Forward di-hadron back-to-back correlations: Interplay of CGC and Sudakov resummation

Cyrille Marquet

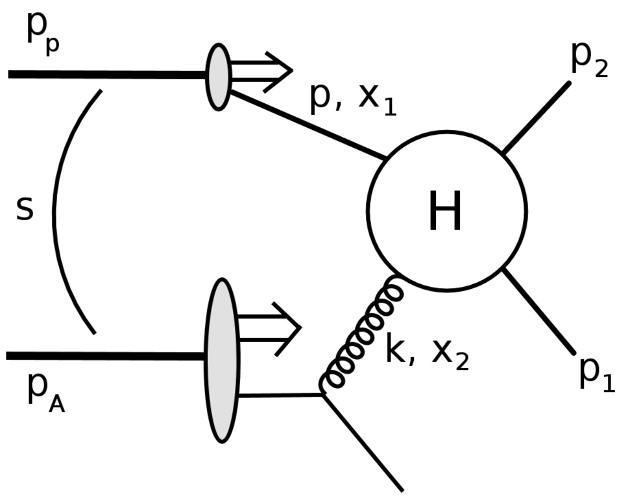
Centre de Physique Théorique

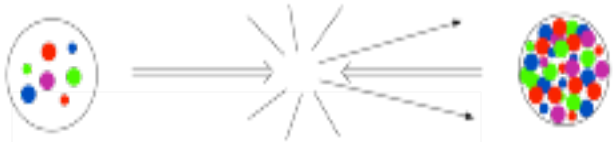
Ecole Polytechnique & CNRS

based on Albacete, Giacalone, CM and Matas, PRD 99 (2019) 1, 014002
Giacalone, CM, Matas and Wei, in preparation

The context: forward di-hadrons

- large-x projectile (proton) on small-x target (proton or nucleus)





$\langle k_{1t} \rangle \sim \Lambda_{QCD}$ $\langle k_{2t} \rangle \sim Q_s(x_2)$

$Q_s(x_2) \gg \Lambda_{QCD}$

so-called "dilute-dense" kinematics

Incoming partons' energy fractions:

$$\begin{aligned} x_1 &= \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}\right) \\ x_2 &= \frac{1}{\sqrt{s}}\left(|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}\right) \end{aligned} \xrightarrow{y_1,y_2 \gg 0} \begin{aligned} x_1 &\sim 1 \\ x_2 &\ll 1 \end{aligned}$$

CM (2007)

Gluon's transverse momentum (p_{1t}, p_{2t} imbalance):

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}|\cos\Delta\phi$$

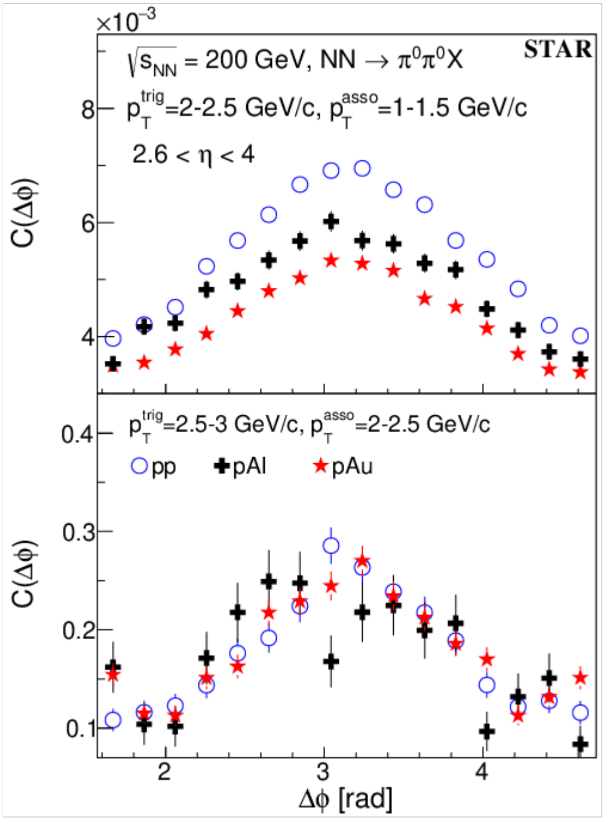
prediction: modification of the k_t distribution in p+Pb vs p+p collisions

