New insights on α_s extractions from Soft Collinear Effective Theory

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Outline

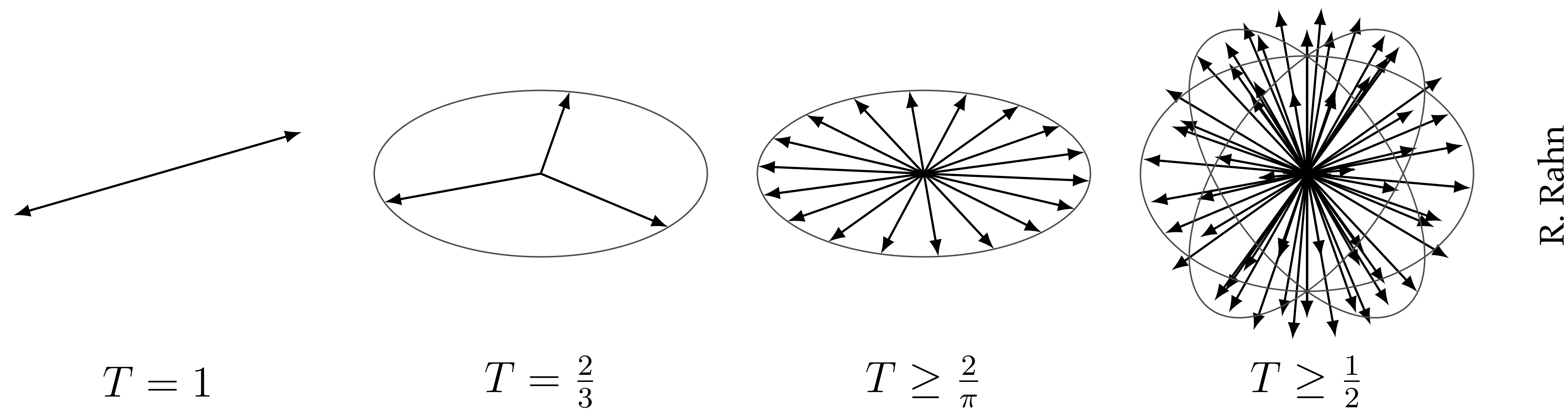
Event shape distributions and α_s in SCET

Revisiting NP Phenomenology

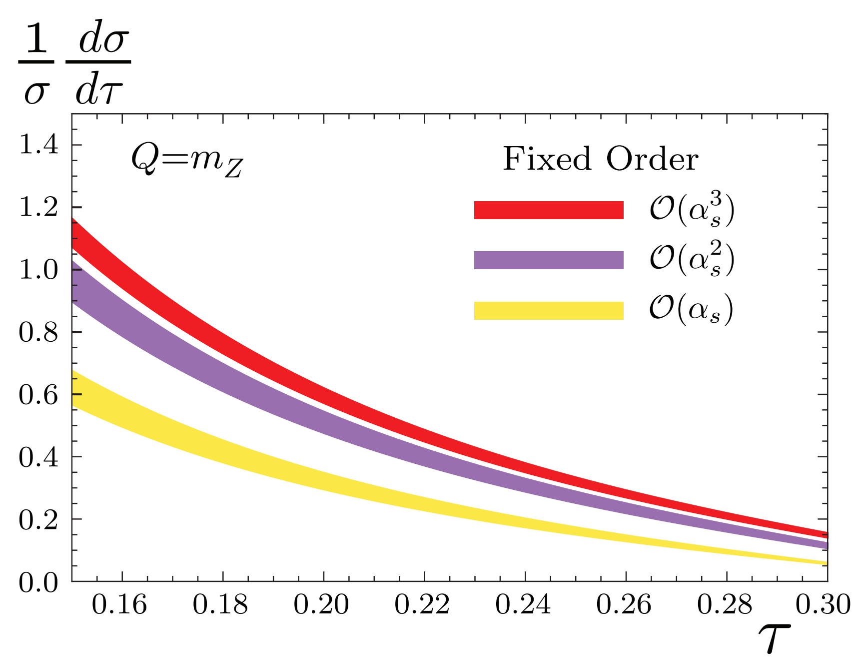
Some Preliminary Fit Results

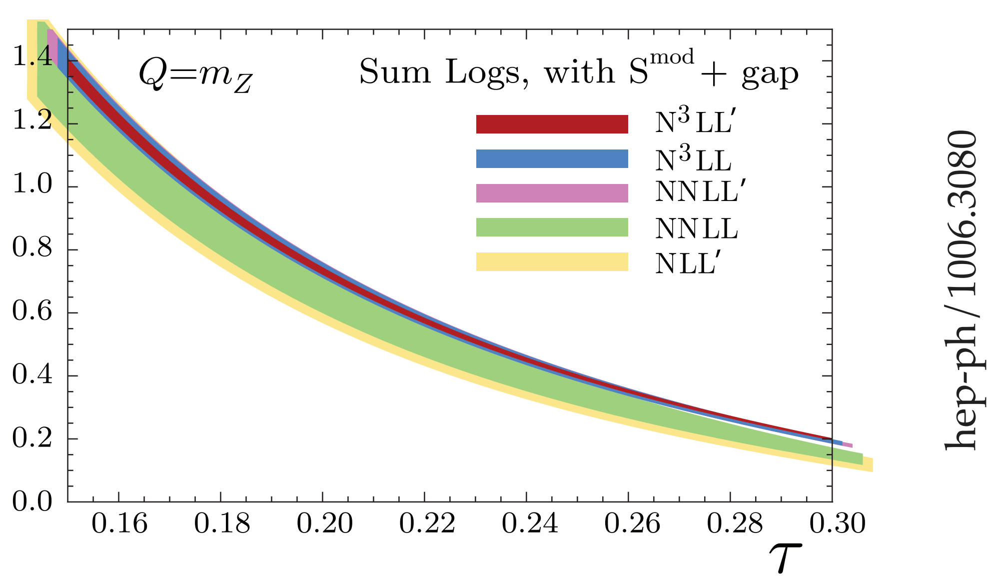
Event shape distributions: thrust

- The classic example is *Thrust:* $\tau \equiv 1 - T = 1 - \frac{1}{Q} \max_{\hat{\mathbf{t}}} \sum_{i \in X} |\hat{\mathbf{t}} \cdot \mathbf{p_i}|$ [Farhi, PRL 39 (1977)]

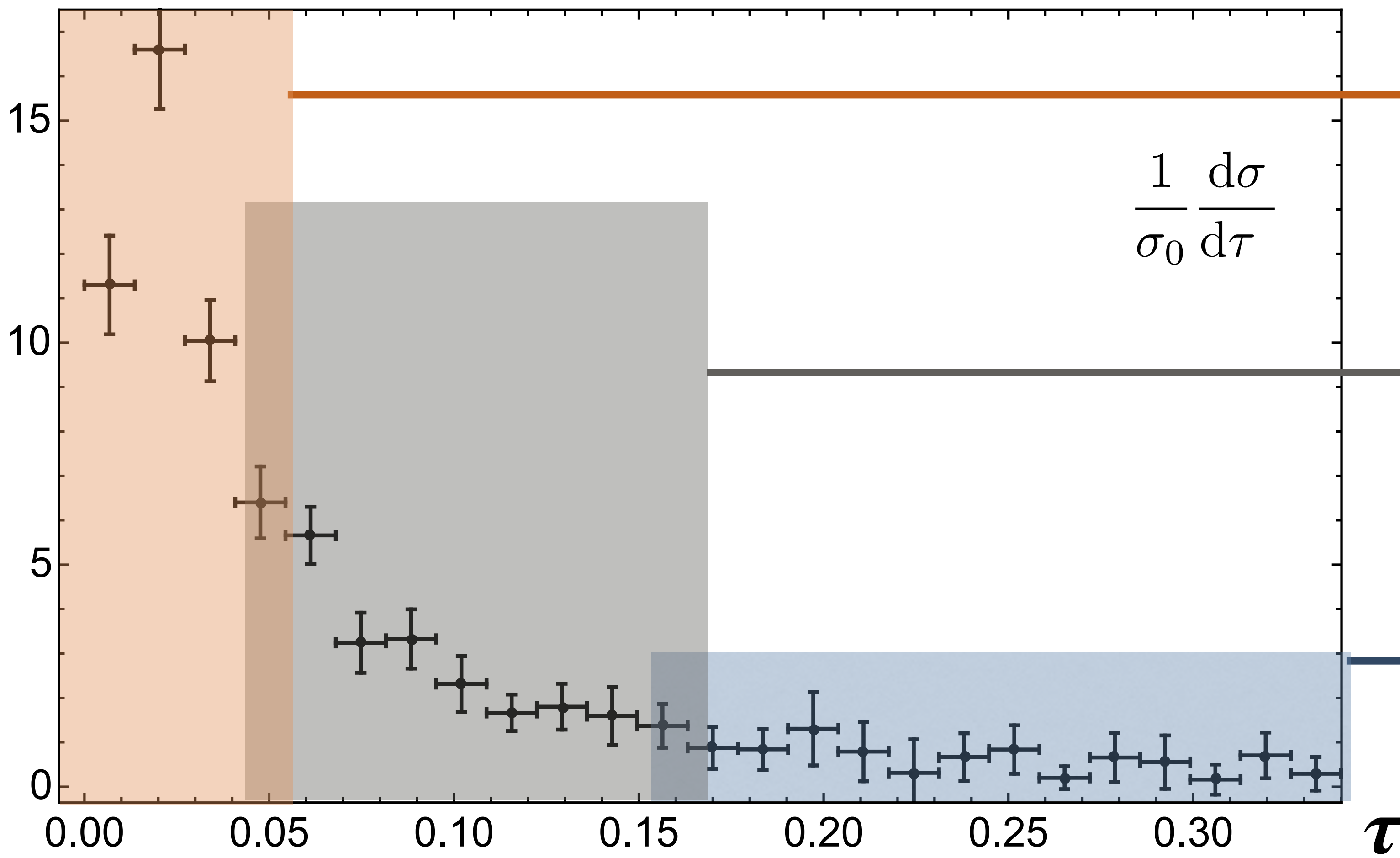


- The fixed order distribution can readily be computed in QCD, though state of the art is a N^{3}LL′ + $O(\alpha_s^3)$ resummation — readily achieved with **Soft Collinear Effective Theory**.





