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Dijet production in DIS at one-loop in the CGC

International Workshop on Deep Inelastic Scattering
and Related Subjects

Santiago de Compostela
May 3rd, 2022

Farid Salazar

Paul Caucal, FS, and Raju Venugopalan. [2108.06347](https://arxiv.org/abs/2108.06347) [*JHEP 11 (2021) 222*]



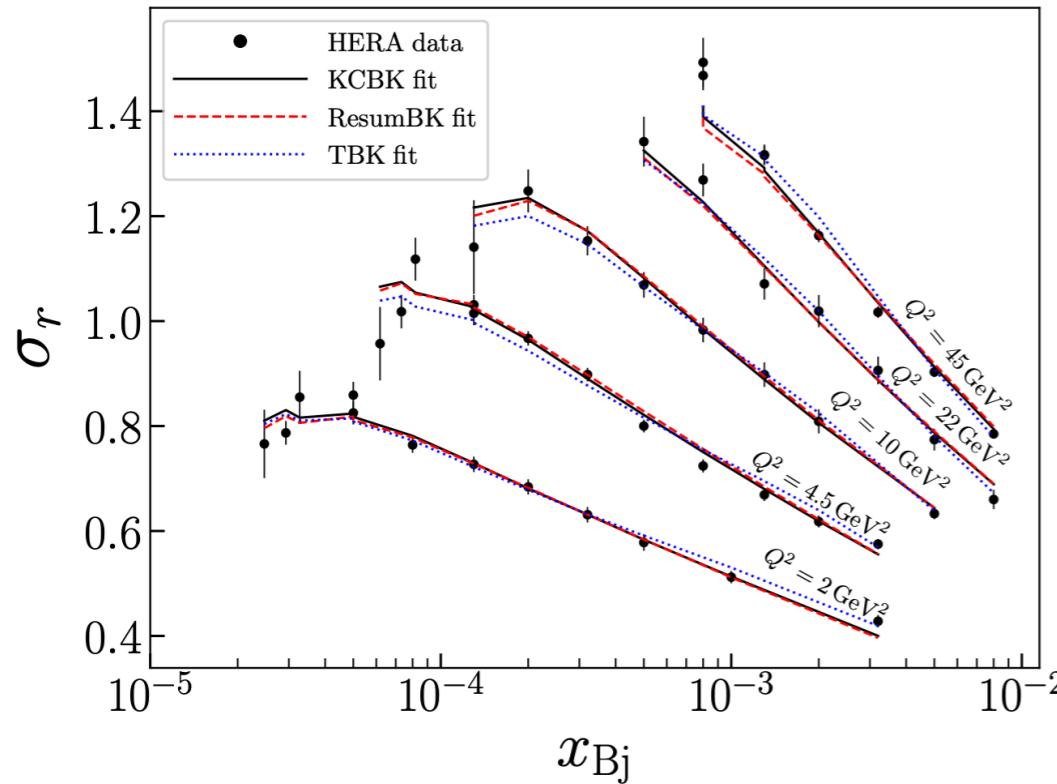
+ some work in progress
PC, FS, Björn Schenke, and RV

Outline

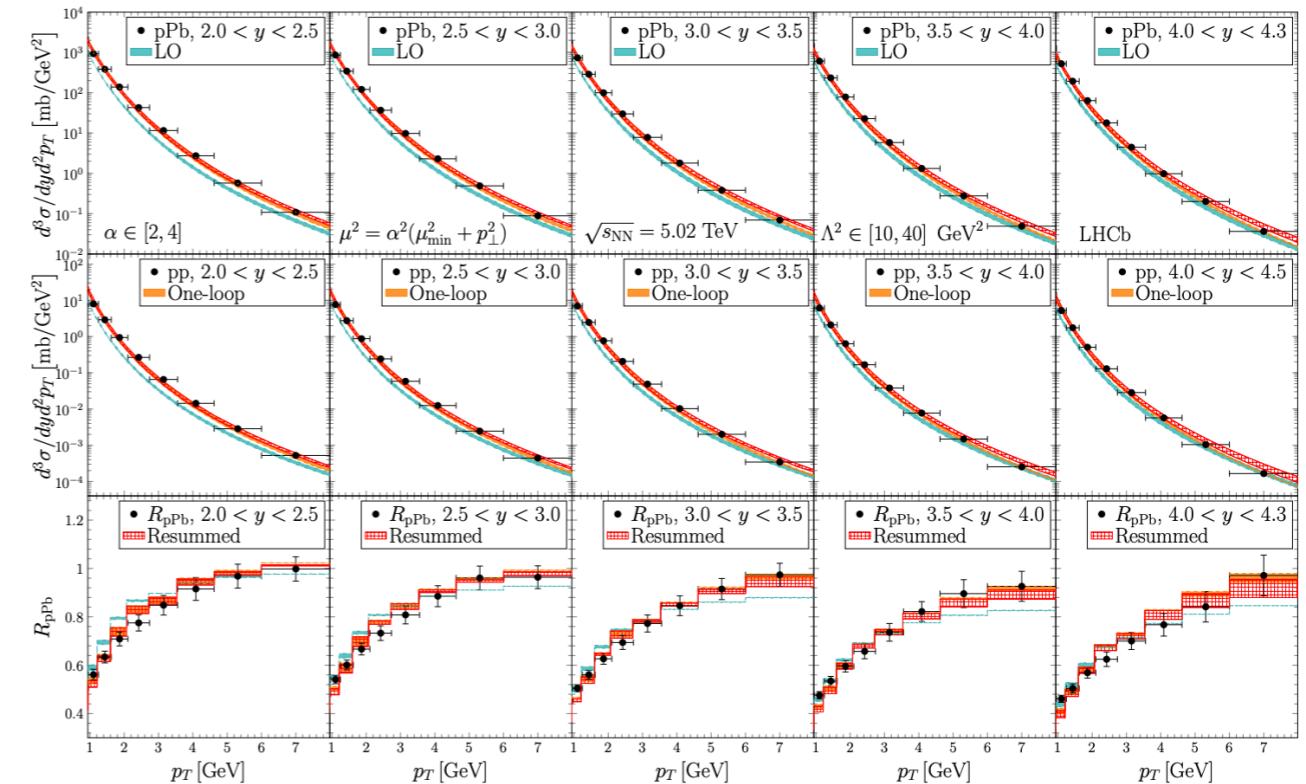
- Motivation
Saturation Physics at NLO. Why inclusive dijets in DIS? Review of Leading Order
- One-loop corrections to dijet production in DIS
Framework. NLO Amplitudes. Cancellation of divergences. JIMWLK factorization
- Back-to-back limit
Sudakov suppression. The need for kinematic constraint
- Summary & Outlook

Saturation physics at NLO

[see Bowen Xiao's plenary talk](#)



Beuf, Lappi, Hänninen, Mäntysaari (PRD 2020)



Shi, Wang, Wei, Xiao (2021)

Many developments presented at this very conference!

Structure functions with massive quarks
[\(see Tuomas Lappi's talk\)](#)

Exclusive vector meson production in DIS
[\(see Jani Penttala's talk\)](#)

Dihadron production in DIS
[\(see Jamal Jalilian-Marian's talk\)](#)

Single inclusive jet production in pA
[\(see Hao-yu Liu's talk\)](#)

JIMWLK with massive quarks
[\(see Lin Dai's talk\)](#)

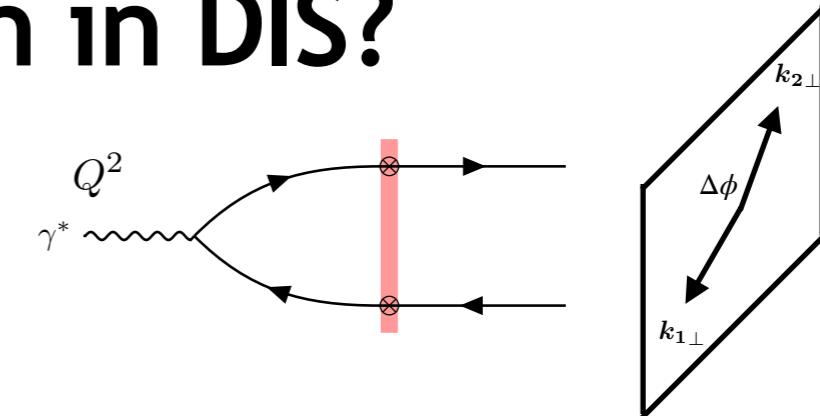
This talk: Inclusive dijet in DIS

Why inclusive dijet production in DIS?

Complementary process to fully inclusive

Study dependence on kinematic variables, correlations, etc

Kharzeev, Levin, McLerran (Nuc.Phys. A 2005) Marquet (Nuc.Phys. A 2007)

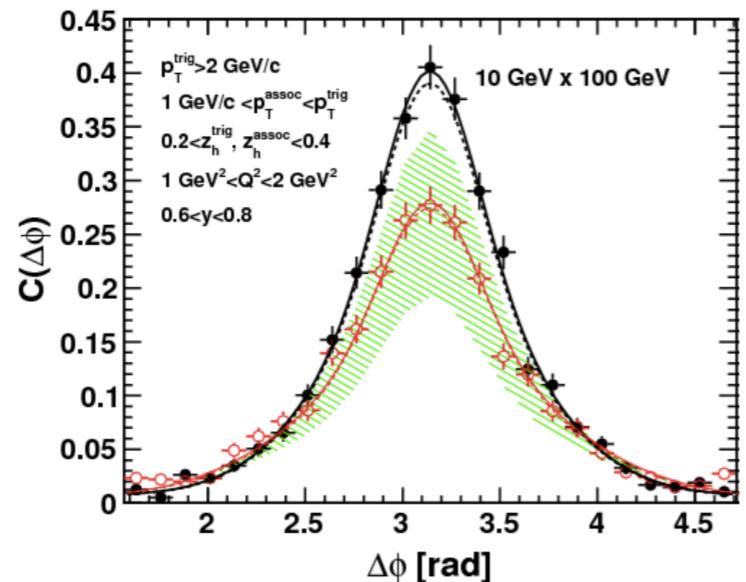


$$Q_Y(x_\perp, y_\perp; y'_\perp, x'_\perp) = \frac{1}{N_c} \langle \text{Tr} [V(x_\perp)V^\dagger(y_\perp)V(y'_\perp)V^\dagger(x'_\perp)] \rangle_Y$$

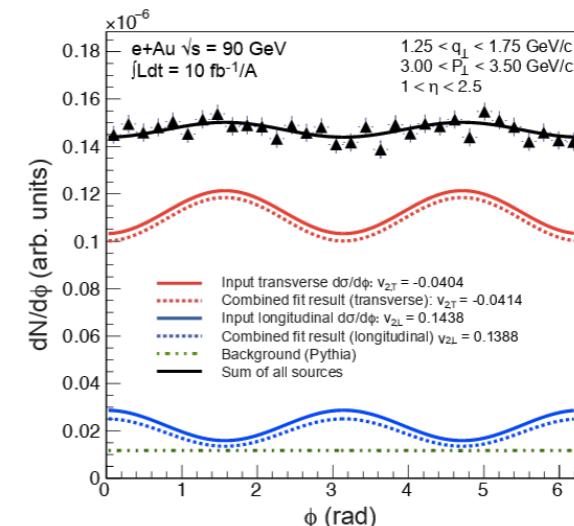
In the back-to-back limit contact with the
(transverse momentum dependent) TMD formalism

Weizsäcker-Williams TMD

Dominguez, Marquet, Xiao, Yuan (PRD 2011)



Simplest observable featuring the quadrupole.
Validity of JIMWLK factorization beyond the dipole



Dumitru, Skokov, Ullrich (PRC 2019)

Suppression of back-to-back peak potential
signature of gluon saturation (dihadrons)

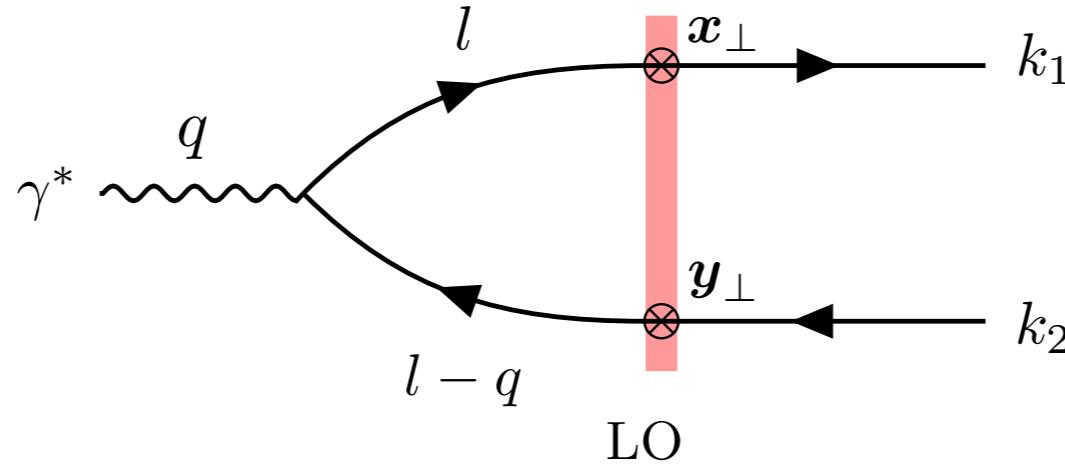
Zheng, Aschenauer, Lee, Xiao (PRD 2014)

See also [Cyrille Marquet's talk](#) and [Sanjin Benic's talk](#)

Jets are better proxies of hard partons (than hadrons)

Leading order

Anatomy of the cross-section



Amplitude:

$$\mathcal{M}_{\text{LO},ij,\sigma_1\sigma_2}^\lambda \propto [V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) - \mathbb{1}]_{ij} \mathcal{N}_{\sigma_1,\sigma_2}^\lambda(Q, z_1, \mathbf{x}_\perp - \mathbf{y}_\perp)$$

q̄q interaction with nucleus γ^* splitting to *q̄q*

Unpolarized differential cross-section:

