

The 3D structure of pions at future electron-ion colliders

Based on: 2110.09462 [RPD] and 2110.06052 [PRL]

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Deep-Inelastic Scattering and Related Subjects*

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Universidad
de Huelva

Introduction

Introduction: GPDs and hadron's structure

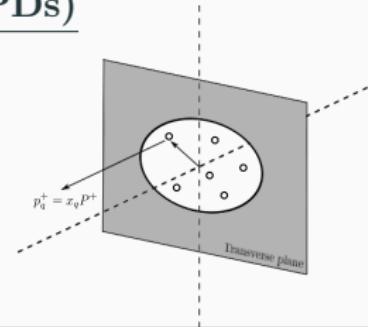
Question: *How can we gain insights into hadron's structure?*

Generalized parton distributions (GPDs)

Probabilistic interpretation:

Probability amplitude of finding a parton at a given position in transverse plane carrying a momentum fraction “ x ” of the hadron’s averaged light-cone momentum.

[M.Burkardt-PRD:071503(62)2020]



“3D picture” of hadrons

Properties:

1. PDFs as forward limit.
2. Electromagnetic and gravitational FFs as Mellin moments.
3. **Parametrize DVCS amplitudes through CFFs.**

[X.Ji-PRL:610(78)1997]

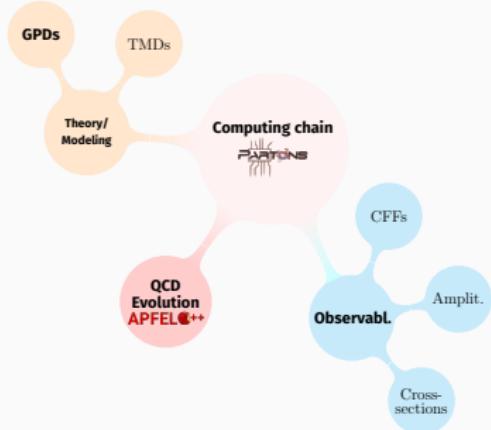
Introduction

1. Generalized parton distributions:

- “3D picture” of hadrons.
- Connection with QCD (GFFs).
- Experimentally accessible.

2. Pions: DCSB Nambu-Goldstone bosons:

- Clear window onto emergence of hadronic mass.



One main question guides this talk

Can we access pion's GPDs through experiment?

(Pion structure through the Sullivan process)

[D.Amrath et al.-EPJC:179(58)2008]

Phenomenology of pion GPDs

Phenomenology of pion GPDs: Sullivan process

Question: Can we access pion's GPDs through experiment?

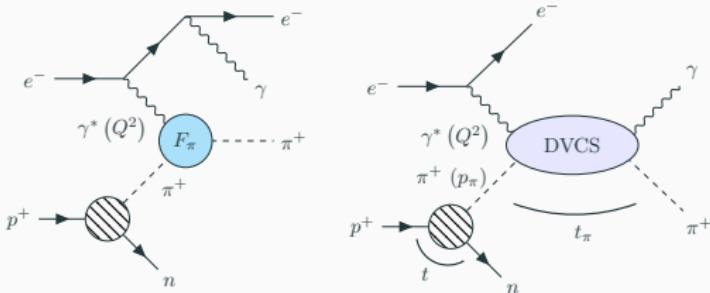
[D.Amrath et al.-EPJC:179(58)2008]

Sullivan process [J.D.Sullivan-PRD:1732(5)1972]

Deep inelastic electron-proton scattering with πn fixed final states.

One-pion-exchange approximation: [D.Amrath et al.-EPJC:179(58)2008]

- $|t|^{Max.} = 0.6 \text{ GeV}^2$
 - Factorization: $\sigma_L^{\gamma^*} \gg \sigma_{\perp}^{\gamma^*}$
- } Met at EIC [EICYR:phys.ins-det/2103.05419]



DVCS amplitudes
are parametrized
by hadron GPDs.
[X.Ji-PRD:7114(55)1997]

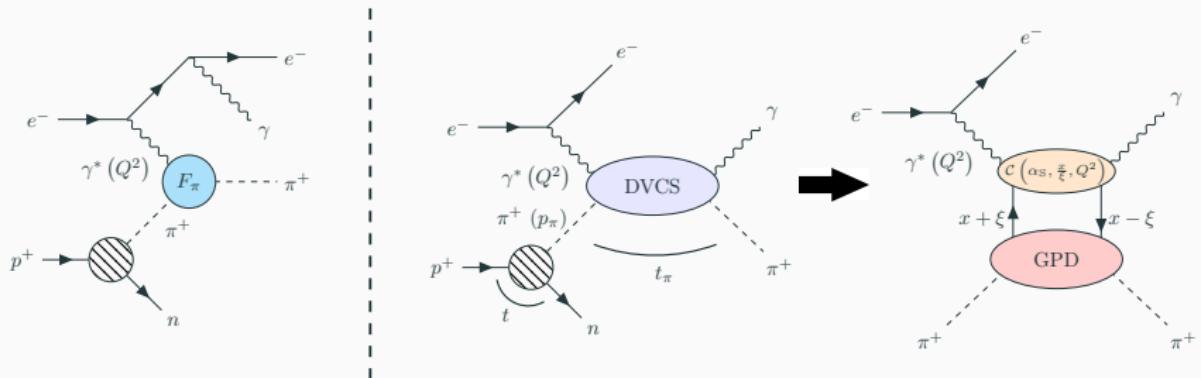
Employed for EFFs.

[G.M.Huber et al.-PRC:045203(78)2008]

Can we probe DVCS contribution through experiment?

[D.Amrath et al.-EPJC:179(58)2008]

Phenomenology of pion GPDs: Sullivan process and DVCS



$$\mathcal{M}_{e\pi} = \mathcal{M}_{\text{BH}} + \mathcal{M}_{\text{DVCS}}$$

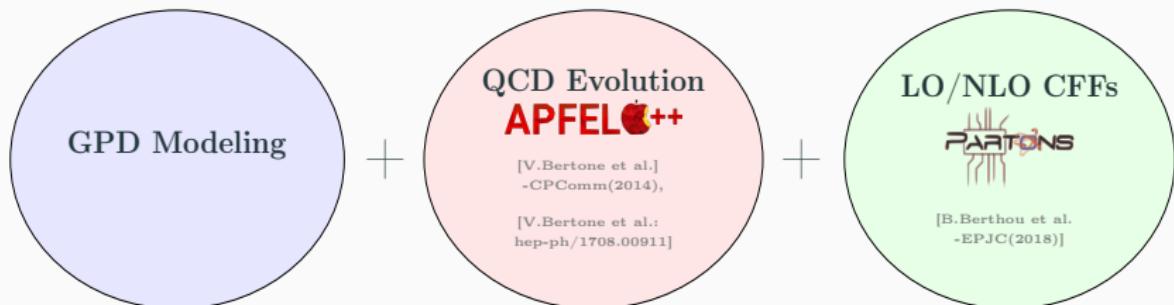
DVCS amplitudes are parametrized by hadron GPDs through CFFs.
[X.Ji-PRD:7114(55)1997]

$$\mathcal{M}_{\text{DVCS}} \propto \mathcal{H}_\pi(\xi, t; Q^2) = \sum_{p=q,g} \int_{-1}^1 \frac{dx}{\xi} \mathcal{K}^p \left(\frac{x}{\xi}, \frac{Q^2}{\mu_F^2}, \alpha_s(\mu_F^2) \right) H_\pi^p(x, \xi, t; \mu_F^2)$$

Phenomenology of pion GPDs: the path towards DVCS

Goal: Compute DVCS Compton form factors (CFFs)

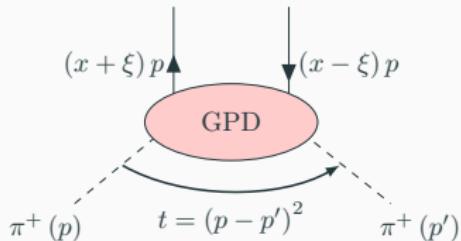
1. Start from state-of-the-art **models for pion GPDs**.
2. Leading order **scale evolution** for GPDs.
3. Convolution with coefficient functions: **CFFs**.



Phenomenology of pion GPDs:

GPD modeling

[Reminder] GPDs: definition and properties



x : Momentum fraction of p .

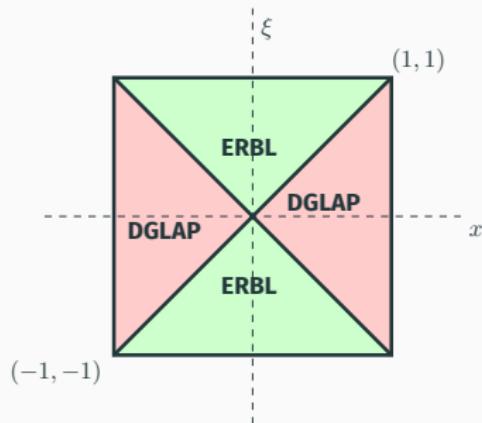
ξ : Fraction of momentum longitudinally transferred.

t : Momentum transfer.

Kinematics:

[M.Diehl-Phys.Rept.:41(388)2003]

- **DGLAP** ($|x| > |\xi|$):
Emits/takes a quark ($x > 0$)
or antiquark ($x < 0$).
- **ERBL**: ($|x| < |\xi|$):
Emits pair quark-antiquark.

