

# DIS2022

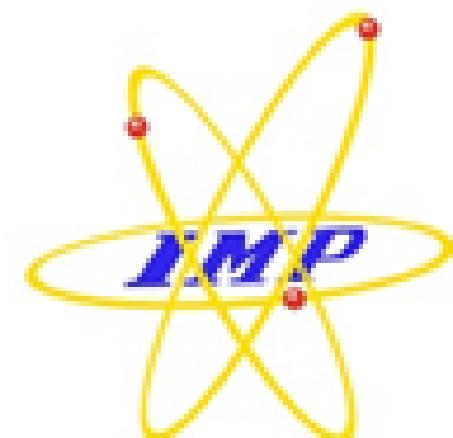
**XXIX International Workshop on Deep-Inelastic Scattering and Related Subjects**

Santiago de Compostela, 2–6 May 2022

## Electromagnetic structure of heavy baryons

(  $\Lambda(\Sigma^0, \Sigma^+, \Sigma^-)$  and  $\Lambda_c(\Sigma_c^+, \Sigma_c^{++}, \Sigma_c^0)$  )

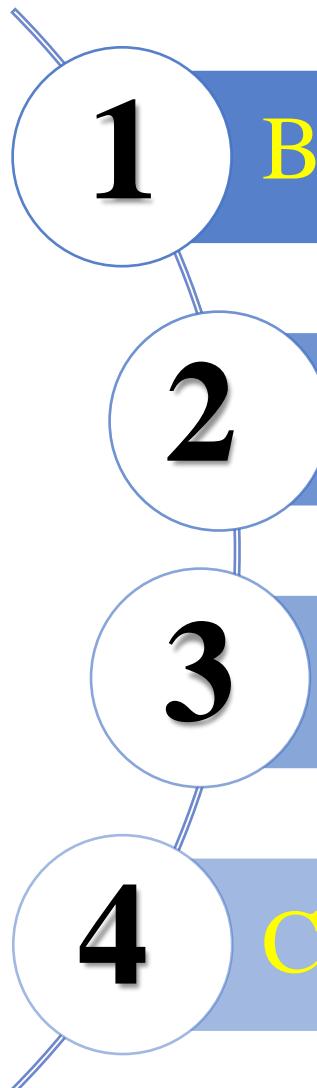
**Speaker:** Tian-cai Peng



Collaborators: Zhimin Zhu, Siqi Xu, Chandan Mondal, Xingbo Zhao and James P. Vary

May 5th , 2022

# Outline

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- 1** Basis Light-front quantization(BLFQ)
  - 2**  $\Lambda$  and  $\Lambda_c$  PFDs from BLFQ
  - 3**  $\Lambda$  and  $\Lambda_c$  Electromagnetic structure
  - 4** Conclusions

# Basis Light-front quantization (BLFQ)

# 1.Basis Light-front quantization

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
$x^1, x^2, x^3$	$x^- = x^0 - x^3, \quad x^\perp = x^{1,2}$
$P^0, \vec{P}$	$p^- = p^0 - p^3,$ $p^+ = p^0 + p^3,$ $p^\perp = p^{1,2}$
$i\frac{\partial}{\partial t}  \varphi(t)\rangle = H  \varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}  \varphi(x^+)\rangle = \frac{1}{2} P^-  \varphi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + p_\perp^2}{P^+}$

[Dirac, 1949]

1.Frame-independent wave function

2.No square root in Hamiltonian  $P^-$

# 1.Basis Light-front quantization

First, we solve the time-independent (at fixed light-front time) Schrodinger equation:

$$H_{\text{eff}}|\Phi\rangle = M^2 |\Phi\rangle$$

Here, we adopt an **effective light-front Hamiltonian** :

$$H_{\text{eff}} = H_{k.E} + H_{\text{trans}} + H_{\text{longi}} + H_{\text{int}}$$

The light-front wave function is obtained by solving light-front schrodinger equation and then expanding it to the basis vector of the multi-particle Fock sector. For example,

$$|\Lambda\rangle_{\text{phys}} = \color{red}{a}|uds\rangle + \color{green}{b}|udsg\rangle + \color{blue}{c}|udsq\bar{q}\rangle + \dots$$

$$|\Lambda c\rangle_{\text{phys}} = \color{red}{a}|udc\rangle + \color{green}{b}|udcg\rangle + \color{blue}{c}|udcq\bar{q}\rangle + \dots$$

For the Fock space, we can not involve all the Fock sector directly, so we adopt a truncation.

The current studying on  $\Lambda$  and  $\Lambda c$  only considers **valence Fock sector**.

# 1.Basis Light-front quantization

We represent the state of a parton by four quantum numbers ( $n, m, k, \lambda$ ).

The longitudinal basis vector are plane wave solutions:

$$\Psi_k(x^-) = \frac{1}{2L} e^{i \frac{\pi}{L} k x^-}$$

The transverse basis vector are two dimensional harmonic oscillator solutions:

$$\phi_{nm} = \frac{\sqrt{2}}{b(2\pi)^{\frac{3}{2}}} \sqrt{\frac{n!}{(|m| + n)!}} \left(\frac{P_\perp}{b}\right)^{|m|} e^{\frac{-P_\perp^2}{2b^2}} L_n^{|m|} \left(\frac{P_\perp^2}{b^2}\right) e^{im\varphi}$$

The many-body basis states have well defined values of the total angular momentum projection  $M_J = \sum_i (m_i + \lambda_i)$ .

In order to calculate, except the Fock space truncation, for each Fock sector, we also need other truncation to reduce the basis to a finite dimension.

In the longitudinal direction, We truncate the infinite basis by a truncation parameter  $K$ .

As well as, in the transverse direction, we require the total transverse quantum number.

$$P^+ = \frac{2\pi}{L} K \quad P^+ = \sum_i p_i^+$$

$$\sum_i (2n_i + |m_i| + 1) = N_\alpha \leq N_{Max}$$

## 2. $\Lambda$ (uds) and $\Lambda_c$ (udc) PDFs from BLFQ

# PDFs(Parton distribution functions)

$$\Phi^{\Gamma(q)}(x) = \frac{1}{2} \int \frac{dz^-}{4\pi} e^{ip^+ z^-/2} \times \langle P, \Lambda | \bar{\psi}_q(0) \Gamma \psi_q(z^-) | P, \Lambda \rangle \Big|_{z^+ = \vec{z}_\perp = 0}$$

With different Dirac structure  $\Gamma = \gamma^+, \gamma^+ \gamma^5, i\sigma^j \gamma^5,$

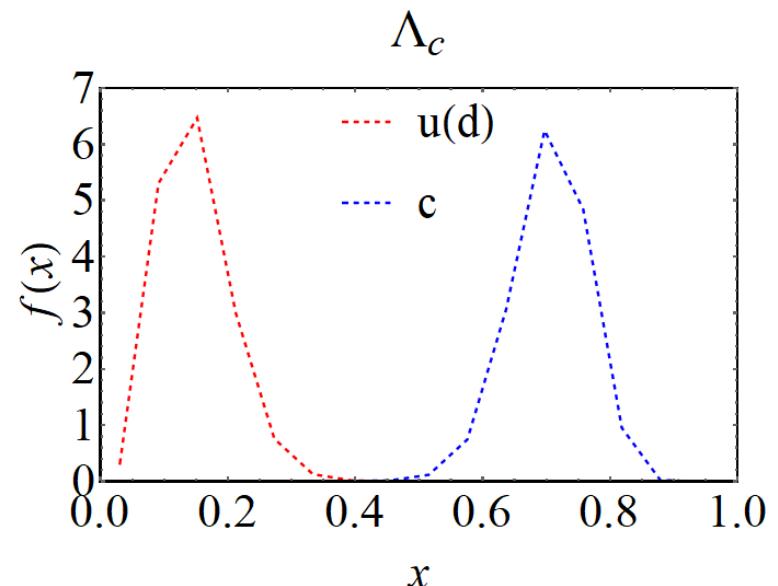
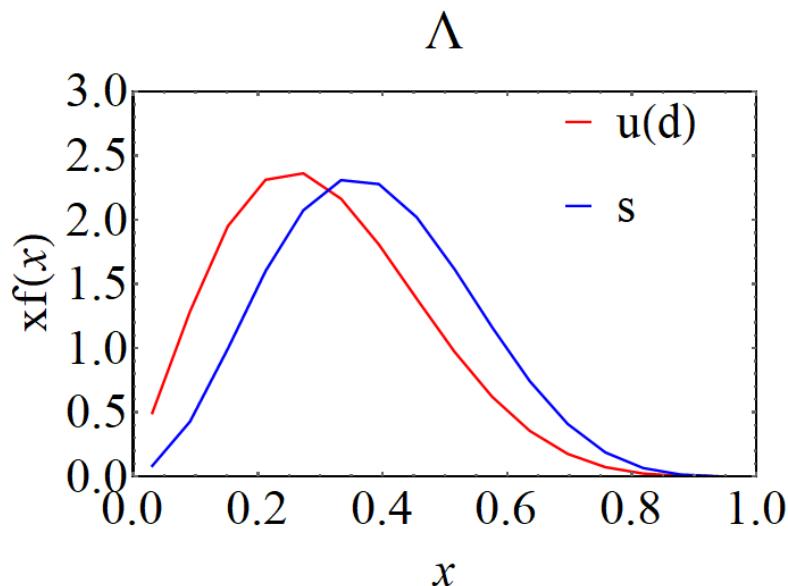
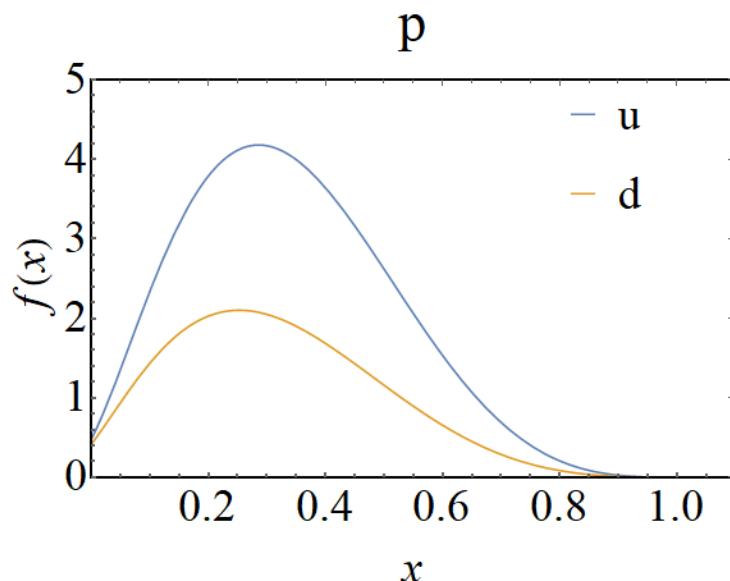
we can get Unpolarized ( $f_1$ ), Helicity( $g_1$ ) and Transversity ( $h_1$ ) PDFs.

