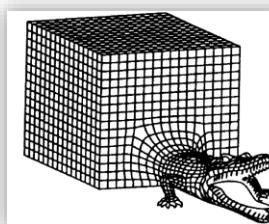
Gluon PDF

§ Pioneering first glimpse into gluon PDF using LaMET

- ❖ Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- ❖ Promising results using coordinate-space comparison, but signal does not go far in z
- ❖ Hard numerical problem to be solved

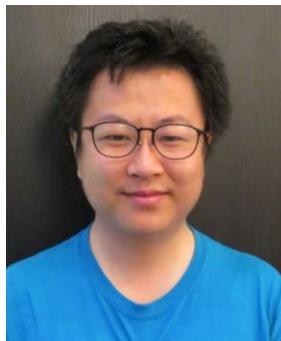


Fan et al, Phys.Rev.Lett. 121, 242001 (2018)

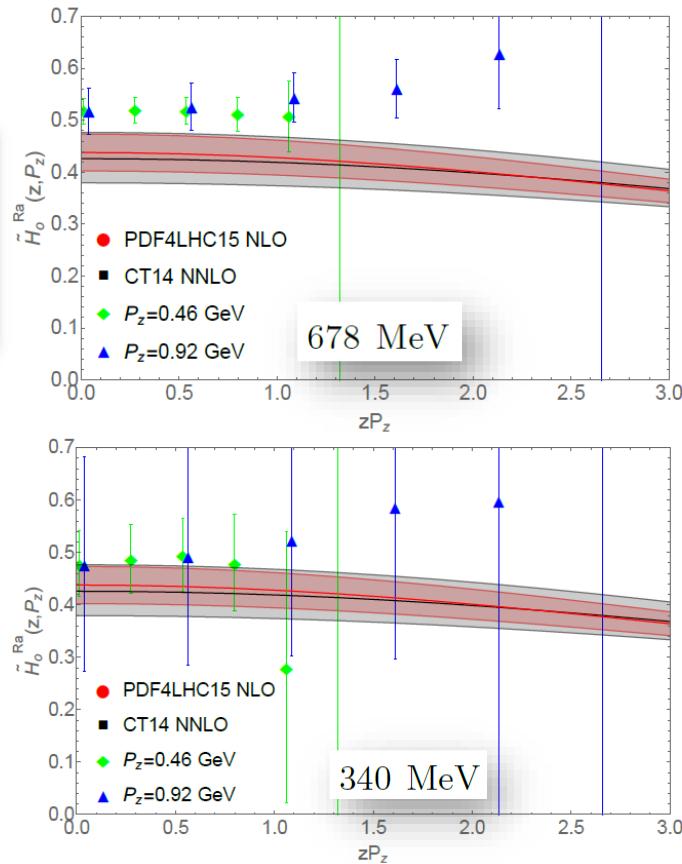


G: Zhouyou Fan

G: Adam Antony



P: Yi-Bo Yang



Gluon PDF in Nucleon

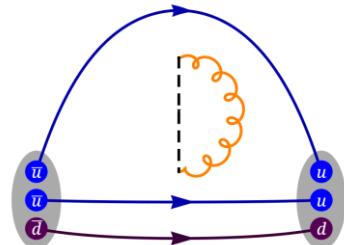
§ Gluon PDF using pseudo-PDF

❖ Lattice details: clover/2+1+1 HISQ 0.12 fm,

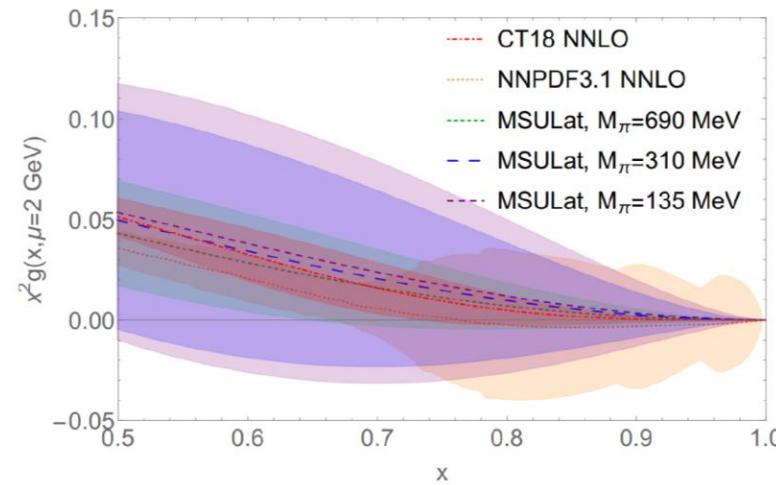
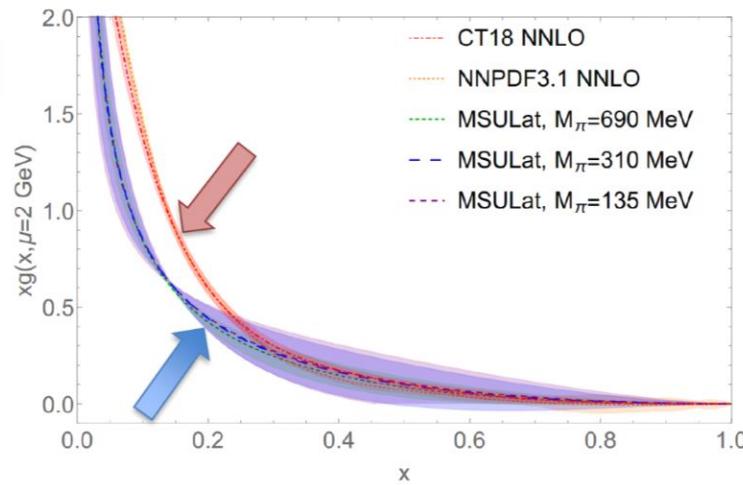
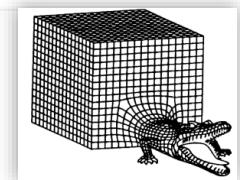
310-MeV sea pion

❖ Study strange/light-quark

Z. Fan. et al (MSULat),
2007.16113



The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2 \text{ GeV}$ in the $\overline{\text{MS}}$ scheme.