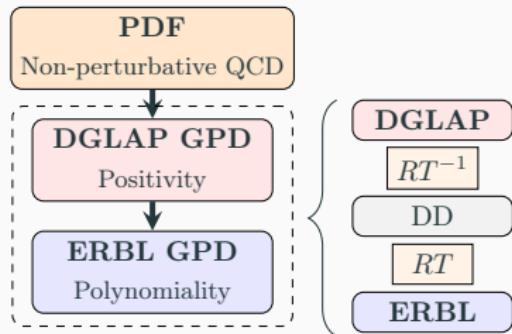


Phenomenology of pion GPDs: GPD modeling

[JMMC et al.-PRD:DK13155 (2022)]

Every candidate GPD model
must fulfill these properties

GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Pobyl.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

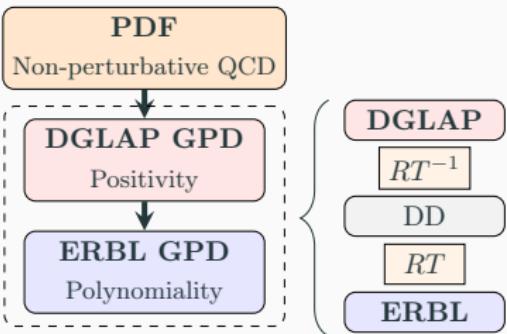


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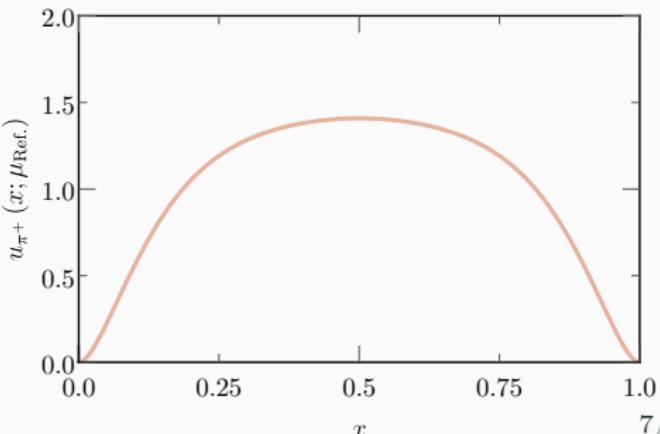
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1. PDF: DSE methods

[Ding et al.-PRD:054014(101)2020]

$$q(x) = \mathcal{N}x^2(1-x)^2 \left[1 + \gamma x(1-x) + \rho \sqrt{x(1-x)} \right]$$

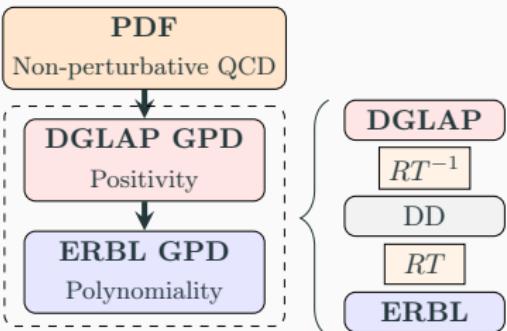


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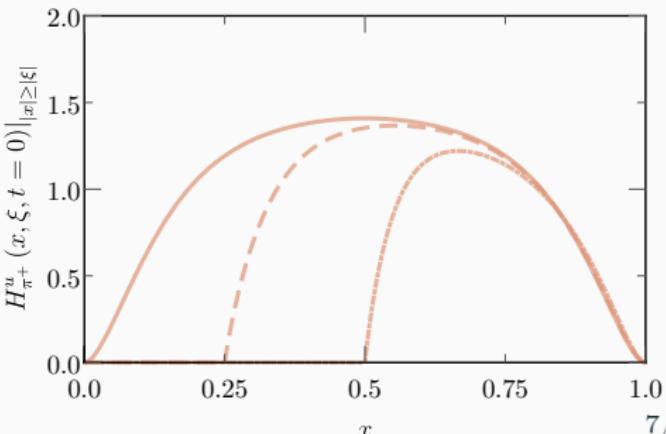
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2. Positive DGLAP GPD

$$H_{\pi^+}^q(x, \xi, t) = \sqrt{q} \left(\frac{x+\xi}{1+\xi} \right) q \left(\frac{x-\xi}{1-\xi} \right)$$

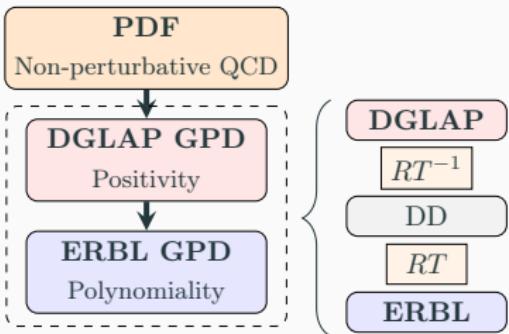


Phenomenology of pion GPDs: GPD modeling

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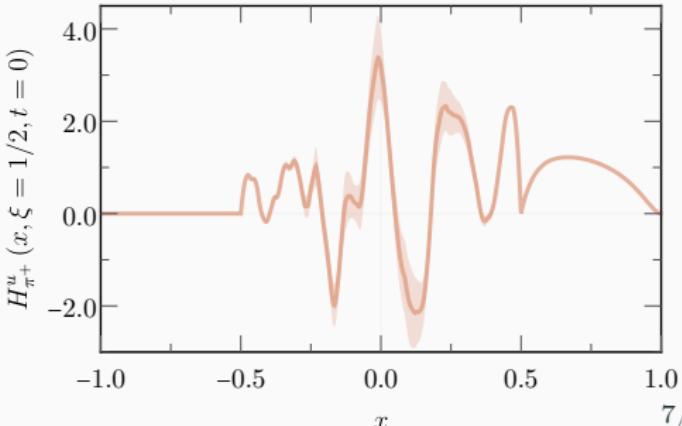
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3. ERBL GPD

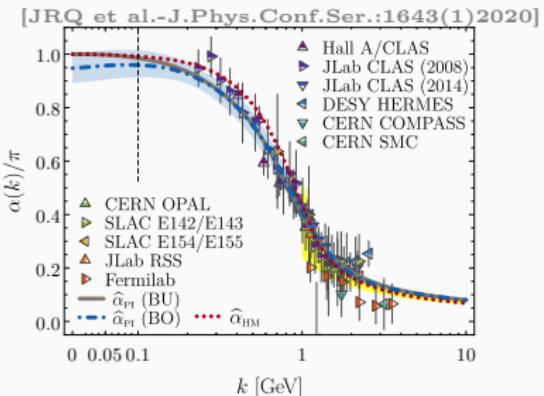


Phenomenology of pion GPDs: QCD evolution

[Reminder] GPDs: scale evolution

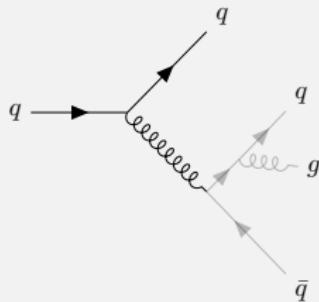
Reference scale: $\mu_{\text{Ref.}} = 331 \text{ MeV}$

No gluons nor sea quarks



Parton splitting (DGLAP)

[AltarelliParisi:Nucl.Phys.B:298(126)1977]

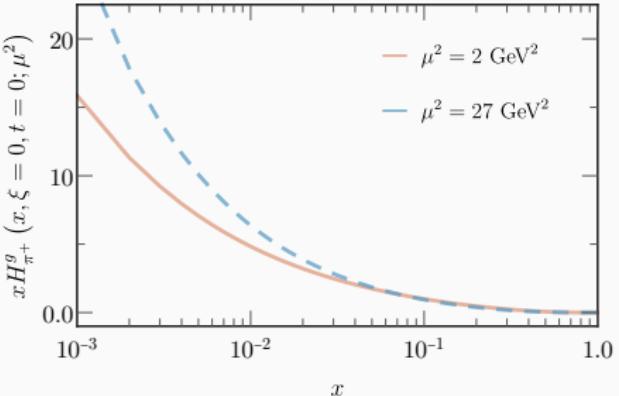
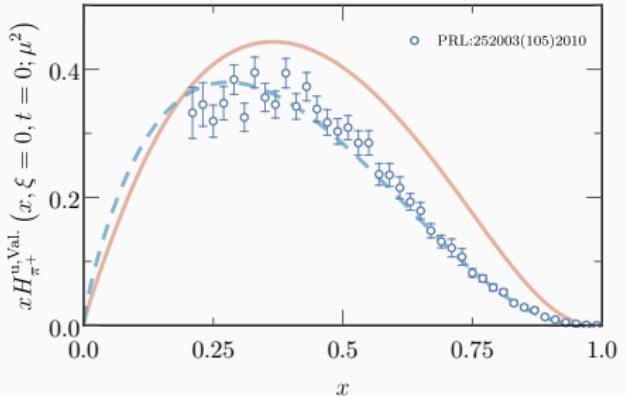


Renormalization group equations for GPD operators yield:

[X. Ji-PRD:7114(55)1997, V.Bertone-Private comm.]

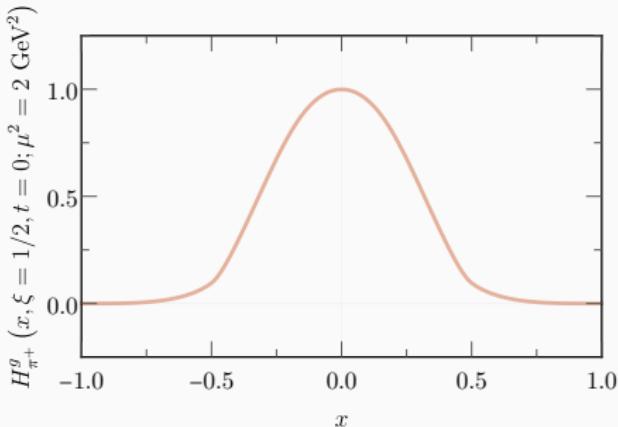
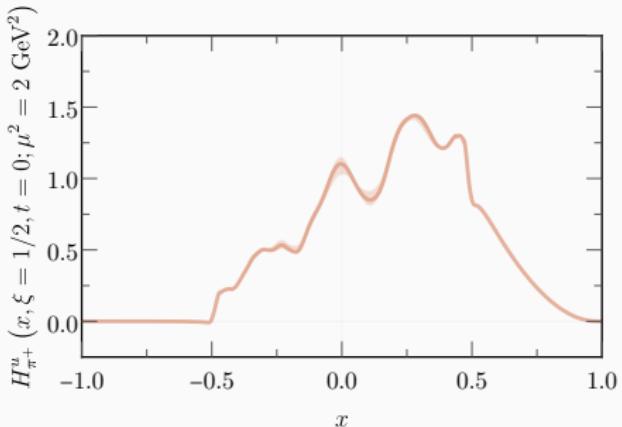
$$\frac{dH^{(\pm)}(x, \xi; \mu^2)}{d \log \mu^2} = \frac{\alpha_s(\mu^2)}{4\pi} \int_x^\infty \frac{dy}{y} \mathcal{P}^{(\pm)}\left(\frac{x}{y}, \frac{\xi}{x}, \alpha_s(\mu^2)\right) H^{(\pm)}(y, \xi; \mu^2)$$

Phenomenology of pion GPDs: QCD evolution



- Non-zero gluon distribution generated by scale evolution.