Parameter set:

	$\alpha_s$	$s_k/s_{oge} [\text{GeV}]$	$l_k/l_{oge} [\text{GeV}]$	$K_L = K_T [\text{GeV}]$	Mass [GeV]
$\Lambda$	1.06	0.390/0.290	0.300/0.200	0.377	1.115
	$\alpha_s$	$c_k/c_{oge} [\text{GeV}]$	$l_k/l_{oge} [\text{GeV}]$	$K_L = K_T [\text{GeV}]$	Mass [GeV]
$\Lambda_c$	0.57	1.580/1.480	0.300/0.200	0.377	2.286

# Unpolarized PDFs at initial scale

$$f^q(x) = \sum_{\{\lambda_i\}} \int [d\chi d\mathcal{P}_\perp] \times \Psi_{\{x_i, \vec{p}_{i\perp}, \lambda_i\}}^{\uparrow*} \Psi_{\{x_i, \vec{p}_{i\perp}, \lambda_i\}}^{\uparrow} \delta(x - x_q)$$



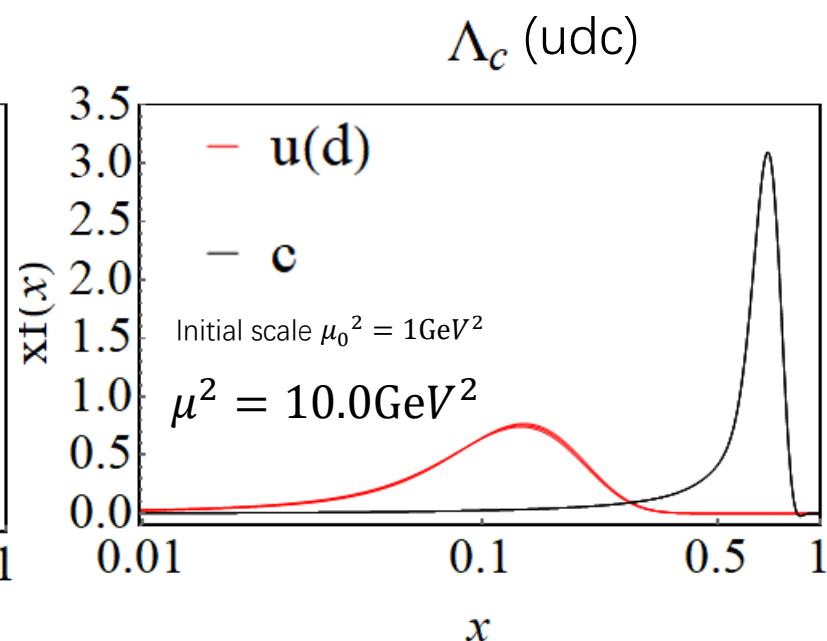
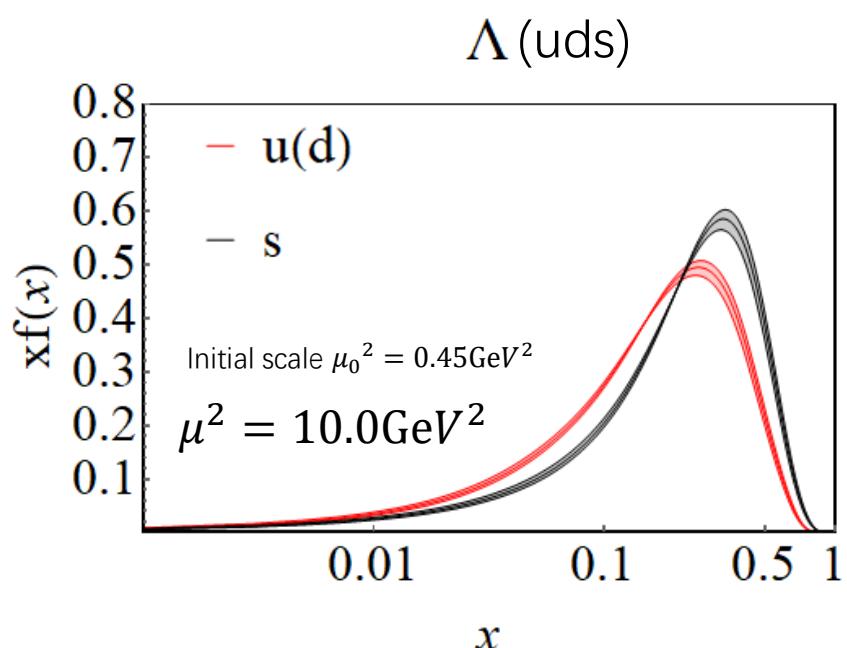
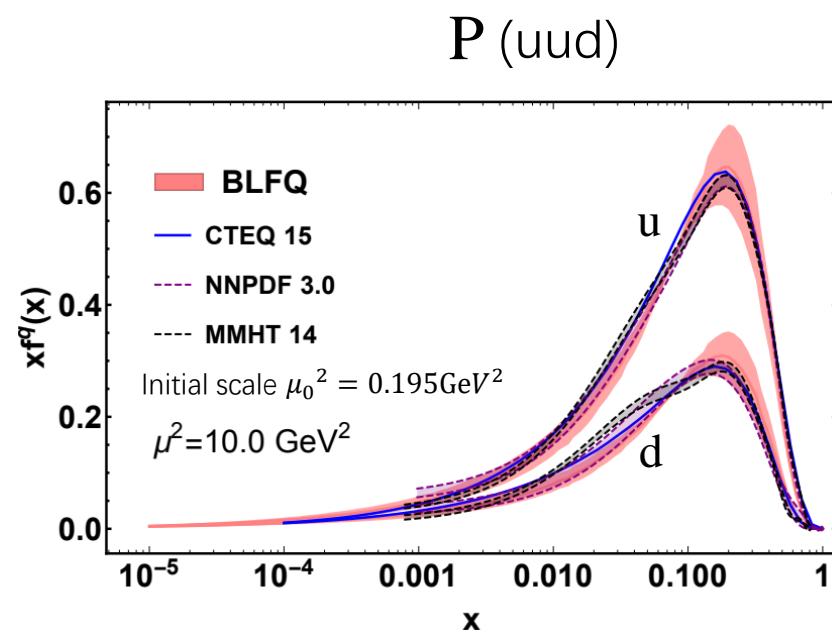
How to connect QCD quarks and gluons to the observed hadrons and leptons?

Here needs fundamentals of QCD factorization and **QCD evolution**.

# Unpolarized PDFs $xf_1$ at a higher scale compare with proton

Tiancai Peng et al.  
In preparing

With QCD evolution equation, we can get the partons distribution functions at a higher scale.



