

One-loop light cone wave functions with massive quarks

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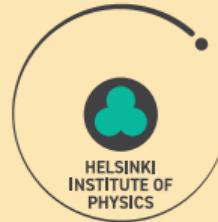
DIS2022, Santiago de Compostela



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Outline

Massive quarks in NLO dipole factorization for DIS

G. Beuf, T.L. and R. Paatelainen:

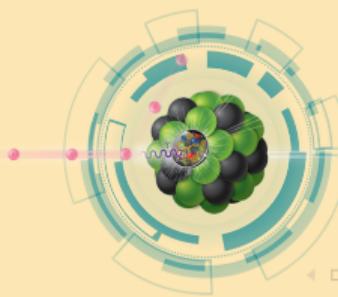
- ▶ Phys. Rev. D **104** (2021) no.5, 056032, [[arXiv:2103.14549 \[hep-ph\]](https://arxiv.org/abs/2103.14549)]. \Rightarrow longitudinal photon
- ▶ [arXiv:2204.02486 \[hep-ph\]](https://arxiv.org/abs/2204.02486). \Rightarrow transverse photon
- ▶ [arXiv:2112.03158 \[hep-ph\]](https://arxiv.org/abs/2112.03158). \Rightarrow short version with just results

Outline of this talk

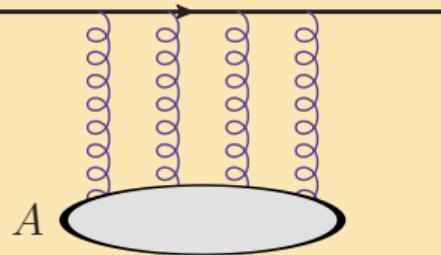
- ▶ Eikonal scattering: DIS in dipole picture
- ▶ Light Cone Perturbation Theory (LCPT)
- ▶ Mass renormalization
- ▶ Wave function and total DIS cross section with quark masses at one loop

Process of interest

DIS cross section at high energy



Eikonal scattering off target of glue



How to measure small- x glue?

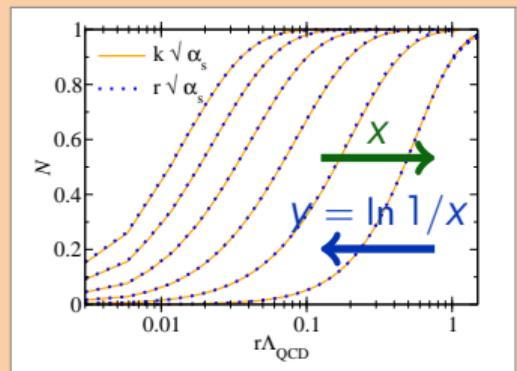
- ▶ Dilute probe through target color field
- ▶ At high energy interaction is **eikonal**, \perp coordinate conserved
(T -matrix diagonal in \perp coordinate space)

- ▶ Amplitude for quark: **Wilson line**

$$\mathbb{P} \exp \left\{ -ig \int^{x^+} dy^+ A^-(y^+, x^-, \mathbf{x}) \right\} \underset{x^+ \rightarrow \infty}{\approx} V(\mathbf{x}) \in \text{SU}(N_c)$$

- ▶ Amplitude for color dipole

$$\mathcal{N}(r = |\mathbf{x} - \mathbf{y}|) = 1 - \left\langle \frac{1}{N_c} \text{Tr } V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$

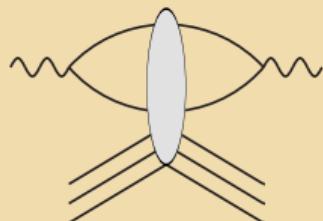


- ▶ $r = 0$: color transparency, $r \gg 1/Q_s$: saturation , nonperturbative!

