Intrinsic Charged-Current DIS at NNLO QCD

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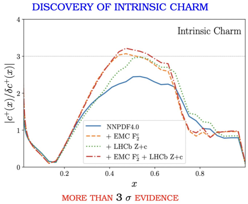
- Towards 1% PDFs theoretical uncertainties [Juan Rojo, Monday]
- Global fit $\rightarrow$ methodology matters [Roy Stegeman, Tuesday]
- DIS module `YADSIM` [Felix Hekhorn, Wednesday; talks by Jun Gao]
- Global fit $\rightarrow$ DIS module $\rightarrow$ VFNS (ACOT, FONLL,etc) $\rightarrow$ heavy flavor mass effects
- Heavy flavor mass effects $\rightarrow$ Collins, ACOT, Forte, Ball, etc.
- Most of the structure functions are computed at α_s^2
- Not all matching coefficients are computed at α_s^2

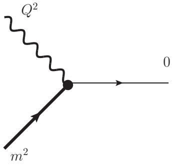


Charm in the Proton [Giacomo Magni, poster]

Intrinsic Structure Functions and Matching Coefficients

Structure functions of DIS of virtual W^- boson on quark c with mass m producing massless quark s **do not exist in the literature**.

$$W^-(p_1) + c(p_2) \to s(p_3) + X,$$
$$p_1^2 = Q^2,\ p_2^2 = m^2,\ p_3^2 = 0.$$



Reference tree-level partonic diagram of the process we study.

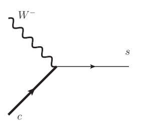
Since, the computations are **fully analytic**, these structure functions can be used to derive at the moment **unknown intrinsic matching functions/coefficients** [talk: YADISM, Felix Hekhorn], [R. Ball, M. Bonvini, L. Rottoli, 2015;S. Forte, E. Laenen, P. Nason and J. Rojo, 2010]

Structure Functions: Standard Formulae

Massive Factorization [J.C. Collins,1998]

$$\Omega^{\mu\nu} = \int \frac{\mathrm{d}\xi}{\xi} \omega^{\mu\nu} Q(\xi, \mu^2) \Big|_{p_+ = \xi P_+}$$

DIS of virtual W^- boson on quark c with mass m producing massless quark s.



Tensor decomposition and normalization [S. Kretzer, I. Schienbein,1998]

$$\begin{aligned}\omega_X^{\mu\nu} &= -\omega_{1,X}^Q g^{\mu\nu} + \omega_{2,X}^Q p^\mu p^\nu \\ &+ i\omega_{3,X}^Q \varepsilon^{\mu\nu}_{\alpha\beta} p^\alpha q^\beta + \omega_{4,X}^Q q^\mu q^\nu \\ &+ \omega_{5,X}^Q \left(q^\mu p^\nu + q^\nu p^\mu\right),\end{aligned}$$

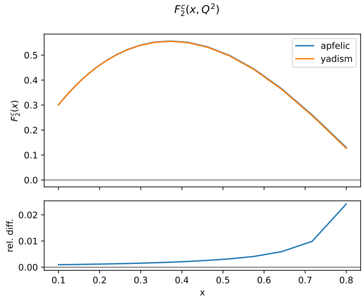
$$F_i^X = \int \frac{\mathrm{d}\xi}{\xi} f_i Q(\xi) \int \mathrm{d}\Pi_{X-\mathrm{LIPS}} \omega_{i,X}^Q$$

f_i is normalization from Ref. [S. Kretzer, I. Schienbein,1998]

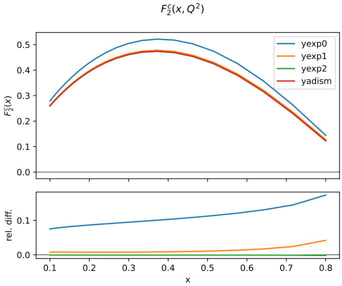
$$F_i = F_{i,0} + F_{i,\mathrm{NLO}} + F_{i,\mathrm{NNLO}} + \mathcal{O}(\alpha_s^3)$$

$$F_{i,0} = \mathrm{tree},\, F_{i,\mathrm{NLO}} = F_i^{\mathrm{R}} + F_i^{\mathrm{V}},\, F_{i,\mathrm{NNLO}} = F_i^{\mathrm{RR}} + F_i^{\mathrm{RV}} + F_i^{\mathrm{VV}}$$

Structure Functions @ NLO



Comparison of the structure function F_2 as implemented in APFEL and YADISM. Regards Alex & Felix



Expanding in a small parameter. "Intrinsic" contribution only. Regards Alex & Felix.

