QCD at the future circular $e^+e^-\text{ collider (FCC-ee)}$

DIS2022
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Eduardo Ploerer (on behalf of the FCC collaboration)

Many thanks to David d’Enterria for input material
QCD = Key piece at future ee, pp colliders

- QCD is crucial for many pp, ee measurements (signals & backgrounds):
  - High-precision $\alpha_s$: Affects all $x$-sections & decays (esp. Higgs, top, EWPOs).
  - $N^n$LO, $N^n$LL resummations: Affects all pQCD $x$-sections & decays.
  - High-precision PDFs: Affects all precision $W,Z,H$ (mid-$x$) measurements & all searches (high-$x$) in pp collisions.
  - Heavy-Quark/Quark/Gluon separation (jet substructure, boosted topologies..): Needed for all precision SM measurements & BSM searches with final jets.
  - Non-perturbative QCD: Affects final-states with jets: Colour reconnection, $e^+e^- \to Z,WW$, $ttbar \to l+4j,6j...$ ($m_W, m_{top}$ extractions). Parton hadronization,
**Precision QCD in $e^+e^-$ collisions**

- $e^+e^-$ collisions provide an extremely clean environment with fully-controlled initial-state to very precisely probe q,g dynamics:

**Advantages compared to p-p collisions:**
- QED initial-state with known kinematics
- Controlled QCD radiation
- Well-defined quark, gluon jets
- Smaller non-pQCD uncertainties:
  - no PDFs, no QCD “underlying event”,…

Direct clean parton fragmentation & hadroniz.

\[\sqrt{s} \sim 91 \text{ GeV}\]
\[\sqrt{s} \sim 160 \text{ GeV}\]
\[\sqrt{s} \sim 240 \text{ GeV}\]
Precision QCD in $e^+e^-$ collisions

$e^+e^-$ collisions provide an extremely clean environment with fully-controlled initial-state to very precisely probe q,g dynamics:

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Direct clean parton fragmentation & hadroniz.
Future $e^+e^-$ colliders under discussion

- FCC-ee features lumis a few times larger than other machines over 90–300 GeV
- Unparalleled Z, W, jets, $\tau$, … data sets: Negligible stat. uncertainties
QCD physics at FCC-ee

(1) QCD coupling

(2) Quark-gluon tagging & jet substructure

(3) Non-perturbative QCD

NOTE: Only UNIQUE QCD measurements, inaccessible at any current machine, are covered.
**QCD coupling \( \alpha_s \)**

- Determines **strength of the strong interaction** between quarks & gluons.
- Determined at a ref. scale \( Q=m_Z \), decreases as \( \alpha_s \sim \ln(Q^2/\Lambda^2)^{-1} \) \( \Lambda \sim 0.2 \text{ GeV} \)

Least precisely known of all interaction couplings!

\[
\delta \alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \ll 10^{-5} \ll \delta \alpha_s \sim 10^{-3}
\]
World $\alpha_s$ determination (PDG today)

- Determined today by comparing 7 experimental observables to pQCD NNLO, N$^3$LO predictions, plus global average at the Z pole scale:

1. $\tau$ decays
2. Lattice
3. QQbar
4. PDFs
5. $e^+e^-$ jets (shapes, rates)
6. $Z, W$ decays
7. $pp\to t\bar{t}$ collisions

$$\alpha_s(M_Z^2) = 0.1179 \pm 0.0010$$
\( \alpha_s \) from hadronic \( \tau \)-lepton decays

- Computed at N\(^3\)LO:
  \[
  R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = S_{EW} N_C (1 + \sum_{n=1}^{4} c_n \left( \frac{\alpha_s}{\pi} \right)^n + \mathcal{O}(\alpha_s^5) + \delta_{np})
  \]

- Experimentally: \( R_{\tau,\text{exp}} = 3.4697 \pm 0.0080 \) (±0.23%)

- Various pQCD approaches (FOPT vs CIPT) & treatment of non-pQCD corrections, yield different results.

Uncertainty slightly increased:
2013 (±1.3%) → 2019 (±1.5%)

\[ \alpha_s(m_\tau) = 0.1187 \pm 0.0018 \) (±1.5%)

- Future:
  - TH: Better understanding of FOPT vs CIPT differences.
  - Better spectral functions needed (high stats & better precision):
    - B-factories (BELLE-II)?
  - High-stats: \( \mathcal{O}(10^{11}) \) from \( Z \rightarrow \tau\tau \) at FCC-ee(90)
\( \alpha_s \) from e\(^+\)e\(^-\) event shapes & jet rates

- Computed at \( N^{2,3}\text{LO} + N^{(2)}\text{LL} \) accuracy.

