A Future Muon-Ion Collider at Brookhaven National Laboratory

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There has been significant discussion in the community regarding a future $\mu^+\mu^-$ collider. While such a facility is still decades away from realization, it is also understood that significant technological development and feasibility demonstrations are necessary at lower beam energies. Here we propose such a possibility coupled with a rich physics program. We propose a future Muon-Ion Collider that would serve as a natural extension to the EIC program currently planned for the 2030’s and 40’s. We envision this collider would be implemented as an upgrade to the EIC, with $\mu$-beam energies between 18 GeV and 200 GeV. In this presentation we discuss the physics reach of such a collider, which could reach $x \approx 10^{-5}$ with a luminosity approaching $5 \times 10^{32}$ cm$^{-2}$ s$^{-1}$. We argue that the physics reach of such a program is excellent and comparable to the LHeC, and it will facilitate accelerator technology development for the future muon collider.


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I. INTRODUCTION

The future Electron-Ion Collider (EIC) aims to collide polarized and unpolarized electron and ion beams to provide an unprecedented look into the structure of nuclei. The EIC will measure electron-proton and electron-ion collisions in the Deep Inelastic Scattering (DIS) regime up to a center-of-mass energy ($\sqrt{s}$) of 140 GeV [1]. In the longer term the Large Hadron-electron Collider aims to measure electron-ion collisions at $\sqrt{s} \approx 1$ TeV [2], and the Future Circular Collider plans to measure electron-hadron collisions at $\sqrt{s} \approx 3.5$ TeV [3].

Independent of the EIC effort, there has been significant discussion in the high-energy particle physics community in favor of a $\mu^+\mu^-$ collider. The physics program for a muon collider has had significant discussion and justification [4–7]. It has been estimated that an $O(10)$ TeV muon collider could have the same discovery potential of an $O(100)$ proton-proton collider [8] but in a significantly smaller footprint and reduced cost. However, the technological challenges in achieving such a collider are immense, requiring significant investment in research and development before such an accelerator complex can be realized. The reference $\mu^+\mu^-$ collider aims to have $\sqrt{s} = 10$ TeV and luminosity $O(10^{34})$ cm$^{-2}$s$^{-1}$ [9].

Here we propose a future muon-Ion Collider ($\mu$IC) that could be constructed at Brookhaven National Laboratory (BNL) at the end of the EIC science program. This collider would serve three important functions: First, it would be an excellent candidate for R&D of the accelerator technologies necessary for achieving a $\mu^+\mu^-$ collider, with less demanding design requirements. Second, it would have an extensive physics reach in its own right, by providing an increased kinematic reach to the EIC, and probing a low-$x$ region not covered by existing lepton-hadron accelerator facilities. Third, it will also provide fertile ground and opportunities potentially comparable to or beyond the needs of the future colliders discussed above for detector R&D to realize its science. With a muon beam radiative corrections can be reduced, so we believe this $\mu$IC could cover a similar kinematic regime as the EIC but with potentially improved systematic uncertainties.

II. A MUON-ION COLLIDER

We envision this collider to be constructed as an upgrade to the EIC, starting with a muon frontend at an energy of 18 GeV to replicate the kinematics of the EIC, followed by increases in muon beam energy up to 200 GeV. Proceeding with upgrades in this manner will provide invaluable insight in the construction challenges of a muon collider, and at the highest energy will provide an order of magnitude improvement in the $x$ reach compared to the EIC. For this work we do not suggest any significant upgrades to the ion beam provided by the EIC.

There have been similar proposals for a $\mu$IC with significantly higher beam energies [10], however we believe that there is a serious need to prove the R&D challenges can be overcome at lower energies before advancing to a high-energy collider.

A. Generating Muon Beams

The generation of muons is of crucial importance to the $\mu$IC. There are two widely discussed methods of generating muons when considering a high energy muon collider.

One method is a proton driven method, where protons impinge on a target material, e.g. carbon, and produce charged pions. These pions are generated with a momentum of a few hundred MeV/c and with a relatively large emittance and transverse momentum. The muons produced from the pion decays would need to be cooled and focused before being accelerated to the $\mu$IC design energies, making it difficult to achieve high luminosities. The MICE experiment recently demonstrated that muons produced via this method can be cooled and potentially be the source for a high-brightness muon beam [11]. However, a muon source capable of producing $10^{11}$ muons per bunch with a final beam energy above 100 GeV, and a luminosity sufficient to meet the $\mu^+\mu^-$ collider requirements has not been demonstrated in the lab.

It is worth noting that muons generated from this method are naturally polarized. The possibility of maintaining this polarization should be a high priority for future muon colliders. In the case of the $\mu$IC, this could allow for the replication of the EIC doubly polarized beams at higher lepton beam energies.