- F_2^c is defined in FONLL [Ball, Bonvini, Rottoli]
- dummy PDFs
- $Q^2 = 4\,\mathrm{GeV}^2$
- effect $2-5\%$ in the soft region
- Correct soft behavior

- $Q^2 = 10\,\mathrm{GeV}^2$
- charm-mass "effects" $\mathcal{O}(1\%)$.
- $\{F_i^{\mathrm{NLO}}\}$ are implemented in YADISM

Structure Functions @ NNLO: general remarks

$$F_{i,\mathrm{NNLO}} = F_i^{\mathrm{RR}} + F_i^{\mathrm{RV}} + F_i^{\mathrm{VV}}$$

$$F_i^X = \int \frac{\mathrm{d}\xi}{\xi} f_i Q(\xi) \hat{F}(\xi, \ldots),$$

$$\hat{F}_{i,X}(\xi, \ldots) = \int \mathrm{d}\Pi_{X-\mathrm{LIPS}} \omega_{i,X}^Q$$

We use the "canonical" method to evaluate NNLO corrections, i.e.

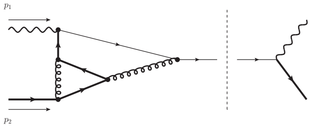
- Feynman Amplitudes $\mathcal{M}$ are generated in `QGRAF` [P. Nogueira,1993]
- All possible "algebras" (Clifford, Color,etc.) are implemented in `FORM` [J. A. M. Vermaseren , 2000] to compute $\omega_X^{\mu\nu}$.
- We written a procedure to handle traces that involve γ_5 based on [D. Kreimer, 1989; Körner, Kreimer and Schilcher, 1991.]
- $\hat{F}_{i,X}$ takes the form

  $$\hat{F}_{i,X} = \sum_k C_k(\{s_{ij}\}; d = 4 - 2\epsilon) \cdot I_k,$$

  where C_k is a rational coefficient and I_k is scalar Feynman integral.
- IBPs $0 = \int \mathrm{d}^d p \frac{\partial}{\partial p} f(p, \ldots)$ [K. Chetyrkin,F.V. Tkachov, 1981]; we use `REDUZE2` [Manteuffel, Studerus,2012]

Structure Functions @ NNLO: Pure Virtual Case

	fam1	fam2
D_1	$(L_1 - p_2)^2$	L_2^2
D_2	$(L_2 + p_1)^2$	$(L_1 - p_2)^2$
D_3	$L_1^2 - m^2$	$(L_2 + p_1)^2$
D_4	$(L_1 + p_1)^2$	$L_1^2 - m^2$
D_5	$(L_2 - p_2)^2$	$(L_1 + p_1)^2$
D_6	$(L_1 - L_2)^2$	$(L_2 - p_2)^2 - m^2$
D_7	$(L_1 - L_2 - p_1)^2 - m^2$	$(L_1 - L_2)^2 - m^2$



$$s = (p_1 + p_2)^2,\, p_1^2 = Q^2,\, p_2^2 = m^2$$

- Dimension regularization $d \to 4 - 2\epsilon$
- IBPs $\to$ 18 master integrals
- one-scale problem $y = \frac{m^2}{-Q^2}$
- "Finite" integrals

$$I_{\mathrm{VV}}(a_1, a_2, \ldots, a_7; \{y, \epsilon\}) = \int \frac{\mathrm{d}^d L_1}{\mathrm{i}(\pi)^{d/2}} \frac{\mathrm{d}^d L_2}{\mathrm{i}(\pi)^{d/2}} \frac{1}{D_1^{a_1} \cdot D_2^{a_2} \cdots D_7^{a_7}}$$