Dissecting dijets — constructing the curve



$d\sigma \sim H \cdot \mathcal{J} \otimes \mathcal{J} \otimes \mathcal{S}$

'Far Tail' Region: fixed-order, multi-jet region. QCD MATCHING

'Tail' Region: resummation region. PERTURBATIVE SCET PREDICTIONS

'Peak' Region: non-perturbative, soft region. NON-PERTURBATIVE MODELING

See 'Introduction to SCET' by Becher, Broggio, & Ferroglia

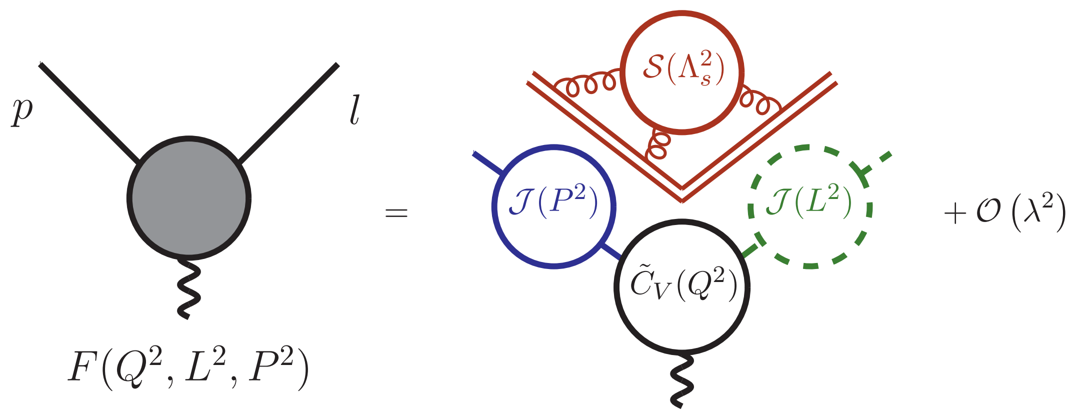
Original SCET papers: [0011336][0005275][0109045][0206152]

SCETching thrust: perturbative regime

[0801.4569]
[0901.3780]

- SCET permits all-orders derivations of factorization theorems, with individual components resummed via RG evolution:

$$d\sigma \sim H \cdot \mathcal{J} \otimes \mathcal{J} \otimes \mathcal{S} \qquad \ln \frac{\mu^2}{Q^2}, \quad \ln \frac{\mu^2}{\tau Q^2}, \quad \ln \frac{\mu^2}{\tau^2 Q^2}$$

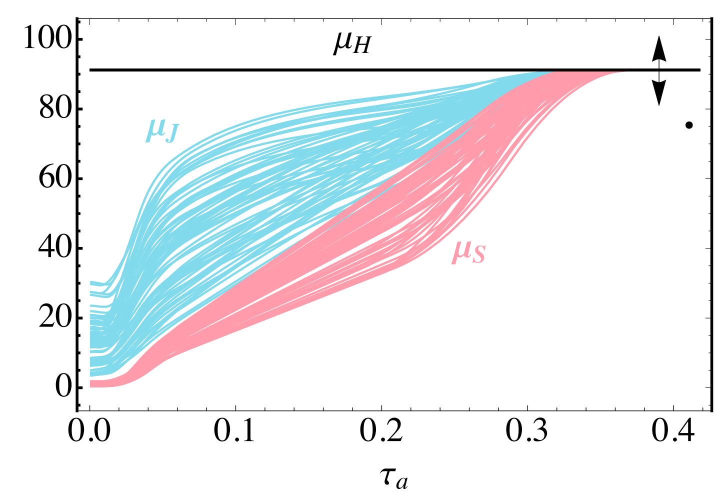


$$\frac{dH(Q^2,\mu)}{d\ln\mu} = \left[2\Gamma_{cusp}\ln(\frac{Q^2}{\mu^2}) + 4\gamma_H(\alpha_s)\right] H(Q^2.\mu)$$

$$H(Q^2,\mu) = H(Q^2,\mu_h)\; U_h(\mu_h,\mu)$$

This cookbook changes at 'primed' accuracies, and of course when considering matching to QCD!

Accuracy	$\Gamma_{\rm cusp}$	$\gamma_F, \gamma_\Delta^\mu, \gamma_R$	β	$H, \tilde{J}, \tilde{S}, \delta_a$
LL	α_s	1	α_s	1
NLL	α_s^2	α_s	α_s^2	1
NNLL	α_s^3	α_s^2	α_s^3	α_s
N^{3}LL	α_s^4	α_s^3	α_s^4	α_s^2



Note that there also is freedom in scale-setting choices -> *'profiles'*

- Results for $O(\alpha_s^{(2,3)})$ **matching** obtained from **EVENT2 / EERAD3:**

$$\underline{\frac{\sigma_c(\tau_a)}{\sigma_0}} - \underline{\frac{\sigma_{c,\rm sing}(\tau_a)}{\sigma_0}} = r_c(\tau_a) = \theta(\tau_a)\left\{\frac{\alpha_s(Q)}{2\pi}\underline{r_c^1(\tau_a)} + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 \underline{r_c^2(\tau_a)}\right\} \quad +$$

QCD distribution **SCET distribution** **Remainder**

SCETching thrust: non-perturbative regime

- A treatment of **non-perturbative effects** is critical in e^+e^- *-> hadrons…*
- When dominant power corrections come from the soft function, NP effects can be parameterized into a shape function f_{mod}:

$$S(k,\mu) = \int dk'\, S_{\rm PT}(k-k',\mu)\, f_{\rm mod}(k'-2\overline{\Delta}_a) \qquad f_{\rm mod}(k) = \frac{1}{\lambda}\left[\sum_{n=0}^{\infty} b_n\, f_n\left(\frac{k}{\lambda}\right)\right]^2$$

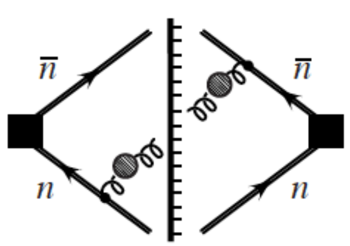
[0709.3519]
[0807.1926]

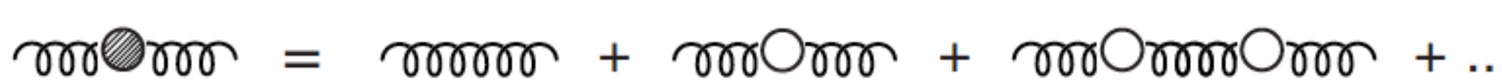
- The leading impact of this shape function correction is to shift the overall perturbative distribution:

[9408222]
[9504219]
[9806537]
[9902341]
[0611061]

a = 0 (Thrust) $\quad \frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow[\rm NP]{} \frac{d\sigma}{d\tau_a}\left(\tau_a - c_{\tau_a}\frac{\overline{\Omega}_1}{Q}\right) \qquad \frac{2\overline{\Omega}_1}{1-a} = 2\overline{\Delta}_a + \int dk\, k\, f_{\rm mod}(k)$

- However, both the gap parameter Δ_{bar} and the soft function S_PT have a renormalon ambiguity!





see e.g. Beneke (9807443)

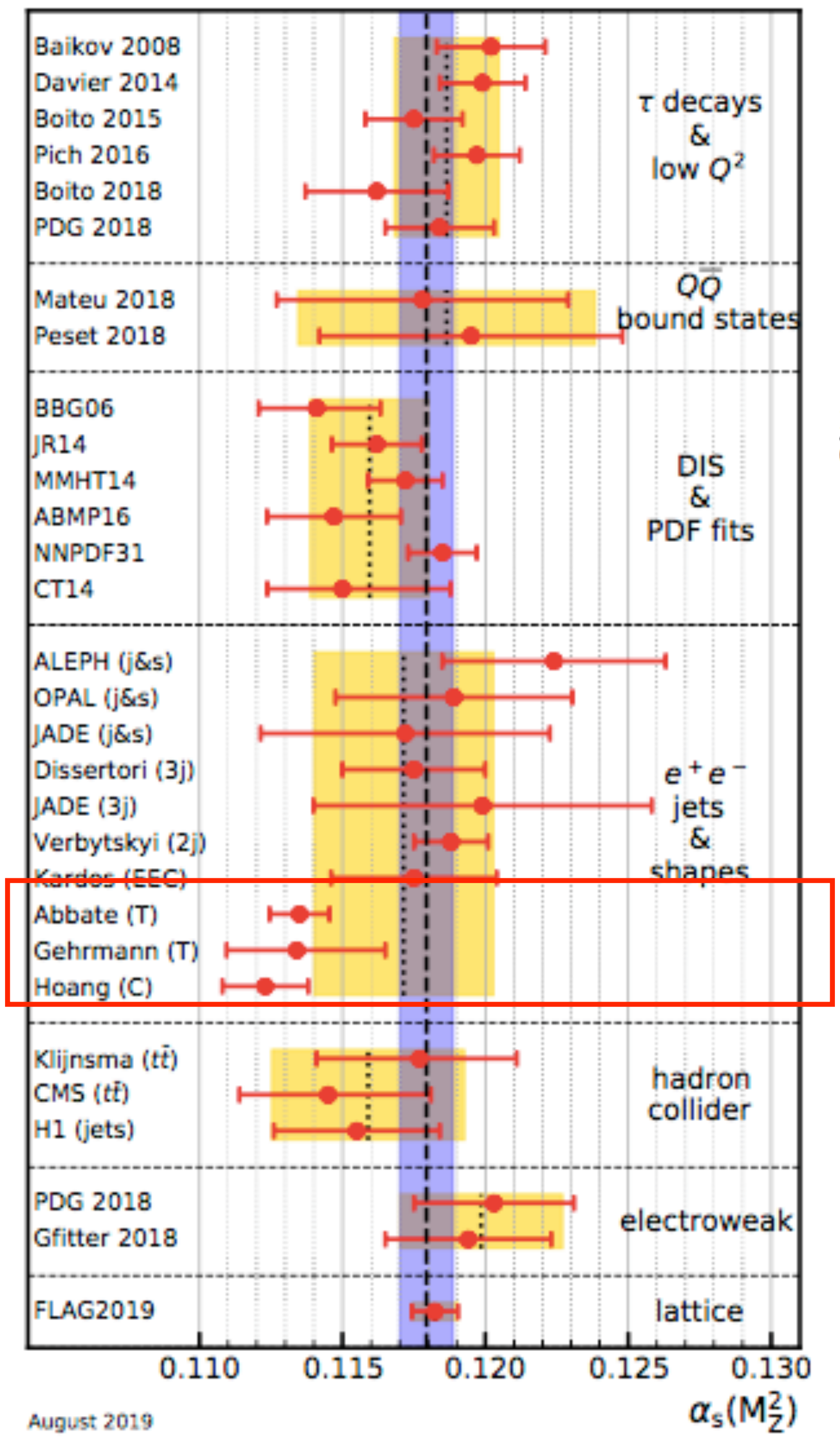
- Solution: subtract a series with a compensating/cancelling ambiguity:

[0003179]
[0709.3519]
[0803.4214]
[0806.3852]
[0801.4743]
[0908.3189]

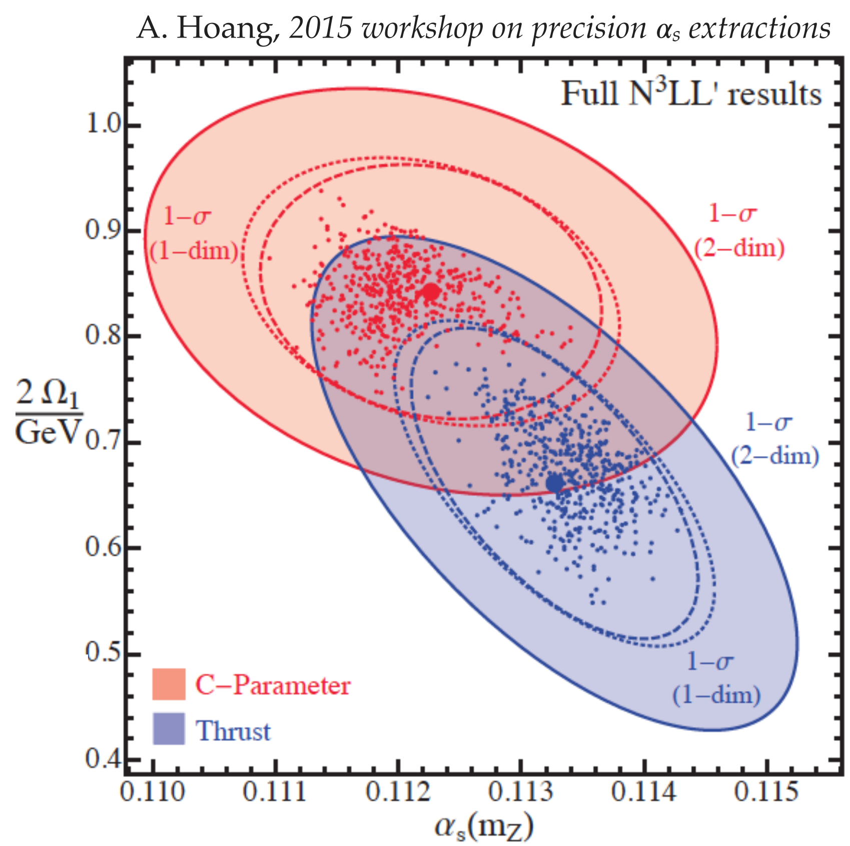
$$\overline{\Delta}_a = \Delta_a(\mu) + \delta_a(\mu) \longrightarrow \widetilde{S}(\nu,\mu) = \left[e^{-2\nu\Delta_a(\mu)}\widetilde{f}_{\rm mod}(\nu)\right]\left[e^{-2\nu\delta_a(\mu)}\widetilde{S}_{\rm PT}(\nu,\mu)\right]$$

- The highest precision SCET extractions have done so with a very particular scheme.