- Experimentally (LEP):
  - Thrust, C-parameter, jet shapes
  - n-jet x-sections

- Results sensitive to non-pQCD (hadronization) accounted for via MCs or analytically (with some disagreement)

\[
\tau = 1 - \max_{\hat{n}} \frac{\sum i |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}
\]

\[
C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}
\]

\( \alpha_s(m_Z) = 0.1171 \pm 0.027 \) (\( \pm 2.6\% \))

- Future:
  - FCC-e\(^+\)e\(^-\): Lower-\( \sqrt{s} \) (ISR) for shapes, higher-\( \sqrt{s} \) for jet rates
  - TH: Improved \( (N^{2,3}\text{LL}) \) resummation for rates, hadronization for shapes

\( \delta \alpha_s / \alpha_s < 1\% \)
\( \alpha_s \) from hadronic Z decays (FCC-ee)

- QCD coupling extracted from:
  (i) Combined fit of 3 Z pseudo-observ:
  (ii) Full SM fit (with \( \alpha_s \) free parameter)

- FCC-ee:
  - Huge Z pole stats. (\( \times 10^5 \) LEP)
  - Exquisite systematic/parametric precision (stat. uncert. much smaller):
  - TH uncertainty reduced by \( \times 4 \) computing missing \( \alpha_s^5, \alpha^3, \alpha\alpha_s^2, \alpha\alpha_s, \alpha_2 \alpha_s \) terms

- 10 times better precision than today:
  \( \delta \alpha_s / \alpha_s \sim \pm 0.2\% \) (tot), \( \pm 0.1\% \) (exp)
  Strong (B)SM consistency test.

\[ \alpha_s(m_Z) = 0.1203 \pm 0.0028 \ (\pm 2.3\%) \]
**\( \alpha_s \) from hadronic W decays (FCC-ee)**

- QCD coupling extracted from **new** **\( N^3\)LO** fit of combined \( \Gamma_w, R_w \) pseudo-observ.:  
  
  - **Very imprecise extraction:**  
    - Large parametric uncert. from poor \( V_{cs} \) exp. precision (±2%):  
      - QCD coupling unconstrained: 0.04±0.05  
    - Imposing CKM unitarity: **large exp. uncertainties** from \( \Gamma_w, R_w \) (0.9–2%):  
      - QCD extracted with ~27% precision  
    - Propagated TH uncertainty much smaller today: ~1.5%

- **FCC-ee extraction:**  
  - Huge W pole stats. (×10⁴ LEP-2).  
  - Exquisite syst./parametric precision:  
  - TH uncertainty reduced by ×10 after computing missing \( \alpha_s^5, \alpha^2, \alpha^3, \alpha^2 \alpha_s, \alpha_s^2, \alpha^2 \alpha_s \) terms

\[
\alpha_s(m_z) = 0.101 \pm 0.027 \text{ (±27%)}
\]

\[
\alpha_s(m_z) = 0.11790 \pm 0.00023 \text{ (±0.2%)}
\]
QCD physics at FCC-ee

(1) QCD coupling

(2) Quark-gluon tagging & jet substructure

(3) Non-perturbative QCD

**NOTE:** Only UNIQUE QCD measurements, inaccessible at any current machine, are covered.
Quark-gluon discrimination

- Exciting but challenging prospect in pp collisions
  - Enhance quark signal at hadron colliders
    (e.g. VBF, ttH hadronic W’s, hadronic W/Z+jets)
  - Multijet BSM final states

- Several handles exist to separate quarks and gluons (in principle):
  - Gluons radiate more $C_F = 4/3 < C_A = 3$
  - Spin correlations in subjet location
  - $p_T$-weighted jet charge

- ML approaches have already found success
  - unclear how much we can trust gluon disc. presently

[Cornelis CMS, arxiv: 1409.3072]

[see J.Gallicchio & M.Schwartz, 1211.7038 [hep-ph]]

[F.Bedeschi, L.Gouskos, M.Selvaggi, 2202.03285 [hep-ex]]
Jet substructure

- Need for state-of-the-art jet substructure studies based on angularities

- Variables of jet constituents: multiplicity, LHA, width/broadening, mass/thrust, C-parameter,...