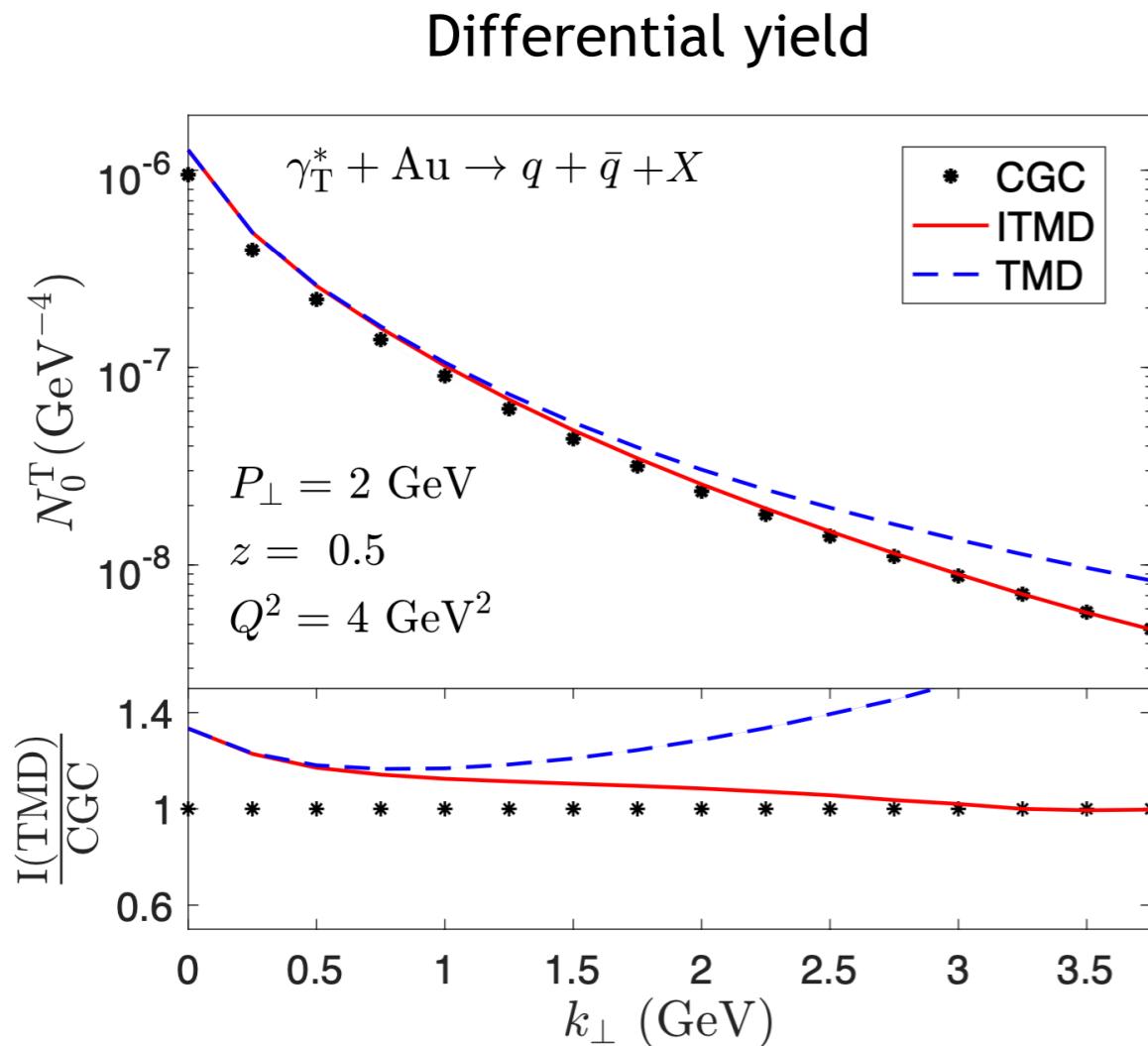
$$\frac{d\sigma^{\gamma_\lambda^* + A \rightarrow q\bar{q} + X}}{d^2 k_{1\perp} d^2 k_{2\perp} d\eta_1 d\eta_2} \propto \int d^8 X_\perp e^{-i k_{1\perp} \cdot (x_\perp - x'_\perp)} e^{-i k_{2\perp} \cdot (y_\perp - y'_\perp)} \\ \times \langle \Xi_{\text{LO}}(x_\perp, y_\perp; y'_\perp x'_\perp) \rangle_Y \mathcal{R}^\lambda(x_\perp - y_\perp, x'_\perp - y'_\perp)$$

$$\Xi_{\text{LO}}(x_\perp, y_\perp; y'_\perp x'_\perp) = 1 - S^{(2)}(x_\perp, y_\perp) - S^{(2)}(y'_\perp, x'_\perp) + S^{(4)}(x_\perp, \underset{\substack{\text{dipoles} \\ \uparrow}}{y_\perp}; \underset{\substack{\text{quadrupole} \\ \uparrow}}{y'_\perp, x'_\perp})$$

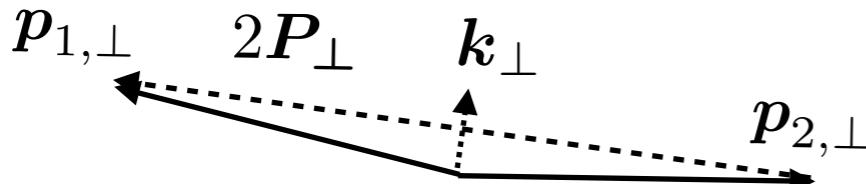
Leading order

Kinematic and genuine power corrections to the TMD limit

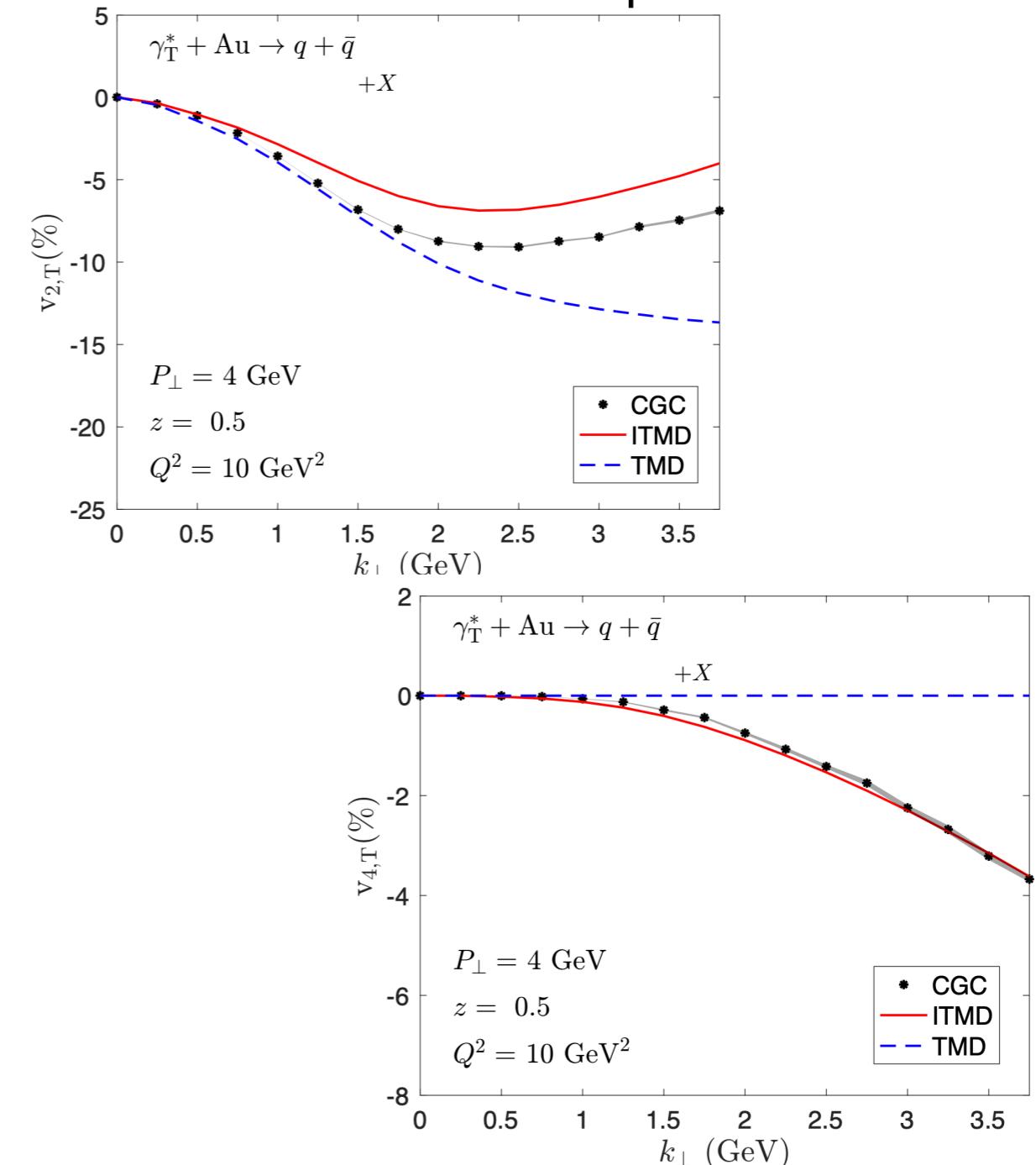
Mäntysaari, Mueller, FS, Schenke (PRL 2020)
 Boussarie, Mäntysaari, FS, Schenke (JHEP 2021)



CGC shows further suppression relative to TMD at back-to-back limit



Momentum imbalance azimuthal anisotropies

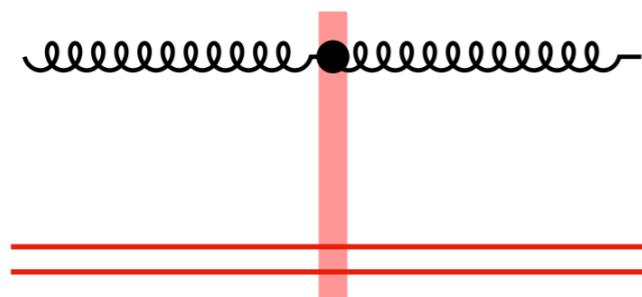


Anisotropies modified in ITMD and CGC

One-loop corrections

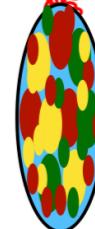
General remarks

- Covariant PT (in light-cone gauge) with effective Feynman rules for eikonal multiple scattering



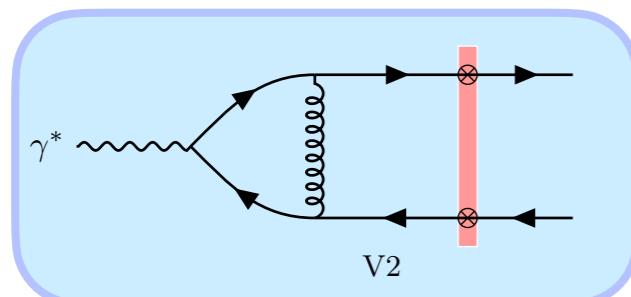
$$\mathcal{T}_g^{\mu\nu,ab}(l, l') = -(2\pi)\delta(l^- - l'^-)(2l^-)g^{\mu\nu}\text{sgn}(l^-)$$

$$\times \int d^2 z_\perp e^{-i(l'_\perp - l_\perp) \cdot z_\perp} U_{ab}^{\text{sgn}(l^-)}(z_\perp)$$

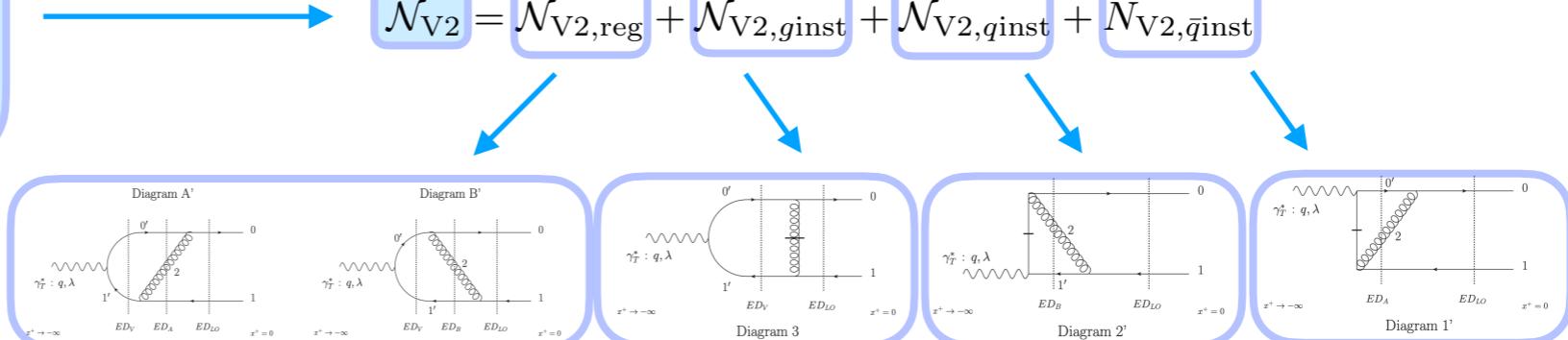


Adjoint Wilson line

- Separate regular and instantaneous pieces in the Dirac-Lorentz structures



Inspired from spinor-helicity techniques and LCPT



- Regularization schemes

Dimensional regularization for transverse integrals
+ sharp cut-off Λ_0^- in longitudinal momentum