SCET extractions @ N^{3}LL + O(α^3) accuracy



C-parameter versus Thrust Tail Global Fit



2020 PDG world average: .1179 +- .0010

hep-ph/0803.0342 (BS)
hep-ph/1006.3080 (AFHMS)
hep-ph/1501.04111 (HKMS)

(Q1) Why are SCET results so discrepant with PDG?

(Q2) What can break the α_s - Ω degeneracy? (not today, unfortunately)

Revisiting NP Phenomenology

R-Gap scheme

$$\widetilde{S}(\nu,\mu) = \Big[e^{-2\nu\Delta_a(\mu)}\widetilde{f}_{\rm mod}(\nu)\Big]\Big[e^{-2\nu\delta_a(\mu)}\widetilde{S}_{\rm PT}(\nu,\mu)\Big]$$

[0803.4214]
[0806.3852]
[0801.4743]
[0908.3189]
[1006.3080]

- After redefining gap, one can choose the **R-Gap scheme** to cancel the leading renormalon,

$$Re^{\gamma_E}\frac{d}{d\ln\nu}\Big[\ln\widehat{S}_{\rm PT}(\nu,\mu)\Big]_{\nu=1/(Re^{\gamma_E})} = 0 \longrightarrow \delta_a(\mu,R) = \frac{1}{2}Re^{\gamma_E}\frac{d}{d\ln\nu}\Big[\ln\widetilde{S}_{\rm PT}(\nu,\mu)\Big]_{\nu=1/(Re^{\gamma_E})},$$

$$\widehat{S}_{\rm PT}(\nu,\mu) = e^{-2\nu\delta_a(\mu)}\widetilde{S}_{\rm PT}(\nu,\mu)$$

All of these objects can be defined perturbatively!

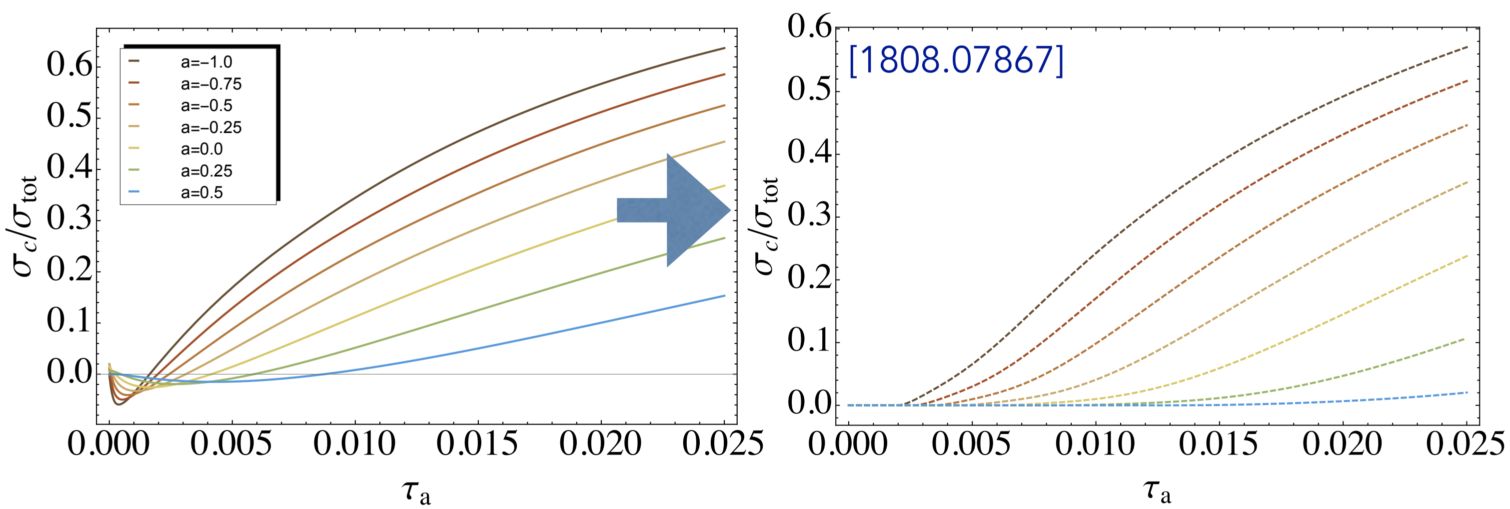
- and accounting for R and μ evolution,

$$\frac{d}{dR}\Delta_a(R,R) = -\frac{d}{dR}\delta_a(R,R) \equiv -\gamma_R[\alpha_s(R)] \qquad \mu\frac{d}{d\mu}\Delta_a(\mu,R) = -\mu\frac{d}{d\mu}\delta_a(\mu,R) \equiv \gamma_\Delta^\mu[\alpha_s(\mu)]$$

- one obtains the final soft function -> cross section:

Final cross section is expanded order-by-order in bracketed term

$$\frac{1}{\sigma_0}\sigma(\tau_a) = \int dk\,\sigma_{\rm PT}\Big(\tau_a - \frac{k}{Q}\Big)\Big[e^{-2\delta_a(\mu_S,R)\frac{d}{dk}}f_{\rm mod}(k - 2\Delta_a(\mu_S,R))\Big]$$



Also results in better convergence than shape function alone!

However, fits with this scheme implemented amongst LOWEST in global PDG table…

Effective non-perturbative shifts

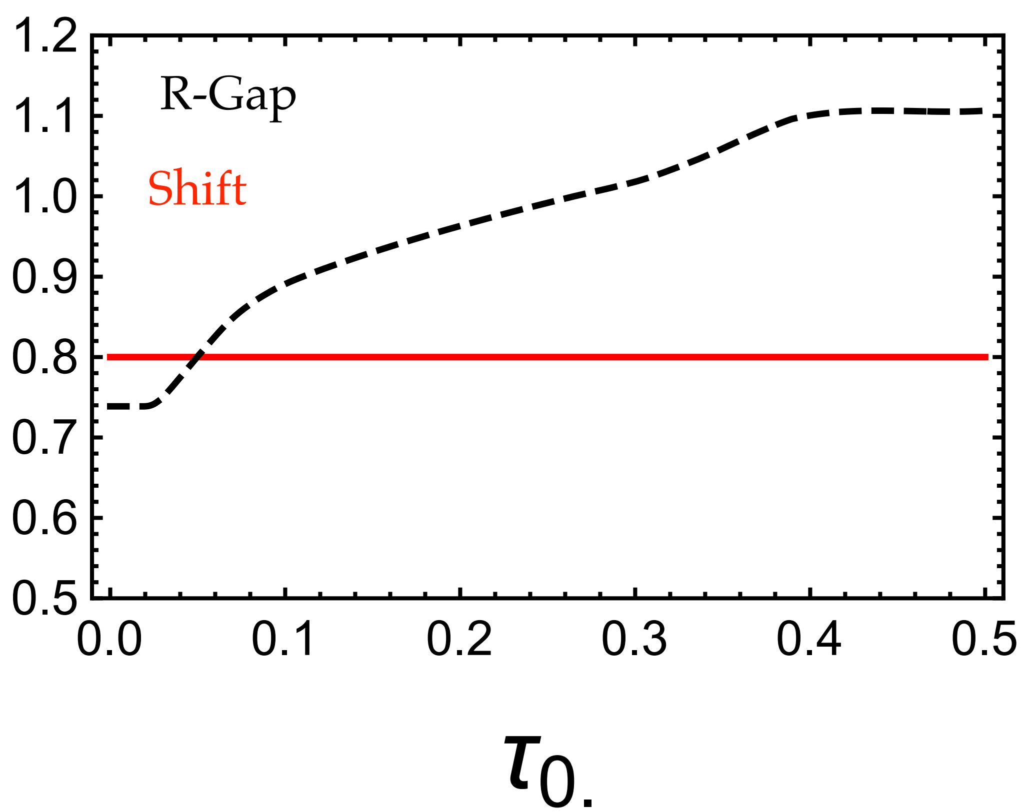
- Before considering gapped renormalons, the leading-order NP effect is a constant shift:

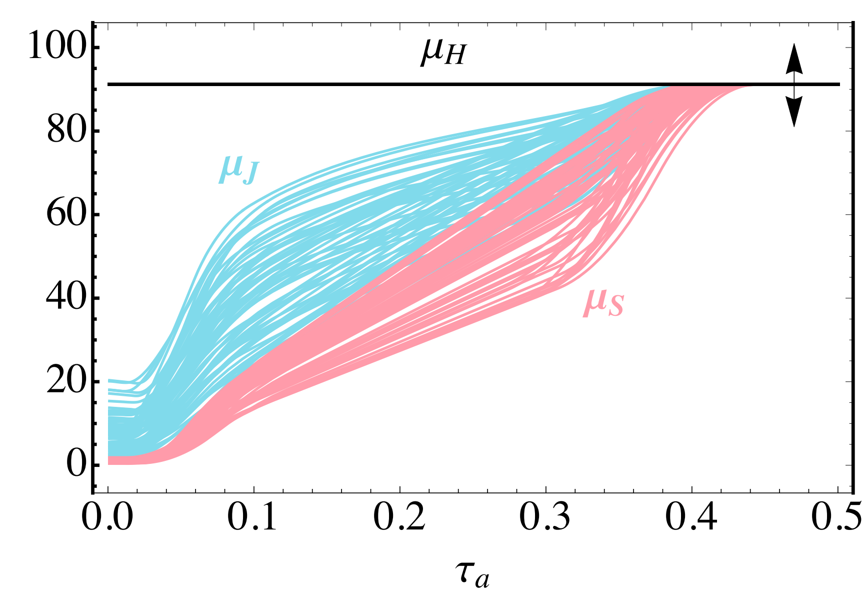
$$\frac{d\sigma}{d\tau_a}(\tau_a) \underset{\mathrm{NP}}{\longrightarrow} \frac{d\sigma}{d\tau_a}\Big(\tau_a - c_{\tau_a}\frac{\overline{\Omega}_1}{Q}\Big) \qquad \frac{2\overline{\Omega}_1}{1-a} = 2\overline{\Delta}_a + \int dk\, k\, f_{\mathrm{mod}}(k)$$

- But what is the 'effective shift' of the distribution in the R-Gap scheme?

$$\int dk\, k\, e^{-2\delta_a(\mu_S,R)\frac{d}{dk}} f_{\mathrm{mod}}\left(k - 2\Delta_a\left(\mu_S, R\right)\right) = \int dk\, k\left[\sum_i f_{\mathrm{mod}}^{(i)}\left(k - 2\Delta_a\left(\mu_S, R\right)\right)\right]$$

$$\begin{aligned} f_{\mathrm{mod}}^{(0)}(k - 2\Delta_a(\mu_S,R)) &= f_{\mathrm{mod}}(k - 2\Delta_a(\mu_S,R))\,, \\ f_{\mathrm{mod}}^{(1)}(k - 2\Delta_a(\mu_S,R)) &= -\frac{\alpha_s(\mu_S)}{4\pi}\, 2\delta_a^1(\mu_S,R) R e^{\gamma_E} f'_{\mathrm{mod}}(k - 2\Delta_a(\mu_S,R))\,, \\ f_{\mathrm{mod}}^{(2)}(k - 2\Delta_a(\mu_S,R)) &= \Big(\frac{\alpha_s(\mu_S)}{4\pi}\Big)^2\Big[-2\delta_a^2(\mu_S,R) R e^{\gamma_E} f'_{\mathrm{mod}}(k - 2\Delta_a(\mu_S,R)) \\ &\quad + 2(\delta_a^1(\mu_S,R) R e^{\gamma_E})^2 f''_{\mathrm{mod}}(k - 2\Delta_a(\mu_S,R))\Big]\,, \end{aligned}$$





Why does this effect <u>grow</u> as one moves toward the fixed-order regime?