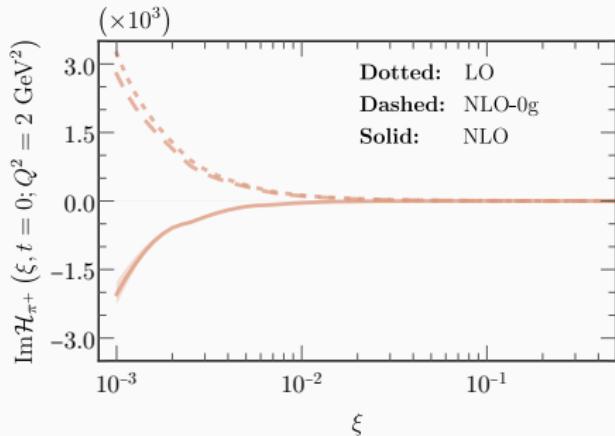
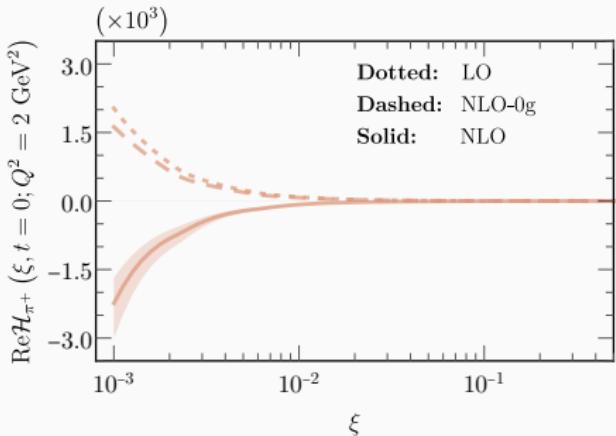
Phenomenology of pion GPDs: QCD evolution



- Non-zero gluon distribution generated by scale evolution.
- Uncertainty band narrowed.
- Continuity along $x = \xi$ lines.

Phenomenology of pion GPDs: Compton form factors

Phenomenology of pion GPDs: Compton Form Factors



- Small effect of NLO corrections to quark amplitudes.
- Dominant effect of gluons.

Gluon dominance makes essential at least NLO accuracy in any phenomenological analysis of DVCS at an EIC.

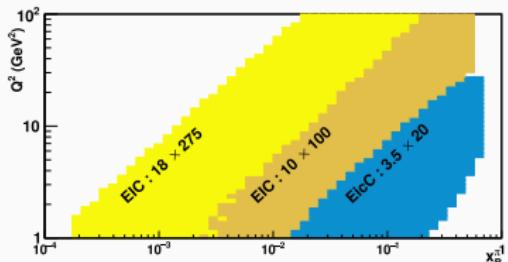
Phenomenology of pion GPDs:

Sullivan process

Phenomenology of pion GPDs: Generating events

Monte Carlo event generation [JMMC et al.-PRL:128, LK18061 (2022)]

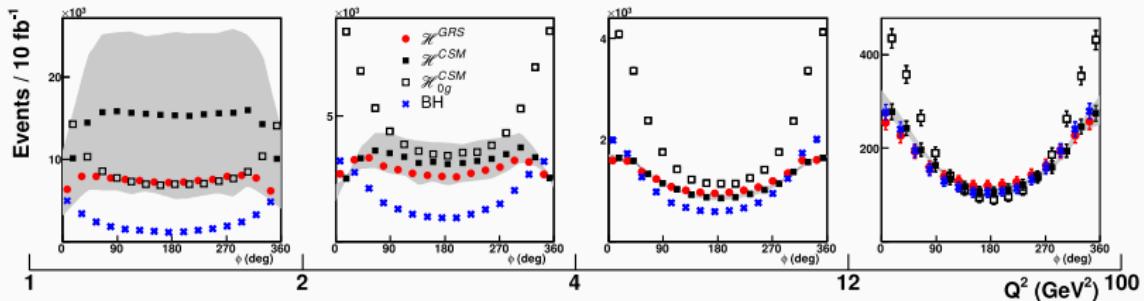
- Pick events compatible with detector geometry and performance:
EIC and EicC. [EICYR:phys.ins-det/2103.05419, EICCCWP:Nucl-ex/2102.09222]
- Add kinematical cuts: [D.Amrath et al. EPJC:179(58)2008]
 - DVCS kinematics and one pion exchange approximation:
 - $s_\pi^{\text{Min}} = 4 \text{ GeV}^2$
 - $|t_\pi|^{\text{Max.}} = 0.6 \text{ GeV}^2$
 - Reduce contamination of resonances (Δ): $M_{n\pi}^2 \gtrsim 4 \text{ GeV}^2$
- Integrated one-year luminosity.



[JMMC et al.-PRL:128, LK18061 (2022)]

Phenomenology of pion GPDs: EIC event-rates

[JMMC et al.-PRL:128, LK18061 (2022)]



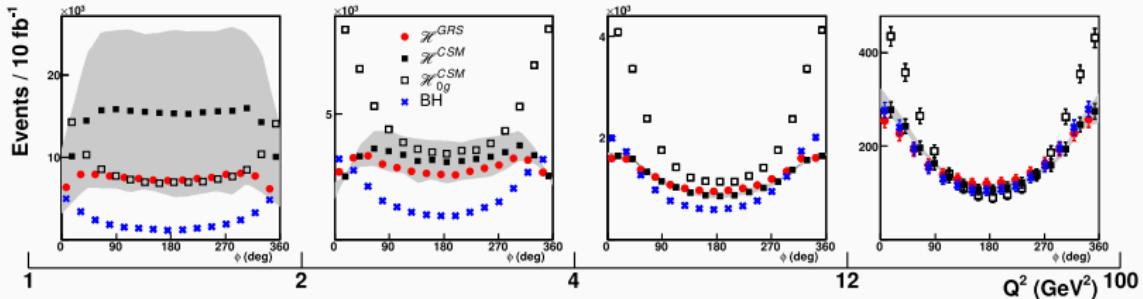
The cross-section can be written as:

[A.V. Belitsky et al.-PRD:014017(79)2009]

$$\sigma^{e\pi}(\lambda, \pm e) = A_{\text{BH}} F_\pi^2 + \frac{B_{\text{DVCS}} |\mathcal{H}_\pi|^2}{Q^2} + e \frac{C_{\text{Int}}}{Q} (C_{\text{Int}}^{\cos} \cos \phi_\pi \text{Re}(\mathcal{H}_\pi) + \lambda C_{\text{Int}}^{\sin} \sin \phi_\pi \text{Im}(\mathcal{H}_\pi))$$

Phenomenology of pion GPDs: EIC event-rates

[JMMC et al.-PRL:128, LK18061 (2022)]



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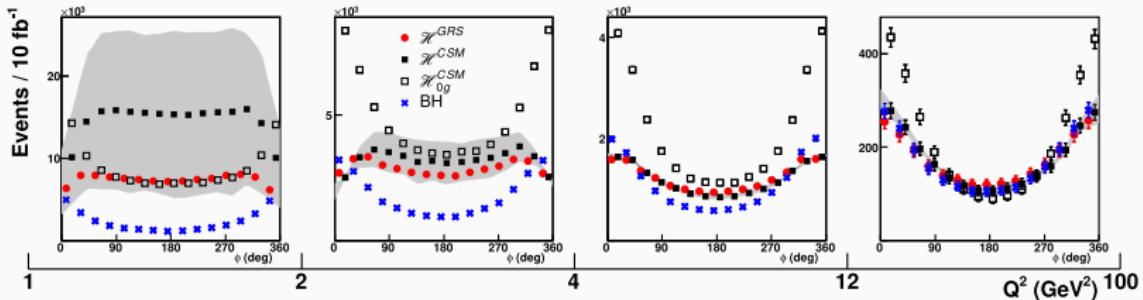
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$$|\mathcal{H}_\pi|^2 = \text{Re}^2(\mathcal{H}_\pi) + \text{Im}^2(\mathcal{H}_\pi) \sim \text{Re}^2(|\mathcal{H}_\pi^{\text{q,LO}}| - |\mathcal{H}_\pi^g|) + \text{Im}^2(|\mathcal{H}_\pi^{\text{q,LO}}| - |\mathcal{H}_\pi^g|)$$

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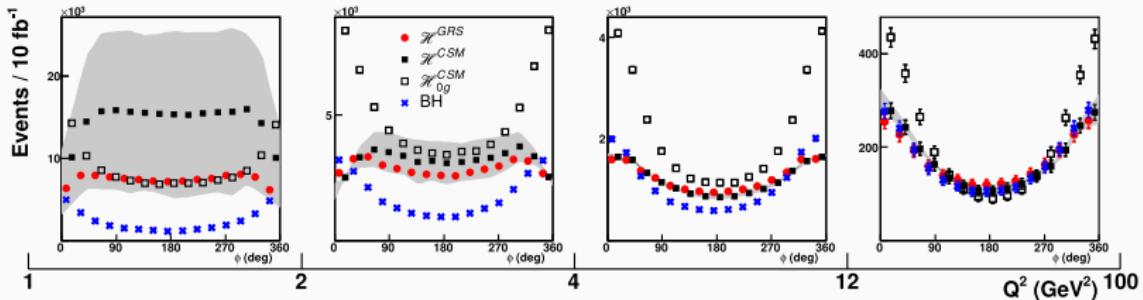
Low energy, $Q^2 < 2$ GeV²

Intermediate energies

$$|\mathcal{H}_\pi|_{Q^2 < 2 \text{ GeV}^2}^2 \sim |\mathcal{H}_\pi^g|_{Q^2 < 2 \text{ GeV}^2}^2 > |\mathcal{H}_\pi^{q,\text{LO}} - \mathcal{H}_\pi^g|_{2 < Q^2 < 12 \text{ GeV}^2}^2 \sim |\mathcal{H}_\pi|_{2 < Q^2 < 12 \text{ GeV}^2}^2$$

Phenomenology of pion GPDs: EIC event-rates

[JMMC et al.-PRL:128, LK18061 (2022)]



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Low energy, $Q^2 < 2 \text{ GeV}^2$