+ Small R cone algorithm

$$\int d^{2-\varepsilon} l_\perp \int_{\Lambda_0^-} \frac{dl^-}{l^-}$$

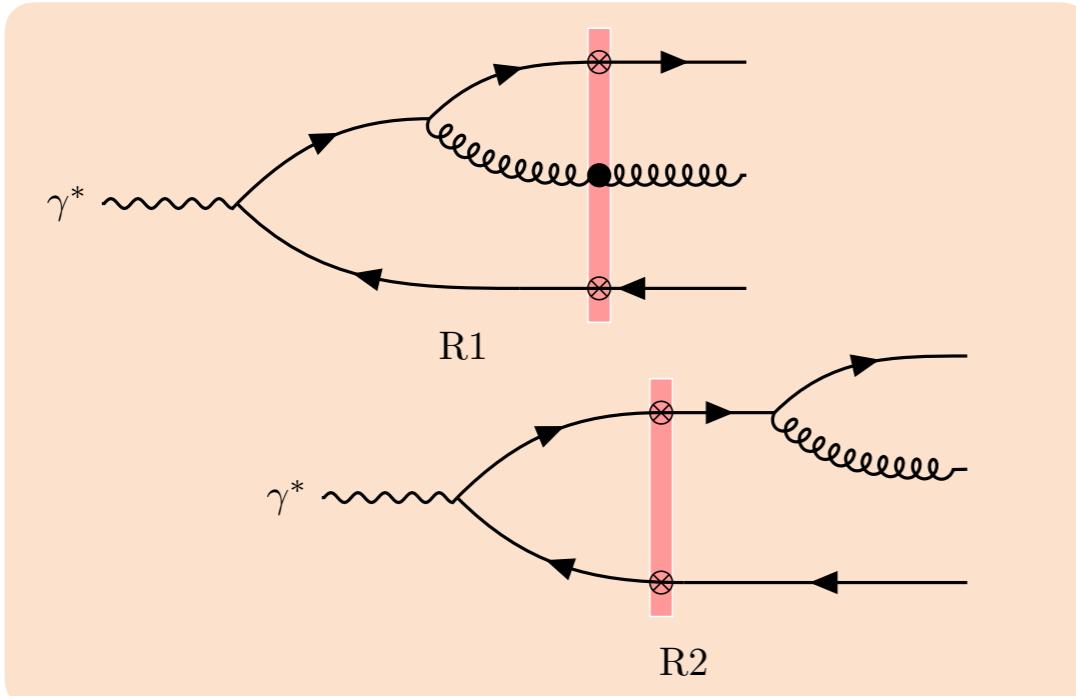
Similar regularization procedure:

Boussarie, Grabovsky, Szymanowski, Wallon (JHEP 2016)

Roy, Venugopalan (PRD 2019)

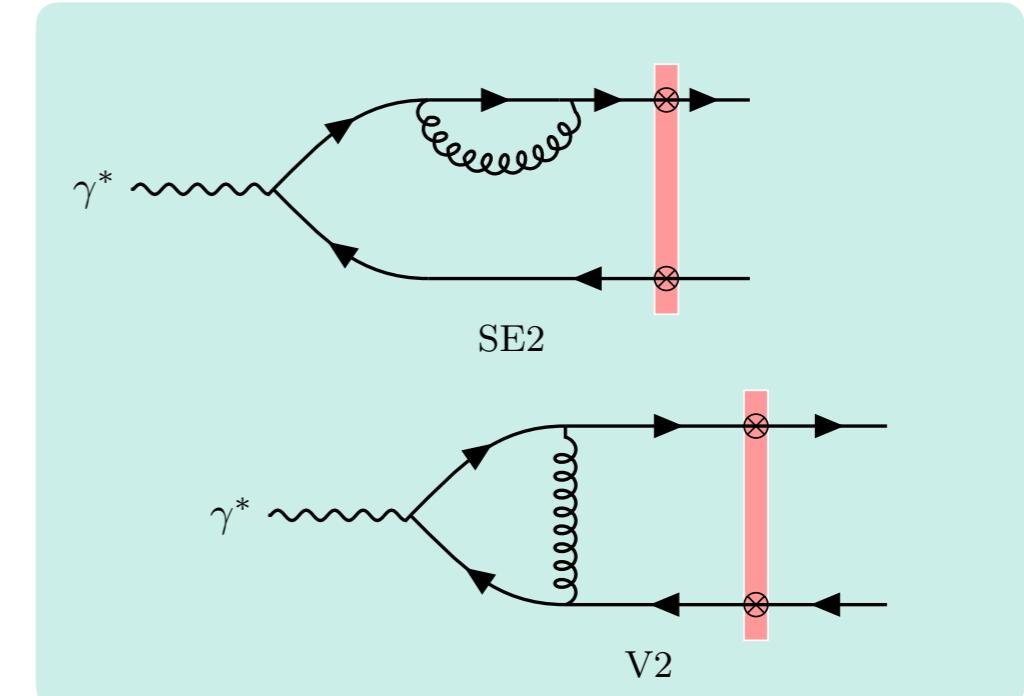
One-loop corrections

Amplitudes



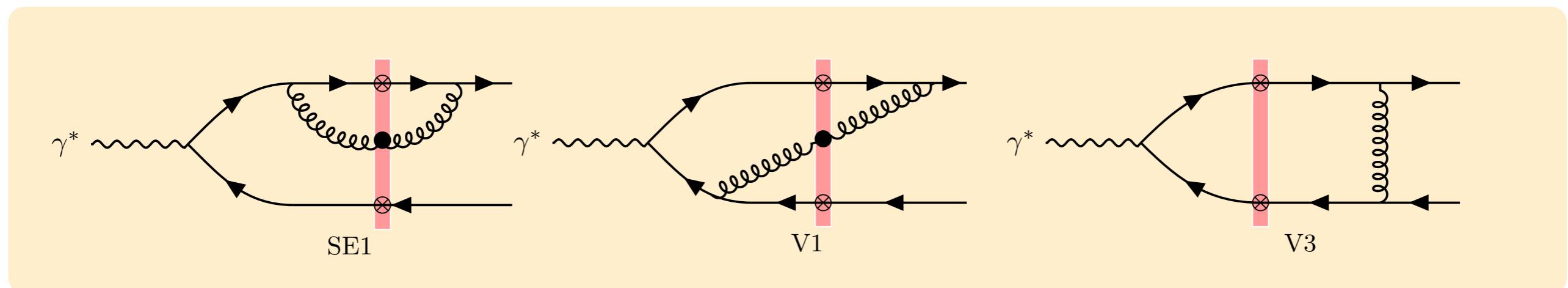
Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans
(Nuc.Phys. B 2017)

Altinoluk, Boussarie, Marquet, Tael (JHEP 2020)
with LCPT and $Q^2 = 0$



Beuf (PRD 2016) with LCPT

Hänninen, Lappi, and Paatelainen (Annals Phys. 2017) with LCPT



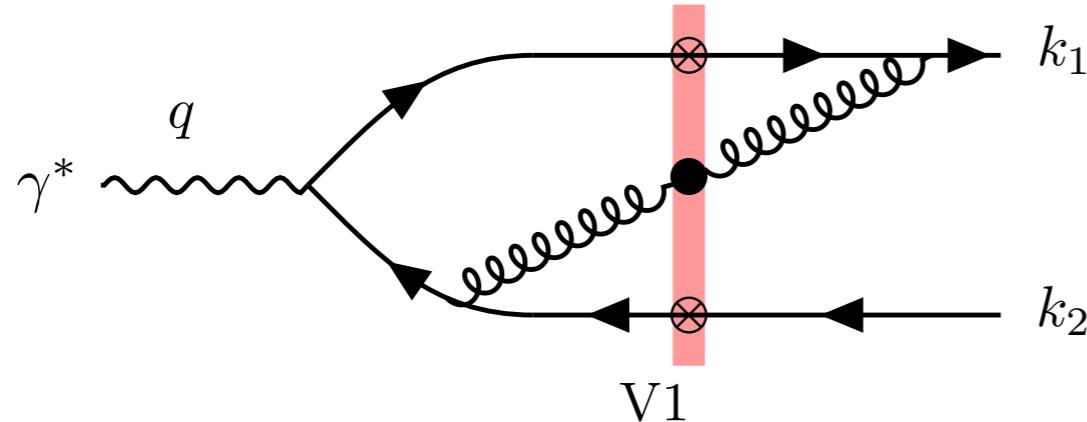
Boussarie, Grabovsky, Szymanowski, Wallon (JHEP 2016) for exclusive dijets

Tael, Altinoluk, Marquet, Beuf (2022) with LCPT and $Q^2 = 0$

Caucal, FS, Venugopalan (JHEP 2021)
Computed all diagrams within covariant PT for
inclusive dijets

One-loop corrections

Example I: Vertex with gluon crossing SW



$$\mathcal{C}_{V1,ij}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp) = [t^a V(\mathbf{x}_\perp) V^\dagger(\mathbf{z}_\perp) t_a V(\mathbf{z}_\perp) V^\dagger(\mathbf{y}_\perp) - C_F]_{ij}$$

$$\mathcal{M}_{V1,ij,\sigma_1\sigma_2}^\lambda = \frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp)} \mathcal{C}_{V1,ij}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp) \mathcal{N}_{V1,\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{zy})$$

Perturbative factor:

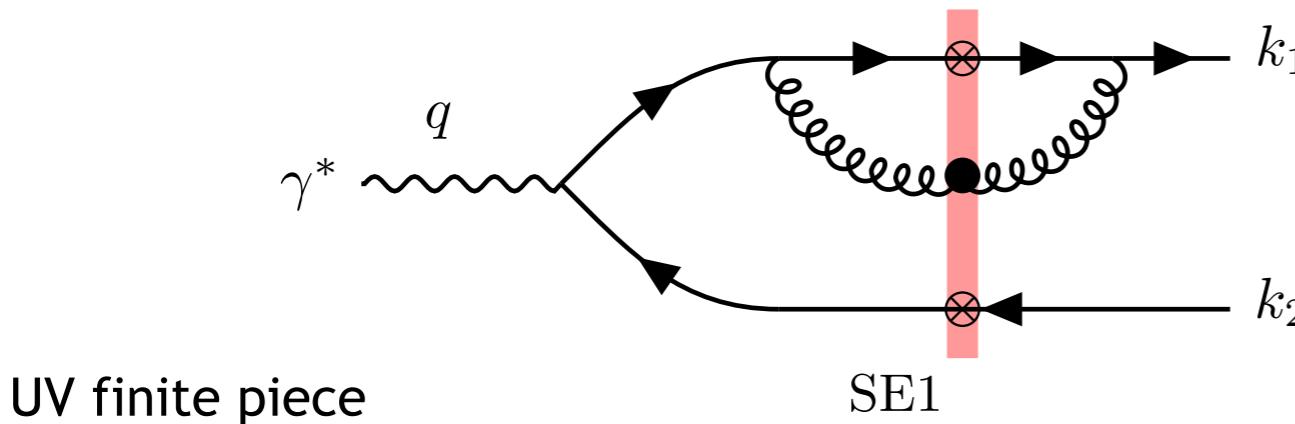
$$\begin{aligned} \mathcal{N}_{V1,\sigma_1\sigma_2}^{\lambda=0}(\mathbf{r}_{xy}, \mathbf{r}_{zy}) &= \frac{\alpha_s}{\pi^2} \int_{z_0}^{z_1} \frac{dz_g}{z_g} e^{-i\frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{zx}} 2(z_1 z_2)^{3/2} Q K_0(Q X_V) \delta_{\sigma_1, -\sigma_2} \\ &\times \left\{ \left(1 - \frac{z_g}{z_1}\right) \left(1 + \frac{z_g}{z_2}\right) \left[1 - \frac{z_g}{2z_1} - \frac{z_g}{2(z_2 + z_g)}\right] \frac{\mathbf{r}_{zx} \cdot \mathbf{r}_{zy}}{\mathbf{r}_{zx}^2 \mathbf{r}_{zy}^2} + \dots \right\} \end{aligned}$$

This contribution is UV finite!

$$X_V^2 = z_2(z_1 - z_g)\mathbf{r}_{xy}^2 + z_g(z_1 - z_g)\mathbf{r}_{zx}^2 + z_g z_2 \mathbf{r}_{zy}^2$$

One-loop corrections

Example II: Self energy with gluon crossing SW



$$\mathcal{M}_{\text{SE1,UVfinite},ij,\sigma_1\sigma_2}^\lambda =$$

$$\frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp)} [\mathcal{C}_{\text{SE1},ij}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp) \mathcal{N}_{\text{SE1},\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{zx}) - \mathcal{C}_{\text{UV},ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \mathcal{N}_{\text{SE1,UV},\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{zx})]$$

$$\mathcal{N}_{\text{SE1}}^{\lambda=0,\sigma\sigma'}(\mathbf{r}_{xy}, \mathbf{r}_{zx}) = -\frac{\alpha_s}{\pi^2} \int_{z_0}^{z_1} \frac{dz_g}{z_g} \frac{1}{2} \left[1 + \left(1 - \frac{z_g}{z_1} \right)^2 \right] \frac{e^{-i\frac{z_g}{z_1}\mathbf{k}_{1\perp} \cdot \mathbf{r}_{zx}}}{\mathbf{r}_{zx}^2} 2(z_1 z_2)^{3/2} Q K_0(Q X_V) \delta_{\sigma_1, -\sigma_2}$$

$$\mathcal{N}_{\text{SE1,UV},\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{zx}) = -\frac{\alpha_s}{\pi^2} \int_{z_0}^{z_q} \frac{dz_g}{z_g} \frac{e^{-\frac{\mathbf{r}_{zx}^2}{2\xi}}}{\mathbf{r}_{zx}^2} 2(z_1 z_2)^{3/2} Q K_0(Q \sqrt{z_1 z_2} r_{xy}) \delta_{\sigma_1, -\sigma_2}$$

Hänninen, Lappi,
Paatelainen
(Annals Phys. 2017)

UV divergent piece

$$X_V^2 = z_2(z_1 - z_g)\mathbf{r}_{xy}^2 + z_g(z_1 - z_g)\mathbf{r}_{zx}^2 + z_2 z_g \mathbf{r}_{zy}^2$$

$$\mathcal{M}_{\text{SE1,UV},ij,\sigma_1\sigma_2}^\lambda = \frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp)} \mathcal{C}_{\text{UV},ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \mathcal{N}_{\text{SE1,UV},\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{zx})$$

$$\mathcal{N}_{\text{SE1,UV},\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy}) = \frac{\alpha_s}{2\pi} \left\{ \left(2 \ln \left(\frac{z_1}{z_0} \right) - \frac{3}{2} \right) \left(\frac{2}{\varepsilon} + \ln(2\pi\mu^2\xi) \right) - \frac{1}{2} + \mathcal{O}(\varepsilon) \right\} \mathcal{N}_{\text{LO},\varepsilon,\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy})$$