New STAR data

- forward di-hadron correlation function in p+Au, p+Al and p+p collisions

low pt: away-side peak suppression with increasing A

STAR Collaboration (2022)



high pt: correlation unchanged

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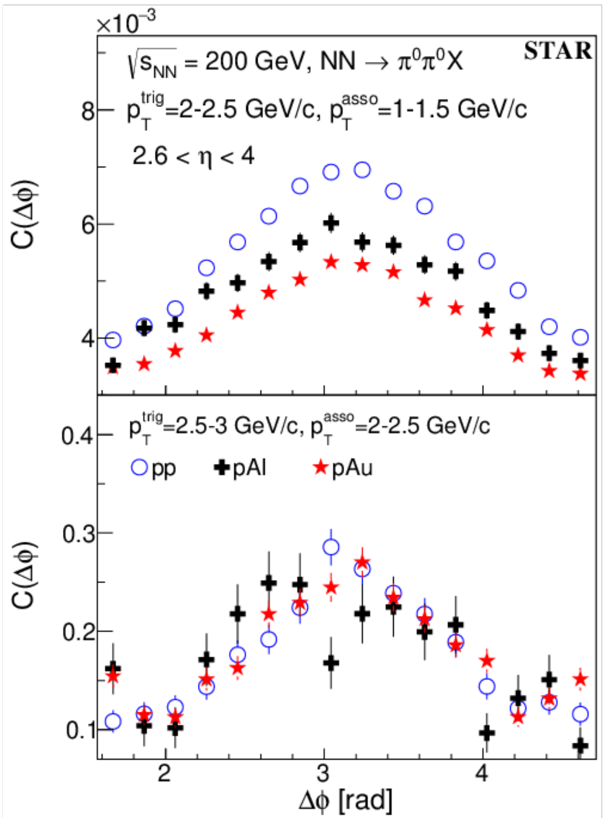
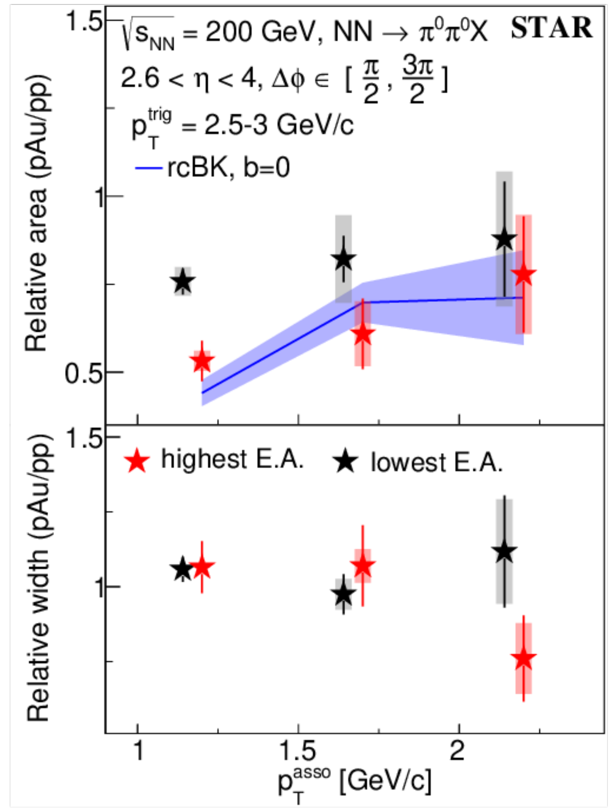




area suppressed

width unchanged

high pt: correlation unchanged

CGC calculation from
Albacete, Giacalone, CM and Matas, (2019)