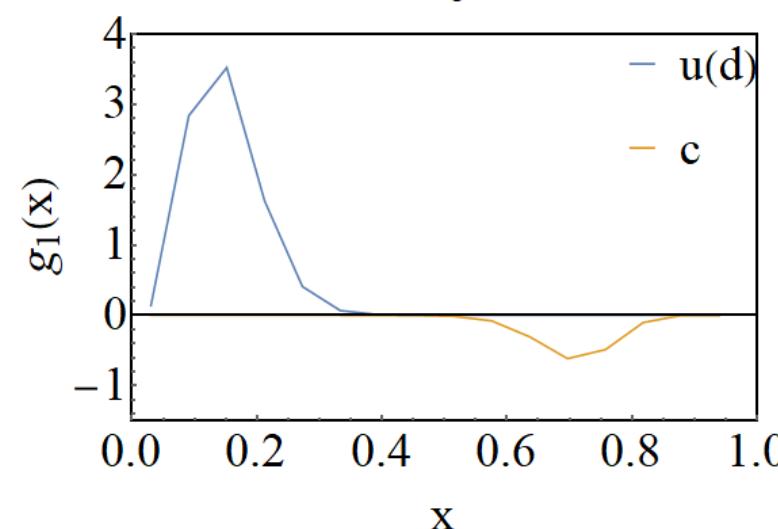
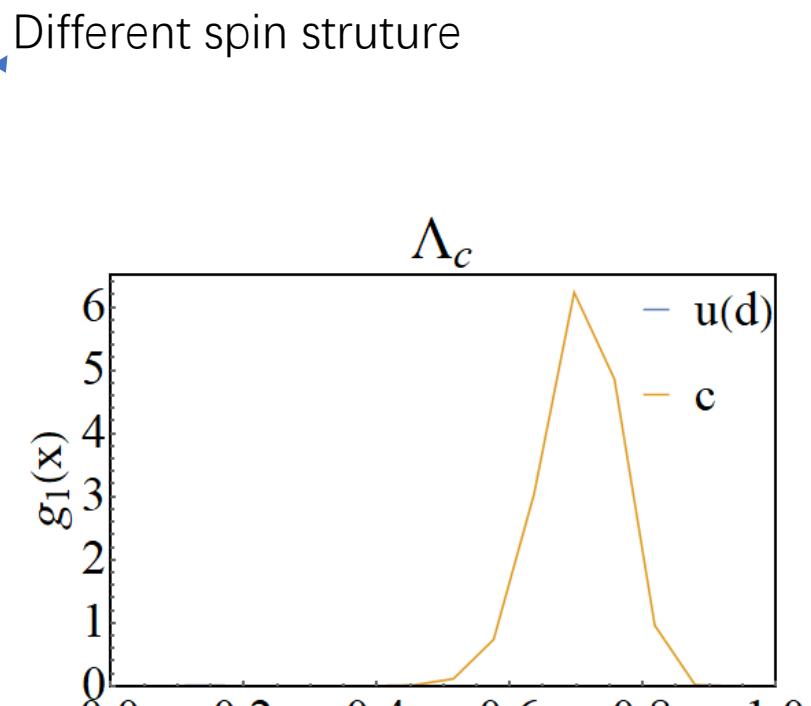
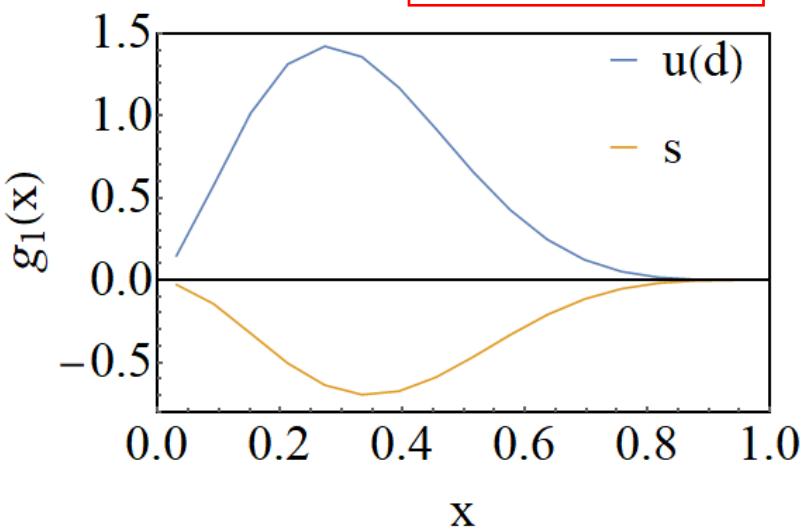
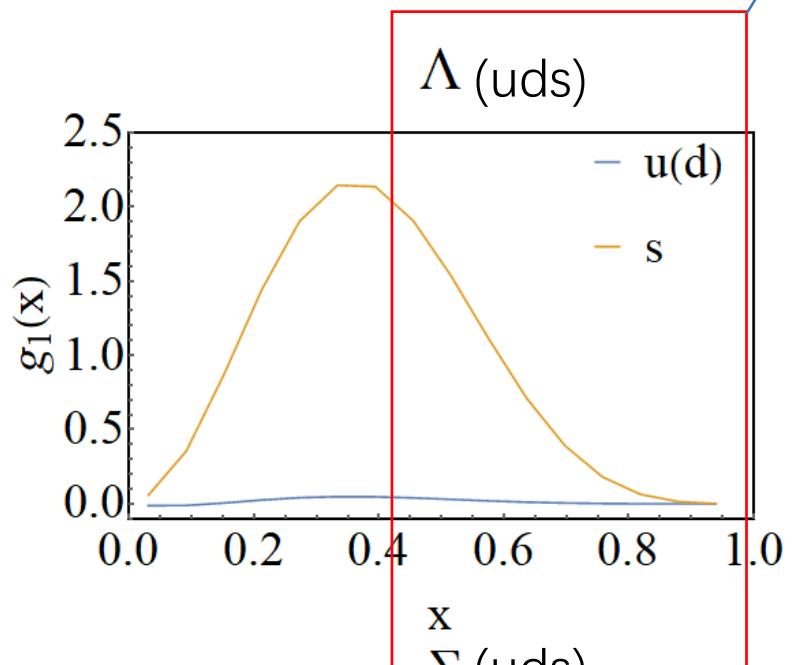
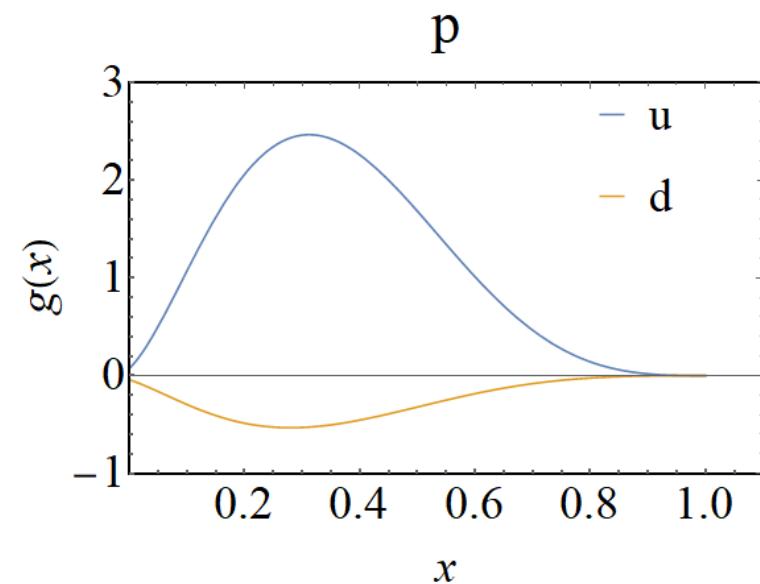
Phys. Rev. D 104, no.9, 094036 (2021)

Charm quark distribution peaks  
at  $x$  bigger than 0.5.

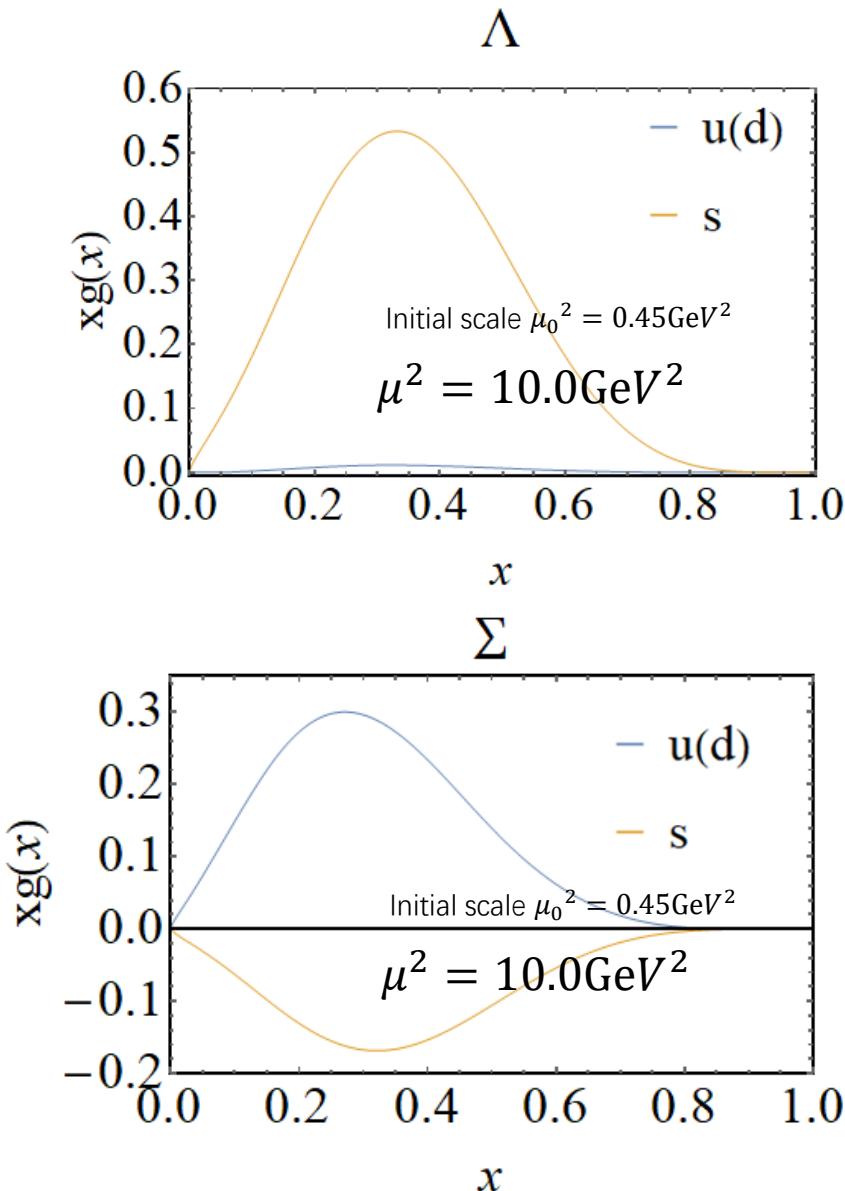
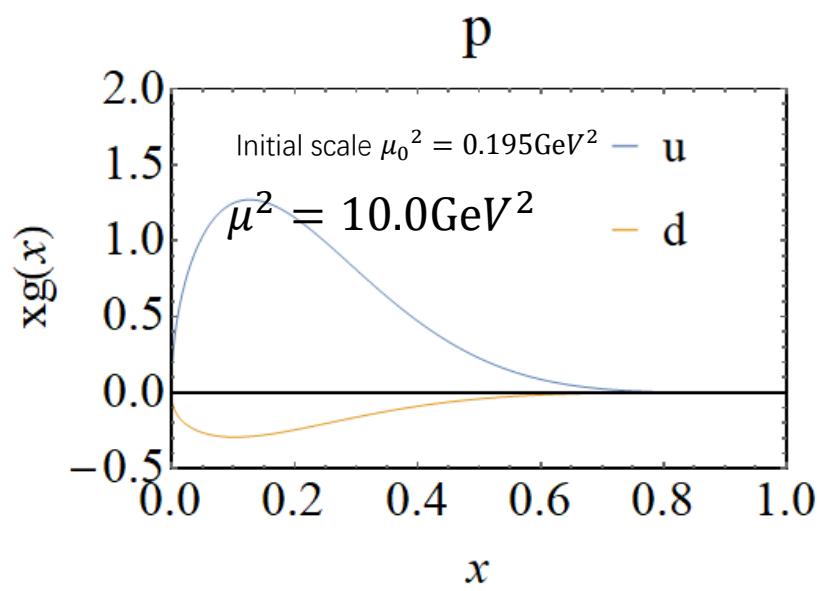
# Helicity PDFs at initial scale