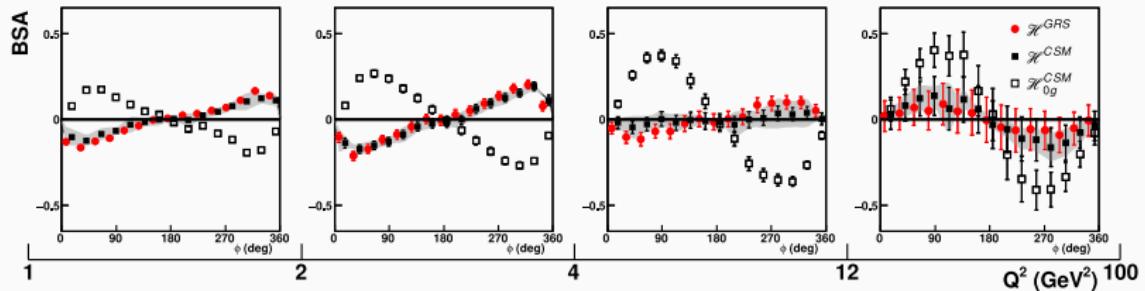
Intermediate energies

$$|\mathcal{H}_\pi|^2_{Q^2 < 2 \text{ GeV}^2} \sim |\mathcal{H}_\pi^g|^2_{Q^2 < 2 \text{ GeV}^2} > |\mathcal{H}_\pi^{q,\text{LO}} - \mathcal{H}_\pi^g|^2_{2 < Q^2 < 12 \text{ GeV}^2} \sim |\mathcal{H}_\pi|^2_{2 < Q^2 < 12 \text{ GeV}^2}$$

Quark-gluon “interference”: modulates expected number of events

Phenomenology of pion GPDs: EIC beam-spin asymmetries

[JMMC et al.-PRL:128, LK18061 (2022)]

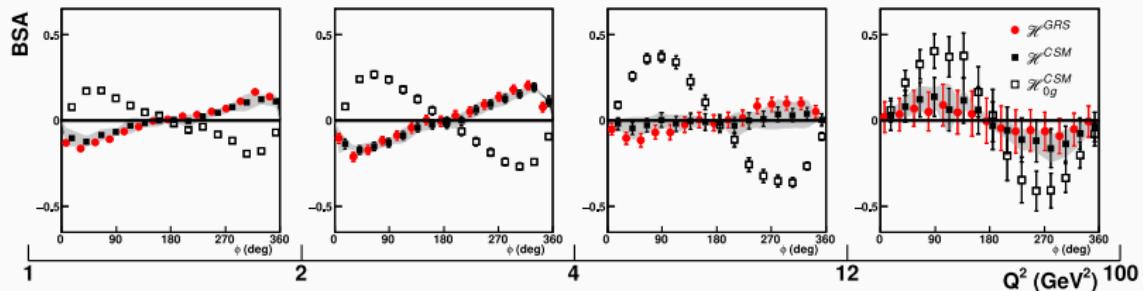


$$\mathcal{A} = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} \sim \mathcal{C}_\sigma \sin \phi_\pi \text{Im} (\mathcal{H}_\pi)$$

$$\text{Im} (\mathcal{H}_\pi) = \text{Im} (\mathcal{H}_\pi^{q,\text{LO}}) + \text{Im} (\mathcal{H}_\pi^{q,\text{NLO}}) + \text{Im} (\mathcal{H}_\pi^g) \sim \text{Im} |\mathcal{H}_\pi^{q,\text{LO}}| - \text{Im} |\mathcal{H}_\pi^g|$$

Phenomenology of pion GPDs: EIC beam-spin asymmetries

[JMMC et al.-PRL:128, LK18061 (2022)]



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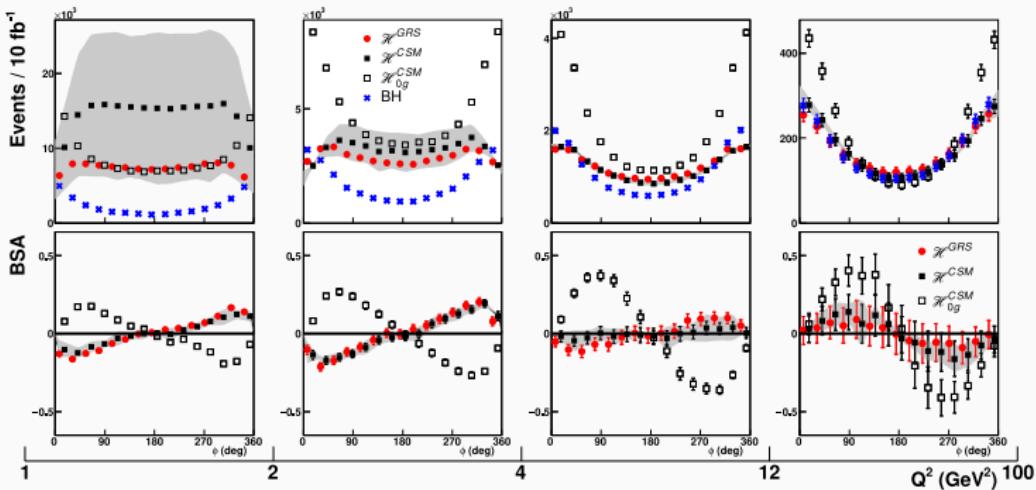
$$\text{Im}(\mathcal{H}_\pi) = \text{Im}(\mathcal{H}_\pi^{q,\text{LO}}) + \text{Im}(\mathcal{H}_\pi^{q,\text{NLO}}) + \text{Im}(\mathcal{H}_\pi^g) \sim \text{Im}|\mathcal{H}_\pi^{q,\text{LO}}| - \text{Im}|\mathcal{H}_\pi^g|$$

Sign inversion: **smoking gun** for gluon dominance

Summary

Summary

- Can we probe pion GPDs in experiment?



- Signal expected at EIC kinematics
- Gluon-quark “destructive interference”
- Gluon dominance: Beam spin asymmetry sign change

Thank you!

Back-up slides

Introduction: why do we focus on pions?

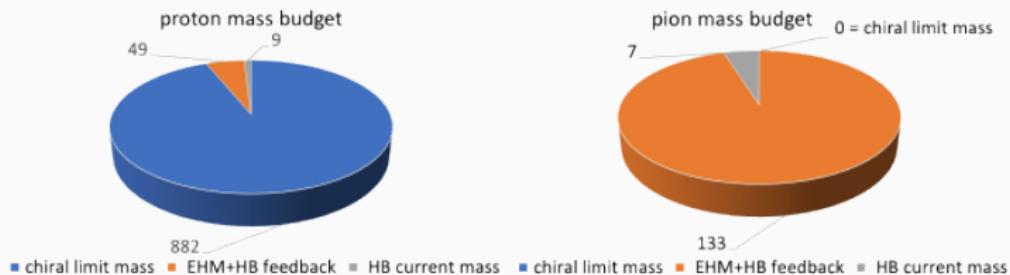
Question: How does hadronic mass emerge from field theory?

$$\mathcal{L}_{\text{QCD}} \xrightarrow{\mathbb{R}^{1,3} \rtimes SO(1,3)} \text{EMT: } T^{\mu\nu}, \quad g_{\mu\nu} \langle h(p) | T^{\mu\nu} | h(p) \rangle = -m_h^2$$

Energy momentum tensor trace

[See e.g.: C.D.Roberts:2102.01765]

$$g_{\mu\nu} T^{\mu\nu} = \frac{1}{4} \beta(\alpha_\mu) G_a^{\mu\nu} G_{\mu\nu}^a + [1 + \gamma(\alpha_\mu)] \sum_i m_{\mu,i} \bar{\psi}_i \psi_i$$



Pions provide us with the clearest window onto EHM.

[C.D.Roberts:2102.01765, A.C.Aguilar-EPJA:10(55)2019]

GPDs: definition and properties

Generalized parton distributions: Fourier transform of non-local hadronic matrix elements of non-local operators evaluated at off-forward kinematics, *e.g.*:

$$H_{\pi^+}^q(x, \xi, t; \mu^2) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi}^q \left(-\frac{\lambda}{2} n \right) \not{p} \psi^q \left(\frac{\lambda}{2} n \right) | p \rangle$$

$$H_{\pi^+}^g(x, \xi, t; \mu^2) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | G_\alpha^\mu \left(-\frac{\lambda}{2} n \right) G_\mu^\beta \left(\frac{\lambda}{2} n \right) | p \rangle n^\alpha n_\beta$$

[D.Müller et al.-Fortsch. Phys.:101(42)1994]

[X.Ji-PRD:7114(55)1997]

[A.V. Radyushkin-PLB:333(385)1996]

[M.Diehl-Phys. Rep.:41(388)2003]

GPDs: definition and properties

- **Support:**

[M.Diehl et al.-PLB:359(428)1998]

$$(x, \xi) \in [-1, 1] \otimes [-1, 1]$$

- **Polynomiality:** Order- m Mellin moments are degree- $(m + 1)$ polynomials in ξ .

[X.Ji-JPG:1181(24)1998, A.Radyushkin-PLB:81(449)1999]

$$\int_{-1}^1 dx x^m H(x, \xi, t) = \sum_{\substack{k=0 \\ k \text{ even}}}^{m+1} c_k^{(m)}(t) \xi^k$$

Lorentz invariance

- **Positivity:**

[P.V.Pobylitsa-PRD:114015(65)2002, B.Pire et al.-EPJC:103(8)1999]

$$|H^q(x, \xi, t=0)| \leq \sqrt{q \left(\frac{x+\xi}{1+\xi} \right) q \left(\frac{x-\xi}{1-\xi} \right)} \quad , \quad |x| \geq \xi$$

Positivity of Hilbert space norm

- **Low energy soft-pion theorem**

[M.V.Polyakov-NPB:231(555)1999, C.Mezrag et al.-PLB:190(741)2015]

PCAC/Axial-Vector WTI

GPD modeling: overlap representation

Question: Can we build pion GPDs fulfilling all these constraints?

Overlap representation - GPDs written as overlap of LFWFs.