Dipole picture of DIS

Limit of small x , i.e. high γ^* -target energy

Leading order

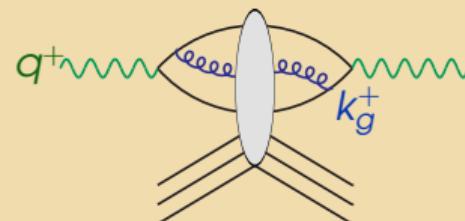


- ▶ $\gamma^* \rightarrow q\bar{q}$ in vacuum
- ▶ $q\bar{q}$ interacts eikonally with target
- ▶ σ^{tot} is $2 \times \text{Im}$ -part of amplitude

"Dipole model": Nikolaev, Zakharov 1991, A. Mueller

Many fits to HERA data, starting with Golec-Biernat,
Wüsthoff 1998

Leading Log: add **soft** gluon



- ▶ Soft gluon: large logarithm

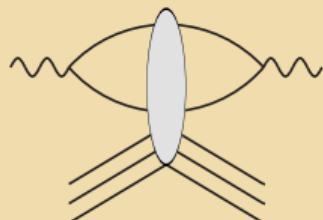
$$\int_{x_{Bj}} \frac{dk_g^+}{k_g^+} \sim \ln \frac{1}{x_{Bj}}$$

Absorb into renormalization of target:
BK equation Balitsky 1995, Kovchegov 1999

Dipole picture of DIS

Limit of small x , i.e. high γ^* -target energy

Leading order

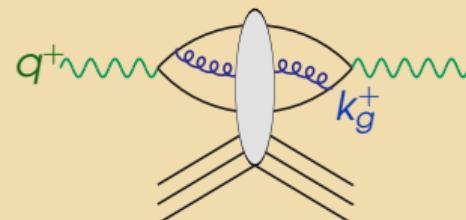


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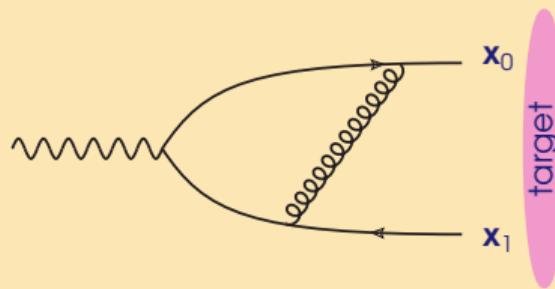
Absorb into renormalization of target:
BK equation Balitsky 1995, Kovchegov 1999

NLO: add the gluon with full kinematics

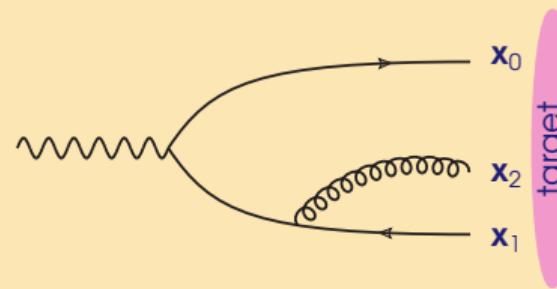
Dipole factorization

Dipole picture: factorize amplitude into 2 parts:

- ▶ $\gamma^* \rightarrow q\bar{q}$ (1 loop) and $\gamma^* \rightarrow q\bar{q}g$ (tree level) — photon wave function
- ▶ Eikonal interaction of (bare) partons with target color field — Wilson lines



$q\bar{q}$ state in γ^*



$q\bar{q}g$ state in γ^*

Why?

- ▶ Perturbative calculation of the perturbative part, no reference to target
- ▶ Including saturation straightforward in eikonal picture

Light cone wave function

Theory framework for calculating Fock states of γ^*

- ▶ Know free particle Fock states: $|\gamma^*\rangle_0$, $|q\bar{q}\rangle_0$, $|q\bar{q}g\rangle_0$ etc.
- ▶ **Interacting** states are superpositions, coefficient=**light cone wave function**

$$|\gamma^*\rangle = (1 + \dots) |\gamma^*\rangle_0 + \psi^{\gamma^* \rightarrow q\bar{q}} \otimes |q\bar{q}\rangle_0 + \psi^{\gamma^* \rightarrow q\bar{q}g} \otimes |q\bar{q}g\rangle_0 + \dots$$

- ▶ Calculate in QM perturbation theory, e.g. ground state $|0\rangle$:

$$\psi^{0 \rightarrow n} = \sum_n \frac{\langle n | \hat{V} | 0 \rangle}{E_n - E_0} + \dots$$

($1/\Delta E \sim$ lifetime of the quantum fluctuation from 0 to n)

- ▶ LC time x^+ — LC energy k^- (energy “not conserved”!)