Slide by Zhouyou Fan@DNP2020

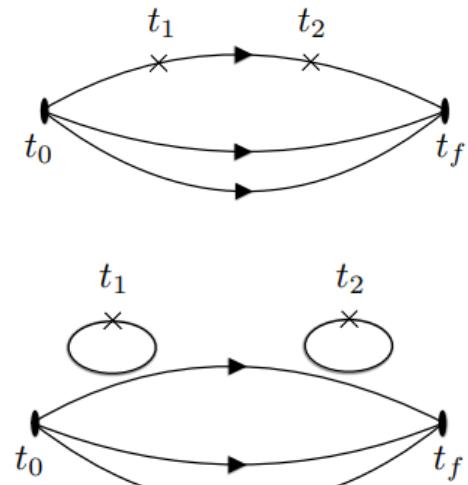
Zhouyou Fan
(MSU)



Lattice Parton Method

§ Short-distance factorization (SDF)

- ❖ Pseudo-PDF method (A. Radyushkin, 2017)
- ❖ Hadronic tensor currents
(Liu et al., hep-ph/9806491, ... 1603.07352)
- ❖ Lattice cross-section method (LCS)
(Y Ma and J. Qiu, 2014, 2017)
- ❖ Euclidean correlation functions
(RQCD, 1709.04325)
- ❖ Compton amplitude approach (QCDSF, 1703.01153)



Lattice Parton Method

§ Differences and similarity



- ❖ Large momentum is needed in the lattice calculations in all methods to reach small- x region
 - ❖ Current projects focus on $x \in [0.3, 0.8]$ (for 2-GeV boosted hadron)
- ❖ Kernel is a complicated object;
mostly current calculations used up to one-loop level
- ❖ SDF suffers Inverse problem to extract the wanted distribution
- ❖ LaMET requires to reach large Wilson-line displacement