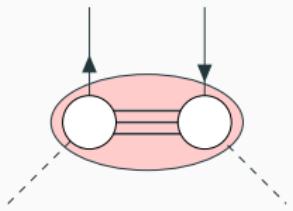
[M. Diehl et al.-NPB:33(569)2001]

Quantizing a quantum field theory on the lightfront allows to expand a hadron state in a Fock-space basis, e.g.:

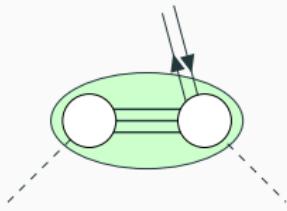
[S. Brodsky et al.-Phys.Rept.299(301)1998]

$$|\pi(p), \lambda\rangle \sim \sum_{\beta} \Psi_{\beta, N=2}^q |q\bar{q}\rangle + \Psi_{\beta, N=4}^q |q\bar{q}q\bar{q}\rangle + \dots$$

whose “coefficients” are lightfront wave functions: $\Psi^q(x, k_{\perp}^2)$.



Same N LFWFs



N and $N + 2$ LFWFs

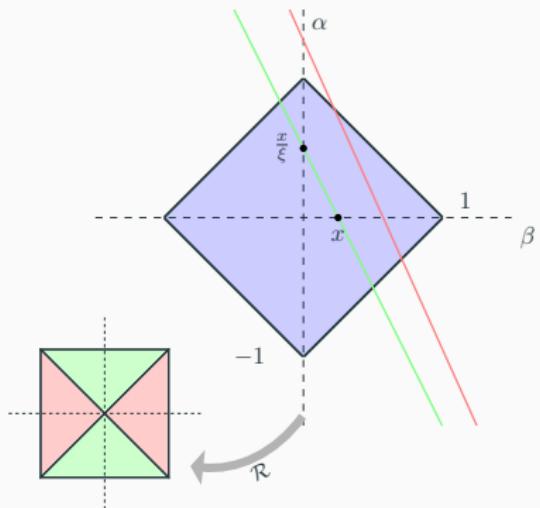
Overlap representation: positivity inbuilt but polynomiality is lost

GPD modeling: double distribution representation

Question: Can we build pion GPDs fulfilling all these constraints?

Double Distribution representation

[D.Müller et al.-Fort.Phys:2(42)1994, JLAB-THY-00-33]



Radon transform

$$H(x, \xi, t) = \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi) h(\beta, \alpha, t)$$

- Polynomiality is explicitly fulfilled:

$$\int_{-1}^1 dx x^n H(x, \xi, t) = \sum_{j=0}^n \binom{n}{j} \xi^j \times \int_{\Omega} d\beta d\alpha \beta^{n-j} \alpha^j h(\beta, \alpha, t)$$

- What about positivity?

GPD modeling: general strategy

Question: Can we build pion GPDs fulfilling all these constraints?

1. Overlap representation

[M.Diehl et al.-NPB:33(569)2001]

Based on LFWFs, $\Psi^q(x, k_\perp^2)$

Polynomiality ?
Positivity ✓

2. Double Distribution representation

[D.Müller et al.-Fort.Phys:2(42)1994, JLAB-THY-00-33]

Relying on Radon transform, \mathcal{R}

Polynomiality ✓
Positivity ?

Problem: Different modeling strategies and different problems

Solution:

Covariant extension: given a DGLAP-GPD, the covariant extension allows for computing the corresponding ERBL-GPD such that polynomiality is satisfied. [N.Chouika et al.-EPJC:906(77)2017]

Covariant extension (I)

Double distribution representation

Given a function $D(\alpha, t)$ with compact support $\alpha \in [-1, 1]$ such that

$$\int_{-1}^1 d\alpha \alpha^m D(\alpha, t) = c_{m+1}^m(t) , \quad \forall t$$

then,

$$\int_{-1}^1 dx x^m \left[H(x, \xi, t) - sgn(\xi) D\left(\frac{x}{\xi}, t\right) \right]$$

is a polynomial of order m in ξ .

Under these conditions, Hertle's theorem guarantees that:

[A.Hertle-Mat.Zeit.:165(184)1983, O.Teryaev-PLB:125(510)2001,

N.Chouika et al.-EPCJ:906(77)2017]

$$H(x, \xi, t) = sgn(\xi) D\left(\frac{x}{\xi}, t\right) + \int_{\Omega} d\beta d\alpha F_D(\beta, \alpha, t) \delta(x - \beta - \alpha\xi)$$

A GPD can always be written as the **Radon transform** of double distributions, thus guaranteeing fullfilment of **polynomiality**.

Covariant extension (II)

Existence and uniqueness theorems

If one writes: $\frac{1}{|\xi|} D\left(\frac{x}{\xi}, t\right) = \mathcal{R}[D(\alpha, t) \delta(\beta)] \equiv \mathcal{R}[G_D(\beta, \alpha, t)].$

$$H(x, \xi, t) = \int_{\Omega} d\beta d\alpha [F_D(\beta, \alpha, t) + \xi G_D(\beta, \alpha, t)] \delta(x - \beta - \alpha\xi)$$

Covariant extension Boman and Todd-Quinto theorem

[N.Chouika et al.-EPJC:906(77)2017, J.Boman et al.-Duke Math.J.:943(55)1987]

If $H(x, \xi, t) = 0 \forall (x, \xi) \in [-1, 1] \otimes [-1, 1] / |x| \geq |\xi| \Rightarrow$
 $\Rightarrow F_D(\beta, \alpha, t) = 0 \forall (\beta \neq 0, \alpha) \in \Omega$

DGLAP region **almost** characterizes the entire GPD.

Except ambiguities along $\beta = 0$.

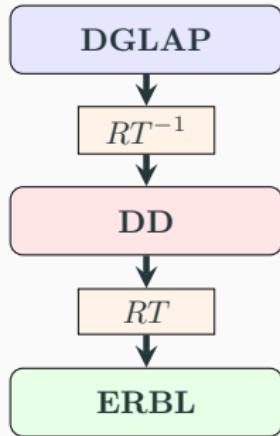
- Ambiguity along $\beta = 0$ comming from $D(x/\xi, t)$
- If $F_D(\beta, \alpha, t)$ is a distribution, further ambiguity: $\delta(\beta) D^+(\alpha)$

GPD modeling: covariant extension

Covariant extension: given a DGLAP-GPD, the covariant extension allows for computing the corresponding ERBL-GPD such that polynomiality is satisfied. [N.Chouika et al.-EPJC:906(77)2017]

$$H(x, \xi, t) = \mathcal{R}[h(\beta, \alpha, t)] + \frac{1}{|\xi|} D^+ \left(\frac{x}{\xi}, t \right) + \text{sign}(\xi) D^- \left(\frac{x}{\xi}, t \right)$$

1. Build positive DGLAP GPD \Rightarrow
How?
2. Covariant extension: ERBL GPD
3. Soft pion theorem: fix $D^\pm(\alpha, 0)$



GPD properties			
Support [Diehl-PLB(1998)]	✓	Positivity [Pobyl-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓

Pion GPDs: from separable LFWFs to positive DGLAP GPDs

Question: How can we build a positive DGLAP GPD?

1. Overlap representation [M.Diehl-NPB:33(569)2001]

$$H^q(x, \xi, t)|_{|x| \geq \xi} = \int \frac{d^2 k_\perp}{16\pi^3} \Psi^{q*}(x_-, k_{\perp,-}^2) \Psi^q(x_+, k_{\perp,+}^2)$$

2. Assume factorisation of the LFWF

[J.-L.Zhang et al.-PLB:136158(815)2021]

$$\Psi^q(x, k_\perp^2) \propto \varphi(x) \phi(k_\perp^2)$$

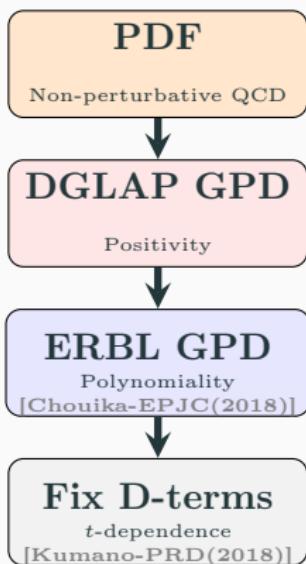
↓
(Overlap rep.)

$$H^q(x, \xi, t)|_{|x| \geq \xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right) q\left(\frac{x+\xi}{1+\xi}\right)} \Phi(x, \xi, t)$$

↓
($t = 0$)

$$H^q(x, \xi, 0)|_{|x| \geq \xi} = \sqrt{q\left(\frac{x-\xi}{1-\xi}\right) q\left(\frac{x+\xi}{1+\xi}\right)}$$

Positivity saturated



Comments on factorized LFWFs (I)

LFWFs are basic building blocks for GPDs. Its factorisation is a desirable property but, *under which conditions can we assume that?*

Remember that LFWFs can be obtained from LF-projections of BSA
[PRL:132001(1102013)]

We can parametrize quark propagator and meson BSA as

[PRL:132001(1102013), PLB:23(737)2014, PLB:190(741)2015, PRD:074021(93)2016]

$$S^q(p) = (-i\gamma \cdot p + m_q) \Delta(p^2, m_q^2)$$

$$\Gamma_\pi(k, P) = i\mathcal{N}_\pi \gamma_5 \int_{-1}^1 dz \rho_\pi(z) \Delta(k_+^2, \Lambda^2) \Lambda^2$$

$$\Delta(k^2, \Lambda^2) = 1/(k^2 + \Lambda^2) \quad , \quad k_+ = k + zP/2$$

and compute the LFWF by the appropriate projection:

$$\Psi_\pi^q(x, k_\perp^2) = \text{tr} \int dk_\parallel \delta_n^x(k) \gamma_5 \gamma \cdot n \chi_\pi(k_-, P)$$

Comments on factorized LFWFs (II)

More explicitly:

[PRD:094014(97)2018, PLB:136158(815)2021]

$$\Psi_\pi^q(x, k_\perp^2) = 12 [m_q(1-x) + m_h x] \chi_\pi(x, \sigma_\perp^2) \quad , \quad \sigma_\perp = k_\perp^2 + \Omega^2$$

$$\chi_\pi(x, \sigma_\perp^2) = \left[\int_{-1}^{1-2x} dw \int_{1+\frac{2x}{w-1}}^1 dv + \int_{1-2x}^1 \int_{\frac{w-1+2x}{w+1}}^1 dv \right] \frac{\rho_\pi(w)}{\sigma_\perp^2} \Lambda^2$$

$$\begin{aligned} \Omega^2 &= vm_q^2 + (1-v)\Lambda^2 + (m_h^2 - m_q^2) \left[x - \frac{1}{2}(1-w)(1-v) \right] + \\ &+ m_\pi^2 \left[x(x-1) + \frac{1}{4}(1-v)(1-w^2) \right] \end{aligned}$$