R*: a new scheme

- Generalized renormalon cancellation schemes can be defined: [2012.12304]

$$\delta_a(\mu) = \frac{R}{2\xi}\frac{d^n}{d(\ln v)^n}\ln \tilde{S}(v,\mu)\Big|_{v=\xi/R} \longrightarrow \delta_a^*(R) = \frac{1}{2}R^* e^{\gamma_E}\frac{d}{d\ln\nu}\left[\ln S_{\rm PT}(\nu,\mu=R^*)\right]_{\nu=1/(R^* e^{\gamma_E})}$$

we are not forced to set $\mu = \mu_S$ in the subtraction series, we can pick $\mu = R$

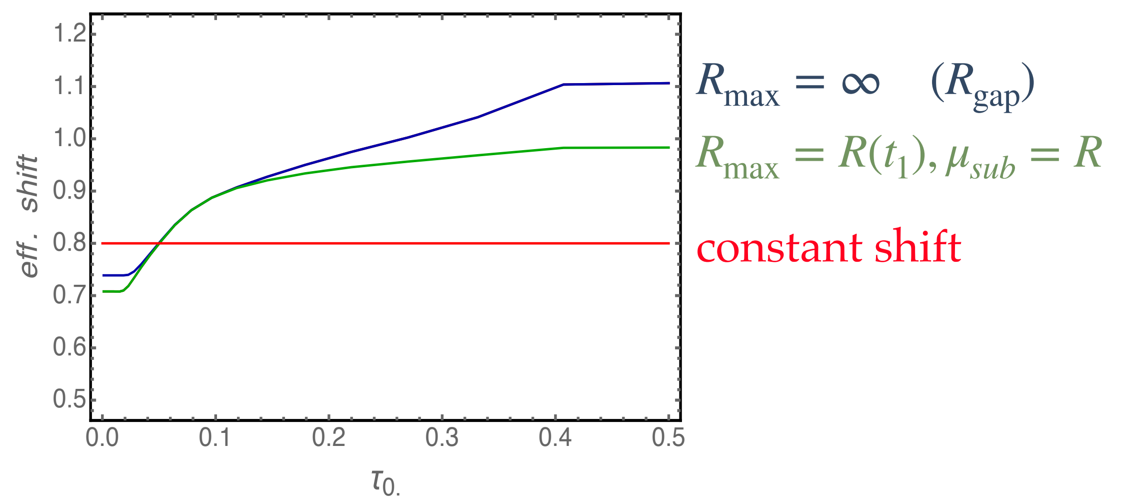
R* Scheme:
(n, ξ, μ) = (1, exp(-γ_E), R*)

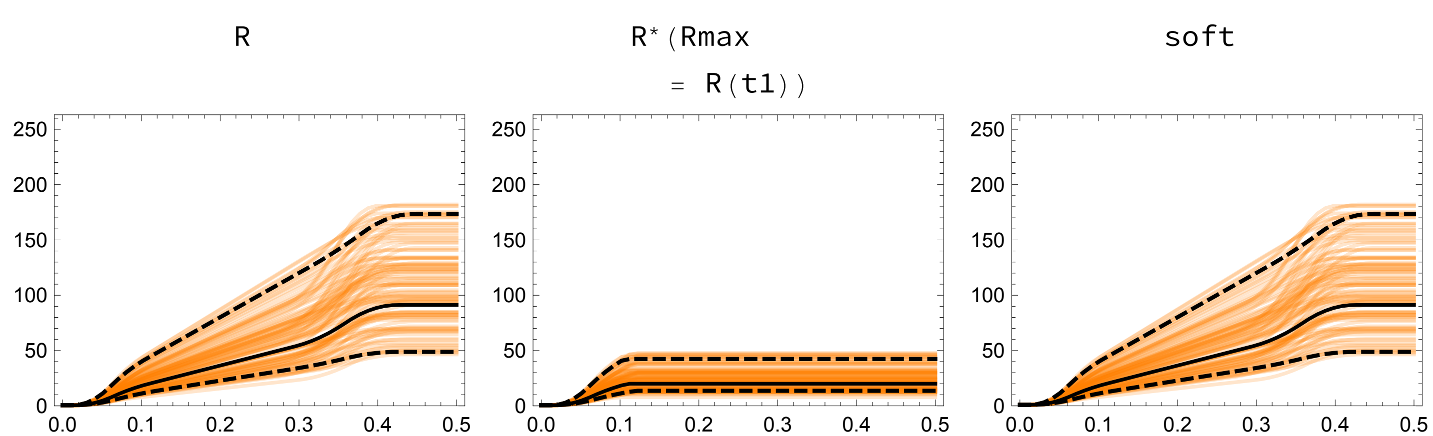
$$R^* \equiv \begin{cases} R & R < R_{\max} \\ R_{\max} & R \geq R_{\max} \end{cases}$$

- Anomalous dimensions, subtractions, turn on at one higher order:

$$\delta_a^\star(R^\star) = \frac{R^\star e^{\gamma_E}}{2}\left[\frac{\alpha_s(R^\star)}{4\pi}\cdot 0 + \left(\frac{\alpha_s(R^\star)}{4\pi}\right)^2\left(\gamma_S^1 + 2c_{\tilde{S}}^1\beta_0\right) + \mathcal{O}(\alpha_s^3)\right]$$

$$\gamma_R^\star = e^{\gamma_E}\left[\frac{\alpha_s(R^\star)}{4\pi}\cdot 0 + \left(\frac{\alpha_s(R^\star)}{4\pi}\right)^2\left(\gamma_S^1 + 2c_{\tilde{S}}^1\beta_0\right) + \mathcal{O}(\alpha_s^3)\right]$$



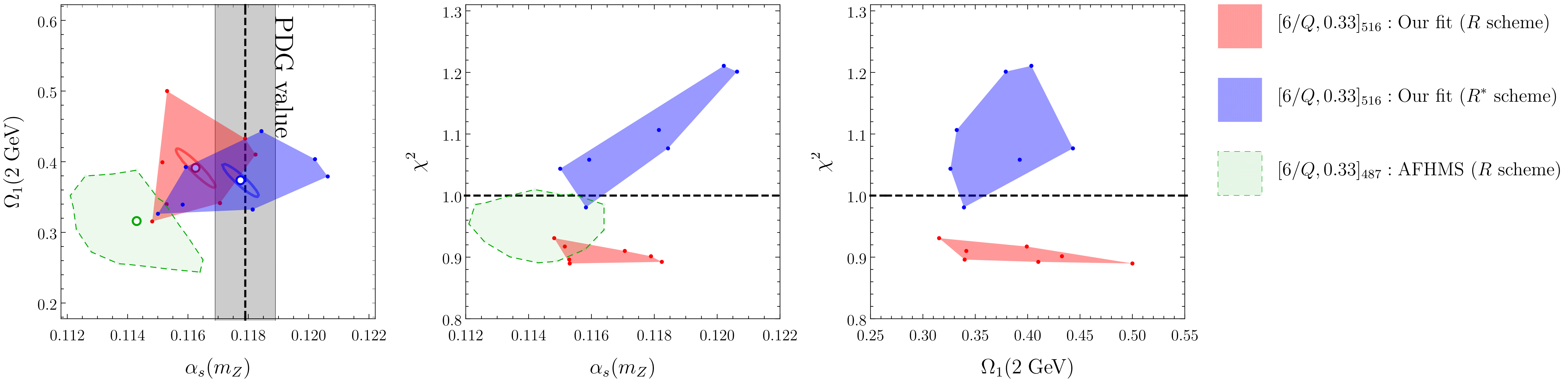


Effective non-perturbative shift flattened, as desired.

Preliminary Results

R(*)-Gap: impact on fits

- Fits at **NNLL′ + $O(\alpha_s^2)$** accuracy:



- Green -> Red: multiple other systematics, including profile parameter choices (dominant effect), b-mass and QED corrections (not included in Red/Blue), global data set, and even binning choices.
- For example, the treatment of **non-singular scale** entering fixed-order matching differs:

$$\mu_{\rm ns} = \begin{cases} \mu_J & \text{default} \\ (\mu_J + \mu_S)/2 & \text{lo} \\ \mu_H & \text{hi} \end{cases} \qquad \mu_{\rm ns} = \begin{cases} \mu_H & \text{default} \\ (\mu_H + \mu_J)/2 & \text{lo} \\ (3\mu_H - \mu_J)/2 & \text{hi} \end{cases}$$

Green　　　　　　　　Red / Blue

- However, note the difference in fit quality between Blue (R*) and Red/Green (R)…

Summary and outlook

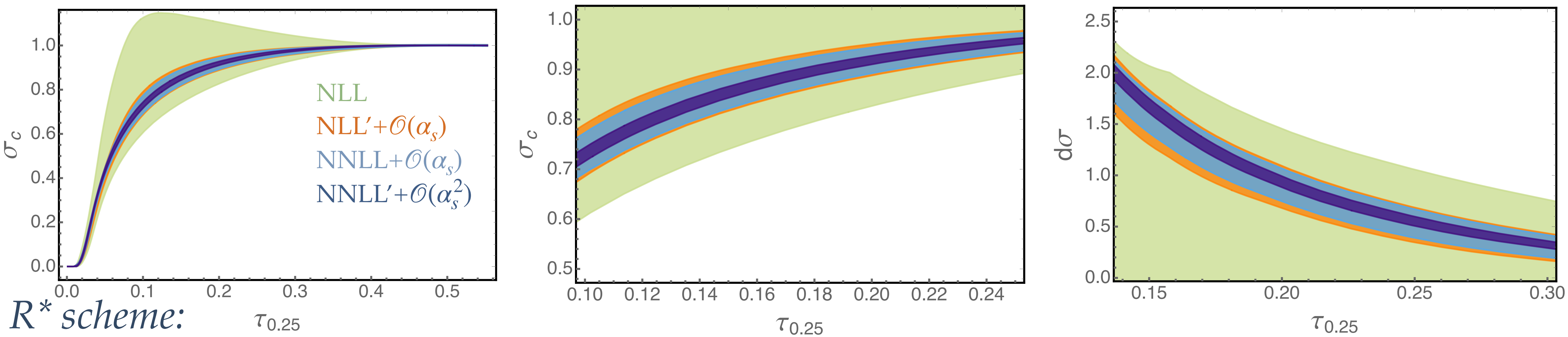
- We have presented **preliminary** results demonstrating the impact of non-perturbative physics on a global SCET extraction of the strong coupling from the Thrust e+e- event shape.
- Our results are valid at **NNLL′ + $O(\alpha_s^2)$ accuracy. WIP: N3LL′ + $O(\alpha_s^3)$ — very close to results.**
- We have also shown how Thrust fit values are sensitive to the **profile parameters** associated to scale setting.
- When the effective shift of the distribution, due to non-perturbative physics, grows less in the multi-jet window, the value of the strong coupling from Thrust approaches the PDG world average…
- **Other WIP**: analyzing a more varied and generic set of renormalon cancellation schemes. Also looking at results from angularities.
- Analytic control over **multi-jet power corrections** would clearly be valuable (also see Luisoni et al. 2012.00622).

