Proton spin, topology and confinement: lessons from QCD_2.

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Outline

- QCD_2 and the Sine-Gordon model
- Constructing a baryon state in QCD_2
- Chirality distributions inside topological baryons and non-topological "mesons" in QCD_2
- Outlook

Motivation

Operator product expansion:

$$J^\mu J^\nu \underset{Q^2\to\infty}{\sim} 2\epsilon^{\mu\nu\lambda\rho}\frac{p_\lambda}{Q^2}\left[C^{NS}\left(j^3_{\rho 5}+\frac{1}{\sqrt{3}}j^8_{\rho 5}\right)+\frac{2\sqrt{2}}{\sqrt{3}}C^S j^0_{\rho 5}\right],$$

$j^a_{\mu 5}=\bar{q}\gamma_\mu\gamma_5 T^a q$ are the QCD axial currents.

In parton model

$$\int_0^1 \mathrm{d}x_B g_1(x_B)=\frac{1}{2}\left[\frac{4}{9}\Delta u+\frac{1}{9}\Delta d+\frac{1}{9}\Delta s\right].$$

So we are interested in

$$2Ms_\mu g_A^{(0)}=\langle \mathrm{p},s|j^0_{\mu 5}|\mathrm{p},s\rangle.$$

QCD_2 in the 't Hooft limit

$$\mathcal{L}_{QCD_2} = -\frac{1}{4}\mathsf{Tr}(F_{\mu\nu}F^{\mu\nu}) + i\bar{q}\gamma^{\mu}(\partial_{\mu} - igA_{\mu})q - m\bar{q}q$$

't Hooft limit: $N \to \infty, \quad g \to 0, \quad N \cdot g^2 \equiv \lambda = const$.

Similarities to QCD_4:

- Confinement: mesons [1] and baryons [2] in the spectrum
- Mass gap generation
- Chiral symmetry breaking

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Leading order in $\frac{1}{N}$ after bosonization is the Sine-Gordon model [2]:

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)^2 - m'^2 \cos\left(\frac{\phi}{f}\right) \ ,$$

where $f = \sqrt{\frac{N}{4\pi}}$, $m' = \sqrt{\frac{N}{\sqrt{\pi}}mg}$.

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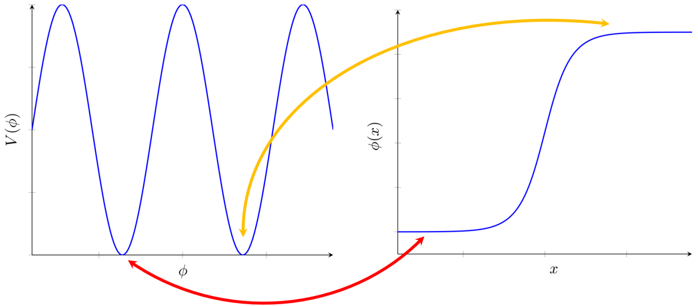
Topological soliton in the Sine-Gordon model



A static soliton "kink" solution of the classical equation of motion in the Sine-Gordon model:

$$\phi_c(x) = \sqrt{\frac{4N}{\pi}} \arctan e^{\sqrt{\frac{4\pi}{N}} m' x}.$$

The mass of the kink is (from the classical Hamiltonian):

$$M_{kink} = 4\sqrt{\frac{N}{\pi}} m'.$$

Baryon as a kink

Bosonization of the quark vector current

$$\bar{\psi}\gamma^{\mu}\psi = j^{\mu} = \sqrt{\frac{N}{\pi}}\epsilon^{\mu\nu}\partial_{\nu}\phi$$

gives the charge of a solution in the Sine-Gordon model

$$Q = \sqrt{\frac{N}{\pi}}\left[\phi(x \to \infty) - \phi(x \to -\infty)\right].$$

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We want to study chirality inside a baryon:

$$\langle kink|j^5_\mu|kink\rangle.$$

Axial current in 1+1:

$$j^5_\mu = \epsilon^{\mu\nu}j_\nu = \sqrt{\frac{N}{\pi}}\partial_\mu\phi.$$

Constructing the quantum soliton state

The defining property of a soliton state [1]:

$$\langle kink|\hat{\phi}(x)|kink\rangle = \phi_c(x).$$

Classical field profile and quantum field operator can be decomposed into Fourier components:

$$\phi_c(x) = \int \frac{dk}{2\pi}\frac{1}{\sqrt{2\omega(k)}}(\alpha_k e^{ikx} + \alpha_k^* e^{-ikx}),$$

$$\hat{\phi}(x) = \int \frac{dk}{2\pi}\frac{1}{\sqrt{2\omega(k)}}(a_k^{sol} e^{ikx} + a_k^{sol\dagger} e^{-ikx})$$

Coherent states: $a_k^{sol}|\alpha_k\rangle = \alpha_k|\alpha_k\rangle$. We define the kink state as

$$|kink\rangle = \bigotimes_k |\alpha_k\rangle,$$

$$\langle kink|a_k^{sol}|kink\rangle = \alpha_k$$

[1] G. Dvali, C. Gomez, L. Gruending, and T. Rug, Nucl. Phys. B 901, 338 (2015)