$$g_1^q(x) = \sum_{\{\lambda_i\}} \int [d\chi d\mathcal{P}_\perp] \times \lambda_1 \Psi_{\{x_i, \vec{p}_{i\perp}, \lambda_i\}}^{\uparrow*} \Psi_{\{x_i, \vec{p}_{i\perp}, \lambda_i\}}^{\uparrow} \delta(x - x_1)$$

The struck quark helicity

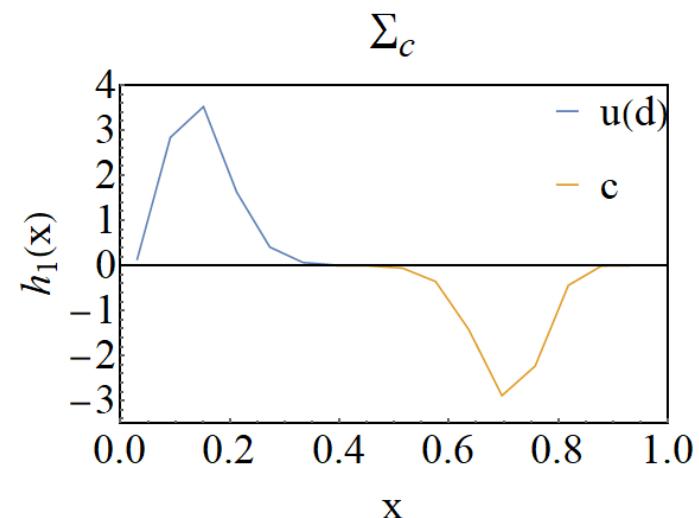
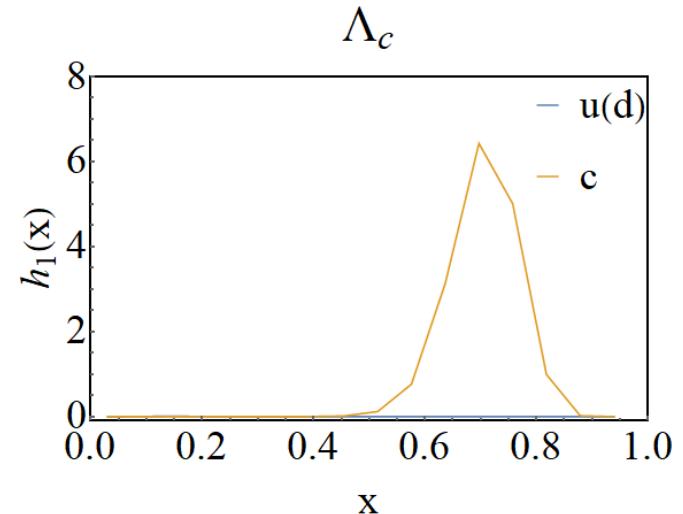
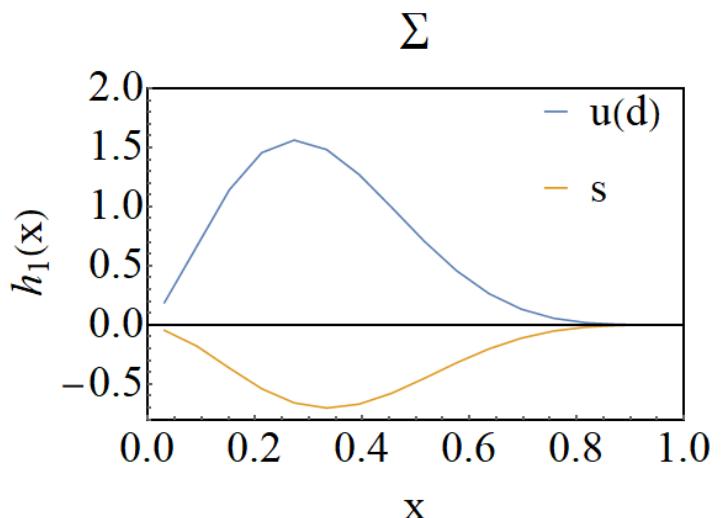
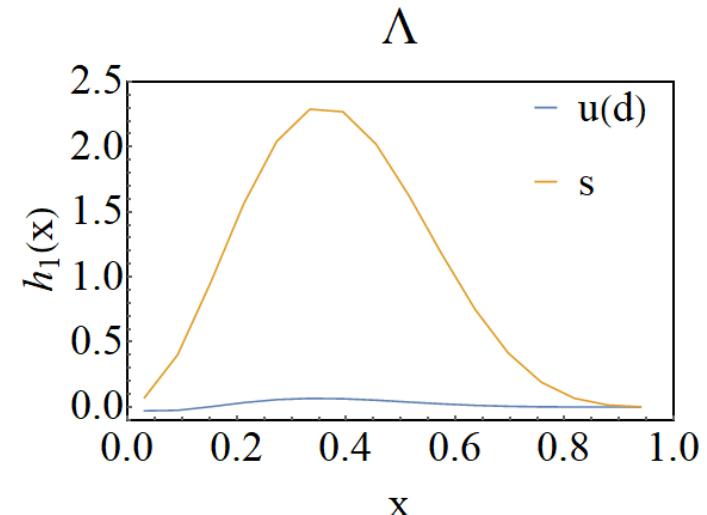
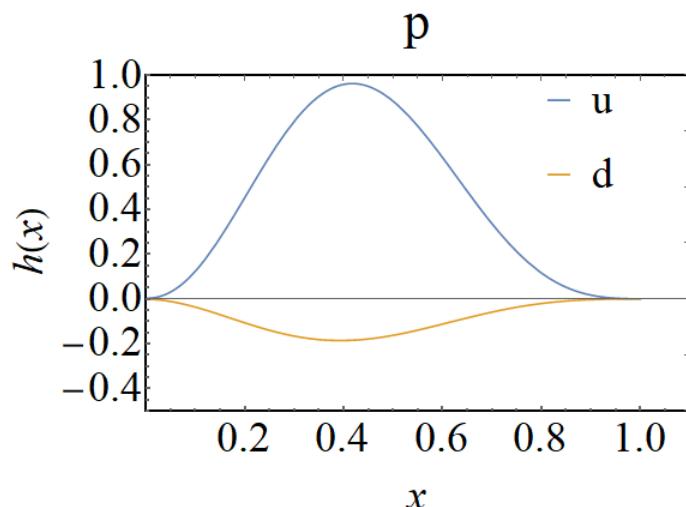