Thanks!

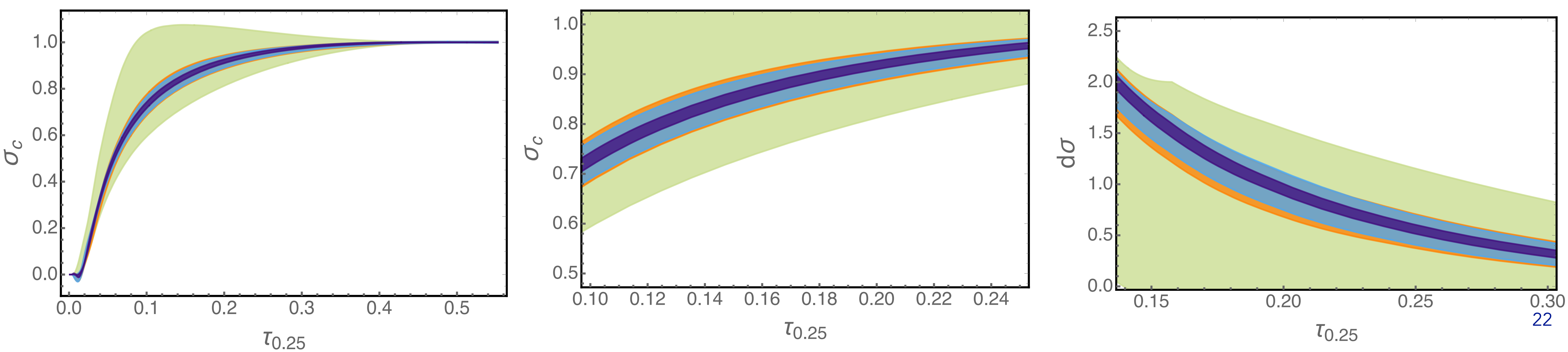
Backup Slides

R vs. R* convergence

R_{gap} scheme:



R^* scheme:



SCET & the PDG table on α_s

hep-ph/0803.0342 (BS)
hep-ph/1006.3080 (AFHMS)
hep-ph/1501.04111 (HKMS)

To be included in the PDG average, a fit must:

- be published in a peer-reviewed journal…
- include $O(\alpha_s^3)$ fixed-order perturbative results…
- include `reliable' error estimates, including NP effects…

2020 PDG world average:
.1179 +- .0010

Thrust at N^{3}LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$

Riccardo Abbate,[1] Michael Fickinger,[2] André H. Hoang,[3] Vicent Mateu,[3] and Iain W. Stewart[1]

hep-ph/1006.3080

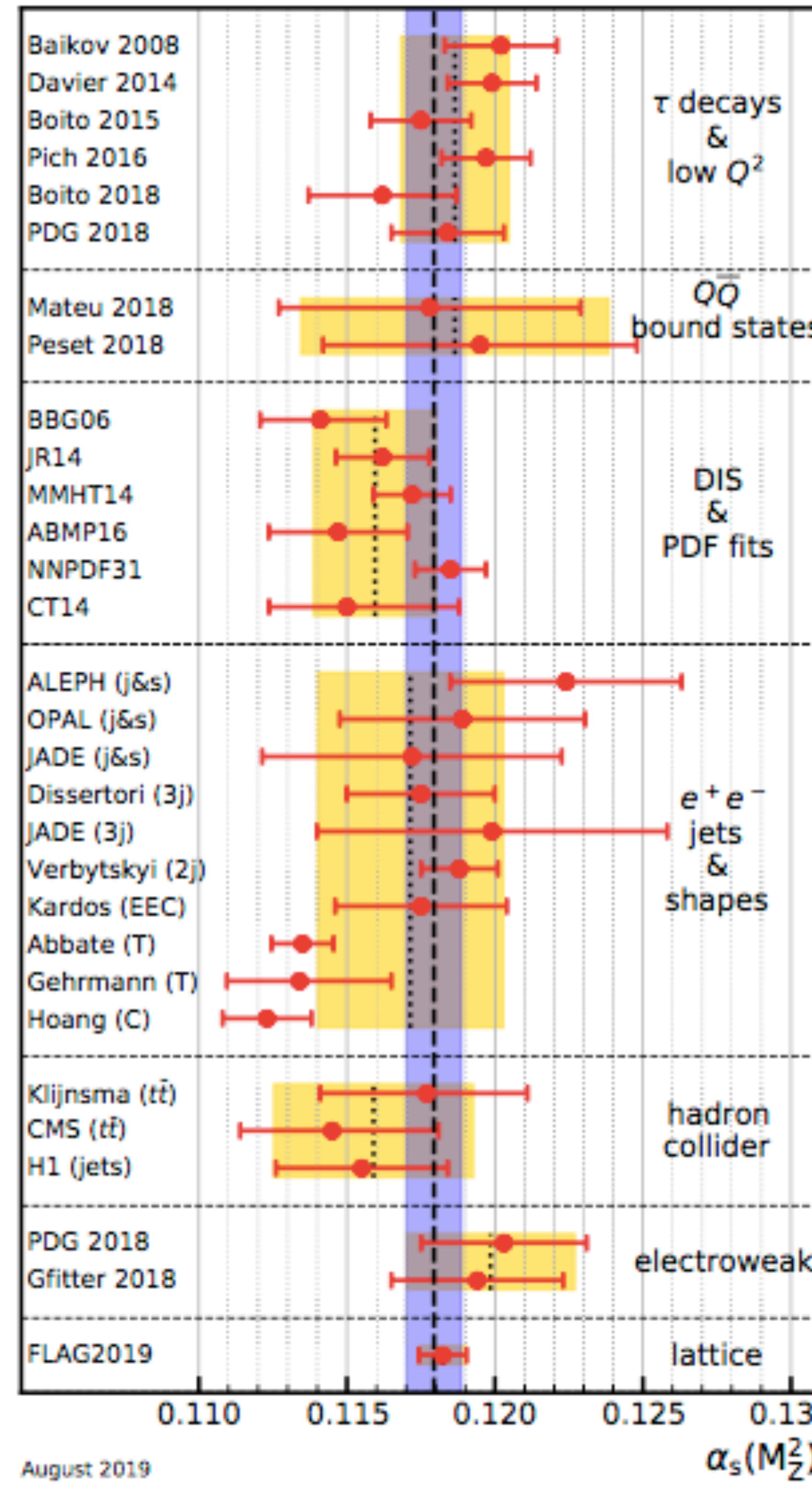
$$\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\rm exp} \pm (0.0005)_{\rm hadr} \pm (0.0009)_{\rm pert}$$

A Precise Determination of α_s from the C-parameter Distribution

André H. Hoang,[1,2] Daniel W. Kolodrubetz,[3] Vicent Mateu,[1] and Iain W. Stewart[3]

hep-ph/1501.04111

$$\alpha_s(m_Z) = 0.1123 \pm 0.0002_{\rm exp} \pm 0.0007_{\rm hadr} \pm 0.0014_{\rm pert}$$



- **2015 C-parameter result ~4σ away from lattice QCD / world average…**

Data sets

▪ For thrust:

```
ALEPH-2004: 133. GeV (7)      L3-2004: 172.3 GeV (12)
ALEPH-2004: 161. GeV (7)      L3-2004: 182.8 GeV (12)
ALEPH-2004: 172. GeV (7)      L3-2004: 188.6 GeV (12)
ALEPH-2004: 183. GeV (7)      L3-2004: 194.4 GeV (12)
ALEPH-2004: 189. GeV (7)      L3-2004: 200. GeV (11)
ALEPH-2004: 200. GeV (6)      L3-2004: 206.2 GeV (12)
ALEPH-2004: 206. GeV (8)      L3-2004: 41.4 GeV (5)
ALEPH-2004: 91.2 GeV (26)     L3-2004: 55.3 GeV (6)
AMY-1990: 55.2 GeV (5)        L3-2004: 65.4 GeV (7)
DELPHI-1999: 133. GeV (7)     L3-2004: 75.7 GeV (7)
DELPHI-1999: 161. GeV (7)     L3-2004: 82.3 GeV (8)
DELPHI-1999: 172. GeV (7)     L3-2004: 85.1 GeV (8)
DELPHI-1999: 89.5 GeV (11)    L3-2004: 91.2 GeV (10)
DELPHI-1999: 93. GeV (12)     OPAL-1997: 161. GeV (7)
DELPHI-2000: 91.2 GeV (12)    OPAL-2000: 172. GeV (8)
DELPHI-2003: 183. GeV (14)    OPAL-2000: 183. GeV (8)
DELPHI-2003: 189. GeV (15)    OPAL-2000: 189. GeV (8)
DELPHI-2003: 192. GeV (15)    OPAL-2005: 133. GeV (6)
DELPHI-2003: 196. GeV (14)    OPAL-2005: 177. GeV (8)
DELPHI-2003: 200. GeV (15)    OPAL-2005: 197. GeV (8)
DELPHI-2003: 202. GeV (15)    OPAL-2005: 91. GeV (5)
DELPHI-2003: 205. GeV (15)    SLD-1995: 91.2 GeV (6)
DELPHI-2003: 207. GeV (15)    TASSO-1998: 35. GeV (4)
DELPHI-2003: 45. GeV (5)      TASSO-1998: 44. GeV (5)
DELPHI-2003: 66. GeV (8)
DELPHI-2003: 76. GeV (9)
JADE-1998: 35. GeV (5)         ------ Summary ------
JADE-1998: 44. GeV (7)         Totlal: 516
L3-2004: 130.1 GeV (11)        Q > 95 : 345
L3-2004: 136.1 GeV (10)        Q < 88 : 89
L3-2004: 161.3 GeV (12)        Q ~ MZ : 82
```

▪ For angularities:

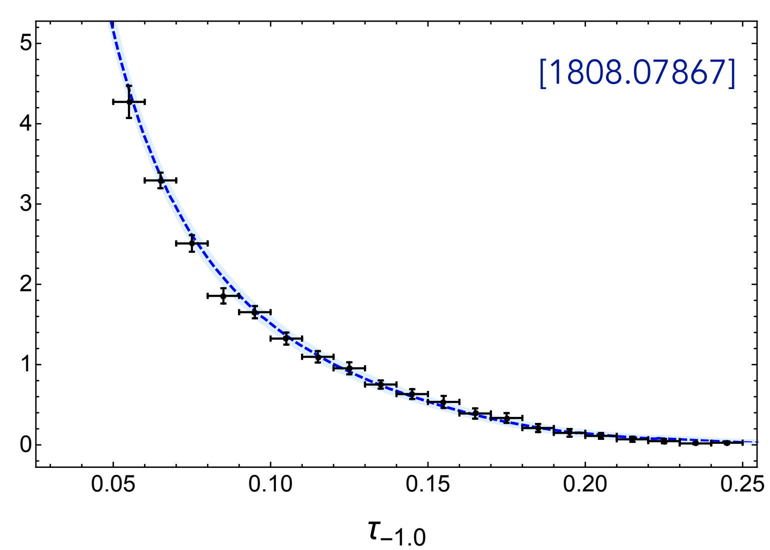
Generalized event shape and energy flow studies in e^+e^- annihilation at $\sqrt{s} = 91.2\text{-}208.0\,\mathrm{GeV}$