Connection to Feynman perturbation theory

- ▶ Matrix elements $\langle n | \hat{V} | m \rangle$ are vertices in Feynman rules
- ▶ LC energy denominators from propagators, integrating over k^- pole

Heavy quarks

- ▶ NLO dipole picture DIS done at $m_q = 0$, successful fit to HERA data
- ▶ But there is data for heavy quarks:
 - ▶ HERA F_2^c
 - ▶ Charm big part of EIC program
- ▶ LO dipole picture with BK evolution: F_2^c problematic in existing fits

Hänninen et al 2007.01645 [hep-ph]

LCPT loops with massive quarks are so much fun!

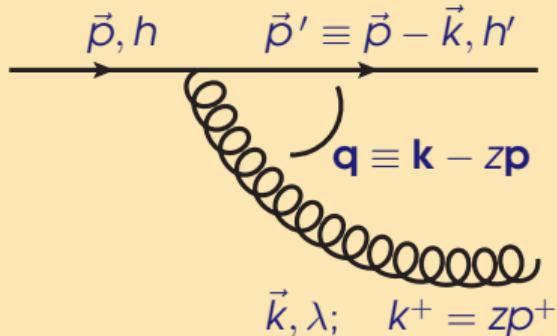
- ▶ Working with fixed helicity states (not Dirac traces=sums) : physics very explicit
- ▶ Masses: new Lorentz structures \implies rotational invariance constraints

Approach for this work: same regularization as in massless case

- ▶ Cutoff in k^+
- ▶ \perp dim. reg.

Mass renormalization turns out to be problematic!

Mass renormalization: elementary vertex



- ▶ h, h' : light cone (z-axis) helicities
- ▶ \mathbf{q} : center-of-mass \perp momentum in splitting
- ▶ polarization λ , with \perp polarization vector ε_λ^{*j}

$$\left[\bar{u}_{h'}(p') \varepsilon_\lambda^*(k) u_h(p) \right] \sim \overbrace{\bar{u}_{h'} \gamma^+ u_h}^{\sim \delta_{h,h'}} \delta^{ij} \mathbf{q}^i \varepsilon_\lambda^{*j} + \overbrace{\bar{u}_{h'} \gamma^+ [\gamma^i, \gamma^j] u_h}^{\sim \delta_{h,h'}} \mathbf{q}^i \varepsilon_\lambda^{*j} + \overbrace{\bar{u}_{h'} \gamma^+ \gamma^j u_h}^{\sim \delta_{h,-h'}} m_q \varepsilon_\lambda^{*j}$$

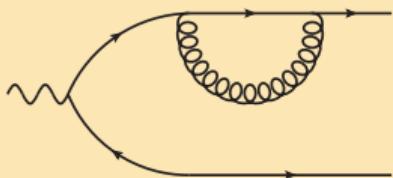
- ▶ New 3rd light-cone-helicity-flip structure $\sim m_q$ (Loops: also 4th $\bar{u}_{h'} \gamma^+ \gamma^i u_h \varepsilon_\lambda^{*j} q^i q^j$)
- ▶ Note: \perp momentum in non-flip \implies flip vertex less UV-divergent

Only 1 mass in Lagrangian, but 2 in Hamiltonian

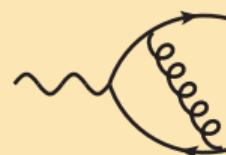
- ▶ Kinetic mass in $k_q^- = (\mathbf{k}_q^2 + m_q^2)/(2k_q^+)$
- ▶ Vertex mass: coefficient of helicity flip amplitude

Mass renormalization: corrections

Propagator corrections: kinetic mass



Vertex corrections: vertex mass



- ▶ 2 flips: UV-divergence $\propto \Psi_{\text{LO}} \frac{m_q^2}{\Delta k_{\text{LO}}^-} \frac{1}{\varepsilon}$
- ▶ Renormalize m_q in energy denominator in LO wavefunction

- ▶ 1 flip: UV-divergence $\propto m_q$
- ▶ Renormalize m_q in LO vertex

Problem: with $D_\perp = 2 - 2\varepsilon$, k^+ -cutoff regularization mass counterterms are different

- ▶ Choose another Lorentz-invariance preserving regularization, **or**
- ▶ Enforce Lorentz-invariance with 2nd renormalization condition:
 \implies this work, use **on-shell form factor**

Calculate loops, Fourier-transform to \perp coordinate ...