UV pole

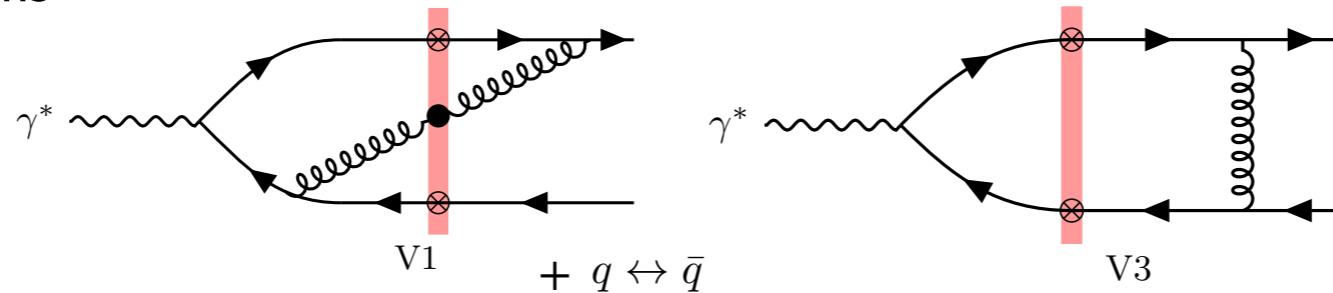
$$\mathcal{C}_{\text{SE1},ij}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp) = [t^a V(\mathbf{x}_\perp) V^\dagger(\mathbf{z}_\perp) t_a V(\mathbf{z}_\perp) V^\dagger(\mathbf{y}_\perp) - C_F]_{ij}$$

$$\mathcal{C}_{\text{UV},ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) = C_F [V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) - \mathbb{1}]_{ij}$$

One-loop corrections

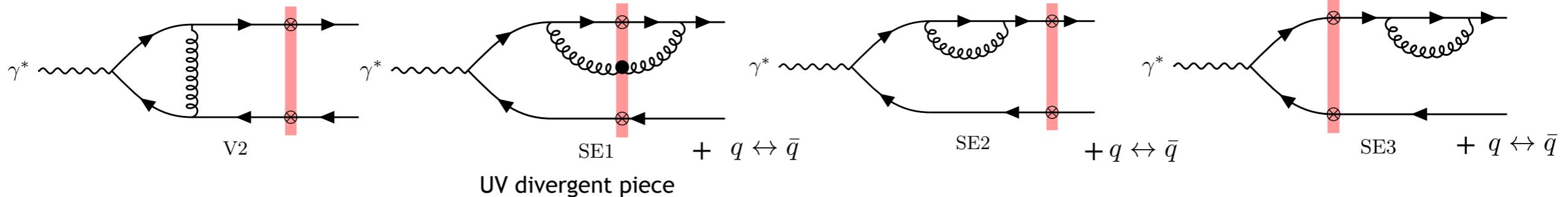
Cancellation of UV divergences

- UV finite diagrams



Real contributions
are UV finite

- UV divergent diagrams



Sum of these contributions:

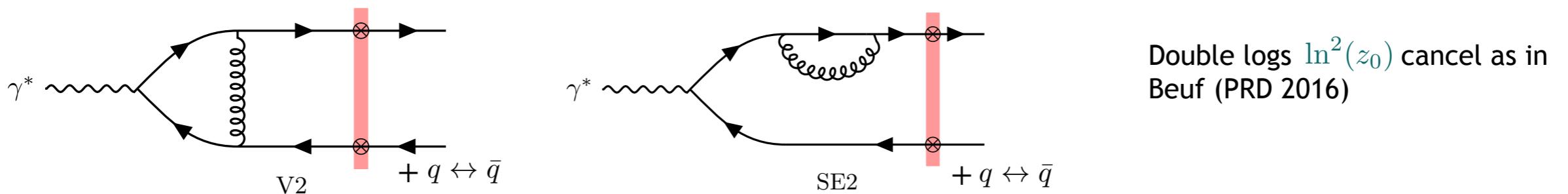
$$\begin{aligned}
 \mathcal{M}_{\text{IR}} &= \mathcal{M}_{V2} + (\mathcal{M}_{\text{SE1,UV}} + \mathcal{M}_{\text{SE2}} + \mathcal{M}_{\text{SE3}} + q \leftrightarrow \bar{q}) \xleftarrow{\text{Contributions proportional to LO color structure}} \\
 &= \frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp)} C_F [V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) - 1]_{ij} \mathcal{N}_{\text{LO}, \varepsilon, \sigma_1 \sigma_2}^\lambda(\mathbf{r}_{xy}) \\
 &\times \frac{\alpha_s}{2\pi} \left\{ \left(\ln \left(\frac{z_q}{z_0} \right) + \ln \left(\frac{z_{\bar{q}}}{z_0} \right) - \frac{3}{2} \right) \left(\frac{2}{\varepsilon} - 2\gamma_E - \ln \left(\frac{\mathbf{r}_{xy}^2 \tilde{\mu}^2}{4} \right) + 2 \ln(2\pi\mu^2 \xi) \right) + \frac{1}{2} \ln^2 \left(\frac{z_{\bar{q}}}{z_q} \right) - \frac{\pi^2}{6} + \frac{5}{2} - \frac{1}{2} \right\}
 \end{aligned}$$

IR pole

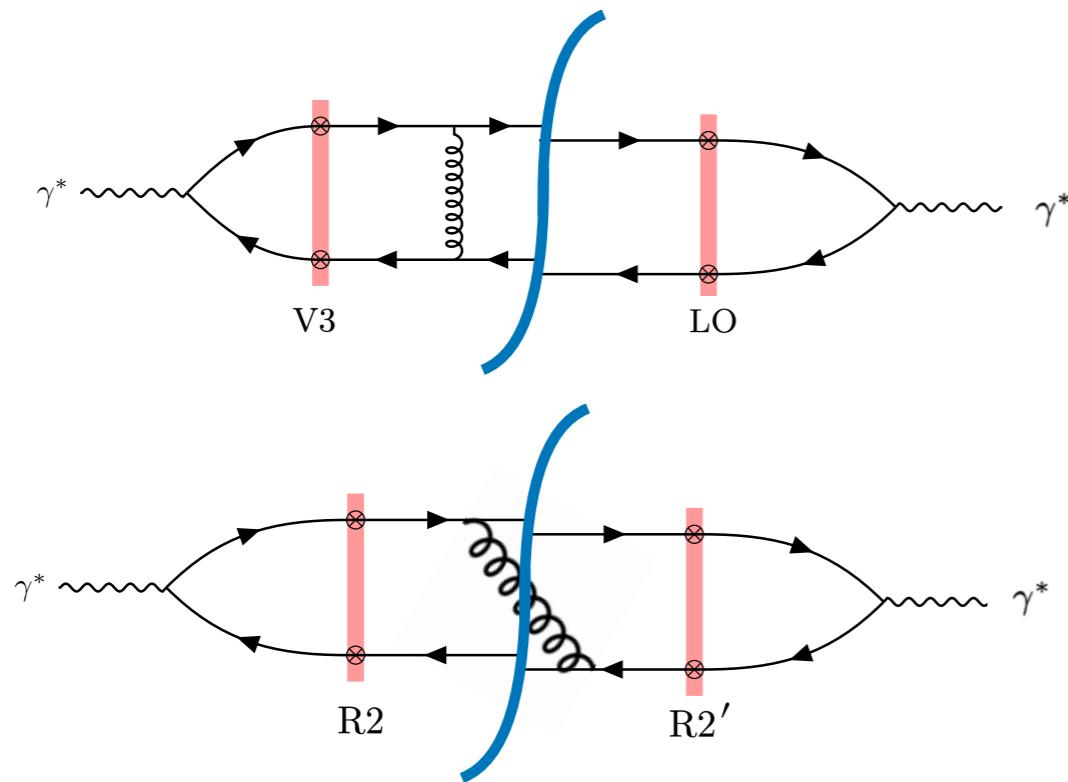
One-loop corrections

Cancellation of IR and collinear divergences

IR divergences manifest as double logs $\ln^2(z_0)$ in our calculation (our regularization scheme)



Double logs also occur in V3xLO and R2xR2' (and c.c.) by examining singular part of z_g



$$\frac{d\sigma_{V3 \times LO, \text{slow}}^\lambda}{d^2 \mathbf{k}_{1\perp} d\eta_1 d^2 \mathbf{k}_{2\perp} d\eta_2} = \frac{\alpha_{\text{em}} e_f^2 N_c}{(2\pi)^6} \delta(1 - z_1 - z_2) \int d\Pi_{\text{LO}} \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \\ \times \frac{(-\alpha_s)}{\pi} \int_{z_0}^{z_f} \frac{dz_g}{z_g} \left[2 \ln \left(\frac{z_g}{2z_1 z_2} \right) + \ln (\mathbf{P}_\perp^2 \mathbf{r}_{xy}^2) + 2\gamma_E \right] \Xi_{\text{NLO},3}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{x}'_\perp, \mathbf{y}'_\perp)$$

$$\frac{d\sigma_{R2 \times R2', \text{slow}}^\lambda}{d^2 \mathbf{k}_{1\perp} d\eta_1 d^2 \mathbf{k}_{2\perp} d\eta_2} = \frac{\alpha_{\text{em}} e_f^2 N_c}{(2\pi)^6} \delta(1 - z_1 - z_2) \int d\Pi_{\text{LO}} \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \\ \times \frac{\alpha_s}{\pi} \int_{z_0}^{z_f} \frac{dz_g}{z_g} \left[2 \ln \left(\frac{z_g}{2z_1 z_2} \right) + \ln (\mathbf{P}_\perp^2 \mathbf{r}_{xy'}^2) + 2\gamma_E \right] \Xi_{\text{NLO},3}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{x}'_\perp, \mathbf{y}'_\perp)$$

Sum V3xLO + R2xR2' is free of double logs

One-loop corrections

Slow gluon limit and JIMWLK factorization

$$d\sigma_{\text{NLO}} = d\tilde{\sigma}_0 \ln \left(\frac{z_f}{z_0} \right) + \int_0^z \frac{dz_g}{z_g} [d\tilde{\sigma}_{\text{NLO}} - d\tilde{\sigma}_0 \Theta(z_f - z_g)] + \mathcal{O}(z_0)$$

Slow gluon piece impact factor

$$\frac{d\sigma_{\text{NLO}}^\lambda}{d^2 k_{1\perp} d\eta_1 d^2 k_{2\perp} d\eta_2} \Big|_{\text{slow}} = \frac{\alpha_{\text{em}} e_f^2 N_c}{(2\pi)^6} \delta(1 - z_q - z_{\bar{q}}) \int d\mathbf{X}_\perp \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \ln \left(\frac{z_f}{z_0} \right)$$

$$\times \frac{\alpha_s N_c}{4\pi^2} \left\langle \int d^2 z_\perp \left\{ \begin{aligned} & \frac{\mathbf{r}_{xy}^2}{\mathbf{r}_{zx}^2 \mathbf{r}_{zy}^2} (2D_{xy} - 2D_{xz}D_{zy} + D_{zy}Q_{y'x',xz} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\ & + \frac{\mathbf{r}_{x'y'}^2}{\mathbf{r}_{zx'}^2 \mathbf{r}_{zy'}^2} (2D_{y'x'} - 2D_{y'z}D_{zx'} + D_{zx'}Q_{xy,y'z} + D_{y'z}Q_{xy,zx'} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\ & + \frac{\mathbf{r}_{xx'}^2}{\mathbf{r}_{zx}^2 \mathbf{r}_{zx'}^2} (D_{zx'}Q_{xy,y'z} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\ & + \frac{\mathbf{r}_{yy'}^2}{\mathbf{r}_{zy}^2 \mathbf{r}_{zy'}^2} (D_{y'z}Q_{xy,zx'} + D_{zy}Q_{y'x',xz} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\ & + \frac{\mathbf{r}_{xy'}^2}{\mathbf{r}_{zx}^2 \mathbf{r}_{zy'}^2} (D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{zx'}Q_{xy,y'z} - D_{zy}Q_{y'x',xz}) \\ & + \frac{\mathbf{r}_{x'y}^2}{\mathbf{r}_{zx'}^2 \mathbf{r}_{zy}^2} (D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{y'z}Q_{xy,zx'} - D_{xz}Q_{y'x',zy}) \end{aligned} \right\} \right\rangle_Y$$

JIMWLK LL Hamiltonian acting on LO color structure $\mathcal{H}_{\text{JIMWLK}} \langle \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp \mathbf{x}'_\perp) \rangle_Y$

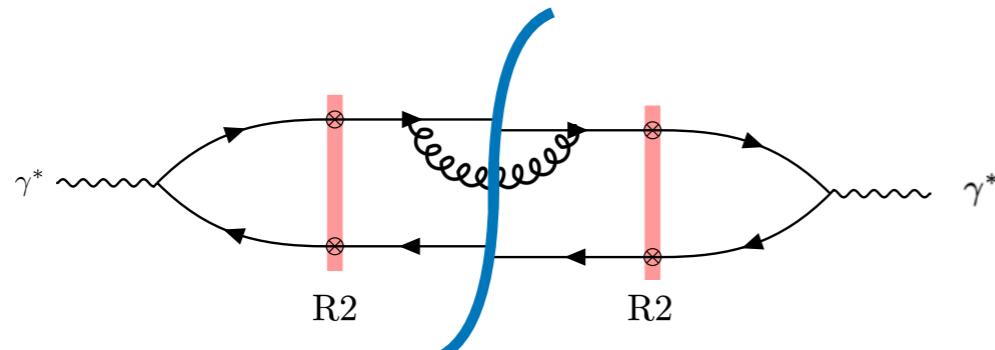
Evolution of quadrupole can be found in
Dominguez, Mueller, Munier, and Xiao (Phys.Lett.B 2011)

Small-x evolution of dipole and quadrupole!