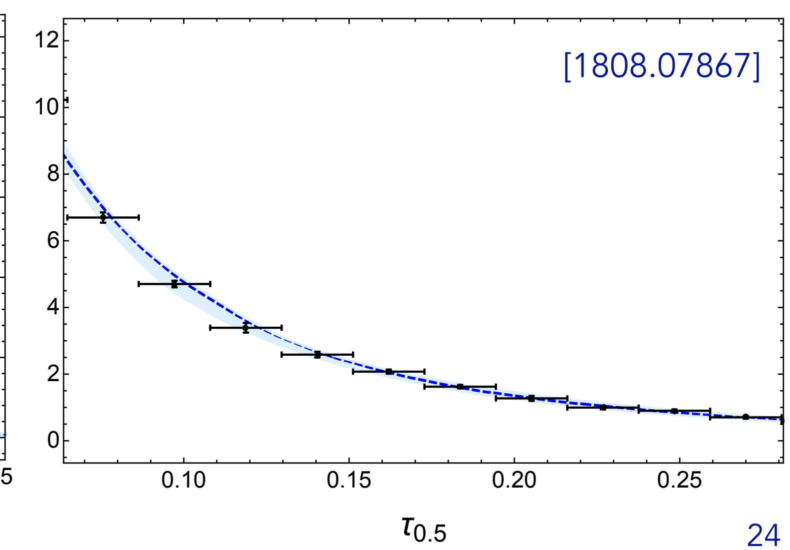
L3 Collaboration

JHEP 10 (2011) 143

Also see thesis by Pratima Jindal, Panjab University, Chandigarh

- Data for a = {**-1.0**, **-0.75**. **-0.5**, **-0.25**, **0.0**, **0.25**, **0.5**, 0.75} at **91.2** and 197 GeV
- Total number of bins = (bins per a) x (number of a) = 25 x 7 = 175 bins @ Q = 91.2 GeV
- e.g. a = -1 and 0.5, Q = 91.2 GeV, compared to our NNLL' prediction:





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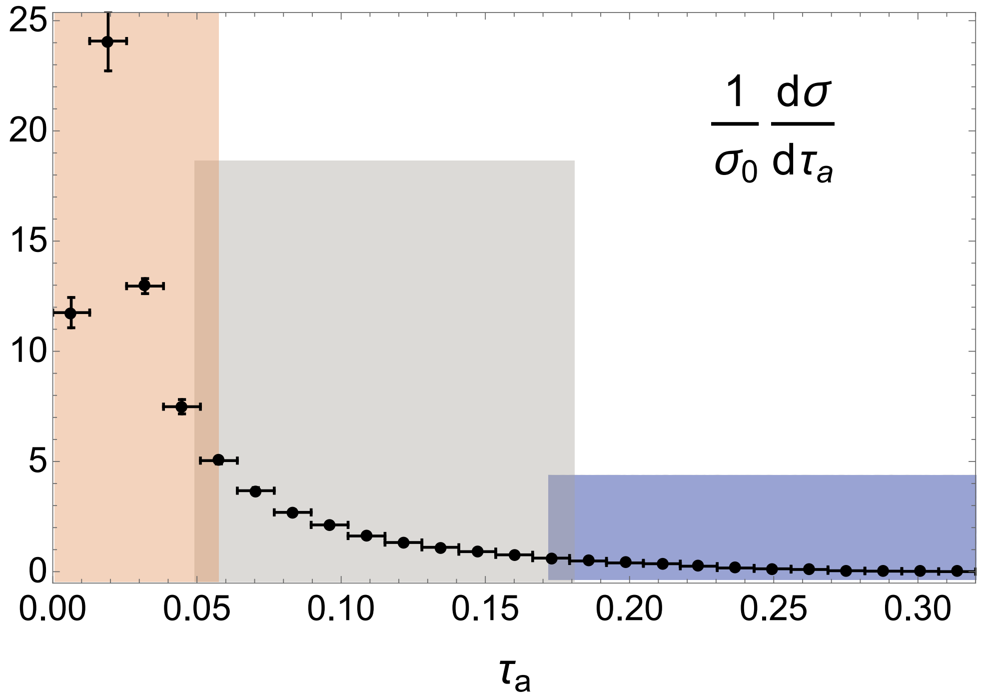
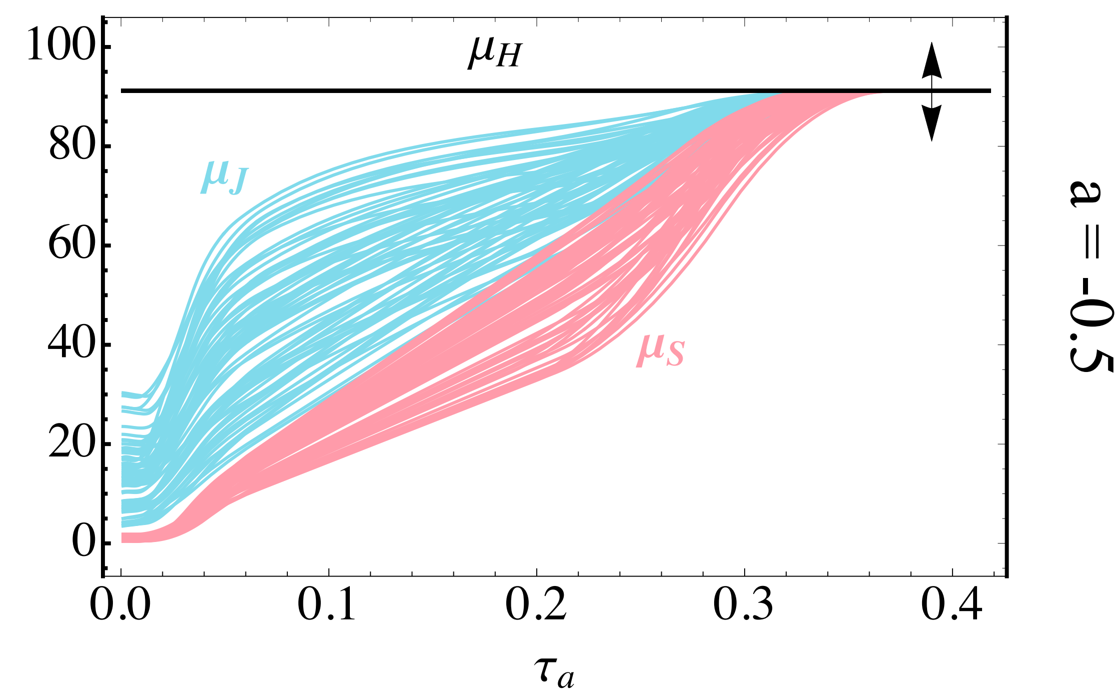
Profiling a fit window

hep-ph/1808.07867

- How can we identify a region sensitive to $\mathcal{A}$ and α_s, and for which our best theory curves are reliable? Look to the profiles!





- Profiles trace scale hierarchies through different regimes of a given distribution:

Peak	$\mu_H \gg \mu_J \gg \mu_S \sim \Lambda_{QCD}$
Tail	$\mu_H \gg \mu_J \gg \mu_S \gg \Lambda_{QCD}$
Far Tail	$\mu_H = \mu_J = \mu_S \gg \Lambda_{QCD}$

Tracks the peak

Turns off resummation

$$t_0 = \frac{n_0}{Q}\, 3^a \qquad t_2 = n_2 \times 0.295^{1-0.637\, a}$$

$$t_1 = \frac{n_1}{Q}\, 3^a \qquad t_3 = n_3\, \tau_a^{\mathrm{sph}}$$

Transitions between NP and PT physics

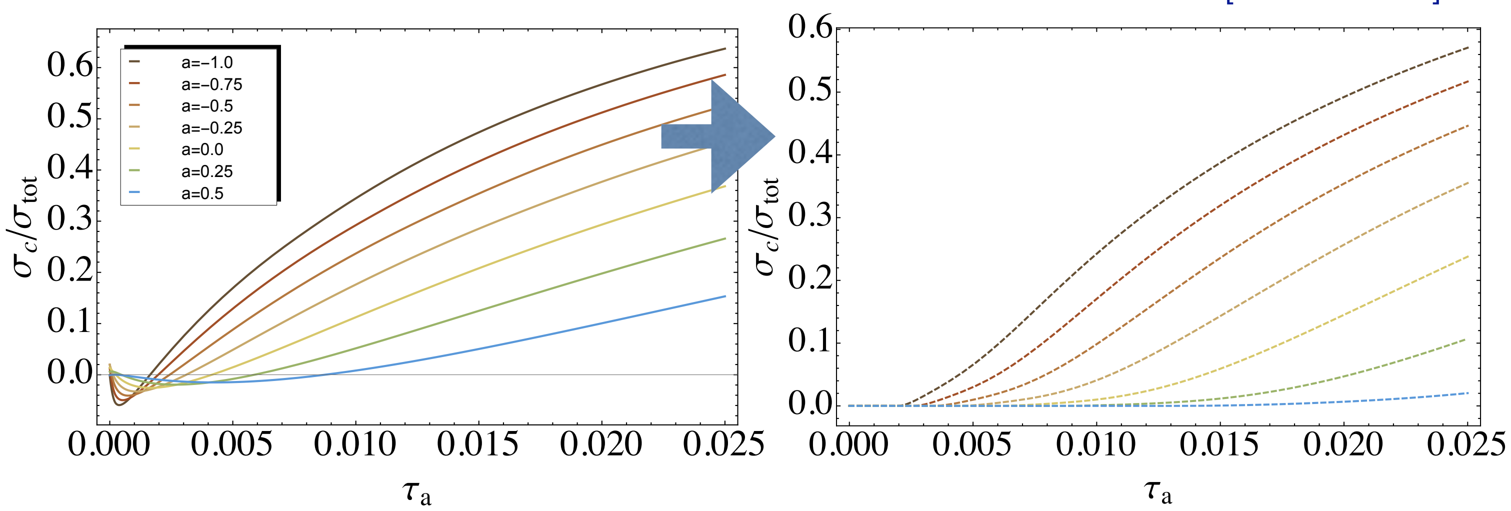
Reverts to fixed-order perturbation theory

- A default fit window will be between **$t_{1'}$ and t_2**, which roughly tracks the tail (former) and far-tail (latter) of the distribution.**