Recent LHCb data

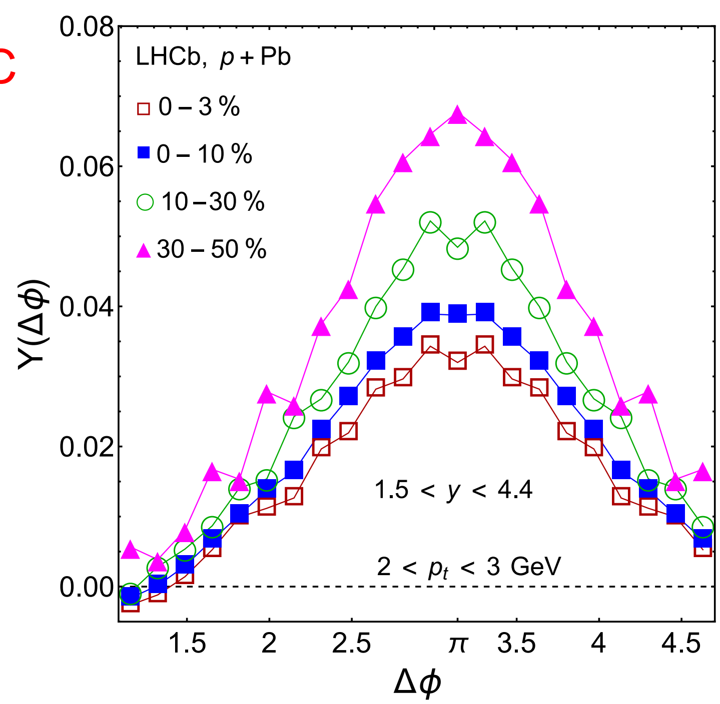
- LHCb measured the di-hadron correlation function at forward rapidities

the delta phi distribution shows:
- a ridge contribution (could be flow, Glasma graphs or something else)

- the remainder of the away-side peak can be qualitatively described in the CGC

suppression of the away-side peak
with increasing centrality seen in the data

without the soft gluon resummation, the
width of the peak cannot be reproduced



Giacalone and CM (2019)

Color Glass Condensate (CGC) calculation of forward di-hadrons

Nearly back-to-back di-hadrons

- having in mind to perform a resummation of soft gluons, we focus on a restricted kinematic window: the vicinity of $\Delta\Phi = \pi$

interestingly, this is also where saturation effects are most important:
this is where the k_t of the small-x_2 gluon in the target is the smallest

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- there, one can take the limit $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$ and simplify the full formula

in practice, we keep only the leading $1/|p_{ti}|$ power,
but we still have all orders in $(Q_s/k_t)^n$

the result is a Transverse Momentum Dependent (TMD) factorization formula

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- this TMD factorization formula for $x_2 \ll x_1 \sim 1$ can be derived in two ways:

from the generic TMD factorization framework (valid up to power corrections): by taking the small-x limit

Bomhof, Mulders and Pijlman (2006)

Kotko, Kutak, CM, Petreska, Sapeta and van Hameren (2015)

from the CGC framework (valid at small-x): by extracting the leading power

Dominguez, CM, Xiao and Yuan (2011)

CM, Petreska, Roiesnel (2016)

TMD factorization

$$
\begin{aligned}
\frac{d\sigma^{pA\to\pi^0\pi^0 X}}{dy_1\ dy_2\ d^2p_{1t}\ d^2p_{2t}} = \frac{\alpha_s^2}{2C_F}\int_{p_{t1}\frac{e^{y_1}}{\sqrt{s}}/(1-p_{t2}\frac{e^{y_2}}{\sqrt{s}})}^{1}\frac{dz_1}{z_1^2}\int_{p_{t2}\frac{e^{y_2}}{\sqrt{s}}/(1-\frac{p_{t1}}{z_1}\frac{e^{y_1}}{\sqrt{s}})}^{1}\frac{dz_2}{z_2^2}\ \frac{z(1-z)}{P_t^4}
\end{aligned}
$$

$$
\begin{aligned}
&\Big\{D_{\pi^0/g}(z_1,\mu^2)\big[x_1u(x_1,\mu^2)\ D_{\pi^0/u}(z_2,\mu^2)+x_1d(x_1,\mu^2)\ D_{\pi^0/d}(z_2,\mu^2)\big]P_{gq}(z)\ \times \\
&\qquad\times\ \Big[(1-z)^2\mathcal{F}_{qg}^{(1)}(x_2,k_t)+\mathcal{F}_{qg}^{(2)}(x_2,k_t)\Big]+ \\
&+\ D_{\pi^0/g}(z_2,\mu^2)\big[x_1u(x_1,\mu^2)\ D_{\pi^0/u}(z_1,\mu^2)+x_1d(x_1,\mu^2)\ D_{\pi^0/d}(z_1,\mu^2)\big]P_{gq}(1-z)\ \times \\
&\qquad\times\ \Big[z^2\mathcal{F}_{qg}^{(1)}(x_2,k_t)+\mathcal{F}_{qg}^{(2)}(x_2,k_t)\Big]+
\end{aligned}
$$