# Helicity PDFs at a higher scale

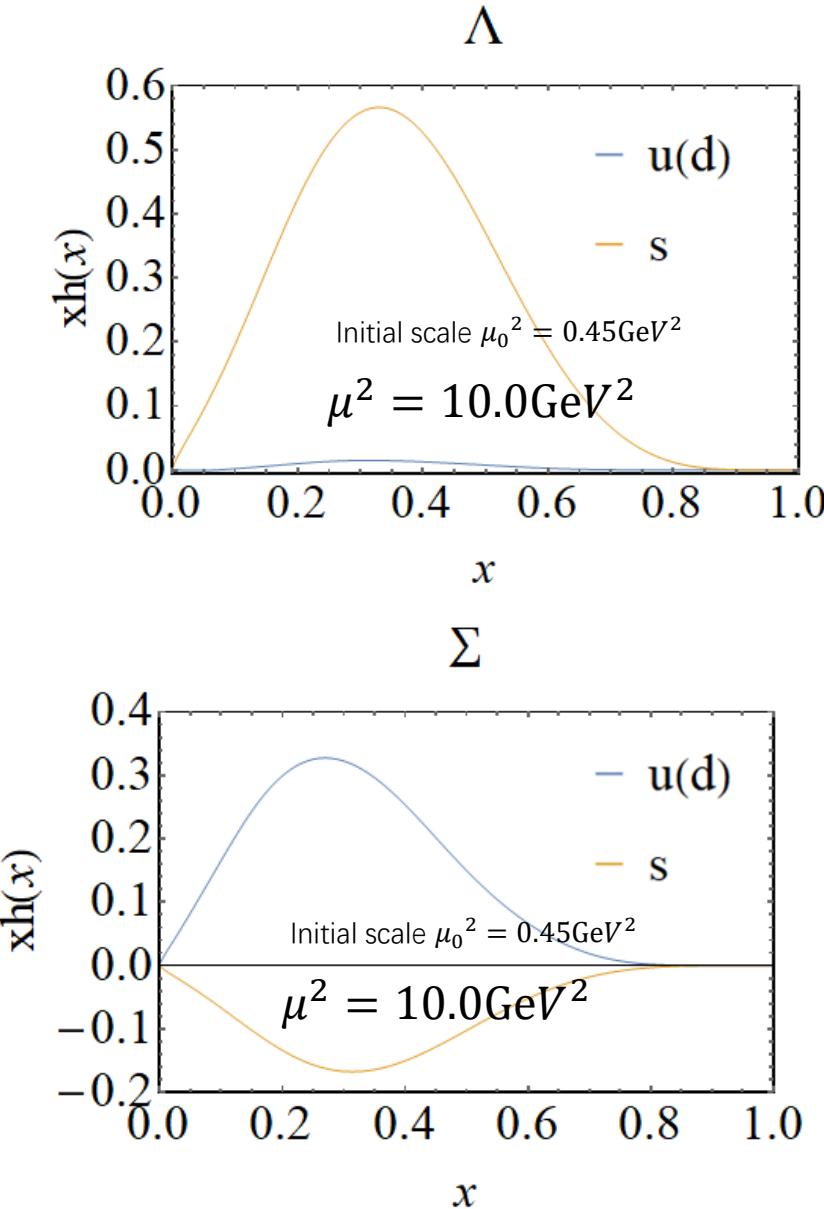
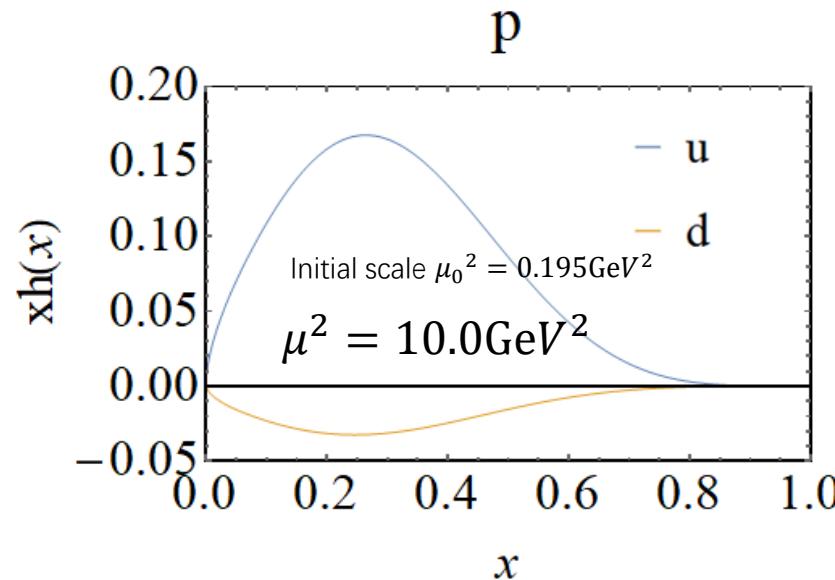


# Transverse PDFs $h_1$ at initial scale

$$h_1^q(x) = \sum_{\{\lambda_i\}} \int [d\chi \, d\mathcal{P}_\perp] \\ \times \left( \Psi_{\{x_i, \vec{p}_{i\perp}, \lambda'_i\}}^{\uparrow*} \Psi_{\{x_i, \vec{p}_{i\perp}, \lambda_i\}}^{\downarrow} + (\uparrow \leftrightarrow \downarrow) \right) \delta(x - x_1)$$