A second method of muon generation is electron-positron annihilation, in which a 45 GeV positron beam impinges on an electron target [12]. This method produces muon pairs with an energy of 22 GeV, small emittance, and a laboratory lifetime of nearly 500 $\mu$s. While this method is still under conceptual study [13], it requires a large, potentially prohibitive, positron bunch charge in order to produce the number of muons necessary for the target luminosity.
Both methods of generating muons provide positive and negatively charged muons from the production physics. In the case of the proton driven method, more positively charged muons are produced. However, if the magnets in the muon beam line are designed to have reversible currents, it would be possible to operate the µIC with both muon charge polarities. This would provide access to physics beyond the current scope of the EIC.

B. Muon Decay

While the advantages of a muon beam are clear, a significant challenge is the beam-induced background. Muon decays in the beamline must be studied in detail, and there will need to be significant upgrades to the EIC interaction region in order to minimize this background. The Muon Accelerator Program suggested a vertical chicane to minimize muon decay byproducts in the interaction region [14]. Additionally a tungsten nozzle is added in the interaction region in order to absorb decay products from the last few meters of beamline after the vertical chicane. While such a decay background is necessarily twice as significant at a µ⁺µ⁻ collider, a µIC would provide a natural proving ground for the construction techniques and background simulation.

At the beam intensities proposed for the µ⁺µ⁻ collider, the radiation dose from neutrinos becomes a hazard to both the accelerator and the detectors. The lower intensity µIC provides a slightly more forgiving environment, but any straight sections of the accelerator tunnel could prove dangerous. It has been suggested [10] that constructing the muon beamline slightly non-parallel to the ground could send the majority of neutrinos into deep space or through the earth.

C. Luminosity

The luminosity of µ-proton interactions can be written as

$$L_{\mu p} = \frac{N_{\mu} N_{p} \min[f_{\mu}, f_{p}]}{4\pi \max[\sigma_{x}, \sigma_{y}] \max[\sigma_{\mu}, \sigma_{p}]}$$

with $\sigma_{x, y} = \sqrt{\beta \cdot \epsilon / \gamma}$ and $f_{\mu, p} = N_{\text{laps}} \cdot f_{\text{rep}}$ [15]. Here $N_{\mu, p}$ is the number of muons or protons in each bunch, $\gamma$ is the boost factor, $\epsilon$ is the beam emittance, $\beta$ is the value of the betatron function at the interaction point, $\sigma_{x, y}$ is the transverse width of the beam, $N_{\text{laps}}$ is the average number of laps made before the particle decays, and $f_{\text{rep}}$ is the repetition rate of the beam. The current estimates indicate that µ beams will generally have a larger transverse width than proton beams due to the larger beam emittance. An estimate of the achievable luminosity is shown in Table I.

The luminosity estimate here is lower than that of the baseline µ⁺µ⁻ collider. This is due to the lower muon beam energy. At higher beam energies the muons are boosted more in the direction of the beamline. A higher energy would corresponds to a larger $\gamma$-factor which reduces the size of the transverse beam distribution. Furthermore, the smaller boost at lower energy corresponds to a shorter muon lifetime in the lab frame. The muon accelerator system would incorporate spin polarization selection which could lower the achievable luminosity by a factor of 20 to achieve 60% polarization [16].

<table>
<thead>
<tr>
<th>$E$ (GeV)</th>
<th>200</th>
<th>275</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\mu, p}$ ($10^{11}$)</td>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2000</td>
<td>275</td>
</tr>
<tr>
<td>$\epsilon$ ($\mu$m)</td>
<td>25</td>
<td>0.2</td>
</tr>
<tr>
<td>$\beta$ (cm)</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>$\sigma_{x, y}$ ($\mu$m)</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Number of laps</td>
<td>680</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$f_{\mu}$ (s⁻¹)</td>
<td>10,350</td>
<td>N/A</td>
</tr>
<tr>
<td>$L_{\mu p}$ (cm⁻²·s⁻¹)</td>
<td>$5 \times 10^{32}$</td>
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TABLE I. Luminosity of unpolarized $\mu p$ collisions.
III. KINEMATIC REACH

In the first stage of the $\mu$IC we would operate with a muon beam energy of 18 GeV. The kinematic reach should be nearly identical to EIC at this beam energy, with the exception of corrections due to the larger muon mass. Here we could investigate backgrounds from muon beams in the most challenging experimental regime, while comparing with existing data. This regime would have the lowest luminosity due to the larger transverse size of the muon beam as the forward boost is smaller, and a shorter muon lab lifetime.