Structure Functions @ NNLO: Finite Integrals

Dimensional recurrence relations [O.V.Tarasov,1997]

$$I_{i,\mathrm{VV}}^{d-2}(y;\epsilon) = \sum_k B_k(y;\epsilon) I_{k,\mathrm{VV}}^{d}(y;\epsilon),$$

Finite Integrals [E. Panzer, 2014; Manteuffel, Panzer, Schabinger, 2014]

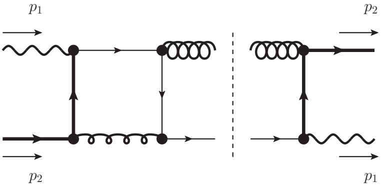
- Rising powers (dots) of propagators $\rightarrow$ "remove" UV-ϵ divergences
- Shifting dimensions to higher ones $\rightarrow$ "remove" IR-ϵ divergences
- A "proper" choice of dots and dim. shifts $\rightarrow$ a `finite` integral $\epsilon \rightarrow 0$.

Linearly reducible integrals with `HyperInt` [E. Panzer,2014]

$$I^{\text{finite}}(y;\epsilon) \propto \int_0^\infty \mathrm{d}x_1 \ldots \int_0^\infty \mathrm{d}x_{\mathrm{N}} \delta\left(\sum x_k - 1\right) \prod_k x_k^{a_k - 1} U^{a-3/2d} F^{d-2}$$
$$= C_0(y) + \epsilon C_1(y) + \epsilon^2 C_2(y) + \ldots,$$

$\{C_i\}$ are linearly reducible $\rightarrow$ can be expressed in terms of *multiple polylogarithm* (discussed later).

Real-Virtual Corrections: Preliminaries



An example of a real-virtual diagram.

Kinematic invariants

$$s = (p_1 + p_2)^2,$$
$$p_1^2 = Q^2 < 0,$$
$$p_2^2 = m^2.$$

Preliminary dimensionless variables

$$x = \frac{s}{-Q^2} \geq 0,\ y = \frac{m^2}{-Q^2} \geq 0.$$

Reverse unitarity [Anastasiou,Melnikov]

$$I = \int \frac{\mathrm{d}^d L_1 \mathrm{d}^d L_2}{\mathrm{i}\pi^{d/2}} \frac{\delta^+ \left[L_2^2\right] \delta^+ \left[(p_1 + p_2 - L_2)^2\right]}{D_1^{a_1} D_2^{a_2} D_3^{a_3} D_4^{a_4} D_5^{a_4}}$$

Cutkosky rules

$$2i\pi\delta\left(p^2 - m^2\right) \to \frac{1}{p^2 - m^2 + i0} - \frac{1}{p^2 - m^2 - i0}$$

we can treat RV integrals in the *same* way as pure virtual ones (e.g. find IBP identities)

Real-Virtual Corrections: Preliminaries

	fam1	fam2	fam3
$D_{1,c}$	L_2^2	L_2^2	L_2^2
$D_{2,c}$	$(p_1+p_2-L_2)^2$	$(p_1+p_2-L_2)^2$	$(p_1+p_2-L_2)^2$
D_3	$(L_1+p_1)^2$	$(L_1+p_1)^2$	L_1^2
D_4	$(L1-p_2)$	$(L1-p_2)$	$(L_2-p_2)^2-m^2$
D_5	$L_1^2-m^2$	$L_1^2-m^2$	$(L_1+p_1)^2-m^2$
D_6	$(L_2-p_2)^2-m^2$	$(L_2-p_2)^2-m^2$	$(L_1-L2+p_1)^2-m^2$
D_7	$(L_1+L_2-p_2)^2$	$(L_1-L_2+p_1)^2$	$(L_1-L_2+p_1+p^2)^2$

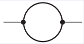
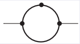
- 21 master integrals
- 2-variables problem

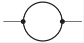
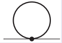
We solve RV and RR integrals with differential equations method. However, we consider only RV integrals in this talk.