$\gamma^* \rightarrow q\bar{q}$ with massive quarks: wavefunction result

$$\begin{aligned} \psi_{\text{NLO}}^{\gamma^* \rightarrow q\bar{q}} &= -\frac{ee_f}{2\pi} \left(\frac{\alpha_s C_F}{2\pi} \right) \left\{ \left[\left(\frac{k_0^+ - k_1^+}{q^+} \right) \delta^{ij} \bar{u}(0) \gamma^+ v(1) + \frac{1}{2} \bar{u}(0) \gamma^+ [\gamma^i, \gamma^j] v(1) \right] \mathcal{F} \left[P_T^i \mathcal{V}^T \right] + \bar{u}(0) \gamma^+ v(1) \mathcal{F} \left[P_T^j \mathcal{N}^T \right] \right. \\ &\quad \left. + m \bar{u}(0) \gamma^+ \gamma^i v(1) \mathcal{F} \left[\left(\frac{P_T^i P_T^j}{P_T^2} - \frac{\delta^{ij}}{2} \right) \mathcal{S}^T \right] - m \bar{u}(0) \gamma^+ \gamma^j v(1) \mathcal{F} \left[\mathcal{V}^T + \mathcal{M}^T - \frac{\mathcal{S}^T}{2} \right] \right\} \epsilon_\lambda^j. \end{aligned}$$

$$\begin{aligned} \mathcal{F} \left[\mathbf{P}^i \mathcal{V}^T \right] &= \frac{i \mathbf{x}_{01}^i}{|\mathbf{x}_{01}|} \left(\frac{\kappa_z}{2\pi |\mathbf{x}_{01}|} \right)^{\frac{D}{2}-2} \left\{ \left[\frac{3}{2} + \log \left(\frac{\alpha}{z} \right) + \log \left(\frac{\alpha}{1-z} \right) \right] \left\{ \frac{(4\pi)^{2-\frac{D}{2}}}{(2-\frac{D}{2})} \Gamma \left(3 - \frac{D}{2} \right) + \log \left(\frac{|\mathbf{x}_{01}|^2 \mu^2}{4} \right) \right. \right. \\ &\quad \left. \left. + 2\gamma_E \right\} + \frac{1}{2} \frac{(D_s-4)}{(D-4)} \right\} \kappa_z K_{\frac{D}{2}-1} (|\mathbf{x}_{01}| \kappa_z) + \frac{i \mathbf{x}_{01}^i}{|\mathbf{x}_{01}|} \left\{ \left[\frac{5}{2} - \frac{\pi^2}{3} + \log^2 \left(\frac{z}{1-z} \right) - \Omega_{\mathcal{V}}^T + L \right] \kappa_z K_1 (|\mathbf{x}_{01}| \kappa_z) + I_{\mathcal{V}}^T \right\} \end{aligned}$$

$$\mathcal{F} \left[\mathbf{P}^j \mathcal{N}^T \right] = \frac{i \mathbf{x}_{01}^j}{|\mathbf{x}_{01}|} \left\{ \Omega_{\mathcal{N}}^T \kappa_z K_1 (|\mathbf{x}_{01}| \kappa_z) + I_{\mathcal{N}}^T \right\}$$