Topology and energy parts in solitonic constituents

$$\phi_c(x) = \int \frac{dk}{2\pi} \frac{1}{\sqrt{2\omega(k)}} (\alpha_k e^{ikx} + \alpha_k^* e^{-ikx}),$$

Fourier coefficients can be represented as a product of topological and non-topological contributions:

$$\alpha_k = t_k c_k \quad , \quad t_k = \frac{i\sqrt{\omega(k)}}{k} \quad , \quad c_k = -\sqrt{\frac{\pi N}{2}} \frac{1}{\cosh \sqrt{\frac{N}{4\pi}} \frac{\pi k}{2m'}}$$

This decomposition amounts to representing the kink as a convolution of topology and "energy" :

$$\sqrt{\frac{4N}{\pi}} \arctan e^{\sqrt{\frac{4\pi}{N}} m' x} = \sqrt{\frac{N}{\pi}} (\mathrm{sign} * \mathrm{sech}) \left(\sqrt{\frac{N}{4\pi}} m' x \right) + \frac{\pi}{2}$$

A baryon state on the light-cone

Boost into the Infinite Momentum Frame:

$$\sqrt{\frac{4N}{\pi}}\arctan e^{\sqrt{\frac{4\pi}{N}}m'\gamma(x+\beta t)} \xrightarrow[\beta\to 1]{} \sqrt{\frac{4N}{\pi}}\arctan e^{\sqrt{\frac{4\pi}{N}}m'\gamma\sqrt{2}x^+}.$$

The Bjorken x is $x_B = \dfrac{k_+}{p_+}$, $p_+ = \sqrt{2}\gamma M_p$.
The occupation number of constituents

$$N_{k_+} = |\alpha_{k_+}|^2 = \frac{\pi N}{\sqrt{2}x_B p_+}\frac{1}{\cosh^2(N x_B)}$$

has a logarithmic divergence at low x_B due to topology.

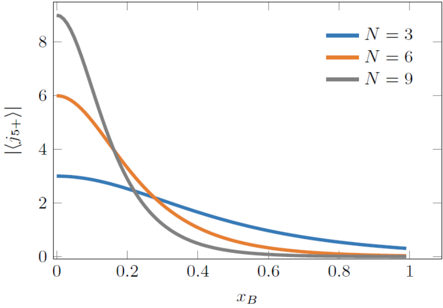
Chirality distribution inside a baryon

Axial current in bosonic language:

$$\bar{\psi}\gamma^{\mu}\gamma^{5}\psi = j_5^{\mu} = \partial^{\mu}\phi.$$

In the momentum space in terms of x_B:

$$\langle kink|j_{5+}|kink\rangle = N\frac{1}{\cosh\left[N x_B\right]}.$$



Non-topological contribution

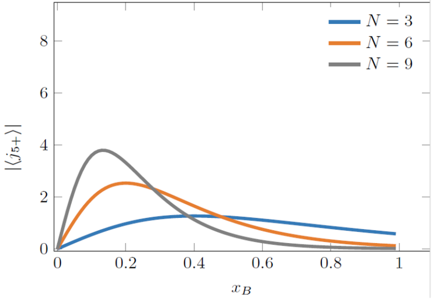
$$\sqrt{\frac{N}{\pi}}(\cancel{\text{sign}} * \text{sech})\left(\frac{\pi p_+ x^+}{2N}\right) \longrightarrow \sqrt{\frac{N}{\pi}}\text{sech}\left(\frac{\pi p_+ x^+}{2N}\right)$$

Eliminate topology be crossing out the sign function. This way we get a non-topological ("meson") state.

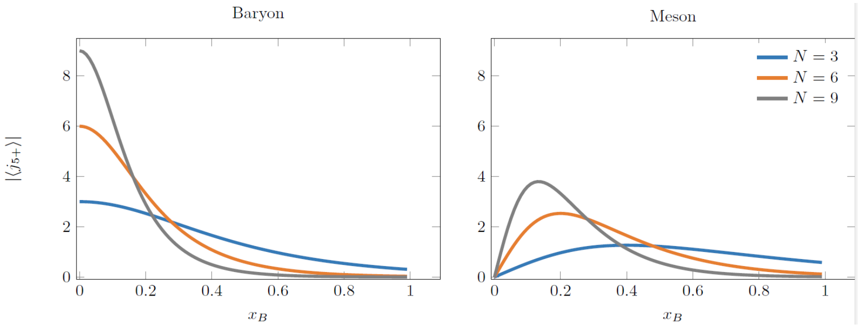
$$\langle non - top.|j_{5+}|non - top.\rangle$$

$$= \frac{2N^2 x_B i}{\pi \cosh(N x_B)}.$$

Chirality vanishes at $x_B \to 0$.



A close comparison of objects with and without topology



- Topology leads to an enhancement of chirality at small x_B
- For large N the $x_B \sim 1$ region is suppressed

Summary and conclusion

- QCD_2 in the 't Hooft limit reduces to the exactly soluble Sine-Gordon model.
- An exact non-perturbative baryon state can be constructed, so the chirality distribution inside a baryon is accessible.
- Topological structure leads to a strong chirality enhancement at small x_B. This enhancement disappears if topology is "switched off". This is an exact result in QCD_2.

This suggests a sharp difference in small-x_B behavior of polarized structure functions of baryons and mesons in QCD_4.