One-loop corrections

Cancellation of soft and collinear divergences

- Implement a jet algorithm* (small cone) excluding slow gluon divergence



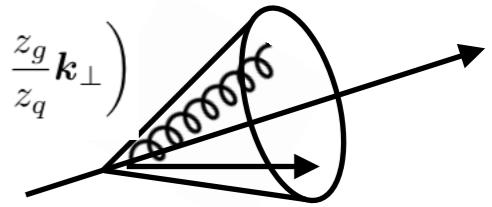
- Collinear divergence cancels against IR divergence left in virtual contributions

Phase space for collinear non-slow gluon

$$\int_{z_f}^{z_j} \frac{dz_g}{z_g} \mu^\varepsilon \int \frac{d^{2-\varepsilon} \mathcal{C}_{qg,\perp}}{(2\pi)^{2-\varepsilon}} \frac{1}{\mathcal{C}_{qg,\perp}^2}$$

Collinearity variable:

$$\mathcal{C}_{qg,\perp} = \frac{z_q}{z_j} \left(\mathbf{k}_{qg\perp} - \frac{z_g}{z_q} \mathbf{k}_{g\perp} \right)$$



Small-cone condition:

$$\mathcal{C}_{qg,\perp}^2 \leq \mathcal{C}_{qg,\perp}^2|_{\max} = R^2 \mathbf{p}_j^2 \min \left(\frac{z_g^2}{z_j^2}, \frac{(z_j - z_g)^2}{z_j^2} \right)$$

$$\begin{aligned} \frac{d\sigma_{R2 \times R2, \text{dijet,in-cone}}^\lambda}{d^2 \mathbf{k}_{1\perp} d\eta_1 d^2 \mathbf{k}_{2\perp} d\eta_2} &= \frac{\alpha_s C_F}{\pi} \frac{d\sigma_{LO,\varepsilon}^\lambda}{d^2 \mathbf{k}_{1\perp} d\eta_1 d^2 \mathbf{k}_{2\perp} d\eta_2} \times \left\{ \left(\frac{3}{4} - \ln \left(\frac{z_{J1}}{z_f} \right) \right) \frac{2}{\varepsilon} \right. \\ &\quad \left. + \ln^2(z_{J1}) - \ln^2(z_f) - \frac{\pi^2}{6} + \left(\ln \left(\frac{z_{J1}}{z_f} \right) - \frac{3}{4} \right) \ln \left(\frac{R^2 \mathbf{p}_{J1}^2}{\tilde{\mu}^2 z_{J1}^2} \right) + \frac{1}{4} + \frac{3}{2} \left(1 - \ln \left(\frac{z_{J1}}{2} \right) \right) \right\} \end{aligned}$$

Collinear pole

Collinear poles from $R2 \times R2$ and $R2' \times R2'$ cancel against IR pole of virtual contributions (see Slide 21)!

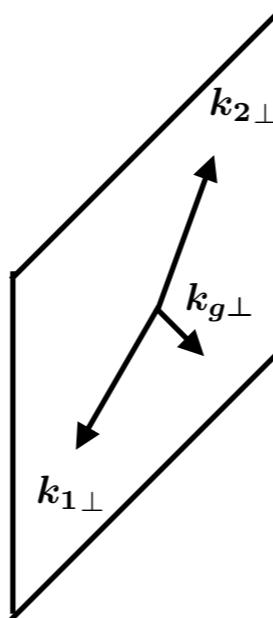
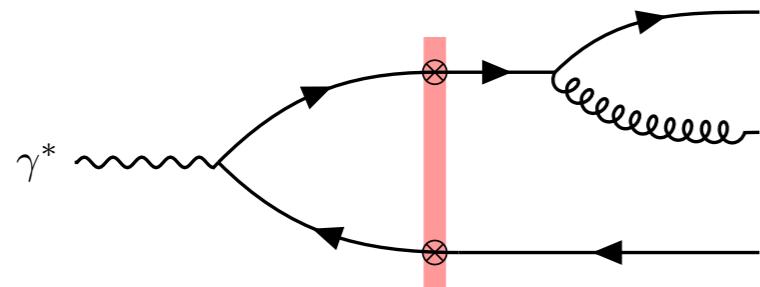
It is possible to show that $\ln^2(z_f)$ cancel with the out-cone contribution.

See Taelts, Altinoluk, Marquet, Beuf (2022) for an explicit cancellation.

Back-to-back limit

Origin of Sudakov double logs

In the back-to-back limit we expect the appearance of double and single Sudakov logarithms



$$k_\perp = k_{1\perp} + k_{2\perp}$$

$$P_\perp = z_2 k_{1\perp} - z_1 k_{2\perp}$$

$$d\sigma \sim \mathcal{H}(P_\perp, Q, z_1) \int d^2 b_\perp d^2 b'_\perp e^{-i \mathbf{k}_\perp \cdot (\mathbf{b}_\perp - \mathbf{b}'_\perp)} e^{-S_{\text{Sud}}(\mathbf{b}_\perp - \mathbf{b}'_\perp, P_\perp)} x G(\mathbf{b}_\perp, \mathbf{b}'_\perp; x)$$

$$= 1 - S_{\text{Sud}}(\mathbf{b}_\perp - \mathbf{b}'_\perp, P_\perp) + \dots$$

one-loop contribution

Sudakov factor

$$S_{\text{Sud}}(\mathbf{b}_\perp - \mathbf{b}'_\perp, P_\perp) = \frac{\alpha_s N_c}{4\pi} \ln^2 \left(\frac{P_\perp^2 (\mathbf{b}_\perp - \mathbf{b}'_\perp)^2}{c_0^2} \right) + \dots \quad \text{Mueller, Xiao, Yuan (PRD 2013)}$$

Single logs depend on jet algorithm... Computed in collinear factorization see Sun, Yuan, Yuan (PRD 2015)

Back-to-back limit

Where is the Sudakov in our computation?

In the back-to-back limit (in-cone+out-cone contribution):

Caucal, Schenke, FS, Venugopalan (2022)

$$\frac{d\sigma_{R2 \times R2}^\lambda}{d^2 \mathbf{P}_\perp d^2 \mathbf{k}_\perp d\eta_1 d\eta_2} = \frac{\alpha_{\text{em}} e_f^2 N_c}{(2\pi)^6} \delta(1 - z_1 - z_2) \mathcal{H}_L(\mathbf{P}_\perp) \int d^2 \mathbf{b}_\perp d^2 \mathbf{b}'_\perp e^{-i\mathbf{k}_\perp(\mathbf{b}_\perp - \mathbf{b}'_\perp)} S^{WW}(\mathbf{b}_\perp, \mathbf{b}'_\perp) \quad \text{Single log}$$

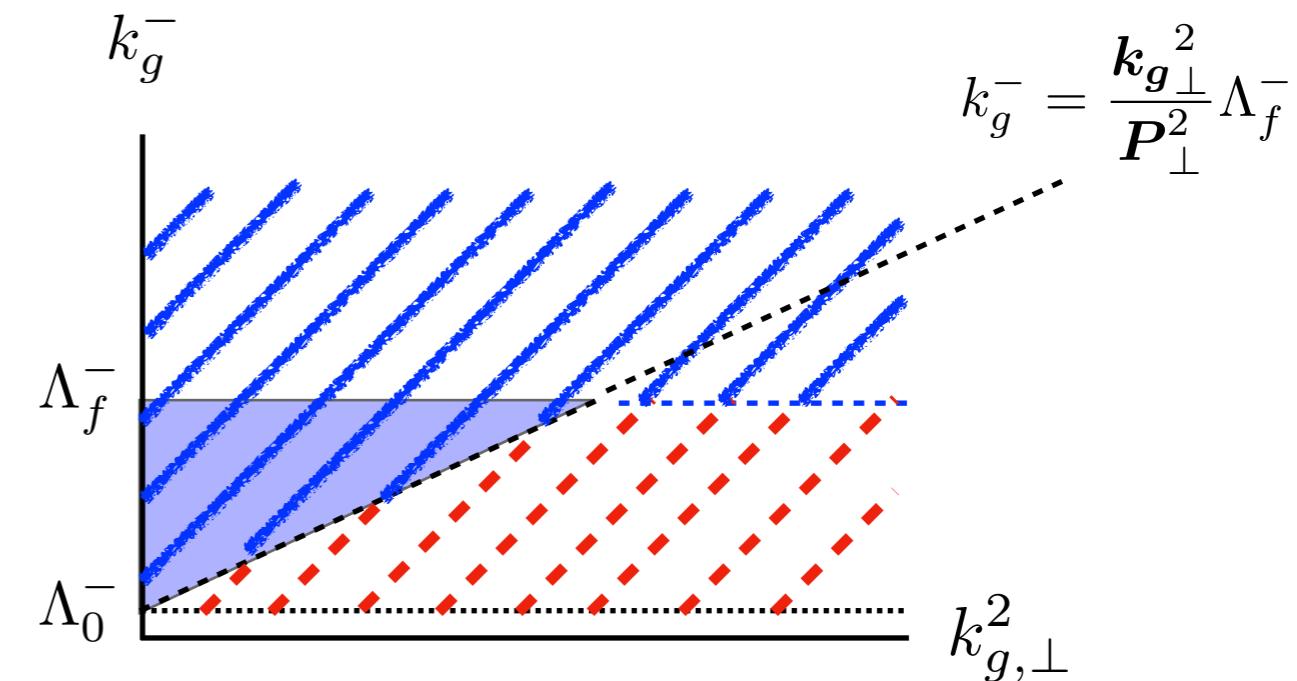
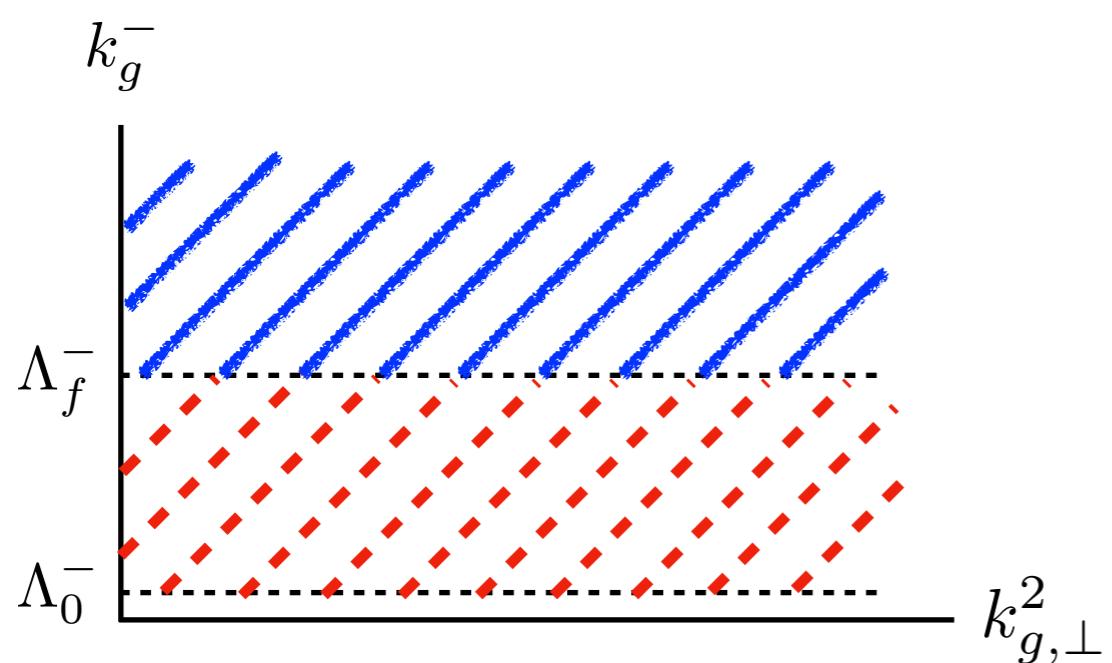
$$\times \frac{\alpha_s C_F}{\pi} \left\{ -\ln \left(\frac{z_1}{z_0} \right) \left(\frac{2}{\varepsilon} + \ln(e^{\gamma_E} \pi \mu^2 \Delta \mathbf{b}_\perp^2) \right) + \boxed{\frac{1}{4} \ln^2 \left(\frac{\mathbf{P}_\perp^2 \Delta \mathbf{b}_\perp^2}{c_0^2} \right)} + \boxed{\ln(R) \ln \left(\frac{\mathbf{P}_\perp^2 \Delta \mathbf{b}_\perp^2}{c_0^2} \right) + \mathcal{O}(1)} \right\}$$

Combining with
R2' x R2', R2 x R2' and R2' x R2

$\frac{\alpha_s N_c}{4\pi} \left(\frac{\mathbf{P}_\perp^2 \Delta \mathbf{b}_\perp^2}{c_0^2} \right)$

Double log but with opposite sign!

Imposing kinematic constraint \longrightarrow Sudakov with correct sign



Recently, beautifully explained in Taelts, Altinoluk, Marquet, Beuf (2022) for photo-production.