$$\begin{aligned} \mathcal{F} \left[\left(\frac{\mathbf{P}^i \mathbf{P}^j}{\mathbf{P}^2} - \frac{\delta^{ij}}{2} \right) \mathcal{S}^T \right] &= \frac{(1-z)}{2} \left[\frac{\mathbf{x}_{01}^i \mathbf{x}_{01}^j}{|\mathbf{x}_{01}|^2} - \frac{\delta^{ij}}{2} \right] \int_0^z \frac{d\chi}{(1-\chi)} \int_0^\infty \frac{du}{(u+1)^2} |\mathbf{x}_{01}| \sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-\chi)} \kappa_\chi^2} \\ &\quad \times K_1 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-\chi)} \kappa_\chi^2} \right) + [z \leftrightarrow 1-z]. \end{aligned}$$

$$\begin{aligned} \mathcal{F} \left[\mathcal{V}^T + \mathcal{M}^T - \frac{\mathcal{S}^T}{2} \right] &= \left(\frac{\kappa_z}{2\pi |\mathbf{x}_{01}|} \right)^{\frac{D}{2}-2} \left\{ \left[\frac{3}{2} + \log \left(\frac{\alpha}{z} \right) + \log \left(\frac{\alpha}{1-z} \right) \right] \left\{ \frac{(4\pi)^{2-\frac{D}{2}}}{(2-\frac{D}{2})} \Gamma \left(3 - \frac{D}{2} \right) + \log \left(\frac{|\mathbf{x}_{01}|^2 \mu^2}{4} \right) \right. \right. \\ &\quad \left. \left. + 2\gamma_E \right\} + \frac{1}{2} \frac{(D_s-4)}{(D-4)} \right\} K_{\frac{D}{2}-2} (|\mathbf{x}_{01}| \kappa_z) + \left\{ 3 - \frac{\pi^2}{3} + \log^2 \left(\frac{z}{1-z} \right) - \Omega_{\mathcal{V}}^T + L \right\} K_0 (|\mathbf{x}_{01}| \kappa_z) + I_{\mathcal{V}\mathcal{M}\mathcal{S}}^T, \end{aligned}$$

$$\begin{aligned} \Omega_{\mathcal{V}}^T &= -\left(1 + \frac{1}{2z} \right) \left[\log(1-z) + \gamma \log \left(\frac{1+\gamma}{1+\gamma-2z} \right) \right] + \frac{1}{2z} \left[\left(z + \frac{1}{2} \right) (1-\gamma) + \frac{m^2}{Q^2} \right] \log \left(\frac{\kappa_z^2}{m^2} \right) + [z \leftrightarrow 1-z] \\ I_{\mathcal{V}}^T &= \int_0^1 \frac{d\xi}{\xi} \left(\frac{2 \log(\xi)}{(1-\xi)} - \frac{(1+\xi)}{2} \right) \left\{ \left[\frac{\kappa_z^2 + \xi(1-z)}{(1-\xi)} m^2 \right] K_1 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + \xi(1-z)} m^2 \right) - [\xi \rightarrow 0] \right\} \\ &\quad - \int_0^1 d\xi \left(\frac{\log(\xi)}{(1-\xi)^2} + \frac{z}{(1-\xi)} - \frac{z}{2} \right) \frac{(1-z) m^2}{\sqrt{\kappa_z^2 + \xi(1-z)} m^2} K_1 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + \xi(1-z)} m^2 \right) \\ &\quad - \int_0^z \frac{d\chi}{(1-\chi)} \int_0^\infty \frac{du}{(u+1)} \frac{m^2}{\kappa_z^2} \left[2\chi + \left(\frac{u}{1-u} \right)^2 \frac{1}{z} (z-\chi)(1-2\chi) \right] \\ &\quad \times \left\{ \sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-\chi)}} \kappa_\chi^2 K_1 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-\chi)}} \kappa_\chi^2 \right) - [u \rightarrow 0] \right\} \\ &\quad - \int_0^z \frac{dx}{(1-x)^2} \int_0^\infty \frac{du}{(u+1)} (z-x) \left[1 - \frac{2u}{1+u} (z-x) + \left(\frac{u}{1+u} \right)^2 \frac{1}{z} (z-x)^2 \right] \\ &\quad \times \frac{m^2}{\sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-x)}} \kappa_\chi^2} K_1 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-x)}} \kappa_\chi^2 \right) + [z \leftrightarrow 1-z]. \end{aligned}$$