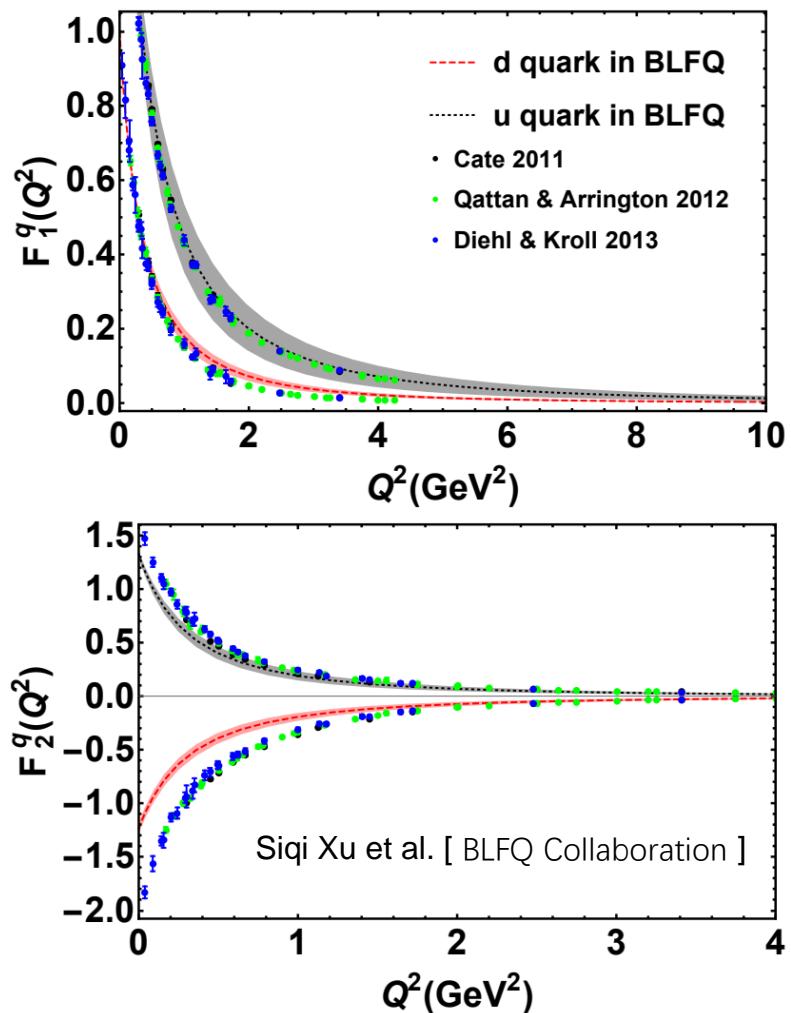


# Transverse PDFs $h_1$ at initial scale



### 3. $\Lambda(\text{uds})$ and $\Lambda\text{c}(\text{udc})$ Electromagnetic structure

# Flavor level Form factors

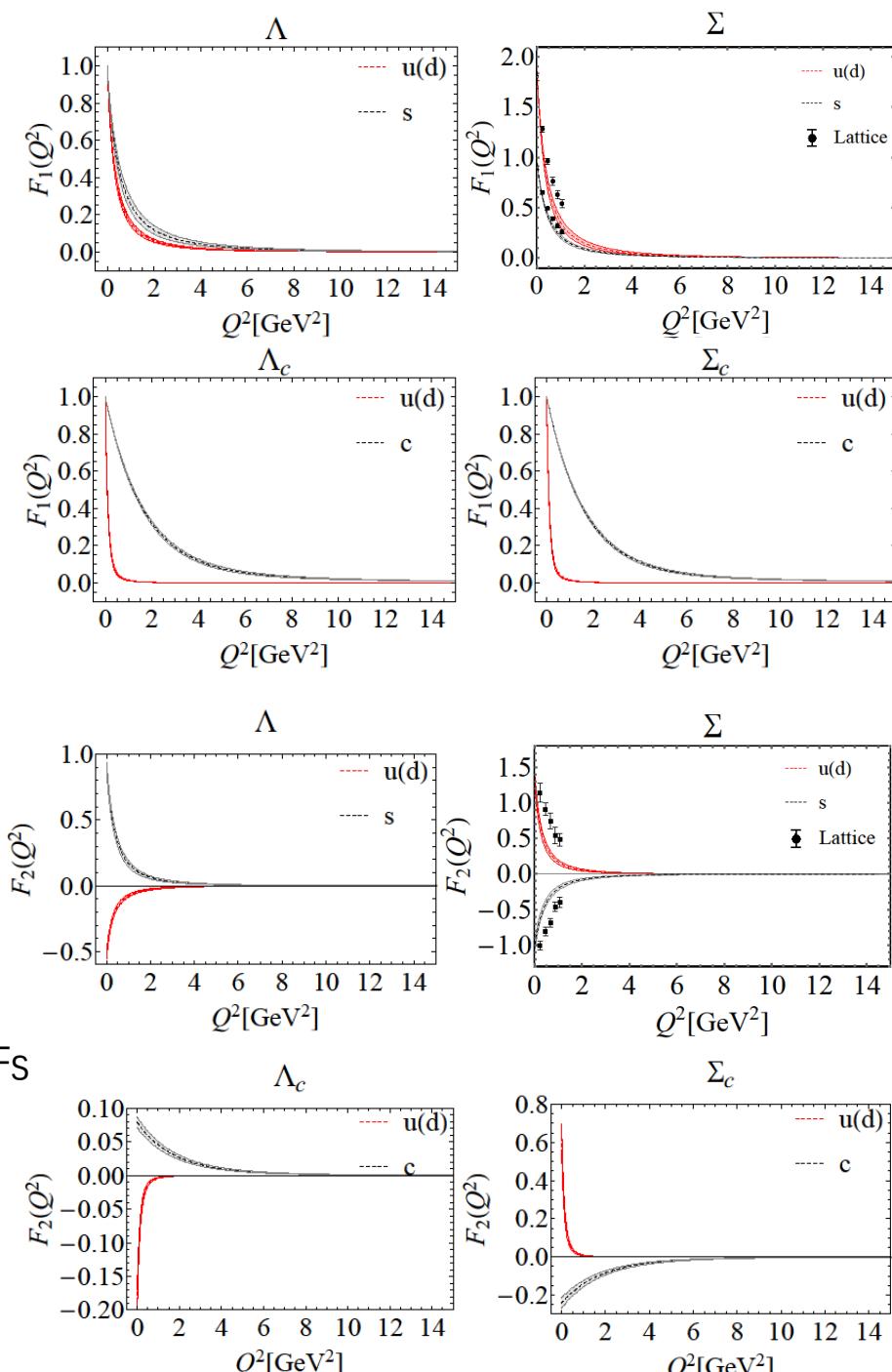


Dirac FFs

Light quarks can give  
large contribution to Pauli FFs

Pauli FFs

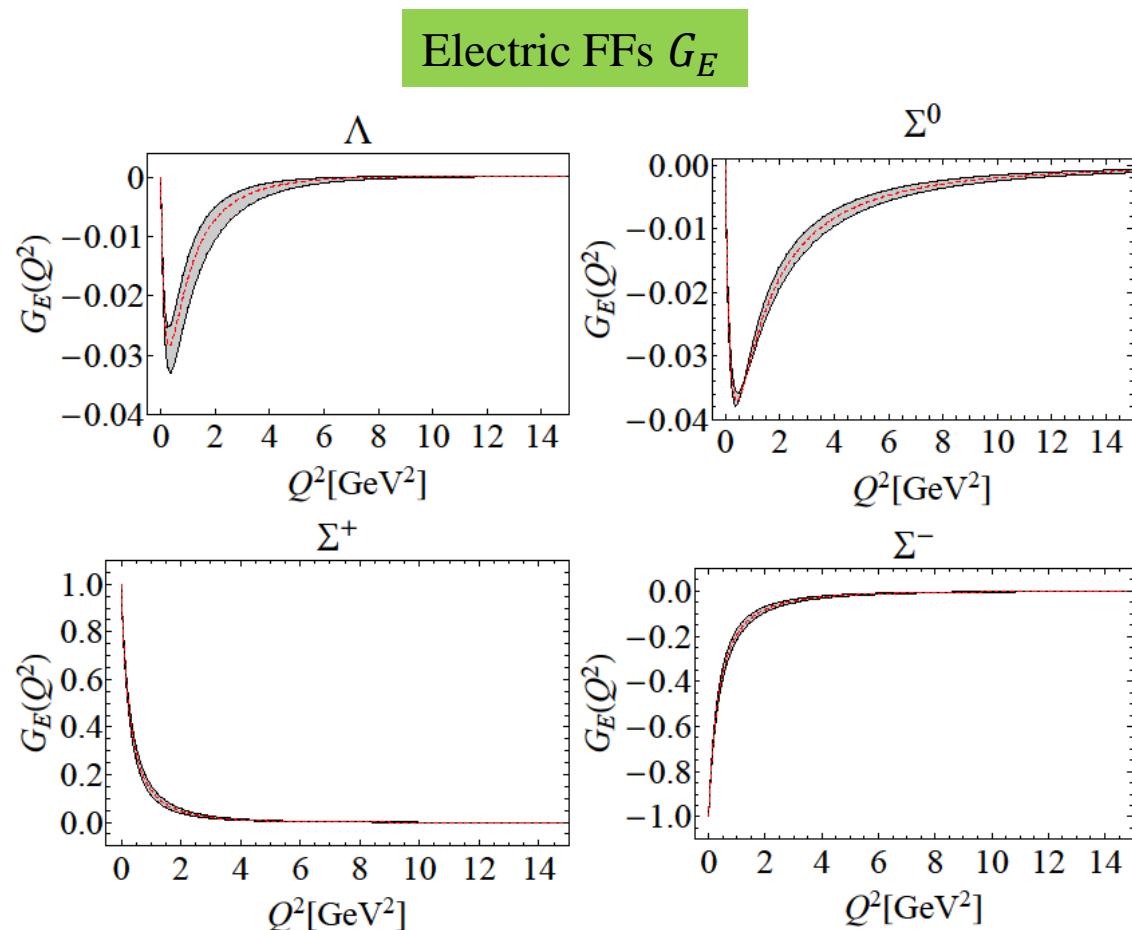
Heavy quark gives  
more contributions to Dirac FFs



# Baryons Electromagnetic FFs

With the quarks charge,  $e_u(e_c) = +\frac{2}{3}$ ,  $e_d(e_s) = -\frac{1}{3}$

we can get the baryons FFs through the flavor FFs

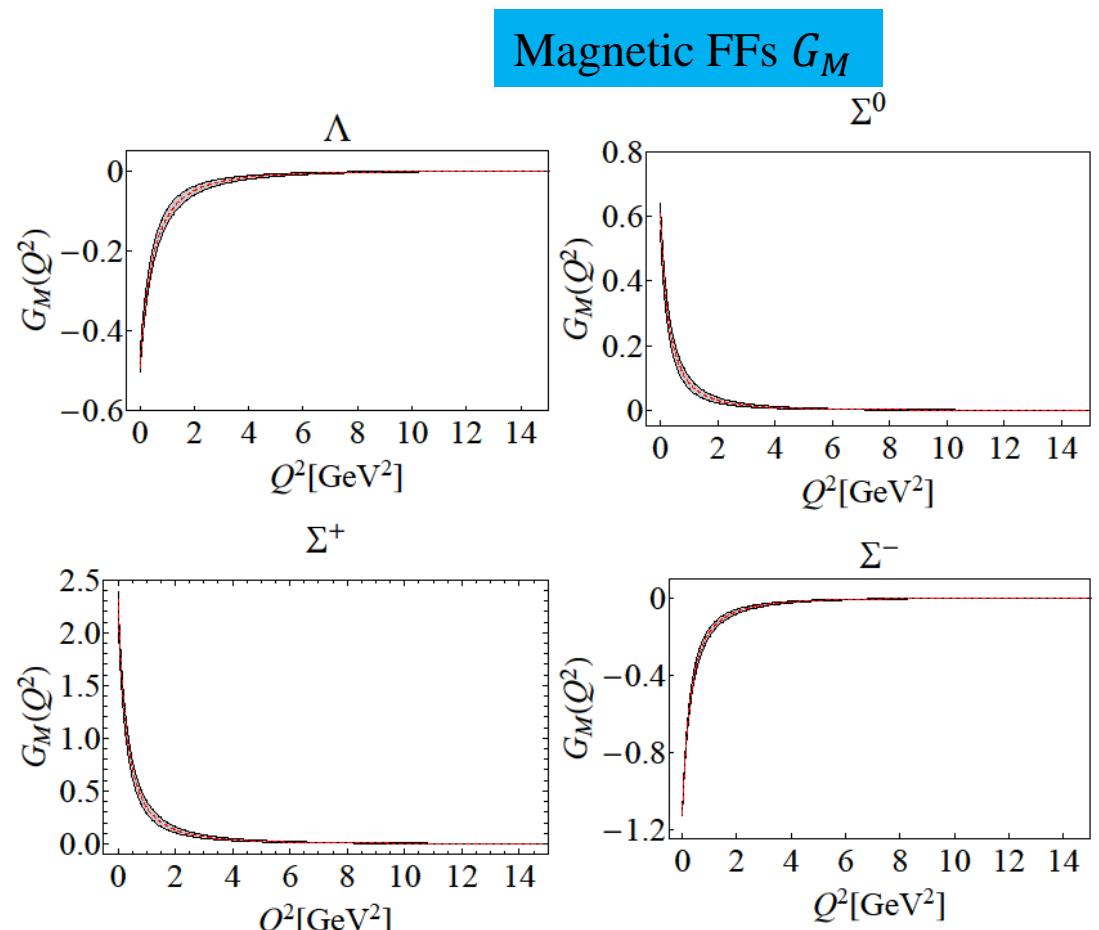


$$F_i^\Lambda = e_u F_i^u + e_d F_i^d + e_s F_i^s$$

$$F_i^{\Lambda_c} = e_u F_i^u + e_d F_i^d + e_c F_i^c$$

$$G_E^{\Lambda(\Lambda_c)}(Q^2) = F_1^{\Lambda(\Lambda_c)}(Q^2) - \frac{Q^2}{4M^2} F_2^{\Lambda(\Lambda_c)}(Q^2)$$

$$G_M^{\Lambda(\Lambda_c)}(Q^2) = F_1^{\Lambda(\Lambda_c)}(Q^2) + F_2^{\Lambda(\Lambda_c)}(Q^2),$$