Need also ordering in k_g^+ (kinematically constrained evolution)

Summary

- Systematic analysis of one-loop corrections to semi-inclusive dijet production in DIS within the CGC
- Proved LL JIMWLK high energy factorization of rapidity divergence, and isolated impact factor
- Need of kinematic constraints in the back-to-back limit to reproduce Sudakov double log

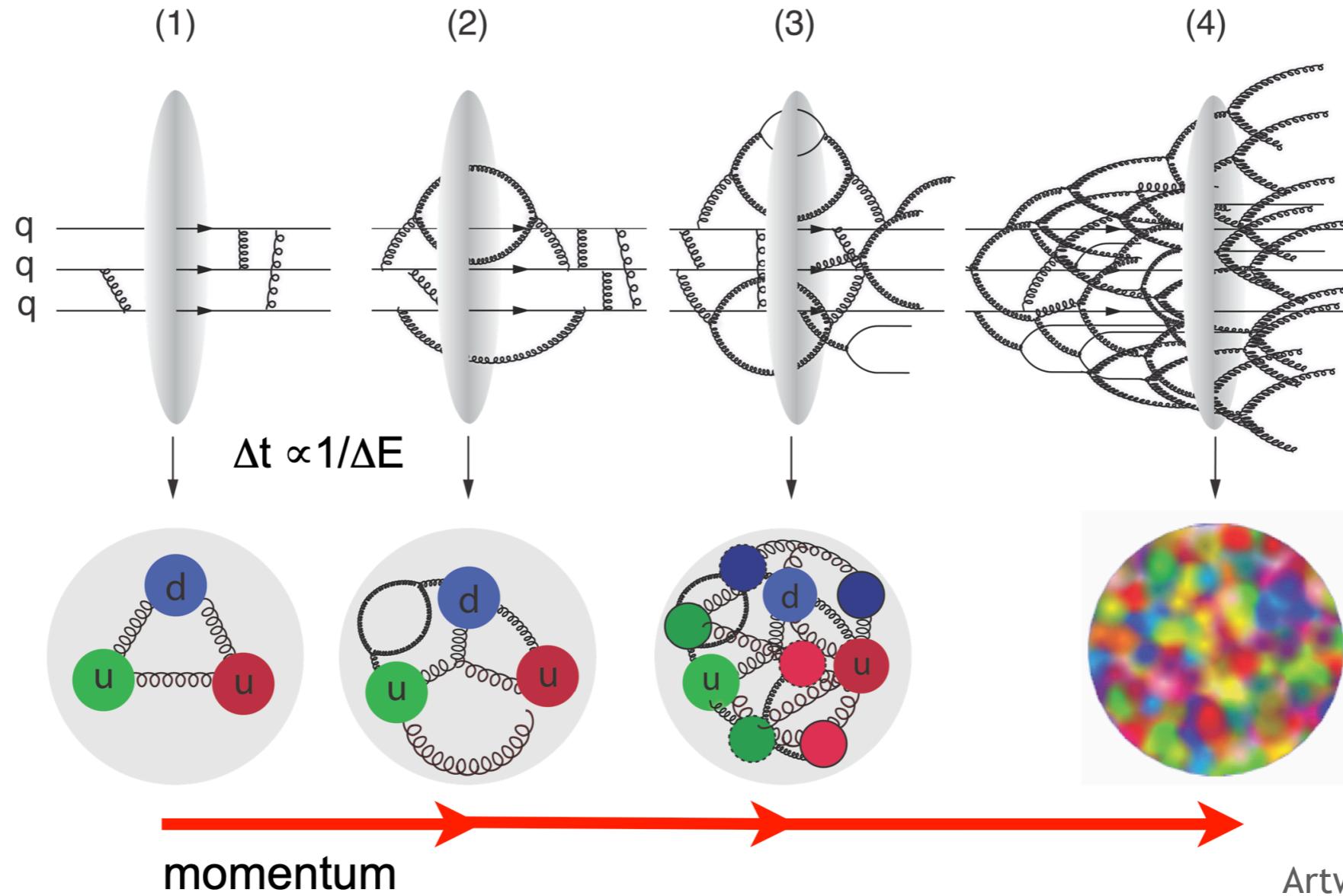
Outlook

- Connection to RHIC and LHC physics via UPCs (photo-production limit)
[See Taels, Altinoluk, Marquet, Beuf \(2022\) for photo-production](#)
- Semi-inclusive dihadron production (more suitable for EIC?)
[See Jamal Jalilian-Marian's talk](#)
- Massive quarks: heavy quark and quarkonia production
[Combine with results from Tuomas Lappi's talk?](#)
- Alternative way to distinguish soft/rapidity divergences using SCET
[See Hao-Yu Liu's talk for jet production in pA](#)
- Numerical predictions for dijets at NLO

Back-up Slides

Anatomy of nuclear matter in the high-energy limit

Gluon dominance at low-x



Emergence of an energy and nuclear specie dependent momentum scale

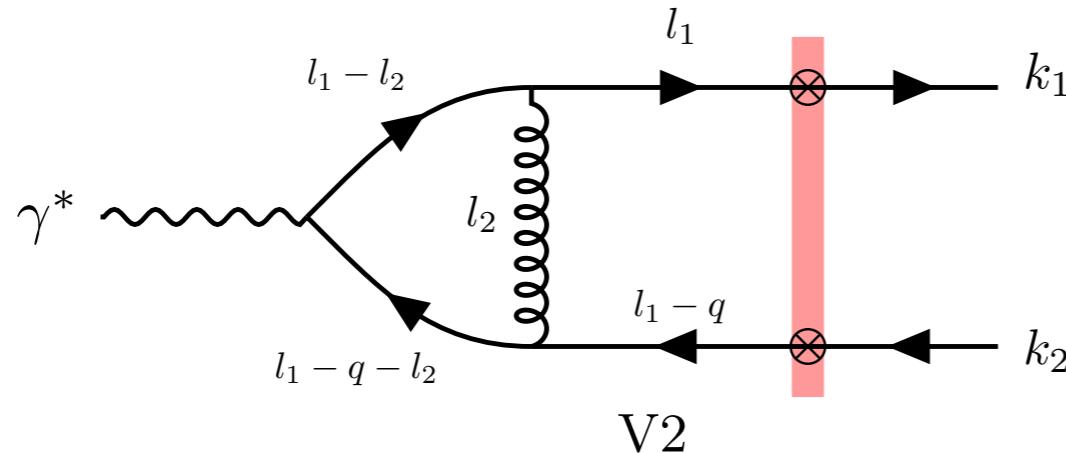
Multiple scattering (higher twist effects)

Non-linear evolution equations (BK/JIMWLK)

$$Q_s^2 \propto A^{1/3} x^{-\lambda}$$

One-loop corrections

Connection to LCPT: an example



$$\begin{aligned} \mathcal{C}_{V2,ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \\ = C_F [V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) - \mathbb{1}]_{ij} \end{aligned}$$

$$\mathcal{M}_{V2,ij,\sigma_1\sigma_2}^\lambda = \frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp)} \mathcal{C}_{V2,ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \mathcal{N}_{V2,\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy})$$

Perturbative factor

$$\mathcal{N}_{V2,\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy}) = g^2 \int_{l_1} \int_{l_2} \frac{(2q^-)\delta(k^- - l_1^-) N_{V2,\sigma_1\sigma_2}^\lambda(l_1, l_2) e^{i\mathbf{l}_{1\perp} \cdot \mathbf{r}_{xy}}}{[l_1^2 + i\epsilon] [(l_1 - l_2)^2 + i\epsilon] [(l_1 - l_2 - q)^2 + i\epsilon] [(l_1 - q)^2 + i\epsilon] [l_2^2 + i\epsilon]}$$

Dirac-Lorentz structure

$$N_{V2,\sigma_1\sigma_2}^\lambda(l_1, l_2) = \frac{1}{(2q^-)^2} [\bar{u}(k_1, \sigma_1) \gamma^- \not{l}_1 \gamma^\mu (\not{l}_1 - \not{l}_2) \not{\epsilon}(q, \lambda) (\not{l}_1 - \not{l}_2 - \not{q}) \gamma^\nu (\not{l}_1 - \not{q}) \gamma^- v(k_2, \sigma_2)] \Pi_{\mu\nu}(l_2)$$

One-loop corrections

Connection to LCPT: an example

Perturbative factor

$$\mathcal{N}_{V2,\sigma_1\sigma_2}^{\lambda}(\mathbf{r}_{xy}) = g^2 \int_{l_1} \int_{l_2} \frac{(2q^-)\delta(k^- - l_1^-)N_{V2,\sigma_1\sigma_2}^{\lambda}(l_1, l_2)e^{i\mathbf{l}_1 \cdot \mathbf{r}_{xy}}}{[l_1^2 + i\epsilon][(l_1 - l_2)^2 + i\epsilon][(l_1 - l_2 - q)^2 + i\epsilon][(l_1 - q)^2 + i\epsilon][l_2^2 + i\epsilon]}$$

Dirac-Lorentz structure (DLS)

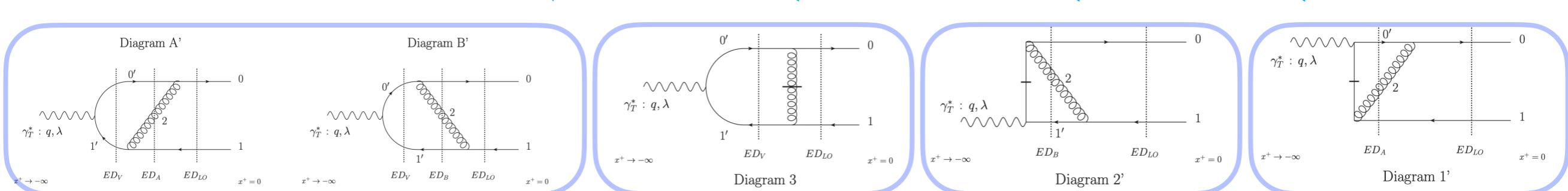
$$N_{V2,\sigma_1\sigma_2}^{\lambda}(l_1, l_2) = \frac{1}{(2q^-)^2} [\bar{u}(k_1, \sigma_1)\gamma^- l_1 \gamma^\mu(l_1 - l_2)\not{e}(q, \lambda)(l_1 - l_2 - q)\not{e}\gamma^\nu(l_1 - q)\gamma^- v(k_2, \sigma_2)] \Pi_{\mu\nu}(l_2)$$

Useful decomposition (dissecting Dirac-Lorentz structure)

$$N_{V2} = N_{V2,\text{reg}} + l_2^2 N_{V2,g\text{inst}} + (l_1 - l_2)^2 N_{V2,q\text{inst}} + (l_1 - l_2 - q)^2 N_{V2,\bar{q}\text{inst}}$$

$$\mathcal{N}_{V2} = \mathcal{N}_{V2,\text{reg}} + \mathcal{N}_{V2,g\text{inst}} + \mathcal{N}_{V2,q\text{inst}} + \mathcal{N}_{V2,\bar{q}\text{inst}}$$

After contour integration l_1^+ and l_2^+ one obtains light-cone energy denominators in LCPT



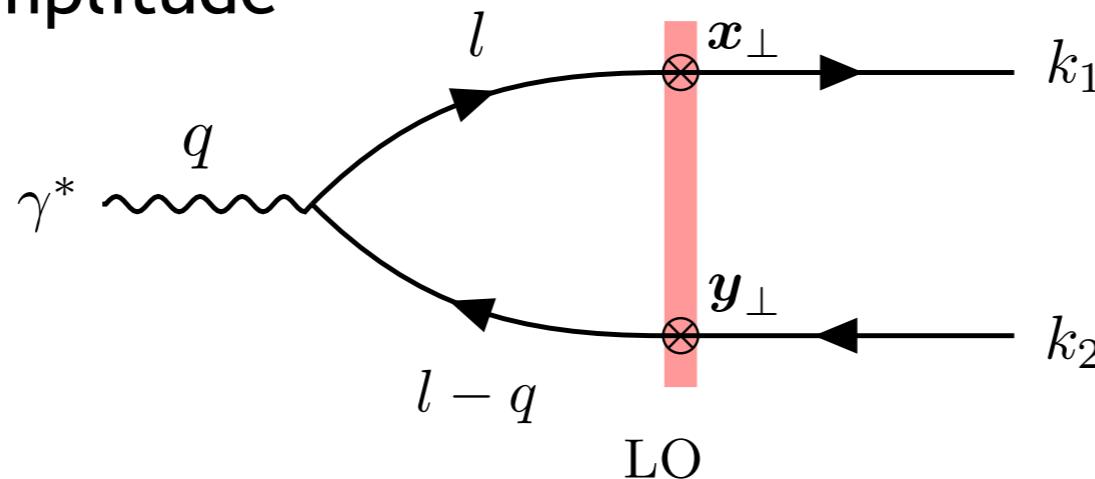
Diagrams from Beuf (2016)

These perturbative factors have been computed in Beuf (2016,2017), Hänninen, Lappi, and Paatelainen (2017)

Review of Leading order

Anatomy of the amplitude

$$q^\mu = \left(-\frac{Q}{q^-}, q^-, \mathbf{0}_\perp \right)$$



$$k_1^\mu = \left(\frac{k_{1\perp}^2}{2z_1 q^-}, z_1 q^-, \mathbf{k}_{1\perp} \right)$$

$$k_2^\mu = \left(\frac{k_{2\perp}^2}{2z_2 q^-}, z_2 q^-, \mathbf{k}_{2\perp} \right)$$

$$\int \frac{d^4 l}{(2\pi)^4} \bar{u}(k_1, \sigma_1) \mathcal{T}^q(k_1, l) S^0(l) (-ie e_f \not{\epsilon}(q, \lambda)) S^0(l - q) \mathcal{T}^q(l - q, -k_2) v(k_2, \sigma_2)$$

*Loop integration variable l since both quark and anti-quark receive momentum from shock-wave.