And taking the **chiral limit**:

$$\Psi_\pi^q(x, k_\perp^2) = \sqrt{q_\pi(x)} \left(4\sqrt{3}\pi \frac{m_q^2}{(k_\perp^2 + m_q^2)^2} \right)$$

Chiral symmetry implies factorisation of the LFWF

Pion GPDs: CSM-inspired models

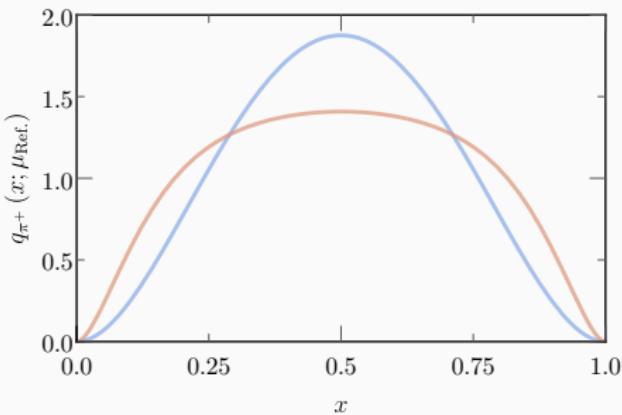
- Under certain PTIR, chiral symmetry allows to factorize LFWF:

[J.-L.Zhang et al.-PLB:136158(815)2021]

$$\Psi_{\pi}^{\lambda_1 \lambda_2}(x, k_{\perp}^2) = \sqrt{q_{\pi}(x)} \frac{i^{\lambda_1 \lambda_2} M^2}{(k_{\perp}^2 + M^2)^2}$$

- Pion GPD saturating positivity

$$H_{\pi}^q(x, \xi, t)|_{\text{DGLAP}} = \frac{\sqrt{q_{\pi}(x_-) q_{\pi}(x_+)}}{(1+z^2)^2} \left[3 + \frac{1-2z}{1+z} \frac{\operatorname{arctanh}\left(\sqrt{\frac{z}{1+z}}\right)}{\sqrt{\frac{z}{1+z}}} \right]$$
$$z = -t(1-x)^2 / 4M^2(1-\xi^2)$$



Two models:

- Algebraic model

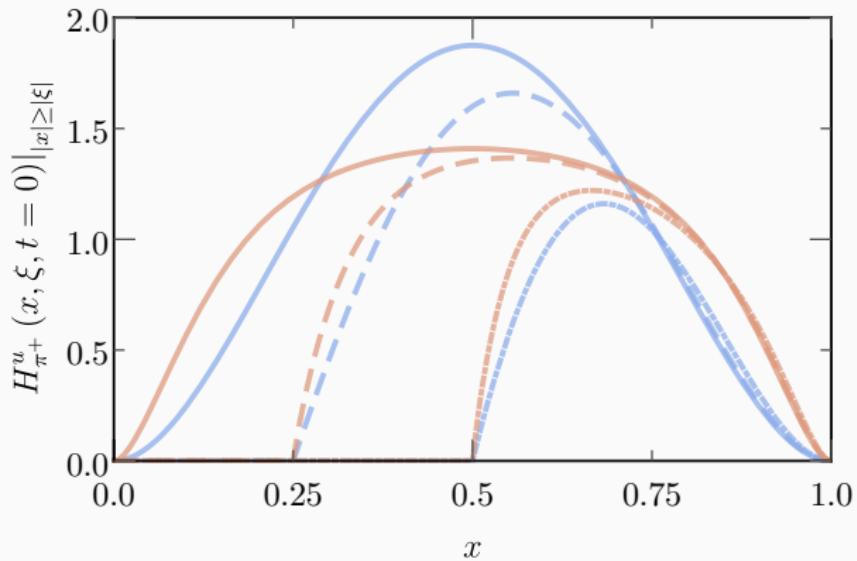
$$q_{\pi}(x) = 30x^2(1-x)^2$$

- Realistic model (DSE)

[M.Ding et al.-PRD:054014(101)2020]

$$q_{\pi}(x) = \mathcal{N}_q x^2(1-x)^2 \times \left[1 + \gamma x(1-x) + \rho \sqrt{x(1-x)} \right]$$

Pion GPDs: CSM-inspired models



Pion GPDs: CSM-inspired models

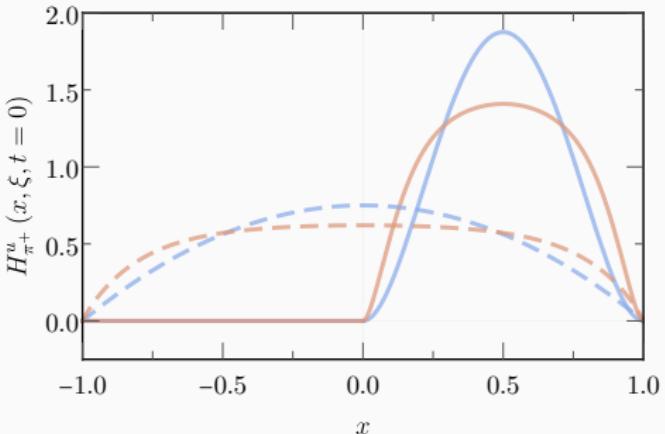
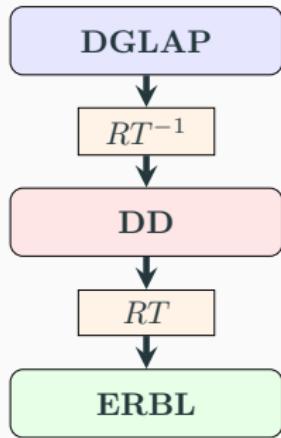
Covariant extension:

$$H^q(x, \xi, t) = \mathcal{R}[h(\beta, \alpha, t)] + \frac{1}{|\xi|} D^+ \left(\frac{x}{\xi}, t \right) + \text{sgn}(\xi) D^- \left(\frac{x}{\xi}, t \right)$$

Fix D-terms with soft pion theorem:

[M.V.Polyakov-NPB:231(555)1999, C.Mezrag et al.-PLB:190(741)2015]

$$\begin{aligned} H_{\pi^+}^{I=0}(x, \xi, t)|_{\xi=1, t=0} &= H_{\pi^+}(x, \xi, t) - H_{\pi^+}(-x, \xi, t)|_{\xi=1, t=0} = 0 \\ H_{\pi^+}^{I=1}(x, \xi, t)|_{\xi=1, t=0} &= H_{\pi^+}(x, \xi, t) + H_{\pi^+}(-x, \xi, t)|_{\xi=1, t=0} = \varphi\left(\frac{1+x}{2}\right) \end{aligned}$$



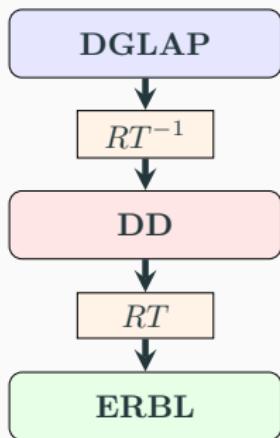
Pion GPDs: CSM-inspired models

GPDs

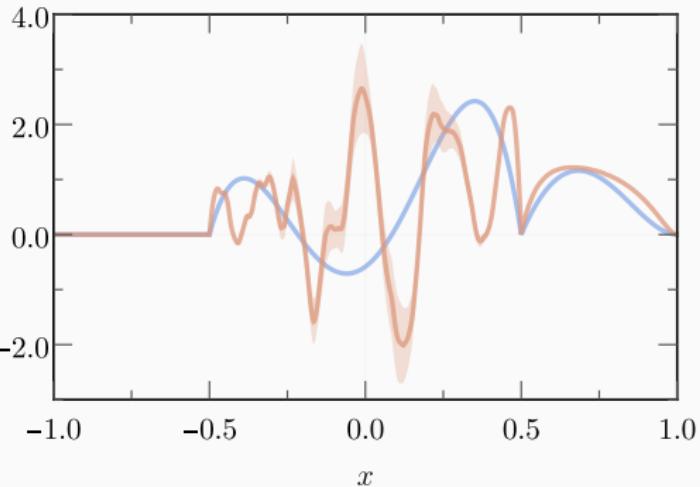
Fundamental properties
+
Pion's phenomenology

GPD properties

Support [Diehl-PLB(1998)]	✓	Positivity [Polyb.-PRD(2002), Pire-EPJC(1999)]	✓
Polynomiality [Ji-JPG(1998), Radyu.-PLB(1999)]	✓	Soft-pion [Poly.-NPB(1999), Mezr.-PLB(2015)]	✓



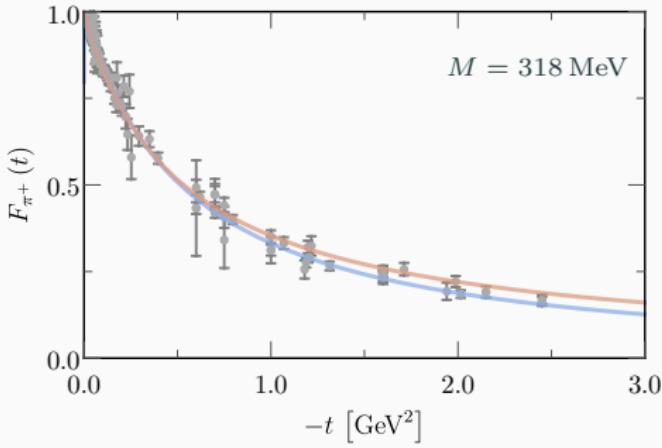
$$H_{\pi^+}^u(x, \xi = 1/2, t = 0)$$



Pion GPDs: Electromagnetic form factor

With GPDs satisfying all the necessary theoretical properties...

$$F_\pi(t) \equiv \int_{-1}^1 H_\pi^q(x, \xi, t) dx = \int_0^1 H_\pi^q(x, 0, t)|_{|x| \geq |\xi|} dx$$