Method of Differential Equasions

[A.V. Kotikov,1991]

Example: massive bubble. Dimensionless variable here $\mu = \frac{m^2}{p^2}$

∂_μ  $=$  $\stackrel{\text{IBP}}{=}$

$-\frac{2\varepsilon - 1}{2(\mu+1)} \cdot$  $+ \frac{2(\varepsilon-1)}{\mu(\mu+1)} \cdot$ 

Canonical system of differential equations [J.M. Henn, 2013]

In our case we have two systems of 21 diff.eqs. each.

$$\partial_x \vec{j} = \hat{M}_x(\{x,y\};\epsilon)\cdot\vec{j}$$
$$\partial_y \vec{j} = \hat{M}_y(\{x,y\};\epsilon)\cdot\vec{j}$$

Bringing diff. eqs. systems to canonical form

$$\vec{j} = \hat{T}\vec{J}, \quad \epsilon\hat{S}_x = \hat{T}^{-1}(\hat{M}_x\hat{T} + \partial_x\hat{T}) \to \partial_x\vec{J} = \epsilon S_x(\{x,y\})\vec{J}$$

An algorithm to find transformation $\hat{T}$ was proposed by Roman Lee, and it was implemented in various programs. For reference, we use package `LIBRA` [R. Lee, 2021]

Iterated Integrals

Consider a simple example

$$\partial_x \vec{J} = \frac{\epsilon \hat{A}}{x-1} \cdot \vec{J},$$

where A is some upper-triangular rational matrix. Choosing some parametrization, i.e. $\gamma : [0,1] \to M : x \in M$, we can rewrite diff.eqs in Pfaffian form

$$\mathbf{d}\vec{J} = \epsilon \hat{A}\, \mathbf{d}\log(W) \cdot \vec{J},$$

$$\gamma^{-1}(\mathbf{d}\log(W)) = \mathrm{d}t \frac{\mathrm{d}\log(f(t)-1)}{\mathrm{d}t}$$

where γ^{-1} is pull-back of one form $\mathbf{d}\log(W)$.
The solution of Pfaffian system $\to$ Picard-iteration $\to$ iterated integrals

$$\vec{J}(x) = T(x, x_0)\vec{J}(x_0),$$

$$T(x, x_0) = \hat{\mathcal{I}} + \sum_{n \geq 1} \int_{x_0 \leq t_1 \leq \ldots \leq t_n \leq x} B(t_n)B(t_{n-1})\ldots B(t_1)\mathrm{d}t_1 \ldots \mathrm{d}t_n,$$

where $B(t) = \epsilon \frac{\mathrm{d}\log(f(t)-1)}{\mathrm{d}t}$

Iterated Integrals and Uniformly Transcendental Form of Solutions

Iterated integrals with kernels of the type

$$B(t) \propto \frac{1}{t-a}$$

are well-known the literature! These are so-called Goncharovs (hyperlogarithms) polylogarithms (GPLs) [A.B. Goncharov, 2001]

$$G(a_1, \ldots, a_n; x) = \int_0^x \frac{\mathrm{d}t}{t-a_1} \circ \frac{\mathrm{d}t}{t-a_2} \circ \ldots \circ \frac{\mathrm{d}t}{t-a_n},$$

$$G(0_1, \ldots, 0_n; x) = \lim_{\varepsilon \to 0} \mathrm{Reg}_\varepsilon \int_\varepsilon^x \left(\frac{\mathrm{d}t}{t}\right)^{\circ n} = \frac{1}{n!} \log^n(x).$$

Goncharov polylogarithms (and iterated integrals), Riemann zeta functions, π constant all have a property called "transcendental weight" $w(f) = n : n \in \mathbb{Z}$

$$w(\pi) \to 1, w(G(a_1, a_2; x)) \to 2, w(\zeta(3)) \to 3, \text{etc.}$$

Uniformly transcendental (UT) functions

$$J_i = C_{i,0} + \epsilon C_{i,1} + \epsilon^2 C_{i,2} + \ldots,$$

where $w(\epsilon^n) = -n$, therefore UT-functions are function of **zero** transcendental weight.