Dissecting amplitude

Color correlator	Perturbative factor!
$[V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) - \mathbb{1}]_{ij}$	$\mathcal{N}_{\sigma_1 \sigma_2}^\lambda(\mathbf{x}_\perp - \mathbf{y}_\perp)$

$$\mathcal{M}_{\text{LO}, ij, \sigma_1 \sigma_2}^\lambda = \frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp} e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp} e^{-i\mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp} [V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) - \mathbb{1}]_{ij} \mathcal{N}_{\sigma_1 \sigma_2}^\lambda(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

$$\mathcal{N}_{\sigma_1 \sigma_2}^\lambda(\mathbf{r}_\perp) = -i(2q^-) \int \frac{d^4 l}{(2\pi)^2} \frac{e^{i\mathbf{l}_\perp \cdot \mathbf{r}_\perp} N_{\sigma_1 \sigma_2}^\lambda(l) \delta(k_1^- - l^-)}{(l^2 + i\epsilon)((l - q)^2 + i\epsilon)}$$

with Dirac/Lorentz structure:

$$N_{\sigma_1 \sigma_2}^\lambda(l) = \frac{1}{(2q^-)^2} [\bar{u}(k_1, \sigma_1) \gamma^- l \not{\epsilon}(q, \lambda) (\not{q} - \not{l}) \gamma^- v(k_2, \sigma_2)]$$

Leading order

Anatomy of the amplitude

- Evaluate Dirac/Lorentz structure with basic gamma matrix manipulations

$$N_{\sigma_1 \sigma_2}^{\lambda=0}(l) = -2Q(z_1 z_2)^{3/2} \delta_{\sigma_1, -\sigma_2} \quad N_{\sigma_1 \sigma_2}^{\lambda=\pm 1}(l) = \mathbf{l}_\perp \cdot \boldsymbol{\epsilon}_\perp^\lambda [z_2 \delta_{\sigma_1}^\lambda - z_1 \delta_{\sigma_2}^\lambda] \delta_{\sigma_1, -\sigma_2}$$

- Evaluate loop integral $d^4 l$

$$\mathcal{N}_{\sigma_1 \sigma_2}^\lambda(\mathbf{r}_\perp) = -i(2q^-) \int \frac{d^2 \mathbf{l}_\perp}{(2\pi)^2} \int dl^+ \boxed{\int dl^- \frac{e^{i\mathbf{l}_\perp \cdot \mathbf{r}_\perp} N_{\sigma_1 \sigma_2}^\lambda(l) \delta(k_1^- - l^-)}{(l^2 + i\epsilon)((l - q)^2 + i\epsilon)}}$$

- Compute l^- integral using eikonal delta function $\delta(k_1^- - l^-)$

$$\mathcal{N}_{\sigma_1 \sigma_2}^\lambda(\mathbf{r}_\perp) = -i(2q^-) \int \frac{d^2 \mathbf{l}_\perp}{(2\pi)^2} e^{i\mathbf{l}_\perp \cdot \mathbf{r}_\perp} N_{\sigma_1 \sigma_2}^\lambda(l) \boxed{\int dl^+ \frac{1}{(l^2 + i\epsilon)((l - q)^2 + i\epsilon)}}$$

- Compute l^+ via contour integration using residues

$$\mathcal{N}_{\sigma_1 \sigma_2}^\lambda(\mathbf{r}_\perp) = \boxed{\int \frac{d^2 \mathbf{l}_\perp}{2\pi} \frac{N_{\sigma_1 \sigma_2}^\lambda(l) e^{i\mathbf{l}_\perp \cdot \mathbf{r}_\perp}}{z_1 z_2 Q^2 + \mathbf{l}_\perp^2}}$$

- Compute \mathbf{l}_\perp with 2D Fourier transform

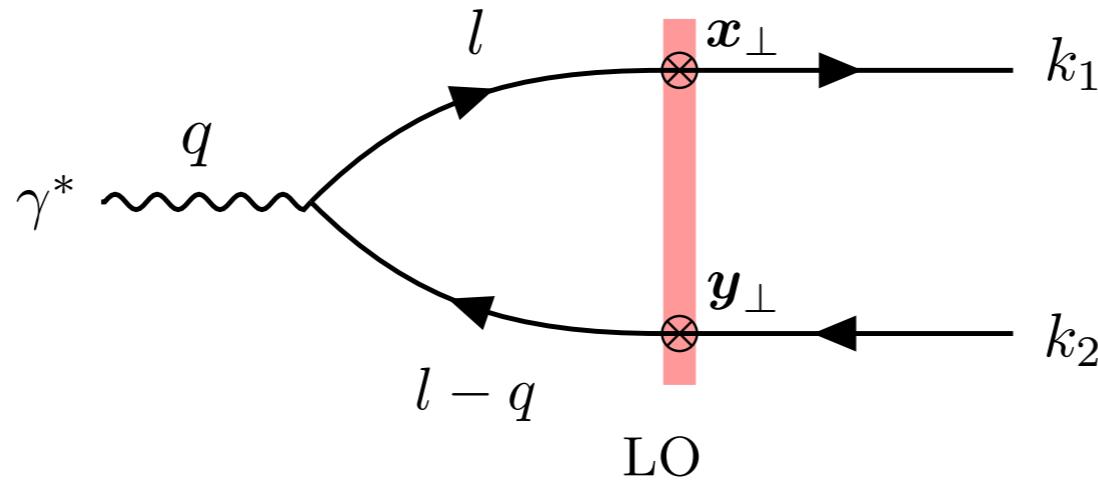
$$\mathcal{N}_{\sigma_1 \sigma_2}^{\lambda=0}(\mathbf{r}_\perp) = -2(z_1 z_2)^{3/2} Q K_0(Q \sqrt{z_1 z_2} r_\perp) \delta_{\sigma_1, -\sigma_2}$$

$$\mathcal{N}_{\sigma_1 \sigma_2}^{\lambda=\pm 1}(\mathbf{r}_\perp) = 2(z_1 z_2)^{3/2} [z_2 \delta_{\sigma_1}^\lambda - z_1 \delta_{\sigma_2}^\lambda] \frac{i Q \mathbf{r}_\perp \cdot \boldsymbol{\epsilon}_\perp^\lambda}{\sqrt{z_1 z_2} r_\perp} K_1(Q \sqrt{z_1 z_2} r_\perp) \delta_{\sigma_1, -\sigma_2}$$

where K_0 and K_1 are Bessel functions (exponential decaying like)

Leading order

Anatomy of the cross-section



$$\mathcal{M}_{\text{LO},ij,\sigma_1\sigma_2}^\lambda =$$

$$\frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp)} \left[V(\mathbf{x}_\perp) V^\dagger(\mathbf{y}_\perp) - \mathbb{1} \right]_{ij} \mathcal{N}_{\sigma_1, \sigma_2}^\lambda(Q, z_1, \mathbf{x}_\perp - \mathbf{y}_\perp)$$

q \bar{q} interaction with nucleus

γ^* splitting to $q\bar{q}$

Unpolarized differential cross-section:

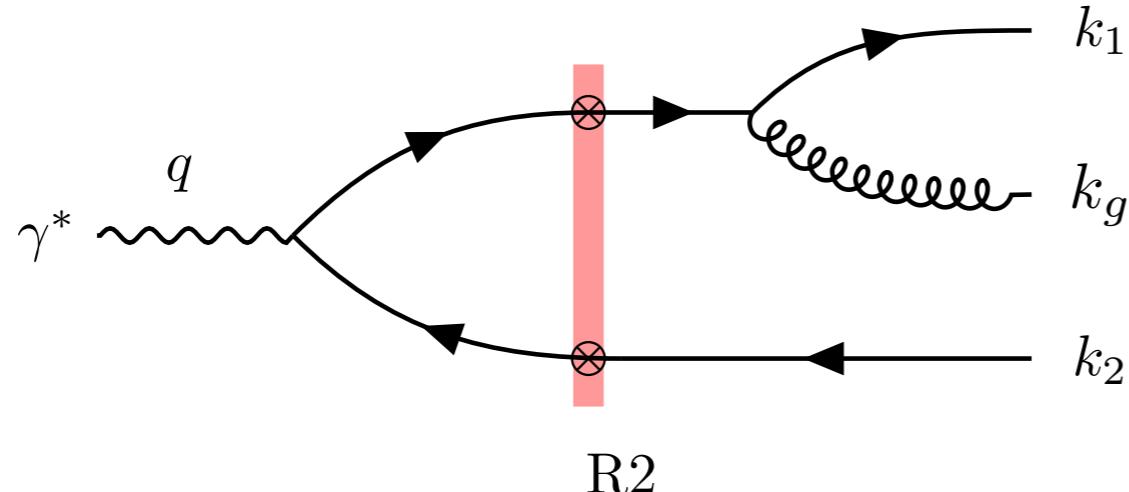
$$\frac{d\sigma^{\gamma_\lambda^* + A \rightarrow q\bar{q} + X}}{d^2\mathbf{k}_{1\perp} d^2\mathbf{k}_{2\perp} d\eta_1 d\eta_2} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta(1 - z_1 - z_2)}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot (\mathbf{x}_\perp - \mathbf{x}'_\perp)} e^{-i\mathbf{k}_{2\perp} \cdot (\mathbf{y}_\perp - \mathbf{y}'_\perp)} \\ \times \langle \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp \mathbf{x}'_\perp) \rangle_Y \mathcal{R}^\lambda(\mathbf{x}_\perp - \mathbf{y}_\perp, \mathbf{x}'_\perp - \mathbf{y}'_\perp)$$

$$\Xi_{\text{LO}}(x_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp x'_\perp) = 1 - S^{(2)}(x_\perp, \mathbf{y}_\perp) - S^{(2)}(\mathbf{y}'_\perp, x'_\perp) + S^{(4)}(x_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, x'_\perp)$$

← dipoles →
↑ quadrupole

One-loop corrections

Real gluon emission after SW



$$\begin{aligned} \mathcal{C}_{R2,ija}(\mathbf{w}_\perp, \mathbf{y}_\perp) \\ = [t_a V(\mathbf{w}_\perp) V^\dagger(\mathbf{y}_\perp) - t_a]_{ij} \end{aligned}$$

$$\mathcal{M}_{R2,ija,\sigma_1\sigma_2}^{\lambda\bar{\lambda}} = \frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp + \mathbf{k}_{g\perp} \cdot \mathbf{z}_\perp)} \mathcal{C}_{R2,ija}(\mathbf{w}_\perp, \mathbf{y}_\perp) \mathcal{N}_{R2,\sigma_1\sigma_2}^{\lambda\bar{\lambda}}(\mathbf{r}_{wy}, \mathbf{r}_{zx})$$

Perturbative factor:

$$\mathcal{N}_{R2,\sigma_1\sigma_2}^{\lambda=0,\bar{\lambda}}(\mathbf{r}_{wy}, \mathbf{r}_{zx}) = 2(z_1 z_2)^{3/2} Q K_0(Q X_{wy}) \delta_{\sigma_1, -\sigma_2} \frac{i g}{\pi} \frac{\mathbf{r}_{zx} \cdot \boldsymbol{\epsilon}_\perp^{\bar{\lambda}*}}{\mathbf{r}_{zx}^2} \frac{[z_1 \delta_{\sigma_1}^{\bar{\lambda}} + (z_1 + z_g) \delta_{\sigma_2}^{\bar{\lambda}}]}{z_1}$$

$$\mathcal{N}_{R2,\sigma_1\sigma_2}^{\lambda=\pm 1,\bar{\lambda}}(\mathbf{r}_{wy}, \mathbf{r}_{zx}) = -2(z_1 z_2)^{3/2} [z_2 \delta_{\sigma_1}^\lambda - z_1 \delta_{\sigma_2}^\lambda] \frac{i Q \mathbf{r}_{wx} \cdot \boldsymbol{\epsilon}_\perp^\lambda}{X_{wx}} K_1(Q X_{wx}) \delta_{\sigma_1, -\sigma_2} \frac{i g}{\pi} \frac{\mathbf{r}_{zx} \cdot \boldsymbol{\epsilon}_\perp^{\bar{\lambda}*}}{\mathbf{r}_{zx}^2} \frac{[z_1 \delta_{\sigma_1}^{\bar{\lambda}} + (z_1 + z_g) \delta_{\sigma_2}^{\bar{\lambda}}]}{z_1}$$

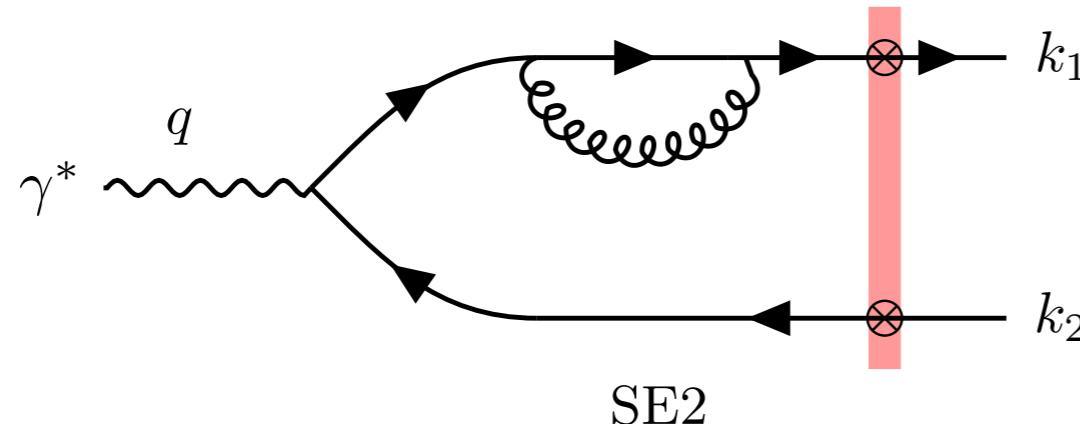
$$\mathbf{w}_\perp = \frac{z_1 \mathbf{x}_\perp + z_g \mathbf{z}_\perp}{z_1 + z_g}$$

$$X_{wy}^2 = z_2 (z_1 + z_g) r_{wy}^2$$

Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans (2017)
with spinor-helicity techniques

One-loop corrections

Self energy with gluon before SW



$$\begin{aligned}\mathcal{C}_{\text{SE2},ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \\ = C_F [V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) - \mathbb{1}]_{ij}\end{aligned}$$

$$\mathcal{M}_{\text{SE2},ij,\sigma_1\sigma_2}^\lambda = \frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp)} \mathcal{C}_{\text{SE2},ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \mathcal{N}_{\text{SE2},\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy})$$

Perturbative factor:

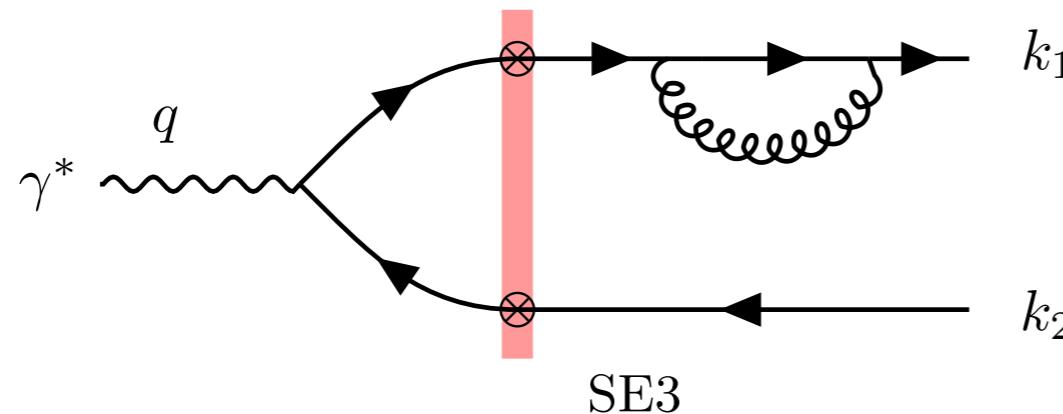
$$\begin{aligned}\mathcal{N}_{\text{SE2},\sigma_1\sigma_2}^{\lambda=0}(\mathbf{r}_{xy}) = \frac{\alpha_s}{2\pi} \left\{ \left(-2 \ln \left(\frac{z_1}{z_0} \right) + \frac{3}{2} \right) \left(\frac{2}{\varepsilon} + \frac{1}{2} \ln \left(\frac{z_1 z_2 Q^2 r_{xy}^2}{4} \right) + \gamma_E - \ln \left(\frac{z_1 Q^2}{\tilde{\mu}^2} \right) \right) \right. \\ \left. + \left(\frac{1}{2} + 3 - \frac{\pi^2}{3} - \ln^2 \left(\frac{z_1}{z_0} \right) \right) + \mathcal{O}(\varepsilon) \right\} \mathcal{N}_{\text{LO},\varepsilon,\sigma_1\sigma_2}^{\lambda=0}\end{aligned}$$

double log

Beuf (2016,2017), Hänninen, T. Lappi, and R. Paatelainen (2017)
Loop corrections light-cone photon wave-function