$qg^* \to qg$ channel

$$
\begin{aligned}
&+\ 2\big[D_{\pi^0/u}(z_1,\mu^2)\ D_{\pi^0/u}(z_2,\mu^2)+D_{\pi^0/d}(z_1,\mu^2)\ D_{\pi^0/d}(z_2,\mu^2)\big]\ x_1g(x_1,\mu^2)\ P_{qg}(z)\ \times \\
&\qquad\times\ \Big[\mathcal{F}_{gg}^{(1)}(x_2,k_t)-2z(1-z)\big(\mathcal{F}_{gg}^{(1)}(x_2,k_t)-\mathcal{F}_{gg}^{(2)}(x_2,k_t)\big)\Big]+
\end{aligned}
$$

$gg^* \to q\bar{q}$ channel

$$
\begin{aligned}
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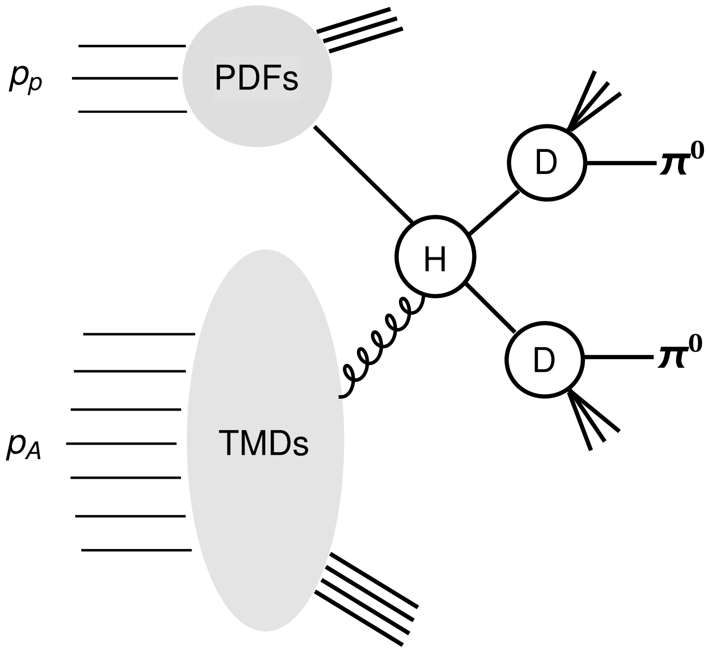
&\qquad\times\ \Big[\mathcal{F}_{gg}^{(1)}(x_2,k_t)-2z(1-z)\big(\mathcal{F}_{gg}^{(1)}(x_2,k_t)-\mathcal{F}_{gg}^{(2)}(x_2,k_t)\big)+\mathcal{F}_{gg}^{(6)}(x_2,k_t)\Big]\Big\},
\end{aligned}
$$