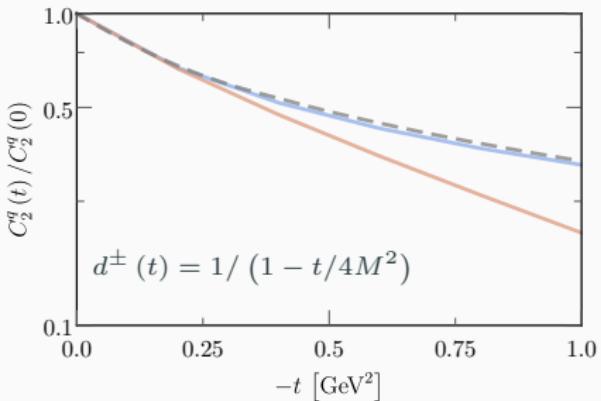
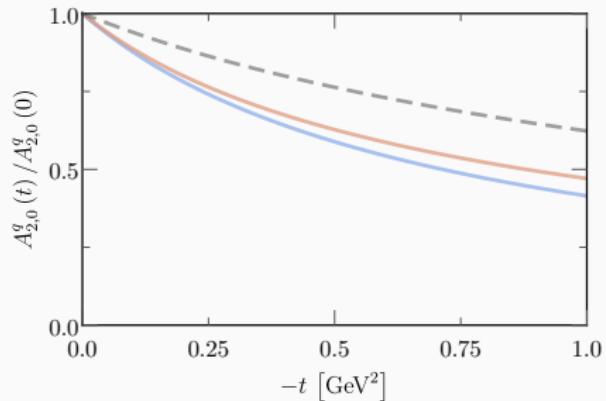


$$r_\pi^2 = -6 \frac{dF_\pi(t)}{dt} \Big|_{t=0} \Rightarrow F_\pi(t) \simeq 1 - \frac{r_\pi^2}{6} t \quad (r_\pi^{\text{PDG}} = 0.672 \pm 0.008 \text{ fm})$$

Pion GPDs: Gravitational form factors

$$\text{EMT: } \langle \pi(p') | T^{\mu\nu} | \pi(p) \rangle = 2P^\mu P^\nu \theta_2(t) + \frac{1}{2} (tg^{\mu\nu} - t^\mu t^\nu) \theta_1(t)$$

$$\text{GPD MM: } \int_{-1}^1 dx x H^q(x, \xi, t) = A_{2,0}^q(t) + 4\xi^2 C_2^q(t)$$



Low- t behaviour in agreement with existing data

Reminder: QCD evolution

$$H(x, \xi, t; \mu^2) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \mathcal{O}(-\lambda n, \lambda n) | p \rangle$$

Light-cone operator product expansion:

$$\mathcal{O}(-\lambda n, \lambda n) \xrightarrow{n^2=0} \sum_i c_i n^{2i} \mathcal{O}_i$$

Renormalization group equations for the operators \mathcal{O}_i yield scale evolution for GPDs:

[X. Ji-PRD:7114(55)1997, V. Bertone-Private comm.]

$$\frac{dH^{(\pm)}(x, \xi, t; \mu^2)}{d \log \mu^2} = \frac{\alpha_S(\mu^2)}{4\pi} \int_x^\infty \frac{dy}{y} \mathcal{P}^{(\pm)}(y, \kappa, \alpha_S(\mu^2)) H^{(\pm)}\left(\frac{x}{y}, \xi, t; \mu^2\right)$$

“Splitting functions”: $\mathcal{P}^{(\pm)}(y, \kappa; \alpha_S(\mu^2)) = \sum_{n=1}^{\infty} \frac{\alpha_S(\mu^2)}{4\pi} \mathcal{P}_n^{(\pm)}(y, \kappa) ; \quad \kappa = \frac{\xi}{x}$

Reminder: QCD evolution

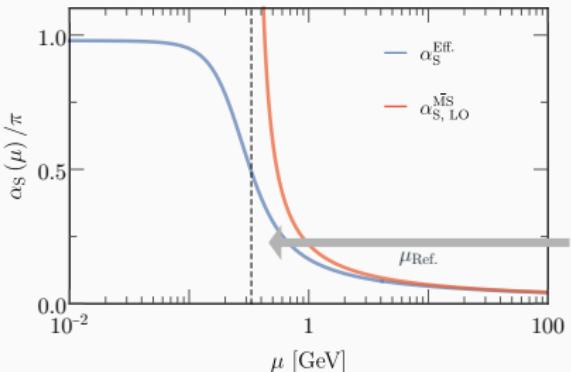
Effective evolution

$$\mathcal{P}^{(\pm)}(y, \kappa, \alpha_S(\mu^2)) = \sum_{n=1}^{\infty} \frac{\alpha_S(\mu^2)}{4\pi} \mathcal{P}_n^{(\pm)}(y, \kappa)$$

Define an **effective** all-orders QCD running

[K.Raya et al.-CPC:26(46)2022]

$$\frac{dH^{(\pm)}(x, \xi, t; \mu^2)}{d \log \mu^2} = \frac{\alpha_S^{\text{Eff.}}(\mu^2)}{4\pi} \int_x^{\infty} \frac{dy}{y} \mathcal{P}_{\text{LO}}^{(\pm)}(y, \kappa) H^{(\pm)}\left(\frac{x}{y}, \xi, t; \mu^2\right)$$



[Z.-F. Cui et al.-EPJC:1064(80)2020]

$$\alpha_S^{\text{Eff.}}(\mu^2) = \frac{4\pi}{\beta_{\text{LO}}^{\text{Nf}=4} \log\left(\frac{\kappa^2(\mu^2)}{\Lambda_{\text{QCD}}^2}\right)}$$

Hadron scale
No sea quarks or gluons

$$\mu_{\text{Ref}} = 0.331 \text{ GeV}$$

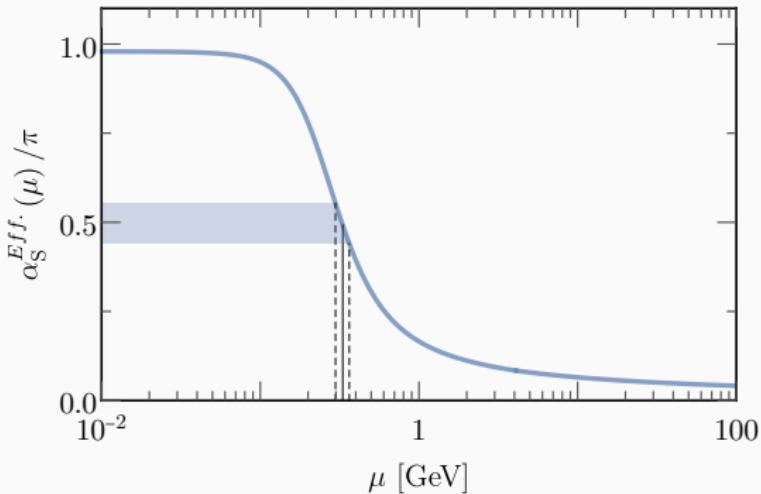
$$\kappa^2(\mu^2) = \frac{a_0 + a_1 \mu^2 + \mu^4}{b_0 + \mu^2}$$

a_0	a_1	b_0
0.104(1)	0.0975	0.121(1)

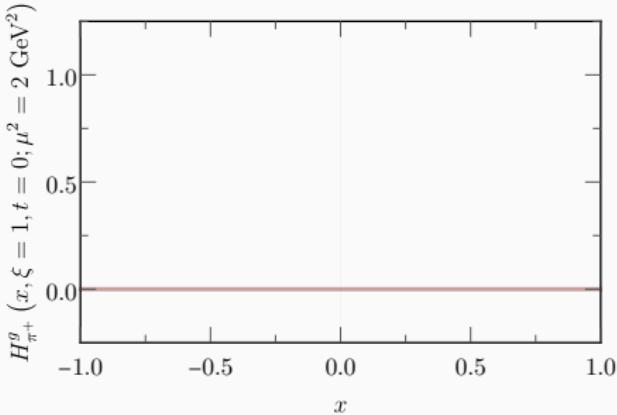
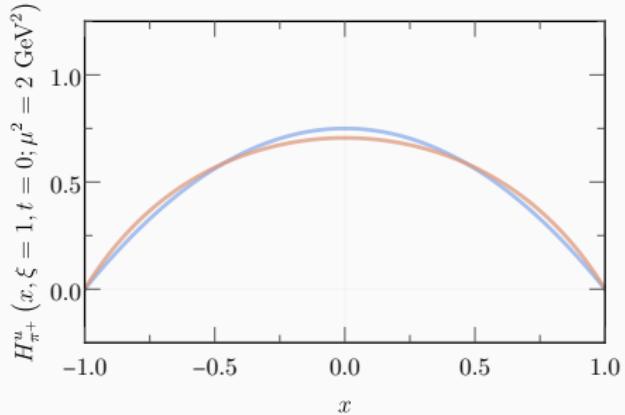
Reminder: QCD evolution uncertainties

Although everywhere analytic and finite, in the “hadron-region” $\alpha_S^{\text{Eff.}}(\mu)$ undergoes large changes with small shifts in the scale.

$$\mu_{\text{Ref}}^{(\pm)} = (1.0 \pm 0.1) \mu_{\text{Ref}}$$

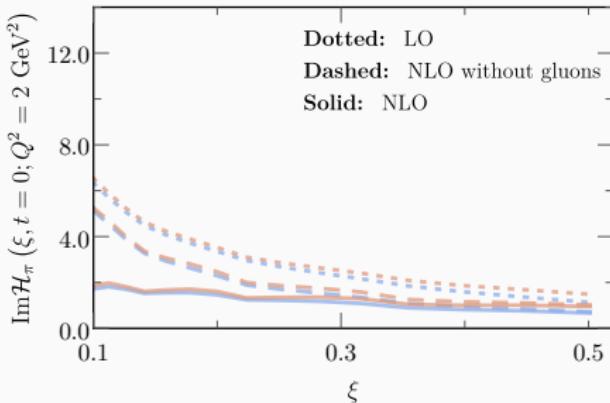
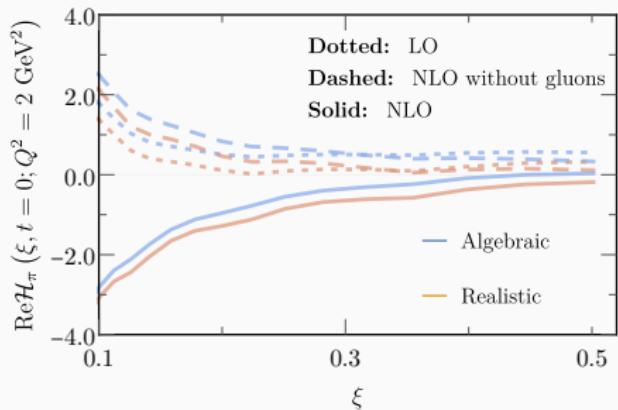


Benchmarking QCD evolution



- Parton distribution amplitudes approach asymptotic DA.
- Chiral symmetry: gluon distribution at $\xi = 1$.