The beam energy would later be upgraded in stages, from 18 GeV to 50, 100, and finally 200 GeV. At the final stage we would have access to a large range in $x$, down to nearly $10^{-5}$. The kinematic range of the $\mu$IC is calculated with $Q^2 = s x y$, where $Q^2$ is the four-momentum transfer, $x$ is the Bjorken-$x$, and $y$ is the inelasticity, taken to be $0.95 > y > 0.01$. The $Q^2 - x$ coverage is shown for the EIC, LHeC, and the $\mu$IC, in Fig. 1 for lepton-proton and lepton-gold interactions.

A particular challenge is detecting the muon after interaction. As can be seen in Fig. 2, muons can scatter from beam particles with $\eta = -6$, which corresponds to a deflection angle of 0.28°. The technical challenges in detecting a particle with such a small scattering angle should not be understated. The technical design for such a detector system must be developed further.

![Fig. 1. A comparison of the $Q^2$ and $x$ reach of the $\mu$IC, EIC, and LHeC for lepton-proton collisions (left) and lepton-gold collisions (right). The blue line corresponds to $Q^2 = 1$.](image1)

![Fig. 2. The $Q^2$ and $x$ reach of the $\mu$IC for lepton-proton collisions (left) and lepton-gold collisions (right). The black curves correspond to constant $\eta$ for the scattered $\mu$. The blue line corresponds to $Q^2 = 1$.](image2)

It is worth emphasizing the benefit that comes from an additional order of magnitude reach in $x$ in the $\mu$IC compared to the EIC. As a brief example, the lever arm in $Q^2$ and smaller $x$ enables tests of the proton polarized structure function $g_1(x, Q^2)$ in a region where it is predicted to be both large and rapidly changing. The EIC will measure $g_1$ where it is predicted to be slowly varying and nearly zero [17]. Measuring at a smaller $x$ that is available at a $\mu$IC would strongly constrain the shape of $g_1$. 
Additionally the charged-current cross section, which grows with $\sqrt{s}$, is experimentally accessible in the large $Q^2 (> 100 \text{ (GeV/c)}^2)$ regime. The exchange of a virtual $W^\pm$-boson provides a theoretically clean access to flavor dependent structure functions. Doubly polarized beams at the $\mu$IC could test polarized flavor dependent structure functions at a $\sqrt{s}$ beyond what was achievable at HERA [18].

A. Saturation Scale

With the proposed muon beam and the future ion beam at the EIC, it will be possible to probe saturation physics. Figure 3 shows the $Q^2_S$ reach of HERA, the EIC, and the $\mu$IC. $Q^2_S$ sets the gluon saturation scale, and defines the scale at which perturbative QCD calculations of nuclear structure functions can be performed in the Color Glass Condensate framework. It is worth noting that the $\mu$IC is able to probe saturation physics in $\mu p$ collisions, using the GBW model [19] for $Q^2_S$, which is likely not possible at the EIC in electron-proton collisions. The difference in the lower bound of the saturation $Q^2$ scale in Fig. 3 comes from the factor $A^{1/3}$ enhancement of saturation in large nuclei; here we consider the case of gold nuclei. Access to saturation inside the proton would allow for a precision test of the universality of gluon interactions in single- and multi-particle systems at very low $x$ [20].

$$x_B \leq 0.01$$

![Diagram](image_url)

FIG. 3. The saturation $Q^2_S$ reach for HERA, the EIC, and the $\mu$IC, for lepton-proton and lepton-gold collisions. The reach is calculated from the GBW model and limits the maximum value $x_B \leq 0.01$.

IV. SUMMARY

The high energy physics community has expressed a significant desire to realize a high energy, high luminosity $\mu^+\mu^-$ collider. Currently there is no source of muons that meets the design requirements of the future $\mu^+\mu^-$ collider. Significant R&D work needs to be done to demonstrate such a source of muons. The site of the future EIC would provide a useful location to perform this R&D work. By replacing the electron beam line with a muon beam line a rich physics program could be developed that would be of significant interest to the nuclear physics community. The EIC interaction region would be significantly modified to remove muon decay backgrounds using a vertical chicane and a tungsten nozzel. Detailed studies need to be performed to determine whether the muon beam line will fit into the RHIC tunnel, or if a new cavity would need to be built.

The $\mu$IC proposed here could reach almost an order of magnitude lower in $x$ than is accessible at the EIC. The enhanced reach into the low-$x$ regime allows for the study of saturation in protons and nuclei, as well as detailed study of the spin structure of the proton and parton distribution functions in collective systems.
V. ACKNOWLEDGEMENTS

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