- $k=1$: IRC-safe computable ($N^n\text{LO}+N^n\text{LL}$) via SCET (but uncertainties from non-pQCD effects)

\[
\lambda_{\beta}^\kappa = \sum_{i \in \text{jet}} z_i^\kappa \theta_i^\beta,
\]
(normalized $E^n \times \theta^n$ products)

[larkoski,salam,thaler,13]
[larkoski,thaler,waalewijn,14]
Les Houches Angularity (LHA) is angularity w/ k=1, B=0.5

Not directly measured at LEP

MC parton showers differ on gluon (less so quark) radiation patterns:

Showering Differences in Generators

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MC parton showers differ on gluon (less so quark) radiation patterns:
High-precision gluon & quark jet studies

- Exploit FCC-ee $H(gg)$ as a "pure gluon" factory: $H \rightarrow gg$ (BR~8% accurately known) provides 120,000 extra-clean digluon events.

- Multiple handles to study gluon radiation & g-jet properties:
  - Gluon vs. quark via $H \rightarrow gg$ vs. $Z \rightarrow qq$
  - Gluon vs. quark via $Z \rightarrow bbg$ vs. $Z \rightarrow qq$

- Multiple high-precision analyses possible:
  - BSM: Improve $q/g/Q$ discrimination tools
  - pQCD: High-precision QCD coupling
  - non-pQCD: Gluon fragmentation, Colour reconnection

Improved MC tuning
QCD physics at FCC-ee

(1) QCD coupling

(2) Quark-gluon tagging & jet substructure

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**NOTE**: Only UNIQUE QCD measurements, inaccessible at any current machine, are covered.
Colour reconnection of partons impacts final state kinematics (shifted angular correlations, invariant mass shifts)

- Exact dynamics poorly understood
- Source of uncertainty in $m_W$, $m_{\text{top}}$, (aGC extractions) in multijet final-states (especially in pp: MPI cross-talk)

- CR impacts all FCC-ee multi-jet final-states:
  - $e^+e^- \rightarrow WW(4j), Z(4j), \text{ttbar,}…$
  - String-drag effect on $W$ mass (hinted at LEP)

- Exploit huge $W$ stats ($\times 10^4$ LEP) to measure $m_W$ leptonically & hadronically and constrain CR in hadronic WW.
Detailed hadronization studies

- High-precision low-$p_T$ PID hadrons in $e^+e^-$ required for detailed studies:
  - Baryon & strangeness production. **Colour string dynamics.**
  - Final-state correlations (spin: BoseEinstein, FermiDirac; momenta; space)
  - Bound state formation: quarkonia, multi-quark states, glueballs, ...

- Understand breakdown of universality of parton hadronization observed at LHC.

- Baseline vacuum $e^+e^-$ studies for high-density QCD
Summary: QCD at future e⁺e⁻ colliders

The precision needed to fully exploit all future ee/pp/ep/eA/AA SM & BSM programs requires exquisite control of pQCD & non-pQCD physics.

Unique QCD precision studies accessible at FCC-ee (CEPC, ILC):

(1) Per-mille $\alpha_s$ via hadronic Z,W,τ decays, evt shapes...

(2) $N^n$LO+$N^n$LL jet substructure

(3) Improved parton showering

(4) High quark-gluon discrimination

(5) <1% control of colour reconnection

(6) High-precision hadronization
Backup slides
High-precision parton FFs

Parton-to-hadron fragment functions evolution known at NNLO at high-z & at NNLO*+NNLL at low-z:

<table>
<thead>
<tr>
<th>Method</th>
<th>Current $\delta \alpha_s(m_Z^2)/\alpha_s(m_Z^2)$ uncertainty (theory &amp; experiment state-of-the-art)</th>
<th>Future $\delta \alpha_s(m_Z^2)/\alpha_s(m_Z^2)$ uncertainty (theory &amp; experiment progress)</th>
</tr>
</thead>
<tbody>
<tr>
<td>soft FFs</td>
<td>$1.8%<em>{\text{th}} \oplus 0.7%</em>{\text{exp}} \approx 2%$ (NNLO* only (+NNLL), npQCD small)</td>
<td>$0.7%<em>{\text{th}} \oplus 0.7%</em>{\text{exp}} \approx 1%$ (~2 yrs), $&lt;1%$ (FCC-ee) (NNLO+NNLL. More precise $e^+e^-$ data: 90–350 GeV)</td>
</tr>
<tr>
<td>hard FFs</td>
<td>$1%<em>{\text{th}} \oplus 5%</em>{\text{exp}} \approx 5%$ (NLO only. LEP data only)</td>
<td>$0.7%<em>{\text{th}} \oplus 2%</em>{\text{exp}} \approx 2%$ (+B-factors), $&lt;1%$ (FCC-ee) (NNLO. More precise $e^+e^-$ data)</td>
</tr>
</tbody>
</table>

FCC-ee (much broader z range) allows for $\alpha_s$ extraction with $\delta \alpha_s < 1\%$
There are few other classes of $e^+e^-$ observables, computed today at lower accuracy (NLO, NNLO*), that can be used to extract the QCD coupling:

1. $\tau$ decays
2. Lattice
3. QQbar
4. PDFs
5. $e^+e^-$ jets (shapes, rates)
6. $Z$, $W$ decays
7. $p\bar{p} \rightarrow t\bar{t}$

Other $\alpha_s$ extractions (not yet in world average)
\( \alpha_s \) from photon QCD structure function (NLO)

- Computed at NNLO:
  \[
  \int_0^1 dx F_2^\gamma(x, Q^2, P^2) = \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \left\{ \frac{4\pi}{\alpha_s(Q^2)} c_{LO} + c_{NLO} + \frac{\alpha_s(Q^2)}{4\pi} c_{NNLO} + \mathcal{O}(\alpha_s^2) \right\}
  \]

- Poor \( F_2^\gamma(x, Q^2) \) experimental measurements:

- Extraction (NLO) with large exp. uncertainties today:
  \[
  \alpha_s(m_Z) = 0.1198 \pm 0.0054 \quad (\pm 4.5\%)
  \]
  [M. Klasen et al., PRL89 (2002)122004]

- Future prospects:
  - Fit with NNLO \( F_2^\gamma \) evolution (ongoing)
  - Better data badly needed: Belle-II?
  - Dedicated simul. studies at ILC exist:
  - Huge \( \gamma\gamma \) (EPA) stats at FCC-ee will lead to: \( \delta \alpha_s / \alpha_s < 1\% \)

[R. Nisius, arXiv:0907.2782]
\( \alpha_s \) extractions from jet fragmentation (NLO, NNLO*)

- **Soft parton-to-hadron FFs (NNLO*+NNLL):**

  \[ \alpha_s(m_Z) = 0.1205 \pm 0.0022 \]  

  (±2%)

- **Hard parton-to-hadron FFs (NLO):**

  \[ \alpha_s(m_Z) = 0.1176 \pm 0.0055 \]  

  (±4.7%)

---

**Combined fit of the jet-energy evolution of the FF moments**

- Multiplicity, peak, width, ...
- with \( \alpha_s \) as single free parameter:

  \[ \alpha_s(m_Z) = 0.1205 \pm 0.0022 \]  

  (±2%)

---

**Figure 3:** Energy evolution of the charged-hadron multiplicity (left) and of the FF peak position (right) measured in \( e^+e^- \) and DIS data fitted to the NNLO*+NNLL predictions. The obtained \( \chi^2_{\text{obs}} \) normalization constant, individual NNLO* \( \alpha_s(m_Z) \) values, and the goodness-of-fit per degree-of-freedom \( \chi^2/\text{ndf} \).
QCD uncertainties on EWK observables

- With $\times 10^5$ more $Z$'s than LEP, EWK uncertainties at FCC-ee will be dominated by syst. (QCD).
  Example: $e^+e^- \rightarrow bb$ forward–backward asymmetry
  - 8 measurements at LEP:
    4 lepton-based, 4 jet-charge-based
  - Exp. observable with largest discrepancy today wrt. the SM: 2.8$\sigma$

- Exp. Uncertainties: ~1.6%
  - Statistical: ±1.5% (~0.05% at FCC-ee)
  - Systematics: ±0.6% (QCD-related: ±0.4%)

- QCD effects on $A_{FB}^{0,b}$ (depending strongly on exp. selection procedure):
  - Gluon splitting (TH control: $\alpha_s^2$ corrections)
  - Smearing of $b$-jet/thrust axis
  - $b$ and $c$ radiation & fragmentation. B and D decay models.
    [Uncertainties estimated by Abbaneo et al., EPJC 4 (1998)]

- We have revisited the impact of QCD effects on $A_{FB}^{0,b}$ implementing original analyses in up-to-date retuned parton-shower+hadronization MCs
Reduced QCD uncertainties on $A_{FB}$ at Z pole

- QCD uncertainties recomputed from PYTHIA8.226 (7 tunes) & VINCIA2.2.

- $\epem \rightarrow b\bar{b}$ forward–backward asymmetry for lepton-based analyses:

- $\epem \rightarrow b\bar{b}$ forward–backward asymmetry for jet-charge-based analyses:

- 2020 vs. 1998 parton shower+hadronization uncertainties:
  - Lepton-based: Consistent for ALEPH, slightly smaller for DELPHI, L3, OPAL.
  - Jet-charge-based: Much smaller for all experiments.