# Baryon Electromagnetic FFs

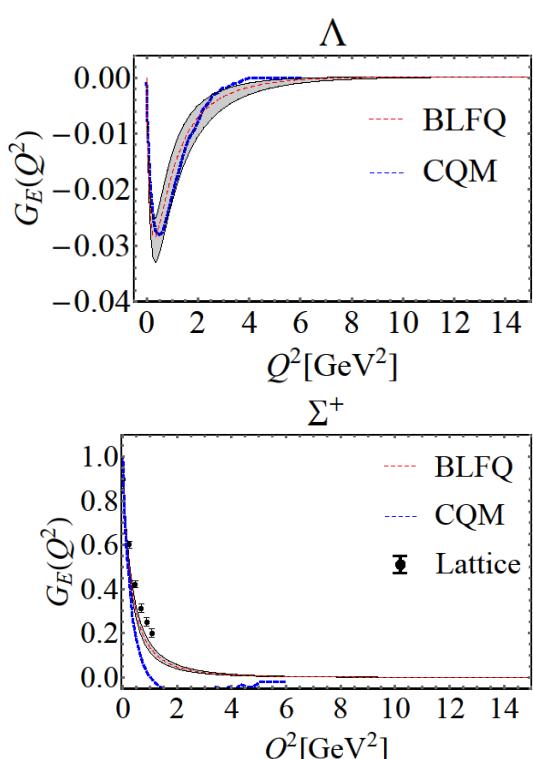
$$F_i^\Lambda = e_u F_i^u + e_d F_i^d + e_s F_i^s$$

$$F_i^{\Lambda_c} = e_u F_i^u + e_d F_i^d + e_c F_i^c$$

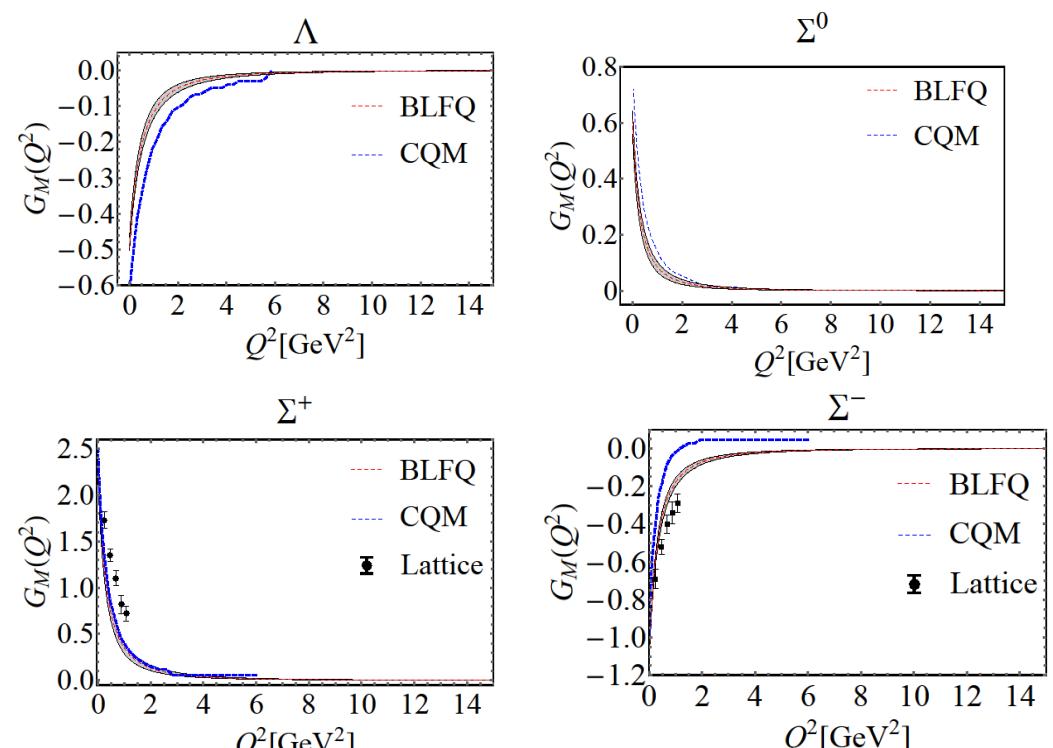
With the quark charge,  $e_u(e_c) = +\frac{2}{3}$ ,  $e_d(e_s) = -\frac{1}{3}$

we can get the baryon FFs through the flavor FFs

Electric FFs  $G_E$

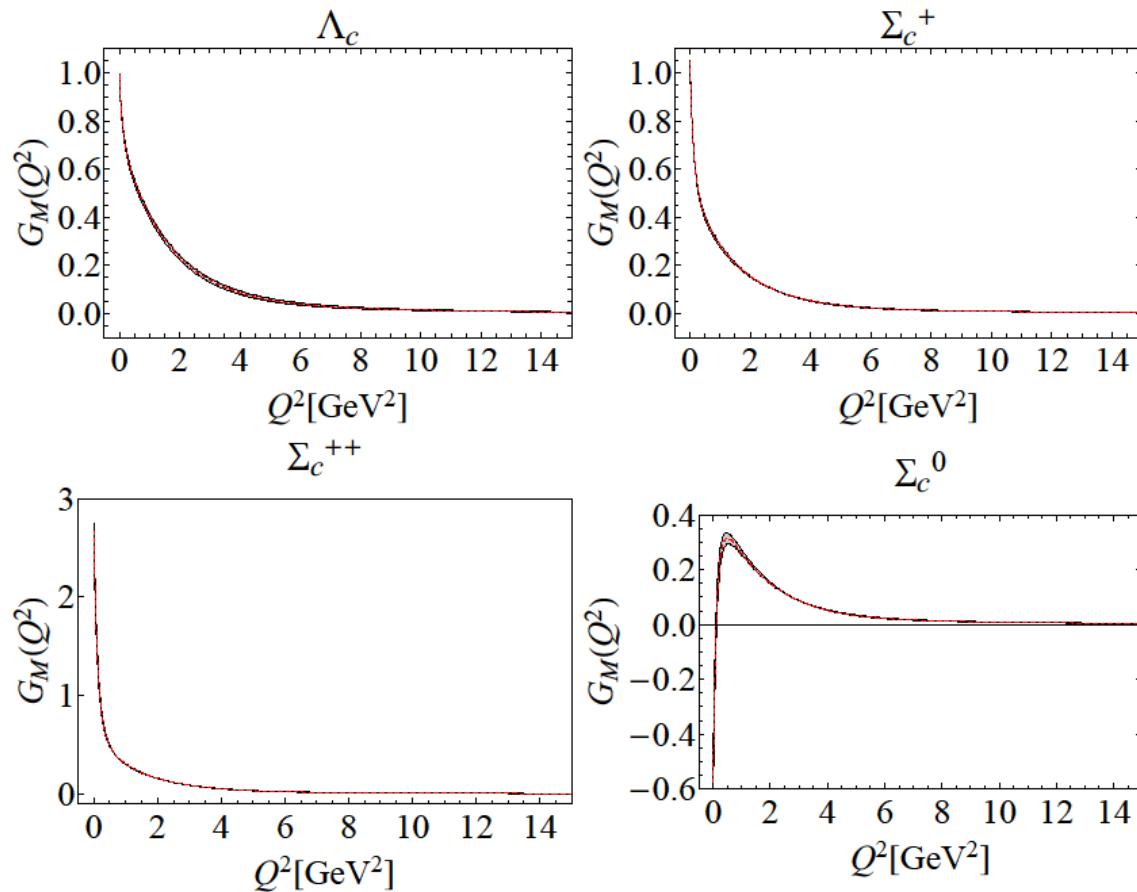


Magnetic FFs  $G_M$

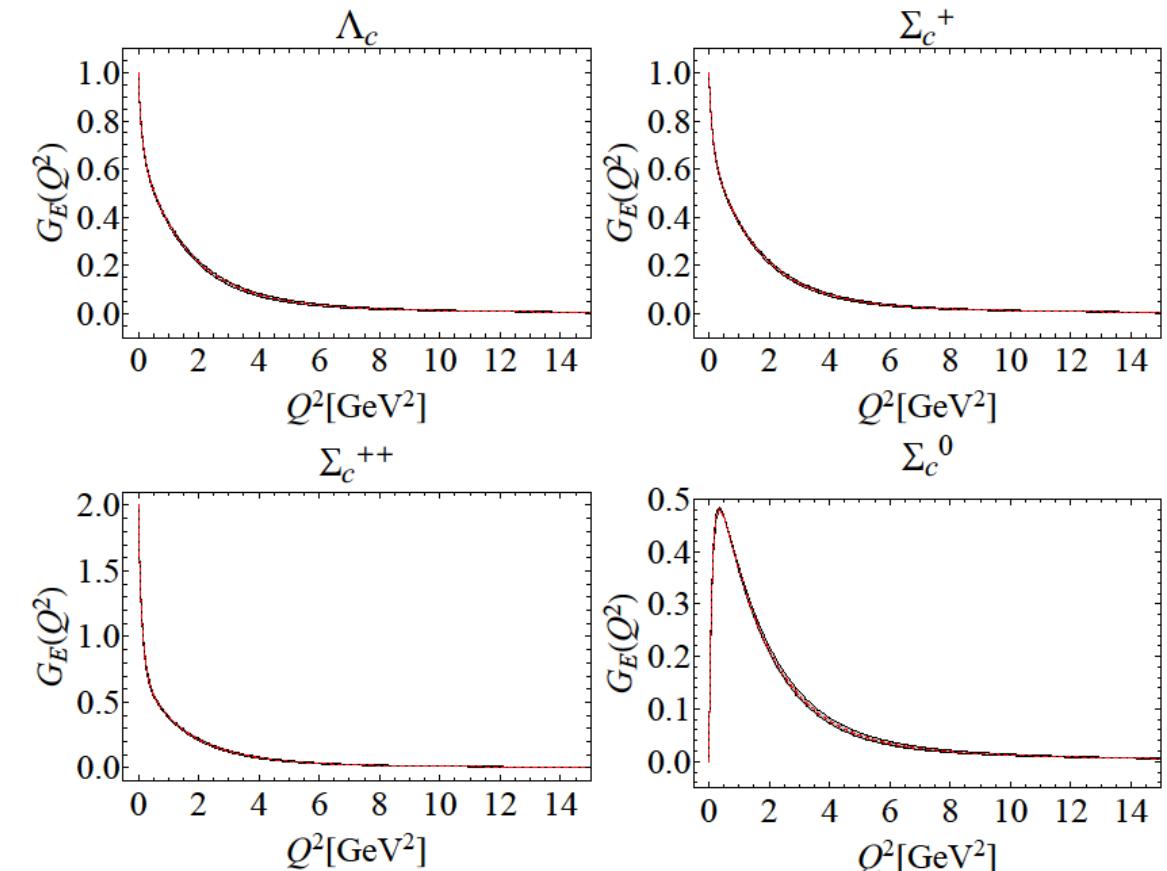


Constituent quark model and lattice QCD.