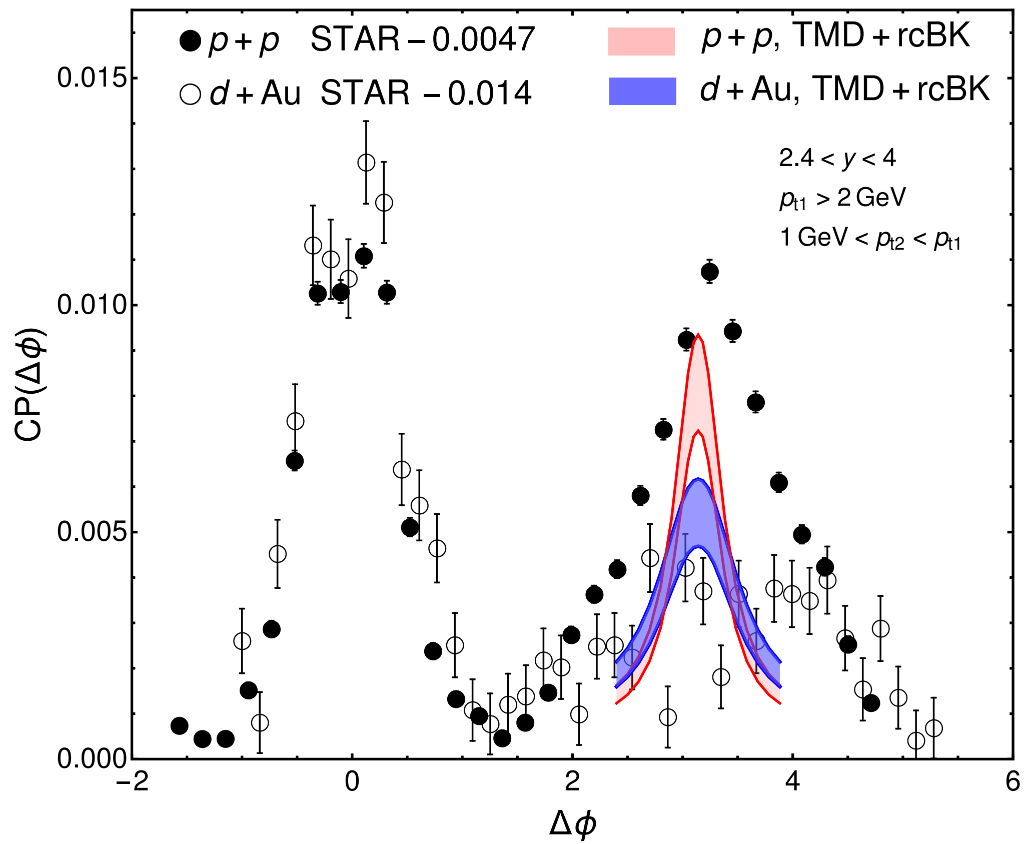
$gg^* \to gg$ channel

it involves *several* transverse-momentum-dependent (TMD) gluon distributions for the target nucleus: $\mathcal{F}_{qg}^{(i)}$ $\mathcal{F}_{gg}^{(i)}$

STAR forward di-hadrons



Albacete, Giacalone, CM and Matas, (2019)



cannot be applied to the overall $\Delta\Phi$ range,

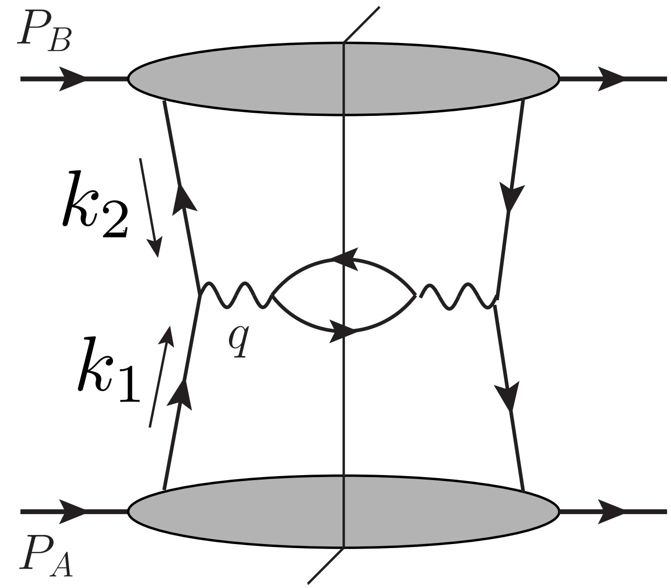
again, without the soft gluon resummation, the width of the peak cannot be reproduced

Soft gluon emissions in hard processes

Drell-Yan/di-jet production

the transverse momentum of the lepton/jet pair q_T is the sum of the transverse momenta of the incoming partons

$$d\hat{\sigma} \propto \delta(k_{1t} + k_{2t} - q_T)$$



so in collinear factorization

$$d\sigma^{p+p\to Z+X} \propto \delta(q_T) + \mathcal{O}(\alpha_s)$$

higher-orders are important at large q_T, the transverse momentum of the pair is then balanced by a recoiling hard parton

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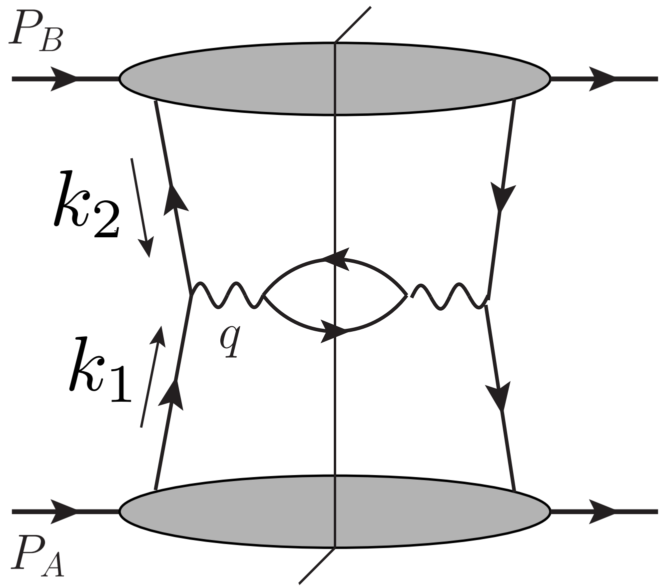