- Improved PS & non-pQCD tunes w/ $\epem$ data needed to reduce syst. uncert.
Ultra-precise W, Z, top physics at FCC-ee

\( \sqrt{s} = 91 \text{ GeV}, 10^{12} \text{ Z's} \)  
\( \sqrt{s} = 161 \text{ GeV}, 10^8 \text{ W's} \)  
\( \sqrt{s} = 350 \text{ GeV}, 10^6 \text{ tops} \)

- Lineshape
  - Exquisite \( E_{\text{beam}} \) (unique!)
  - \( m_{Z^\prime}, \Gamma_{Z^\prime} \) to 10 keV (stat.)
  - \( 100 \text{ keV (syst.)} \)
- Asymmetries
  - \( \sin^2 \theta_w \) to \( 5 \times 10^{-6} \)
- Branching ratios, \( R_{\ell}, R_b \)
  - \( \alpha_s(m_Z) \) to 0.0002
- Predict \( m_{\text{top}}, m_W \) in SM

- Threshold scan
  - \( m_W \) to 500 keV
- Branching ratios \( R_{\ell}, R_{\text{had}} \)
  - \( \alpha_s(m_W) \) to 0.0002
- Radiative returns \( e^+ e^- \rightarrow \gamma Z \) (\( Z \rightarrow \nu \nu, \mu^+ \mu^- \))
  - \( N_\nu \) to 0.001

Mostly thanks to:
(i) Huge statistics
(ii) Threshold scans with \( \delta E_{\text{cm}} \sim 0.1, 0.3, 2., 4. \text{ MeV} \) (Z,W,H,t)
Importance of the QCD coupling $\alpha_s$

- Impacts all QCD x-sections & decays (H), precision top & parametric EWPO:

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sigma$ (pb)</th>
<th>$\delta\alpha_s$ (%)</th>
<th>PDF + $\alpha_s$ (%)</th>
<th>Scale (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ggH</td>
<td>49.87</td>
<td>± 3.7</td>
<td>-6.2 + 7.4</td>
<td>-2.61 + 0.32</td>
</tr>
<tr>
<td>ttH</td>
<td>0.611</td>
<td>± 3.0</td>
<td>± 8.9</td>
<td>-9.3 + 5.9</td>
</tr>
</tbody>
</table>

Summary of future parametric uncertainties:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>FCC-ee</th>
<th>future</th>
<th>param.unc.</th>
<th>Main source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_Z$ [MeV]</td>
<td>0.1</td>
<td>0.1</td>
<td>$\delta\alpha_s$</td>
<td></td>
</tr>
<tr>
<td>$R_b$ [$10^{-5}$]</td>
<td>6</td>
<td>&lt; 1</td>
<td>$\delta\alpha_s$</td>
<td></td>
</tr>
<tr>
<td>$R_t$ [$10^{-3}$]</td>
<td>1</td>
<td>1.3</td>
<td>$\delta\alpha_s$</td>
<td></td>
</tr>
</tbody>
</table>

Channel $M_H$ [GeV] | $\delta\alpha_s$ (%) | $\Delta m_b$ | $\Delta m_c$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$H \rightarrow c\bar{c}$</td>
<td>126</td>
<td>± 7.1</td>
<td>± 0.1%</td>
</tr>
<tr>
<td>$H \rightarrow gg$</td>
<td>126</td>
<td>± 4.1</td>
<td>± 0.1%</td>
</tr>
</tbody>
</table>

(♦ Impacts physics approaching Planck scale: EW vacuum stability, GUT)

$\alpha_i(M_Z) = 0.1184$

$m_t = 171.4$ GeV

$m_t = 175$ GeV

$\alpha_3(M_Z) = 0.1198$

$\alpha_3(M_Z) = 0.117$

$\alpha_1^{-1}$

$\alpha_2^{-1}$

$\alpha_3^{-1}$
\( \alpha_s \) from hadronic Z, W decays

\[ \text{Z \& W observables theoretically known at N}^3\text{LO accuracy:} \]

- The W and Z hadronic widths:

\[
\Gamma_{W,Z}^{\text{had}}(Q) = \Gamma_{W,Z}^{\text{Born}} \left( 1 + \sum_{i=1}^{4} a_i(Q) \left( \frac{\alpha_s(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_s^5) + \delta_{\text{EW}} + \delta_{\text{mix}} + \delta_{\text{np}} \right)
\]

- The ratio of W, Z hadronic-to-leptonic widths:

\[
R_{W,Z}(Q) = \frac{\Gamma_{W,Z}^{\text{had}}(Q)}{\Gamma_{W,Z}^{\text{lep}}(Q)} = R_{W,Z}^{\text{EW}} \left( 1 + \sum_{i=1}^{4} a_i(Q) \left( \frac{\alpha_s(Q)}{\pi} \right)^i + \mathcal{O}(\alpha_s^5) + \delta_{\text{mix}} + \delta_{\text{np}} \right)
\]