Electric FFs  $G_E$



Magnetic FFs  $G_M$



# Magnetic moments and Electromagnetic radii

From the zero point value of the  $G_M$ , we can get the magnetic moment  $\mu = G_M(0)$

$$\langle r_E^2 \rangle = -\frac{6}{G_E(0)} \frac{dG_E(Q^2)}{dQ^2} \Big|_{Q^2=0}$$

From the ratio of the  $G_E$  and  $G_M$  at zero point, we can get the electromagnetic radius like this:

$$\langle r_M^2 \rangle = -\frac{6}{G_M(0)} \frac{dG_M(Q^2)}{dQ^2} \Big|_{Q^2=0}$$

	$\mu_{BLFQ}/\mu_N$	$\mu_{exp}/\mu_N$	$\langle r_E^2 \rangle_{BLFQ}/[fm^2]$	$\langle r_E^2 \rangle_{exp}/[fm^2]$	$\langle r_M^2 \rangle_{BLFQ}/[fm^2]$	$\langle r_M^2 \rangle_{exp}/[fm^2]$
$\Lambda$	$-0.494^{+0.028}_{-0.010}$	$-0.613 \pm 0.004$	$0.07^{+0.01}_{-0.01}$	-	$0.52^{+0.01}_{-0.01}$	-
$\Sigma^0$	$0.610^{+0.032}_{-0.051}$	-	$0.07^{+0.00}_{-0.01}$	-	$0.82^{+0.00}_{-0.01}$	-
$\Sigma^+$	$2.323^{+0.067}_{-0.111}$	$2.458 \pm 0.010$	$0.79^{+0.05}_{-0.05}$	-	$0.79^{+0.00}_{-0.00}$	-
$\Sigma^-$	$-1.124^{+0.011}_{-0.007}$	$-1.160 \pm 0.025$	$0.65^{+0.02}_{-0.02}$	$0.60 \pm 0.08 \pm 0.08$	$0.70^{+0.02}_{-0.02}$	-

	$\mu_{BLFQ}/\mu_N$	$\langle r_E^2 \rangle_{BLFQ}/[fm^2]$	$\langle r_M^2 \rangle_{BLFQ}/[fm^2]$
$\Lambda_c$	$0.99^{+0.00}_{-0.00}$	$0.73^{+0.02}_{-0.02}$	$0.64^{+0.02}_{-0.02}$
$\Sigma_c^+$	$1.05^{+0.01}_{-0.001}$	$0.74^{+0.02}_{-0.02}$	$0.78^{+0.01}_{-0.01}$
$\Sigma_c^{++}$	$2.67^{+0.49}_{-0.08}$	$1.33^{+0.03}_{-0.03}$	$1.54^{+0.01}_{-0.01}$
$\Sigma_c^0$	$-0.58^{+0.07}_{-0.07}$	$-1.19^{+0.04}_{-0.03}$	$3.37^{+0.33}_{-0.27}$

# Magnetic moment

	$\mu_{BLFQ}$	$\mu_{exp}[20]$
$\Lambda$	$-0.494^{+0.028}_{-0.010}$	$-0.613 \pm 0.004$
$\Sigma^0$	$0.610^{+0.032}_{-0.051}$	-
$\Sigma^+$	$2.323^{+0.067}_{-0.112}$	$2.458 \pm 0.010$
$\Sigma^-$	$-1.124^{+0.011}_{-0.007}$	$-1.160 \pm 0.025$

Agree with available experimental data

(Only consider the valence quarks)

Measurement of Short Living Baryon Magnetic Moment  
using Bent Crystals at SPS and LHC.

L. Burmistrov<sup>1</sup>, G. Calderini<sup>2</sup>, Yu. Ivanov<sup>3</sup>, L. Massacrier<sup>1,4</sup>, P. Robbe<sup>1,5</sup>, W. Scandale<sup>1,5</sup>, A. Stocchi<sup>1</sup>.

<sup>1</sup> LAL, <sup>2</sup> LPNHE, <sup>3</sup> PNPI, <sup>4</sup> IPNO, <sup>5</sup> CERN

	$\mu_{BLFQ}$	[36]	[37]	[38]	[39]	[40]	[41]	[42]	[43]	[44]	S-I[45]	S-II[45]
$\Lambda_c$	$0.99^{+0.00}_{-0.00}$	0.41	0.42	0.392	0.341	0.411	-	0.37	0.385	-	0.24	0.24
$\Sigma_c^+$	$1.05^{+0.01}_{-0.01}$	0.65	0.36	0.30	0.525	0.318	-	0.63	0.501	0.46(3)	0.26	0.30
$\Sigma_c^{++}$	$2.67^{+0.49}_{-0.08}$	3.07	1.76	2.20	2.44	1.679	2.1(3)	2.18	2.279	2.15(10)	1.50	1.50
$\Sigma_c^0$	$-0.58^{+0.06}_{-0.07}$	-1.78	-1.04	-1.60	-1.391	-1.043	-1.6(2)	-1.17	-1.015	-1.24(5)	-0.97	-0.91

【36,37】 the relativistic three-quark model    【38】 chiral constituent quark model    【39】 the Dirac equation with a power-law potential    【40】 the bag model    【41】 QCD spectral sum rules    【43】 hyper central model    【44】 chiral quark-soliton model    【45】 chiral perturbation theory

# Electric radius $r_E^2$

	BLFQ	HB[30]		IR[31]		HB $\chi$ PT[32]		RQM[36]		exp.[33]
		$O(q^3)$	$O(q^4)$	$O(q^3)$	$O(q^4)$	$O(1/\Lambda_{chi}^2)$	$O(1/\Lambda_\chi^2 M_N)$	I	II	
Neutral particle	$\Lambda$	$0.07 \pm 0.01$	0.14	0.00	0.05	$0.11 \pm 0.02$	-0.150	-0.050	-0.01	0.02
	$\Sigma^0$	$0.07^{+0.00}_{-0.01}$	-0.14	-0.08	-0.05	$-0.03 \pm 0.01$	-	-	0.02	0.02
	$\Sigma^+$	$0.79 \pm 0.05$	0.59	0.72	0.63	$0.60 \pm 0.02$	1.522	1.366	0.47	0.66
	$\Sigma^-$	$0.65 \pm 0.02$	0.87	0.88	0.72	$0.67 \pm 0.03$	0.977	0.798	0.41	0.64

	BLFQ	[36]		
		Instant	Point	Front
$\Lambda_c$	$0.73^{+0.02}_{-0.02}$	0.5	0.2	0.4
$\Sigma_c^+$	$0.74^{+0.02}_{-0.02}$	0.5	0.2	0.4
$\Sigma_c^{++}$	$1.33^{+0.03}_{-0.03}$	1.7	0.4	1.4
$\Sigma_c^0$	$-1.19^{+0.039}_{-0.03}$	0.7	-0.0	-0.6

[30,32] heavy baryon chiral perturbation theory  
 [31] chiral perturbation theory  
 [36] relativistic quark models

# Magnetic radius $r_M^2$

Chiral perturbative theory

	BLFQ	$O(q^4)HB[30]$	$O(q^4)IR[31]$
$\Lambda$	$0.52 \pm 0.01$	$0.30 \pm 0.11$	$0.48 \pm 0.09$
$\Sigma^0$	$0.82^{+0.00}_{-0.01}$	$0.20 \pm 0.10$	$0.45 \pm 0.08$
$\Sigma^+$	$0.79 \pm 0.00$	$0.74 \pm 0.06$	$0.80 \pm 0.05$
$\Sigma^-$	$0.70 \pm 0.02$	$1.33 \pm 0.16$	$1.20 \pm 0.13$

	$\langle r_M^2 \rangle$
$\Lambda_c$	$0.64^{+0.03}_{-0.03}$
$\Sigma_c^+$	$0.78^{+0.01}_{-0.01}$
$\Sigma_c^{++}$	$1.54^{+0.01}_{-0.01}$
$\Sigma_c^0$	$3.37^{+0.33}_{-0.27}$

# Conclusions

1.  $\Lambda$ ,  $\Lambda_c$  and their isospin triplet ( $(\Sigma^0, \Sigma^+, \Sigma^-)$  and  $(\Sigma_c^+, \Sigma_c^{++}, \Sigma_c^0)$ ) baryons structure from BLFQ.
2. For PDFs, we calculate Unpolarized ( $f_1$ ), Helicity( $g_1$ ) and Transversity ( $h_1$ ) PDFs, and find with addition of a strange quark or charm quark, they will be different from proton.  
For FFs, we find the light quarks will give big contributions to Pauli FFs and heavy quark gives more contributions to Dirac FFs in single heavy flavor baryon.
3. For Electromagnetic properties, the electromagnetic radii and the magnetic moment are in agreement with the available experimental data.

**Thank you for your attention!**