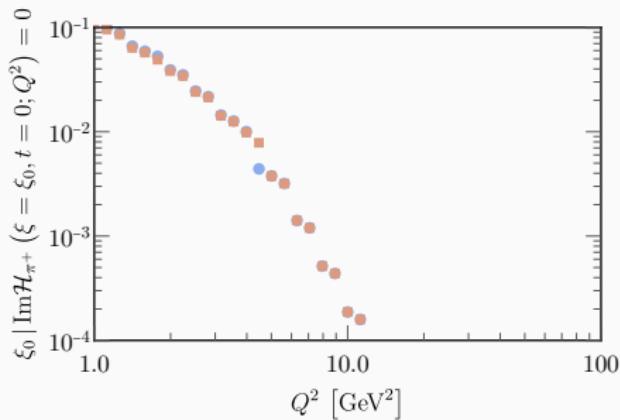
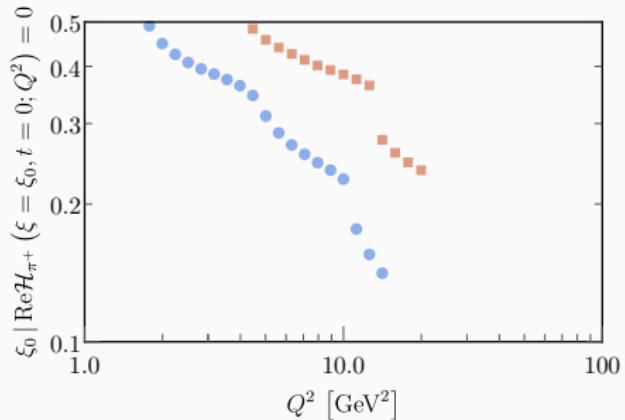
Compton Form Factors: Valence region



- Discrepancy of $\sim 10\%$
- Gluon dominance (change of sign)
- “Destructive interferences” between quark and gluon content

Compton Form Factors: Zero crossings

A final remark on pQCD behaviour



- Importance of NLO contributions decreases with increasing energy, Q^2 . Thus, zero crossing is shifted toward lower ξ -values.
- Coincidence on the imaginary part supports findings on the “deconvolution problem”. [V. Bertone et al.-Phys. Rev. D 11(103)2021]

Zero crossing allows to distinguish between the two models

Phenomenology of pion GPDs: Sullivan process at the EIC

In fact... this has already been advocated in the EIC's yellow report
[EICYR:phys.ins-det/2103.05419]

Science Question	Key Measurement	Key Requirements
What are the quark and gluon energy contributions to the pion mass?	Pion structure function data over a range of x and Q^2 .	<ul style="list-style-type: none">Need to uniquely determine $e + p \rightarrow e' + X + n$ (low $-t$)CM energy range $\sim 10\text{-}100 \text{ GeV}$Charged and neutral currents desirable
Is the pion full or empty of gluons as viewed at large Q^2 ?	Pion structure function data at large Q^2 .	<ul style="list-style-type: none">CM energy $\sim 100 \text{ GeV}$Inclusive and open-charm detection
What are the quark and gluon energy contributions to the kaon mass?	Kaon structure function data over a range of x and Q^2 .	<ul style="list-style-type: none">Need to uniquely determine $e + p \rightarrow e' + X + \Lambda/\Sigma^0$ (low $-t$)CM energy range $\sim 10\text{-}100 \text{ GeV}$CM energy $\sim 100 \text{ GeV}$Inclusive and open-charm detection
Are there more or less gluons in kaons than in pions as viewed at large Q^2 ?	Kaon structure function data at large Q^2 .	<ul style="list-style-type: none">Need to uniquely determine exclusive process $e + p \rightarrow e' + \pi^+ + n$ (low $-t$)$e + p$ and $e + D$ at similar energiesCM energy $\sim 10\text{-}75 \text{ GeV}$
Can we get quantitative guidance on the emergent pion mass mechanism?	Pion form factor data for $Q^2 = 10\text{-}40 \text{ (GeV}/c)^2$.	<ul style="list-style-type: none">Need to uniquely determine exclusive process $e + p \rightarrow e' + K + \Lambda$ (low $-t$)L/T separation at CM energy $\sim 10\text{-}20 \text{ GeV}$$\Lambda/\Sigma^0$ ratios at CM energy $\sim 10\text{-}50 \text{ GeV}$
What is the size and range of interference between emergent-mass and the Higgs-mass mechanism?	Kaon form factor data for $Q^2 = 10\text{-}20 \text{ (GeV}/c)^2$.	<ul style="list-style-type: none">CM energy $\sim 20 \text{ GeV}$ (lowest CM energy to access large-x region)Higher CM energy for range in Q^2 desirable
What is the difference between the impacts of emergent- and Higgs-mass mechanisms on light-quark behavior?	Behavior of (valence) up quarks in pion and kaon at large x .	<ul style="list-style-type: none">Collider kinematics desirable (as compared to fixed-target kinematics)CM energy range $\sim 20\text{-}140 \text{ GeV}$
What is the relationship between dynamically chiral symmetry breaking and confinement?	Transverse-momentum dependent Fragmentation Functions of quarks into pions and kaons.	<ul style="list-style-type: none">Need to uniquely determine exclusive process $e + p \rightarrow e' + J/\Psi + \pi^+ + n$ (low $-t$)High luminosity ($\geq 10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$)CM energy $\sim 70 \text{ GeV}$
More speculative observables		
What is the trace anomaly contribution to the pion mass?	Elastic J/Ψ production at low W off the pion.	<ul style="list-style-type: none">Need to uniquely determine exclusive process $e + p \rightarrow e' + J/\Psi + \pi^+ + n$ (low $-t$)High luminosity ($\geq 10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$)CM energy $\sim 70 \text{ GeV}$
Can we obtain tomographic snapshots of the pion in the transverse plane? What is the pressure distribution in a pion?	Measurement of DVCS off pion target as defined with Sullivan process.	<ul style="list-style-type: none">Need to uniquely determine exclusive process $e + p \rightarrow e' + \gamma + \pi^+ + n$ (low $-t$)High luminosity ($\geq 10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$)CM energy $\sim 10\text{-}100 \text{ GeV}$
Are transverse momentum distributions universal in pions and protons?	Hadron multiplicities in SIDIS off a pion target as defined with Sullivan process.	<ul style="list-style-type: none">Need to uniquely determine SIDIS off pion $e + p \rightarrow e' + h + X + n$ (low $-t$)High luminosity ($10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$)$e + p$ and $e + D$ at similar energies desirableCM energy $\sim 10\text{-}100 \text{ GeV}$

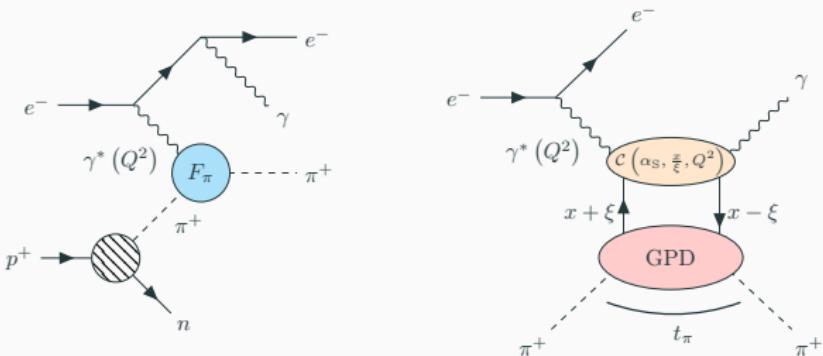
Let us see if that would be feasible in a future electron-ion collider.

Phenomenology of pion GPDs: DVCS and Sullivan process

Can we access pion's GPDs through experiment?

Sullivan process

One pion exchange approximation: $|t_\pi|^{\text{Max.}} = 0.6 \text{ GeV}^2$, $\sigma_L^{\gamma^*} \gg \sigma_\perp^{\gamma^*}$



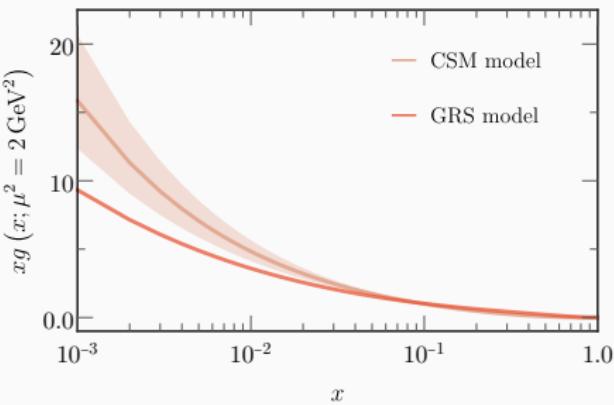
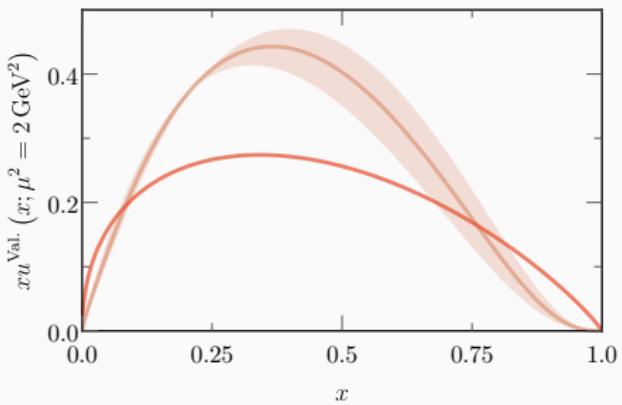
$$\mathcal{M}_{e\pi} = \mathcal{M}_{\text{BH}} + \mathcal{M}_{\text{DVCS}}$$

$$\sigma^{e\pi} (\lambda, \pm e) \propto |\mathcal{M}_{\text{BH}}|^2 + |\mathcal{M}_{\text{DVCS}}|^2 \mp \mathcal{I}$$

[Break] A comparison with global fits

For comparison purposes we include a phenomenological model of the pion's parton content.

[GRS] M. Glück, E. Reya, I. Schienbein. Eur. Phys. J. C, 10, 313-317 (1999).



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