- In the Z boson case, the hadronic cross section at the resonance peak in \( e^+e^- \):

\[
\sigma_{Z}^{\text{had}} = \frac{12\pi}{m_Z} \frac{\Gamma_Z^{\text{tot}}}{(\Gamma_Z^{\text{tot}})^2}
\]

**TH uncertainties:**

- \((\alpha^2, \alpha^3)\) included for Z:
  - \( \pm 0.015–0.03\% \) (Z)
  - \( \pm 0.015–0.04\% \) (W)

**Param. uncerts.:**

- \((m_{Z,W}, \alpha, V_{cs,ud})\):
  - \( \pm 0.01–0.03\% \) (Z)
  - \( \pm 1.1–1.7\% \) (W)
  - \( \pm 0.03\% \) (W, CKM unit)

**Measured at LEP with \( \pm 0.1–0.3\% \) (Z), \( \pm 0.9–2\% \) (W) exp. uncertainties:**

<table>
<thead>
<tr>
<th>( \Gamma_Z^{\text{tot}} ) (MeV)</th>
<th>( R_Z )</th>
<th>( \sigma_{Z}^{\text{had}} ) (ph)</th>
<th>( \text{theory} )</th>
<th>( \text{change} )</th>
<th>( \text{experiment} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>previous</td>
<td>new (this work)</td>
<td>change</td>
<td>previous</td>
<td>new</td>
<td>change</td>
</tr>
<tr>
<td>2494.2 ± 0.8\text{th}</td>
<td>2495.2 ± 0.6\text{par} ± 0.4\text{th}</td>
<td>+0.04%</td>
<td>2495.2 ± 2.3</td>
<td>2495.5 ± 2.3</td>
<td>+0.012%</td>
</tr>
<tr>
<td>20.733 ± 0.007\text{th}</td>
<td>20.750 ± 0.006\text{par} ± 0.006\text{th}</td>
<td>+0.08%</td>
<td>20.767 ± 0.025</td>
<td>20.766 ± 0.0247</td>
<td>−0.040%</td>
</tr>
<tr>
<td>41.490 ± 6\text{th}</td>
<td>41.494 ± 5\text{par} ± 6\text{th}</td>
<td>+0.01%</td>
<td>41.540 ± 37</td>
<td>41.480.2 ± 32.5</td>
<td>−0.144%</td>
</tr>
</tbody>
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**DIS2022**

[31/21]

Eduardo Ploerer (VUB)
\( \alpha_s \) from hadronic Z decays (today)

- **QCD coupling extracted from:**
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  - (ii) Full SM fit (with \( \alpha_s \) free parameter)

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<th>Z boson observable</th>
<th>( \alpha_s(m_Z) )</th>
<th>uncertainties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma_Z^{\text{tot}} )</td>
<td>0.1192 ± 0.0047</td>
<td>±0.0046 ±0.0005 ±0.0008</td>
</tr>
<tr>
<td>( R_Z )</td>
<td>0.1207 ± 0.0041</td>
<td>±0.0041 ±0.0001 ±0.0009</td>
</tr>
<tr>
<td>( \sigma_0^{\text{had}} )</td>
<td>0.1206 ± 0.0068</td>
<td>±0.0067 ±0.0004 ±0.0012</td>
</tr>
<tr>
<td>All combined</td>
<td>0.1203 ± 0.0029</td>
<td>±0.0029 ±0.0002 ±0.0008</td>
</tr>
</tbody>
</table>

**Improved PDG'19:**
\[
\alpha_s(m_Z) = 0.1203 \pm 0.0028 \ (\pm 2.3\%)
\]

**PDG'19:**
\[
\alpha_s(m_Z) = 0.1205 \pm 0.0030 \ (\pm 2.5\%)
\]

**EXP/TH updates lead to better agreement with full SM fit:**
\[
\alpha_s(m_Z) = 0.1202 \pm 0.0028
\]

**PDG'19:**
\[
\alpha_s(m_Z) = 0.1194 \pm 0.0029
\]

**LEP lumi-bias updates lead to much better agreement among \( \Gamma_z, R_z, \sigma_0 \) extractions:**

**Dashed/Full curves: 2018/20**
$\alpha_s$ from hadronic Z decays (FCC-ee)

- QCD coupling extracted from:
  (i) Combined fit of 3 Z pseudo-observables
  (ii) Full SM fit (with $\alpha_s$ free parameter)