$$\begin{aligned} \Omega_{\mathcal{N}}^T &= \frac{z+1-2z^2}{z} \left[\log(1-z) + \gamma \log \left(\frac{1+\gamma}{1+\gamma-2z} \right) \right] - \frac{(1-z)}{z} \left[\frac{2z+1}{2} (1-\gamma) + \frac{m^2}{Q^2} \right] \log \left(\frac{\kappa_z^2}{m^2} \right) - [z \leftrightarrow 1-z] \quad (9) \\ I_{\mathcal{N}}^T &= \frac{2(1-z)}{z} \int_0^1 dx \int_0^\infty \frac{du}{(u+1)} \left[(2+u)uz + u^2 \chi \right] \sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-\chi)}} \kappa_\chi^2 K_1 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-\chi)}} \kappa_\chi^2 \right) \\ &\quad + \frac{m^2}{\kappa_z^2} \left(\frac{z}{1-z} + \frac{X}{1-\chi} [u-2z-2u\chi] \right) \left[\sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-\chi)}} \kappa_\chi^2 K_1 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-\chi)}} \kappa_\chi^2 \right) - [u \rightarrow 0] \right] - [z \leftrightarrow 1-z]. \quad (10) \end{aligned}$$

$$\begin{aligned} I_{\mathcal{V}\mathcal{M}\mathcal{S}}^T &= \int_0^1 \frac{d\xi}{\xi} \left(\frac{2 \log(\xi)}{(1-\xi)} - \frac{(1+\xi)}{2} \right) \left\{ K_0 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + \xi(1-z)} m^2 \right) - [\xi \rightarrow 0] \right\} \\ &\quad + \int_0^1 d\xi \left(\frac{-3(1-z)}{2(1-\xi)} + \frac{(1-z)}{2} \right) K_0 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + \xi(1-z)} m^2 \right) \\ &\quad + \int_0^z \frac{d\chi}{(1-\chi)} \int_0^\infty \frac{du}{(u+1)} \left[-\frac{u}{z} \frac{(z+w\chi)}{z} (\chi-(1-z)) \right] K_0 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-\chi)}} \kappa_\chi^2 \right) \\ &\quad + \int_0^z d\chi \int_0^\infty \frac{du}{(u+1)^2} \left\{ \frac{\kappa_z^2}{u} \left[1 + \frac{u(X-\chi)}{z} \right] - \frac{m^2}{\kappa_z^2} \frac{\chi}{(1-\chi)} \left[\frac{2(u+u)^2}{u} + \frac{u}{z(1-z)} (z-\chi)^2 \right] \right\} \\ &\quad \times \left\{ K_0 \left(|\mathbf{x}_{01}| \sqrt{\kappa_z^2 + u \frac{(1-z)}{(1-\chi)}} \kappa_\chi^2 \right) - [u \rightarrow 0] \right\} + [z \leftrightarrow 1-z]. \end{aligned}$$

Cross section, real and virtual correction

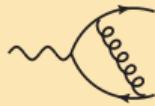
Just like massless quarks, explicit expressions in paper

Evaluate $\gamma^* + A$ total cross section as $\sigma_{L,T}^{\text{NLO}} = \sigma_{L,T}^{\text{LO}} + \sigma_{L,T}^{\text{dip}} + \sigma_{L,T,\text{sub.}}^{qg}$.



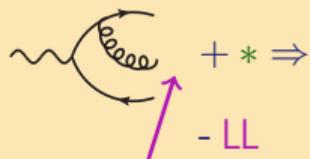
\Rightarrow

$$\sigma^{\text{LO}} \sim \int_0^1 dz_1 \int_{\mathbf{x}_0, \mathbf{x}_1} |\psi_{\gamma^* \rightarrow q\bar{q}}^{\text{LO}}(z_1, \mathbf{x}_0, \mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$$