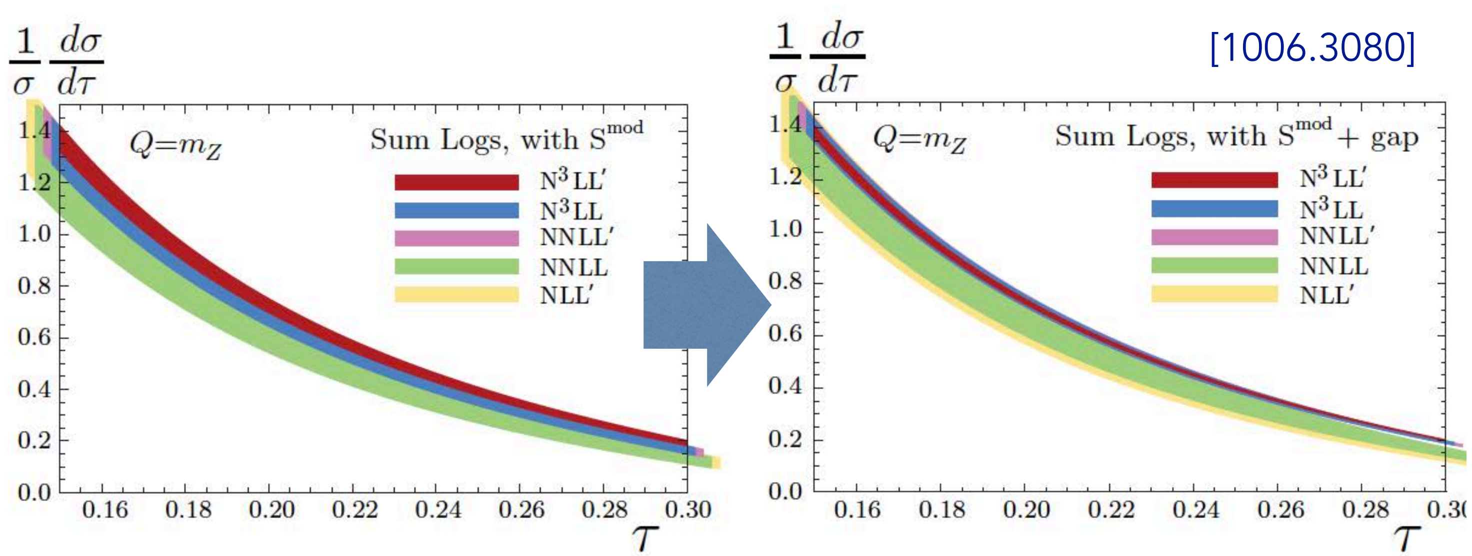
R-Gap phenomenology

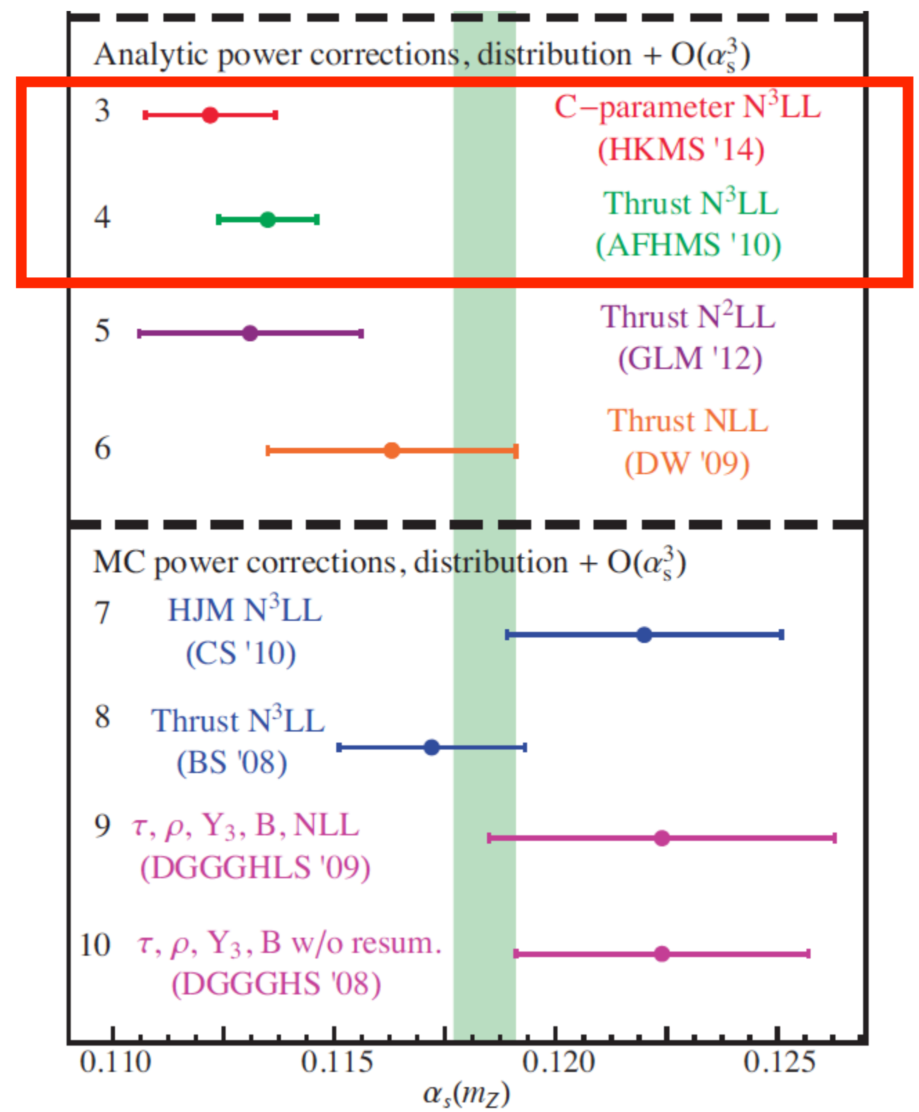
- R-Gap scheme removes unphysical effects in cross-section predictions and gives good qualitative agreement with data:

[1808.07867]



[1006.3080]





see A. Hoang, 2015 workshop on precision α_s

- How non-perturbative effects are implemented (clearly) affects the extraction of the strong coupling!

A naive way to limit the shift…

- Obvious solution is to simply limit the growth of the renormalon scale:

$$\gamma_R \to \theta(R_{\max} - R)\gamma_R \qquad\qquad R = R(\tau)$$

$$\text{need:} \quad \frac{d}{dR}\delta_a(R,R) = \gamma_R[\alpha_s(R)]\theta(R_{\max} - R)$$

$$\text{recall:} \quad \delta_a(R,R) = Re^{\gamma_E}\Big[\frac{\alpha_s(R)}{4\pi}\delta_a^1(R,R) + \Big(\frac{\alpha_s(R)}{4\pi}\Big)^2\delta_a^2(R,R) + \cdots\Big]$$

- Simple solution is to simply set a max value for the R scale:

$$R^* \equiv \begin{cases} R & R < R_{\max} \\ R_{\max} & R \geq R_{\max} \end{cases}$$

$$\delta_a^1(\mu,R) = \Gamma_S^0 \ln\frac{\mu}{R}\,,$$

$$\delta_a^2(\mu,R) = \Gamma_S^0\beta_0 \ln^2\frac{\mu}{R} + \Gamma_S^1 \ln\frac{\mu}{R} + \frac{\gamma_S^1(a)}{2} + c_{\tilde{S}}^1(a)\beta_0$$

Turns off the R-scale at a given (fixed) Rmax (good)

Potentially large logs of μ/R ! (bad)

Angularities: from τ to b

[0303051]
[0801.4569]
[0901.3780]
[1808.07867]

- Consider *Angularities*, which can be defined in terms of the of the rapidity and p_T of a final state particle 'i', with respect to the thrust axis:

IR safe for a ∈ {-∞, 2}!

$$\tau_a = \frac{1}{Q}\sum_i |\mathbf{p}_\perp^i|\, e^{-|\eta_i|(1-a)}$$

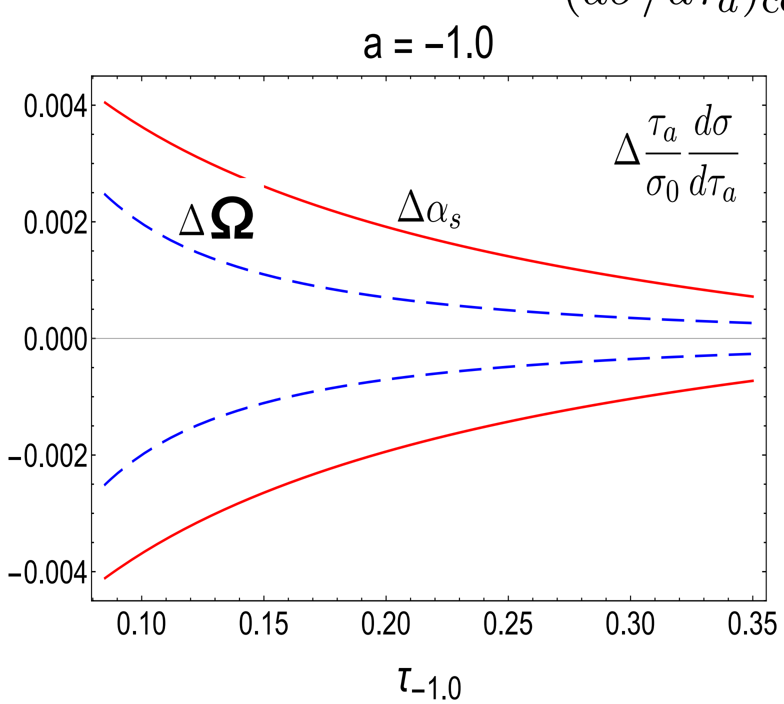
a = 0 <-> `Thrust'

a = 1 <-> `Jet Broadening'

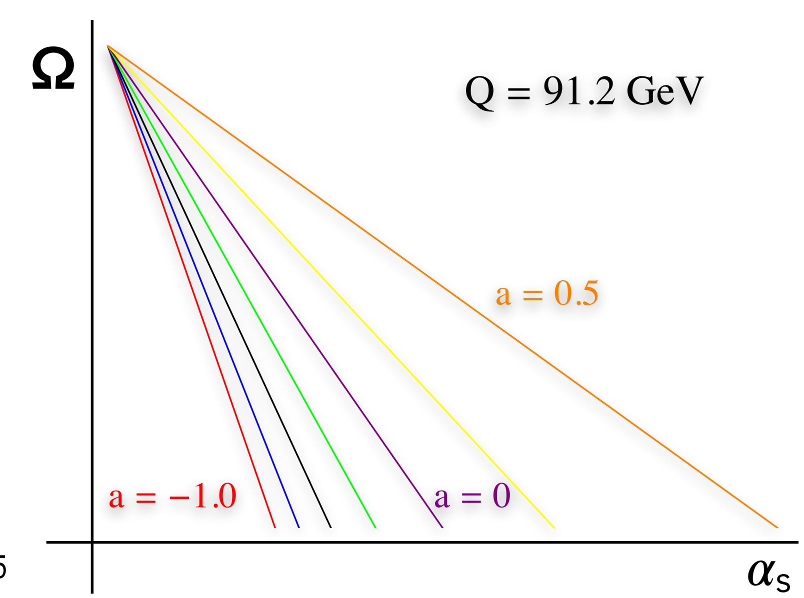
- Leading NP effect is also an (a-dependent (!)) shift of the perturbative distribution:

$$\frac{d\sigma}{d\tau_a}(\tau_a) \xrightarrow[\text{NP}]{} \frac{d\sigma}{d\tau_a}\left(\tau_a - c_{\tau_a}\frac{\Omega_1}{Q}\right) \qquad c_{\tau_a} = \frac{2}{1-a}$$

$(d\sigma/d\tau_a)_{\mathrm{central}} - d\sigma/d\tau_a$



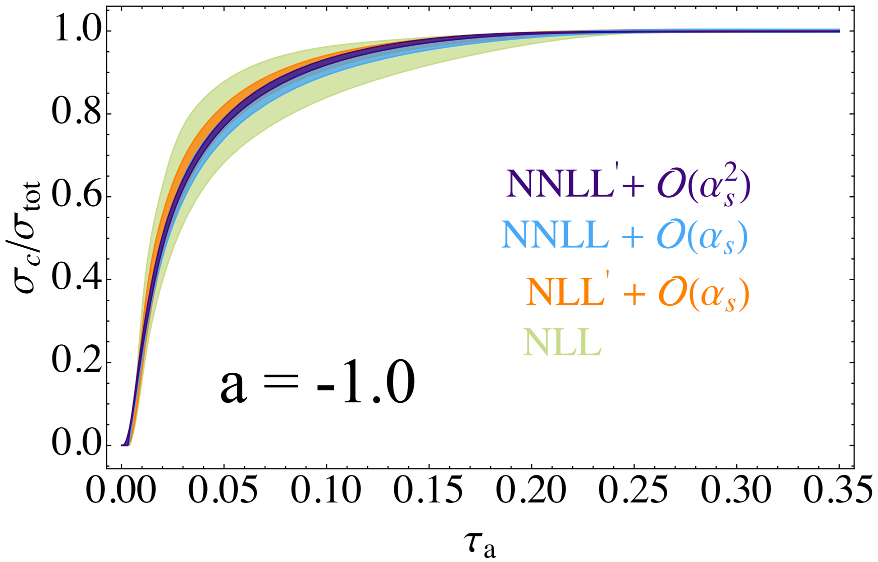
$\Delta slope($ $)$



- Varying $\mathbf{\Omega}$ between 35 and 207 GeV generates same difference as varying a ∈ {-2.0, 0.5} (~6)!!