**FCC-ee:**
- Huge Z pole stats. ($\times 10^5$ LEP)
- Exquisite systematic/parametric precision (stat. uncert. much smaller):
  \[
  \Delta R_Z = 10^{-3}, \quad R_Z = 20.7500 \pm 0.0010 \\
  \Delta \Gamma_{Z}^{\text{tot}} = 0.1 \text{ MeV}, \quad \Gamma_{Z}^{\text{tot}} = 2495.2 \pm 0.1 \text{ MeV} \\
  \Delta \sigma_{Z}^{\text{had}} = 4.0 \text{ pb}, \quad \sigma_{Z}^{\text{had}} = 41494 \pm 4 \text{ pb} \\
  \Delta m_Z = 0.1 \text{ MeV}, \quad m_Z = 91.18760 \pm 0.00001 \text{ GeV} \\
  \Delta \alpha = 3 \cdot 10^{-5}, \quad \Delta \alpha^{(5)}_{\text{had}}(m_Z) = 0.0275300 \pm 0.0000009
  \]
- TH uncertainty reduced by $\times 4$ computing missing $\alpha_s^5$, $\alpha^3$, $\alpha\alpha_s^2$, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms

- 10 times better precision than today:
  $\delta \alpha_s/\alpha_s \sim \pm 0.2\%$ (tot), $\pm 0.1\%$ (exp)
  Strong (B)SM consistency test.

\[
\alpha_s(m_Z) = 0.12030 \pm 0.00028 \ (\pm 0.2\%)
\]
\textbf{\(\alpha_s\) from hadronic W decays (today)}

- QCD coupling extracted from new N\(^3\)LO fit of combined \(\Gamma_w\), \(R_w\) pseudo-observ.: 

<table>
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<tr>
<th>W boson observables</th>
<th>(\alpha_s(m_Z)) extraction</th>
<th>uncertainties</th>
</tr>
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<tr>
<td>(\Gamma^\text{tot}_w, R_w) (exp. CKM)</td>
<td>0.044 ± 0.052</td>
<td>±0.024</td>
</tr>
<tr>
<td>(\Gamma^\text{tot}_w, R_w) (CKM unit.)</td>
<td>0.101 ± 0.027</td>
<td>±0.027</td>
</tr>
<tr>
<td>(\Gamma^\text{tot}_w, R_w) (FCC-ee, CKM unit.)</td>
<td>0.11790 ± 0.00023</td>
<td>±0.00012</td>
</tr>
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</table>

- Very imprecise extraction:
  - Large propagated parametric uncert. from poor \(V_{cs}\) exp. precision (±2%): QCD coupling unconstrained: 0.04±0.05
  - Imposing CKM unitarity: large exp. uncertainties from \(\Gamma_w\), \(R_w\) (0.9–2%): QCD extracted with ~27% precision
  - Propagated TH uncertainty much smaller today: ~1.5%

\(\alpha_s(m_Z) = 0.101 \pm 0.027 \ (±27\%)\)
\( \alpha_s \) from hadronic W decays (FCC-ee)

- QCD coupling extracted from new N\(^3\)LO fit of combined \( \Gamma_w, R_w \) pseudo-observ.:  

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</tr>
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- FCC-ee extraction:  
  - Huge W pole stats. (\( \times 10^4 \) LEP-2).  
  - Exquisite syst./parametric precision:  
    \( \Gamma^\text{tot}_W = 2088.0 \pm 1.2 \) MeV  
    \( R_W = 2.08000 \pm 0.00008 \)  
    \( m_W = 80.3800 \pm 0.0005 \) GeV  
    \( |V_{cs}| = 0.97359 \pm 0.00010 \) \( \sim O(10^{12}) D \) mesons  
  - TH uncertainty reduced by \( \times 10 \) after computing missing \( \alpha_s^5, \alpha^2, \alpha^3, \alpha \alpha_s^2, \alpha_s^2, \alpha^2 \alpha_s \) terms

\( \alpha_s(m_Z) = 0.11790 \pm 0.00023 \) (\( \pm 0.2\% \))
$\alpha_s$ at future $e^+e^-$ colliders (summary)

- World-average QCD coupling at $N_{2,3}^2$LO today:
  - Determined from 7 observables with combined 0.85% uncertainty (least well-known gauge coupling).
  - Impacts all LHC QCD $x$-sections & decays.
  - (Role beyond SM: GUT,) (EWK vacuum stability, (New colored sectors?)

- $e^+e^-$ extractions:
  - Hadronic tau decays: ±1% TH
  - Event shapes, jet rates: ±1% TH
  - Z&W pseudo-observ.: ±0.1% TH

- State-of-the-art Z, W extractions:
  - Z boson: New fit with high-order EW corrections + updated LEP data: ~2.3% (exp.) uncertainty today.
  - W boson: New $N^3$LO fit to $\Gamma_W$, $R_W$ ~27% (exp.) uncertainty today.