Workflow

- Algebraic change of variables is needed to remove square roots

$$x \to \frac{1-\xi}{\xi}(1-\chi^2\xi)$$
$$y \to -\chi^2$$

- First solve ξ-equations asymptotically in the limit $\xi \to 1$ (soft limit) [R. Lee, A. Smirnov, V. Smirnov, 2017; KK, K. Melnikov, C. Wever, 2016]

$$j_i = \sum_{j,k,l} c_{i,j,k,l}(\chi)(1-\xi)^{j-k\epsilon}\log^l(1-\xi) + \mathcal{O}((1-\xi))$$

- We use asymptotic solutions to fix boundary conditions of *exact* ξ differential equations in the $\xi \to 1 \& \chi \to 0$ limits.
- We find all boundary conditions by means of known methods, e.g. method of regions [Beneke, Smirnov, Jantzen], and Mellin-Barnes expansion [Boos, Davydychev,Tausk, Smirnov, Czakon]
- all boundary conditions are brought to UT form with known methods.

Workflow

- Thanks to LIBRA, we find transformation $\hat{T}$ and obtain ϵ-form of differential equations.
- After many judicious transformations we obtain

$$\mathbf{d}\vec{J} = \epsilon \sum_{k=1}^{6} \hat{B}_k \mathbf{d} \log(W_k) \cdot \vec{J}$$

- Our "alphabet" consist of following "letters"

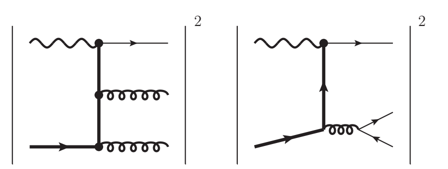
$$\{W_k\} = \{\xi, 1-\xi, 1-\chi^2\xi^2, 1-\chi^2\xi, 1+\chi^2(-2+\xi)\xi, 1+\chi^2(\xi-1)\xi\}$$

Iterated Integrals instead of Goncharov's polylogarithms

Remember that integration kernels has a particular form, i.e. $\frac{1}{t-1}$ Our kernels are not of this form. We can force such a form by rationalizing some algebraic "letters".
We avoid this by using instead a formal definition of iterated integrals with general "letters" [Badger, Hartanto, et al., 2021]! Finally, iterated integrals can be evaluated in GiNaC [Walden, Weinzierl, 2021].
This way, we integrate our integrals up to $\mathcal{O}(\epsilon^6)$.

A Few Words About Double-Real Corrections

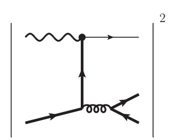
Double-Real: massless final states



$$I = \int \mathrm{d}^d L_1 \mathrm{d}^d L_2 \delta^+ \left[(P - L_1 - L_2)^2 \right] \times \frac{\delta^+ \left[L_1^2 \right] \delta^+ \left[L_2^2 \right]}{D_1^{a_1} D_2^{a_2} D_3^{a_3} D_4^{a_4}}$$

All 24 integrals are formally done!

Double-Real: massive final states



$$I = \int \mathrm{d}^d L_1 \mathrm{d}^d L_2 \delta^+ \left[L_2^2 - m^2 \right] \times \frac{\delta^+ \left[L_1^2 - m^2 \right] \delta^+ \left[(P - L_1 - L_2)^2 \right]}{D_1^{a_1} D_2^{a_2} D_3^{a_3} D_4^{a_4}}$$

There are yet 12 master integrals to compute.

Instead of Conclusions

- We report our progress on the calculations of next-to-next-to leading order correction to intrinsic structure functions
- We computed pure virtual, real-virtual, and partially double-real corrections.

"wish list"

- The last missing contribution to charm structure functions
- This ingredient can be used to derive the last PDF matching coefficient at NNLO.
- It will be nice to perform a comparison of our FONLL against full ACOT at NNLO.