$- * \Rightarrow$

$$\sigma^{\text{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_0, \mathbf{x}_1, z_1} \left[\left| \psi_{\gamma^* \rightarrow q\bar{q}}^{\text{NLO}} \right|^2 - \left| \psi_{\gamma^* \rightarrow q\bar{q}}^{\text{NLO,UV}} \right|^2 \right] \mathcal{N}_{01}(x_{Bj})$$



$$k_g^+ \sim z_2$$

$$\sigma_{\text{sub.}}^{qg} \sim \alpha_s C_F \int_{z_1, z_2, \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2} dz_2 \left[\left| \psi_{\gamma^* \rightarrow q\bar{q}g}(z_1, z_2, \{\mathbf{x}_i\}) \right|^2 \mathcal{N}_{012}(X(z_2)) \right.$$

$$\left. + \text{UV-sub} - \left| \psi_{\gamma^* \rightarrow q\bar{q}g}(z_1, 0, \{\mathbf{x}_i\}) \right|^2 \mathcal{N}_{012}(X(z_2)) \right]$$

* UV-divergence cancellation -LL: leading log, already in BK-evolved \mathcal{N}

► Parametrically $X(z_2) \sim x_{Bj}$, but $X(z_2) \sim 1/z_2$ essential!

(" k_T -factorization" with fixed rapidity scale is unstable @ NLO. Analogous problem in $p + A \rightarrow h + X$)

Conclusions

We have calculated $\psi\gamma^*\rightarrow q\bar{q}$ to one loop **with quark masses**

- ▶ Transverse and longitudinal γ^*
- ▶ Required solving longstanding LCPT problem of mass renormalization
- ▶ Also provides total DIS cross section for F_2^C at NLO in dipole picture
 ➡ can proceed to fits
- ▶ Already being used in other small- x calculations:
 exclusive vector mesons: talk Penttala

Thanks to Risto and Guillaume for the actual work!

Dirac and Pauli form factors

$$\Gamma^\mu(q) = F_D(q^2/m^2) \gamma^\mu + F_P(q^2/m^2) \frac{q_\nu}{2m} i\sigma^{\mu\nu}$$

$$\begin{aligned} \Psi_{\text{LO}}^{\gamma_T^* \rightarrow q\bar{q}} + \Psi_{\text{NLO}}^{\gamma_T^* \rightarrow q\bar{q}} &= \delta_{\alpha_0\alpha_1} \frac{ee_f}{ED_{\text{LO}}} \left\{ \bar{u}(0)\epsilon_\lambda(q)v(1) \left[1 + \left(\frac{\alpha_s C_F}{2\pi}\right) \mathcal{V}^T \right] + \frac{q^+}{2k_0^+ k_1^+} (P_T \cdot \epsilon_\lambda) \bar{u}(0)\gamma^+ v(1) \left(\frac{\alpha_s C_F}{2\pi}\right) \mathcal{N}^T \right. \\ &\quad \left. + \frac{q^+}{2k_0^+ k_1^+} \frac{(P_T \cdot \epsilon_\lambda)}{P_T^2} P_T^j m \bar{u}(0)\gamma^+ \gamma^j v(1) \left(\frac{\alpha_s C_F}{2\pi}\right) \mathcal{S}^T + \frac{q^+}{2k_0^+ k_1^+} m \bar{u}(0)\gamma^+ \epsilon_\lambda(q)v(1) \left(\frac{\alpha_s C_F}{2\pi}\right) \mathcal{M}^T \right\}. \end{aligned}$$

$$-\left(\frac{\alpha_s C_F}{2\pi}\right) \frac{m^2}{P_T^2} \mathcal{S}^T \Big|_{P_T^2 = -\overline{Q}^2 - m^2} = F_P(q^2/m^2)$$

$$-\left(\frac{\alpha_s C_F}{2\pi}\right) \frac{1}{(2z-1)} \mathcal{N}^T \Big|_{P_T^2 = -\overline{Q}^2 - m^2} = F_P(q^2/m^2)$$

$$\left(\frac{\alpha_s C_F}{2\pi}\right) \mathcal{V}^T \Big|_{P_T^2 = -\overline{Q}^2 - m^2} = -1 + F_D(q^2/m^2) + F_P(q^2/m^2)$$

$$\mathcal{M}^T \Big|_{P_T^2 = -\overline{Q}^2 - m^2} = 0$$