Permil uncertainty only possible with a machine like FCC-$e^+e^-$

$\alpha_s(m_Z) = 0.12030 \pm 0.00028 \pm 0.2\%$ (tot), ±0.1% (exp)

$\alpha_s(m_Z) = 0.11790 \pm 0.00023 \pm 0.2\%$ (tot), ±0.1% (exp)

What are the detector design improvements needed to bring propagated syst. uncert. on $W,Z$ pseudo-observ. below 0.1%?
Summary: QCD at future $e^+e^-$ colliders

- The precision needed to fully exploit all future ee/pp/ep/eA/AA SM & BSM programs requires exquisite control of pQCD & non-pQCD physics.
- Unique QCD precision studies accessible at FCC-ee (CEPC, ILC):

1. Per-mille $\alpha_s$ via hadronic $Z,W,\tau$ decays, evt shapes...

2. $N^{nLO}+N^{nLL}$ jet structure
   - High g/q/Q discrimination

3. Reduced PS+hadroniz. uncert. of EWK observ.

4. <1% control of colour reconnection

5. High-precision hadronization:
   - conservation of: baryon number
   - strangeness
   - transverse momentum
High-precision gluon & quark jet studies

- Exploit FCC-ee \( H(gg) \) as a "pure gluon" factory: \( H \rightarrow gg \) (BR\(\approx\)8\% accurately known) provides 120,000 extra-clean digluon events.

- Multiple handles to study gluon radiation & g-jet properties:
  - Gluon vs. quark via \( H \rightarrow gg \) vs. \( Z \rightarrow qq \)
    (Profit from excellent g,b separation)
  - Gluon vs. quark via \( Z \rightarrow bbg \) vs. \( Z \rightarrow qq(g) \)
    (g in one hemisphere recoiling against 2-b-jets in the other).
  - Vary \( E_{jet} \) range via ISR: \( e^+e^- \rightarrow Z^*, \gamma^* \rightarrow jj(\gamma) \)
  - Vary jet radius: small-R down to calo resolution

- Multiple high-precision analyses at hand:
  - BSM: Improve \( q/g/Q \) discrimination tools
  - pQCD: (Check \( N^nLO \) antenna functions.) High-precision QCD coupling.
  - non-pQCD: Gluon fragmentation: (Octet neutralization? (zero-charge gluon jet with rap gaps).) Colour reconnection? (Glueballs? Leading \( \eta \)'s,baryons)?

\[ H \rightarrow gg \]

\[ LH \text{ angularities} \]

\[ 1/N \text{ d}N/\text{d}\lambda^2 \]

\[ \lambda^2 \]

[\text{Pythia8} \quad \text{Herwig7} \quad \text{with mMHT}]
CERN FCC-ee project

- $e^+e^-$ operation before pp at $\sqrt{s} = 90, (125), 160, 240, 350$ GeV

<table>
<thead>
<tr>
<th>Working point</th>
<th>Z, years 1-2</th>
<th>Z, later</th>
<th>WW</th>
<th>HZ</th>
<th>tt</th>
<th>(s-channel H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s}$ (GeV)</td>
<td>88, 91, 94</td>
<td>157, 163</td>
<td>240</td>
<td></td>
<td>340-350, 365</td>
<td>$m_H$</td>
</tr>
<tr>
<td>Lumi/IP ($10^{34}$ cm$^{-2}$s$^{-1}$)</td>
<td>115</td>
<td>230</td>
<td>28</td>
<td>8.5</td>
<td>0.95</td>
<td>1.55 (30)</td>
</tr>
<tr>
<td>Lumi/year ($ab^{-1}$, 2 IP)</td>
<td>24</td>
<td>48</td>
<td>6</td>
<td>1.7</td>
<td>0.2</td>
<td>0.34 (7)</td>
</tr>
<tr>
<td>Physics Goal ($ab^{-1}$)</td>
<td>150</td>
<td>10</td>
<td>5</td>
<td>0.2</td>
<td>1.5</td>
<td>(20)</td>
</tr>
<tr>
<td>Run time (year)</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>4 (3)</td>
</tr>
<tr>
<td>Number of events</td>
<td>$5 \times 10^{12}$ Z</td>
<td>$10^8$ WW</td>
<td>$10^6$ HZ</td>
<td>$10^6$ tt</td>
<td>+200k HZ</td>
<td>+50k WW $\rightarrow H$</td>
</tr>
</tbody>
</table>

# of light-q jets/year: $O(10^{12})$  
# of gluon-jets/year: $O(10^{11})$  
# of heavy-Q jets/yr: $O(10^{12})$