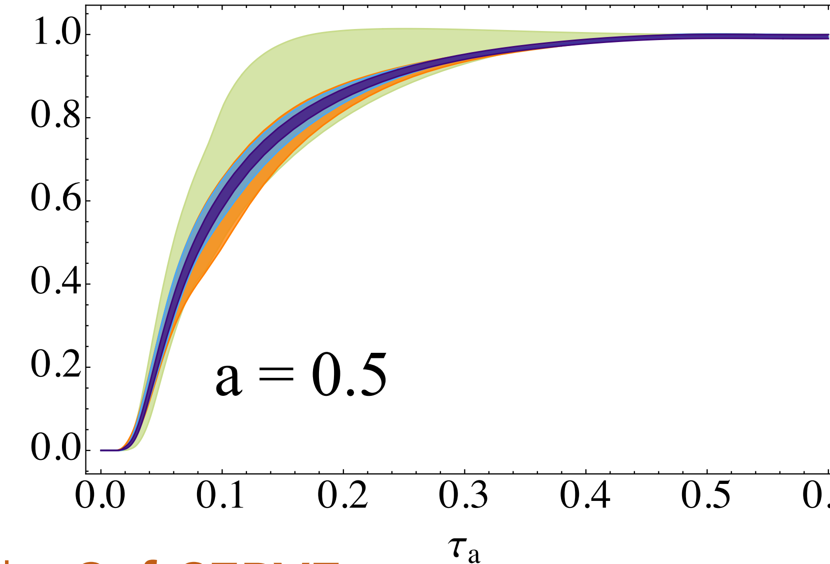
[0901.3780]
[1805.12414]
[1808.07867]
[1812.08690]
[2004.08396]

2018 progress: NLL′ to NNLL′



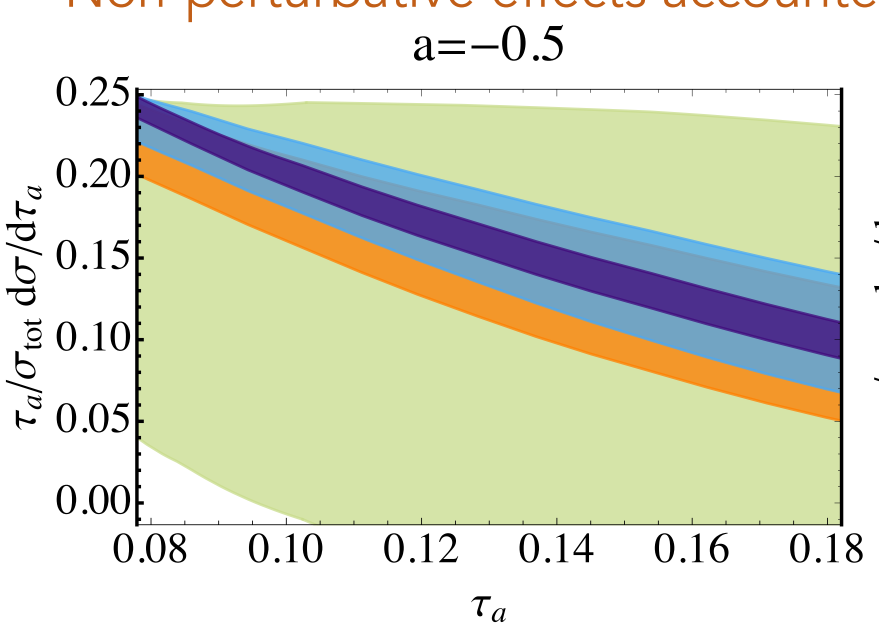
softserve.hepforge.org

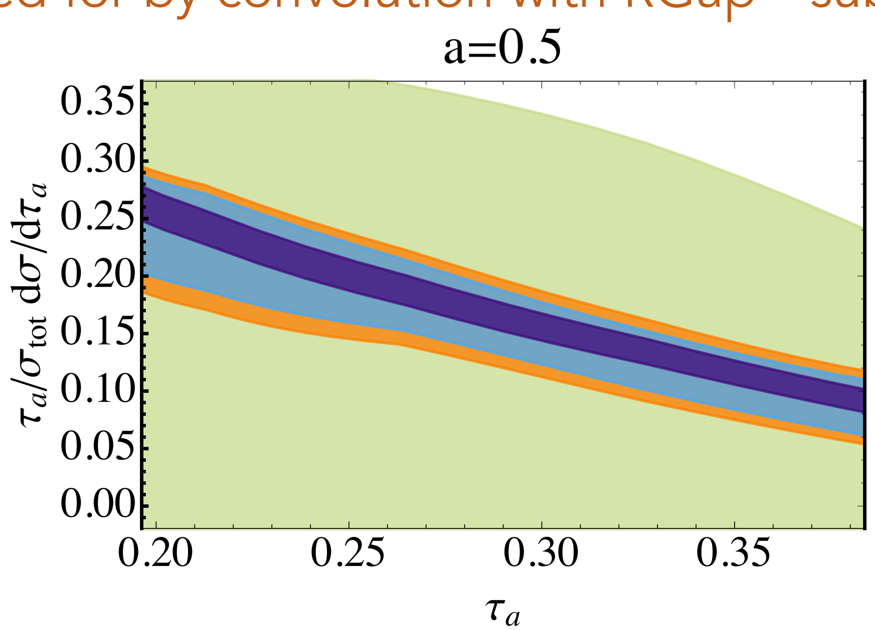
Bell, Rahn & Talbert

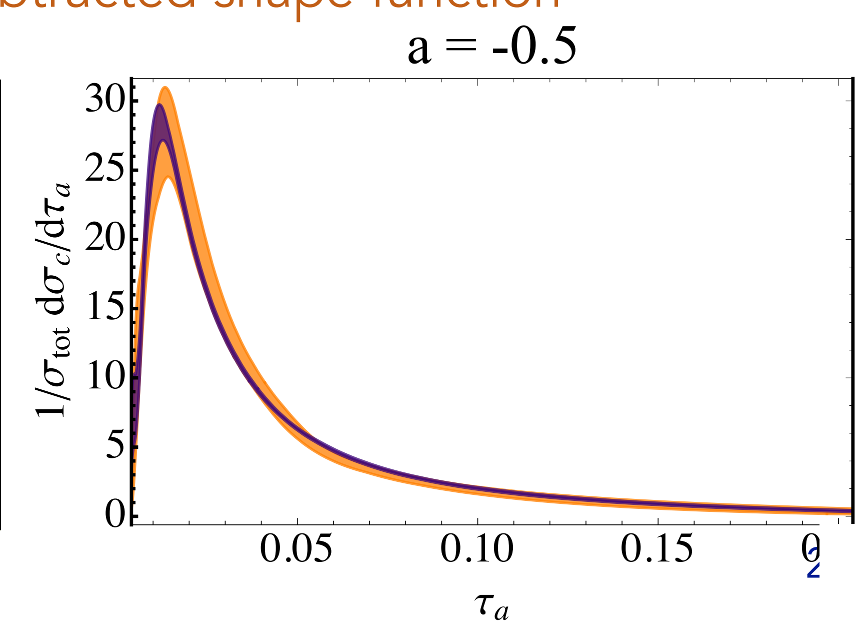


- Two-loop soft anomalous dimensions and singular constants provided by **SoftSERVE**
- Two-loop jet anomalous dimension obtained from consistency relations
- Two-loop singular jet constants extracted from **EVENT2**
- Matching to QCD at $O(\alpha_s^2)$ extracted from **EVENT2 ***
- Includes set of H,J,S, & non-sing. profile scales, tuned for a-dependence, and varied with a random scan over parameters
- Non-perturbative effects accounted for by convolution with RGap—subtracted shape function

Bell, Hornig, Lee & Talbert







The (only) dataset

Generalized event shape and energy flow studies in e^+e^- annihilation at $\sqrt{s} = 91.2\text{-}208.0\,\text{GeV}$

L3 Collaboration

JHEP 10 (2011) 143

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Accepted: *August 24, 2011*
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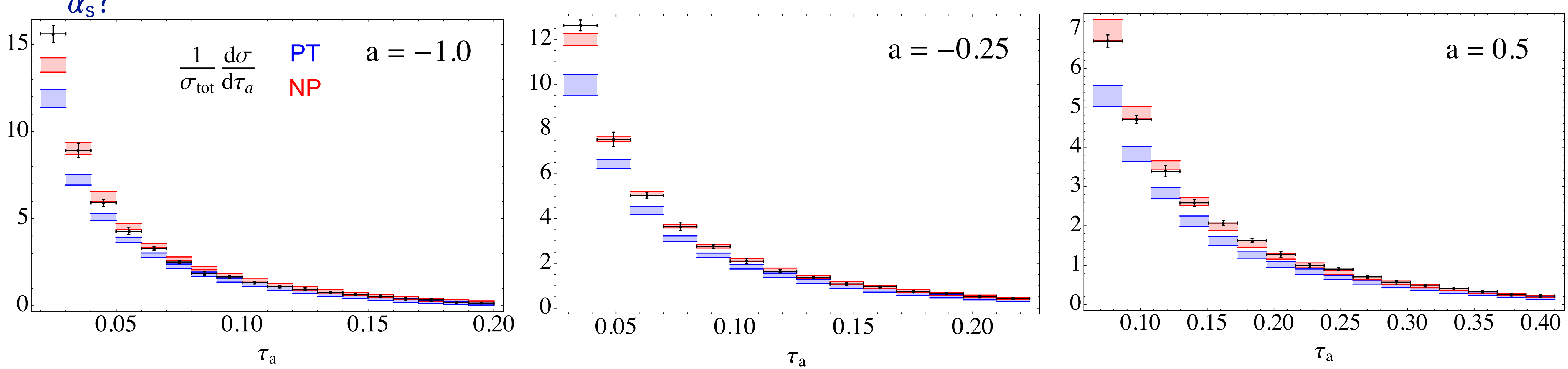
Also see thesis by Pratima Jindal, Panjab University, Chandigarh

- Data for a = {**-1.0**, **-0.75**. **-0.5**, **-0.25**, **0.0**, **0.25**, **0.5**, 0.75} at **91.2** and 197 GeV
- Total number of bins = (bins per a) x (number of a) = 25 x 7 = 175 bins @ Q = 91.2 GeV
- Compare to 404 bins **included** in 2015 C-Parameter fit (across all Q considered)…
- Early theory predictions look good against the data, but what does this translate to for **Ω** and α_s?



BLUE: NNLL′ + $O(\alpha_s^2)$ RED: NNLL′ + $O(\alpha_s^2)$ + NP