One-loop corrections

Self energy with gluon after SW



$$\begin{aligned} \mathcal{C}_{\text{SE3},ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \\ = C_F [V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) - \mathbb{1}]_{ij} \end{aligned}$$

$$\mathcal{M}_{\text{SE2},ij,\sigma_1\sigma_2}^\lambda = \frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp)} \mathcal{C}_{\text{SE3},ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \mathcal{N}_{\text{SE3},\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy})$$

Perturbative factor:

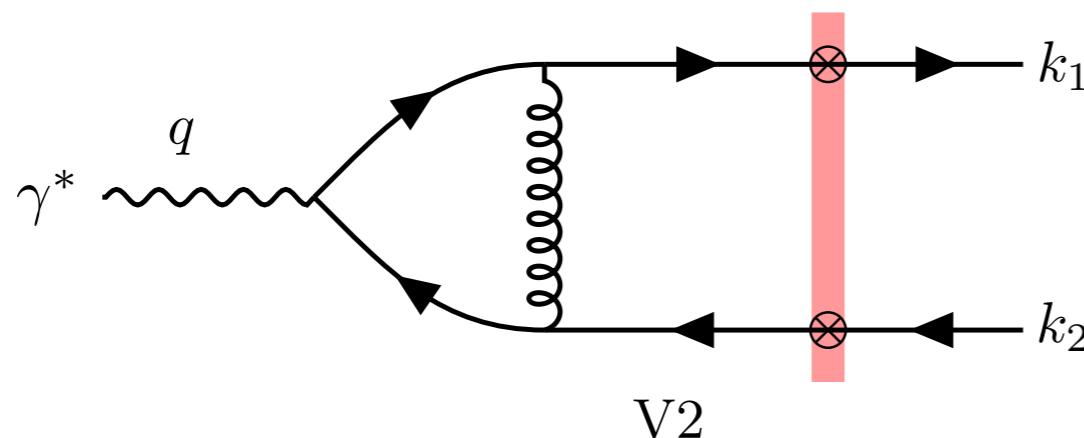
$$\mathcal{N}_{\text{SE3},\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy}) = -\frac{\alpha_s}{2\pi} \mathcal{N}_{\text{LO},\varepsilon,\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy}) \left(\frac{2}{\varepsilon_{\text{UV}}} - \frac{2}{\varepsilon_{\text{IR}}} \right) \left\{ 2 \ln \left(\frac{z_q}{z_0} \right) - \frac{3}{2} \right\}$$

UV pole
IR pole

Self-energy contribution vanishes exactly in dim reg (IR and UV pole cancel each other out)
turns UV divergences into IR (massless quarks)

One-loop corrections

Vertex with gluon before SW



$$\begin{aligned}\mathcal{C}_{V2,ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \\ = C_F [V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) - \mathbb{1}]_{ij}\end{aligned}$$

$$\mathcal{M}_{V2,ij,\sigma_1\sigma_2}^\lambda = \frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp)} \mathcal{C}_{V2,ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \mathcal{N}_{V2,\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy})$$

Perturbative factor*:

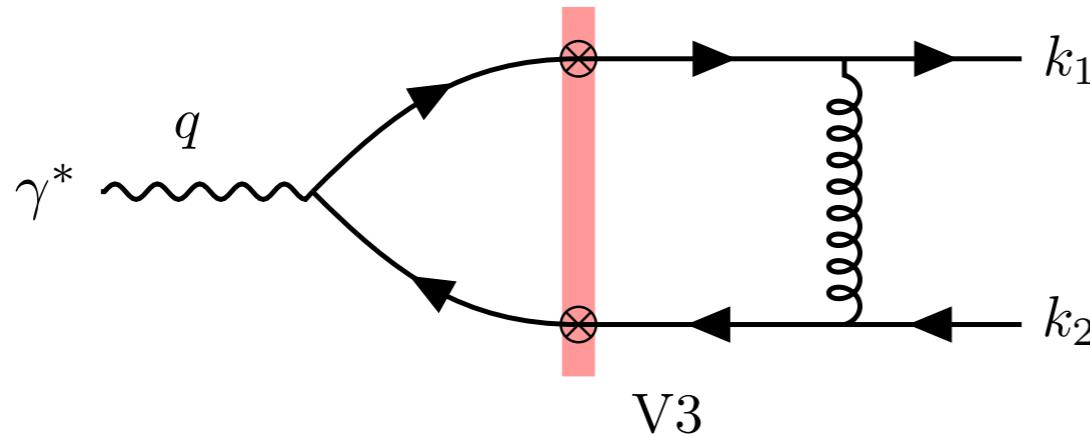
$$\begin{aligned}\mathcal{N}_{V2,\sigma_1\sigma_2}^{\lambda=0}(\mathbf{r}_{xy}) &= \frac{\alpha_s}{2\pi} \left\{ \left(\frac{2}{\varepsilon} + \ln \left(\frac{\tilde{\mu}^2}{z_1 z_2 Q^2} \right) \right) \left[\ln \left(\frac{z_1}{z_0} \right) + \ln \left(\frac{z_2}{z_0} \right) - \frac{3}{2} \right] + \boxed{\ln^2 \left(\frac{z_1}{z_0} \right) + \ln^2 \left(\frac{z_2}{z_0} \right) + \frac{1}{2} \ln^2 \left(\frac{z_1}{z_2} \right) + \frac{\pi^2}{2}} \right. \\ &\quad \left. + \left(2 \ln \left(\frac{z_2}{z_0} \right) - \frac{3}{2} \right) \ln(z_1) + \left(2 \ln \left(\frac{z_1}{z_0} \right) - \frac{3}{2} \right) \ln(z_2) - \frac{7}{2} - \frac{1}{2} + \mathcal{O}(\varepsilon) \right\} \mathcal{N}_{LO,\varepsilon,\sigma_1\sigma_2}^{\lambda=0}(\mathbf{r}_{xy})\end{aligned}$$

*includes both regular and gluon instantaneous contribution

Beuf (2016,2017), Hänninen, T. Lappi, and R. Paatelainen (2017)
Loop corrections light-cone photon wave-function

One-loop corrections

Vertex with gluon after SW



$$\begin{aligned}\mathcal{C}_{V3,ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \\ = C_F [V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) - \mathbb{1}]_{ij}\end{aligned}$$

$$\mathcal{M}_{V3,ij,\sigma_1\sigma_2}^\lambda = \frac{ee_f q^-}{\pi} \int_{\mathbf{x}_\perp, \mathbf{y}_\perp} e^{-i(\mathbf{k}_{1\perp} \cdot \mathbf{x}_\perp + \mathbf{k}_{2\perp} \cdot \mathbf{y}_\perp)} \mathcal{C}_{V3,ij}(\mathbf{x}_\perp, \mathbf{y}_\perp) \mathcal{N}_{V3,\sigma_1\sigma_2}^\lambda(\mathbf{r}_{xy})$$

Perturbative factor*:

$$\begin{aligned}\mathcal{N}_{V3,\sigma_1\sigma_2}^{\lambda=0}(\mathbf{r}_{xy}) = & -\frac{\alpha_s}{\pi} \int_0^{z_g} \frac{dz_g}{z_g} 2(z_1 z_2)^{1/2} (z_1 - z_g)(z_2 + z_g) Q K_0 \left(Q \sqrt{(z_1 - z_g)(z_2 + z_g)} r_{xy} \right) \delta_{\sigma_1, -\sigma_2} \\ & \times \left\{ \left[(1 + z_g) \left(1 - \frac{z_g}{z_1} \right) \right] e^{i(\mathbf{P}_\perp + z_g(\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp})) \cdot \mathbf{r}_{xy}} K_0(-i\Delta_{V3} r_{xy}) \right. \\ & - \left[1 - \frac{z_g}{2z_1} + \frac{z_g}{2z_2} - \frac{z_g^2}{2z_1 z_2} \right] e^{i \frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy}} \boxed{\mathcal{J}_\odot \left(\mathbf{r}_{xy}, \left(1 - \frac{z_g}{z_1} \right) \mathbf{P}_\perp, \Delta_{V3} \right)} \\ & \left. + \sigma \left[\frac{z_g}{z_1} - \frac{z_g}{z_2} + \frac{z_g^2}{z_1 z_2} \right] e^{i \frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy}} \mathcal{J}_\otimes \left(\mathbf{r}_{xy}, \left(1 - \frac{z_g}{z_1} \right) \mathbf{P}_\perp, \Delta_{V3} \right) \right\} + (q \leftrightarrow \bar{q})\end{aligned}$$

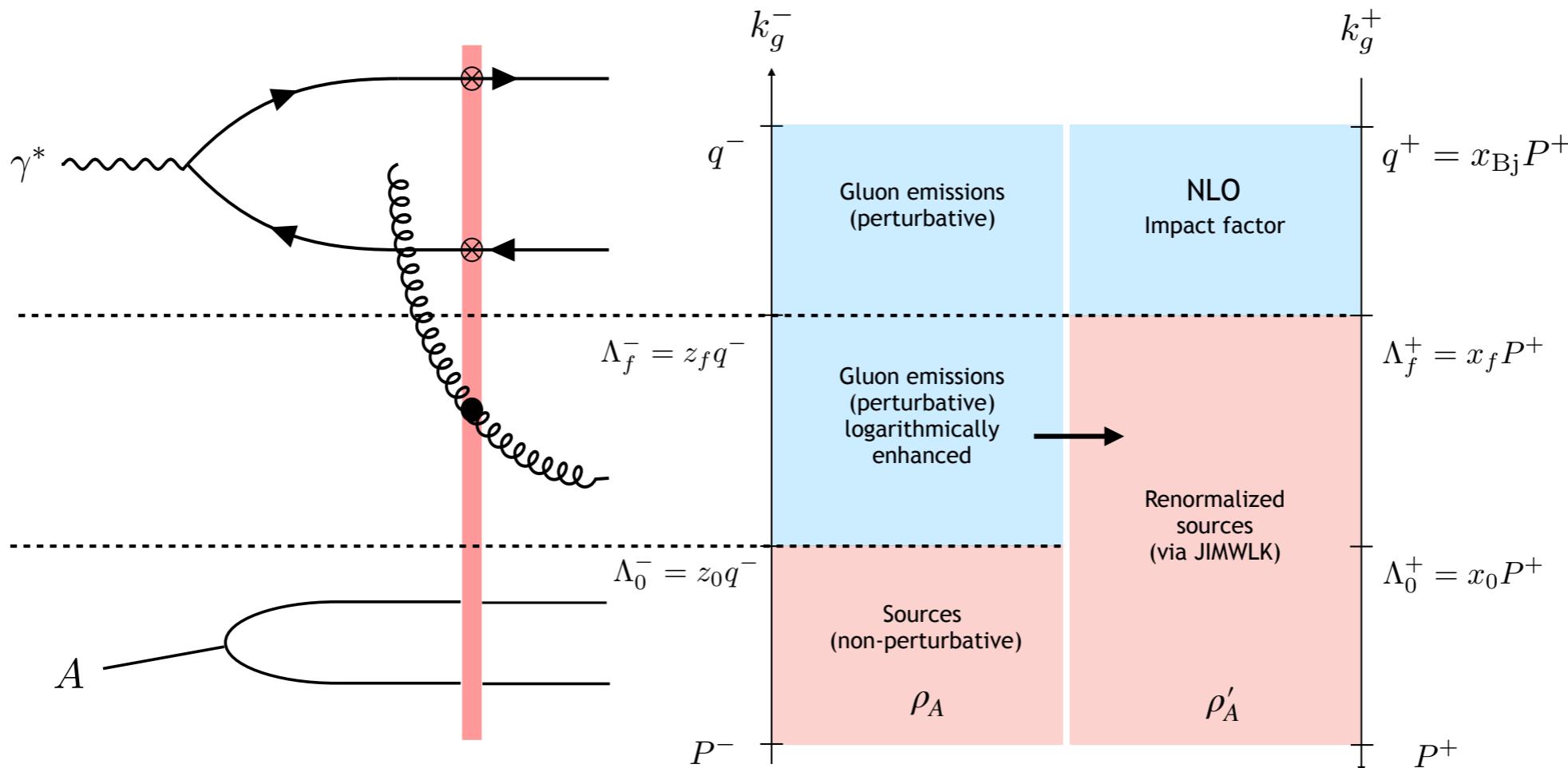
Contains a double log
 $\ln^2(z_0)$

*includes both regular and gluon instantaneous contribution

One-loop corrections

Rapidity (slow gluon) divergences and JIMWLK factorization

$$d\sigma_{\text{NLO,slow}} = \ln \left(\frac{z_f}{z_0} \right) \mathcal{H}_{\text{JIMWLK}} d\sigma_{\text{LO}}$$



Slow gluon radiation $d\sigma_{\text{NLO,slow}} \propto \alpha_s \ln \left(\frac{z_f}{z_0} \right)$ can be large in resummed by redefining distribution of sources:

$$W_{x_0}[\rho_A] \rightarrow W_{x_f}[\rho'_A]$$