at low q_T, the dominant production mechanism is still 2-to-2 scattering, and the transverse momentum of the pair is balanced by soft gluons

the emission of a soft gluon is not suppressed, as it comes with a large logarithm

$$\alpha_s \ln^2\left(\frac{M^2}{q_\perp^2}\right)$$

Soft gluon resummation

$$\sum_{ab}\int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \vec{b}_\perp} x_1 f_a(x_1,\mu_b) x_2 f_b(x_2,\mu_b) \frac{1}{\pi}\frac{d\sigma}{dt} e^{-S(Q^2,b_\perp)}$$

$$S^i(Q,b) = \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[A_i \frac{\alpha_s}{2\pi}\ln\left(\frac{Q^2}{\mu^2}\right) + B_i \frac{\alpha_s}{\pi}\right]$$

the so-called
Sudakov factor
in coordinate space

b*-prescription

$$\mu_b = c_0/b$$

$$\mu_b = c_0/b^* \qquad b^* = b/\sqrt{1+b^2/b_{\max}^2}$$

$$S(Q,b) = S_{\text{perturbative}}(Q,b) + S_{\text{non-perturbative}}(Q,b)$$

double logs and single logs

Universal / Gaussian form / Extracted from experiments

Sudakov & small-x logs together

with saturation effects taken into account in the gluon TMDs,
the soft-gluon resummation is similar

Mueller, Xiao, Yuan (2013)

$$\frac{d\sigma}{d^2p_{T1}d^2p_{T2}} \propto \int \frac{d^2b_\perp}{(2\pi)^2} e^{-i\vec{q}_T\cdot\vec{b}_\perp} f(x_1) \otimes F(x_2, b_\perp) \otimes H \otimes e^{-S_{\mathrm{Sudakov}}}$$

implementation with GBW model — Stasto, Wei, Xiao, Yuan (2018)

implementation with rcBK evolution

CM, Wei, Xiao (2019)
van Hameren, Kotko, Kutak and Sapeta (2020)
Zhao, Xu, Chen, Zhang and Wu (2021)
Benic, Garcia-Montero and Perkov (2022)
Giacalone, CM, Matas and Wei, in preparation

alternative Monte Carlo model — van Hameren, Kotko, Kutak and Sapeta (2014)

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important remark: for the di-jet process, MXY inferred the coefficients in S_{Sudakov} by analogy with Higgs production

the actual calculation from the NLO diagrams shows unexpected intrincacies

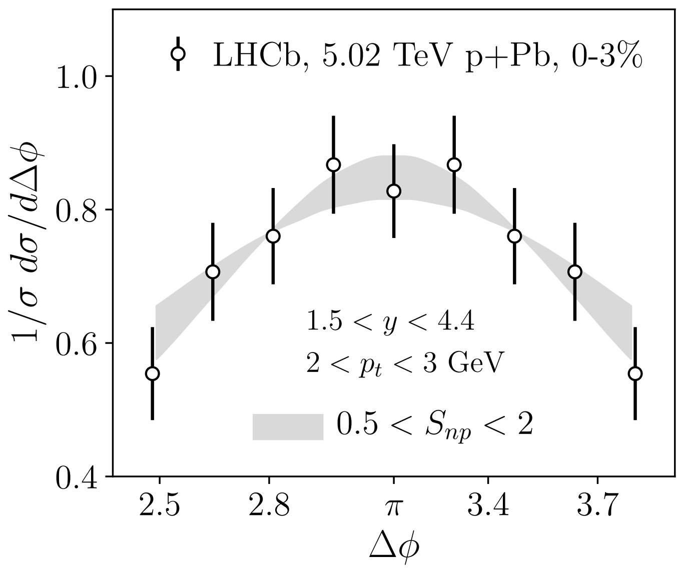
Taels, Beuf, CM and Altinoluk (2022), Caucal, Salazar, Schenke, Venugopalan, in preparation
see also Farid Salazar’s talk yesterday

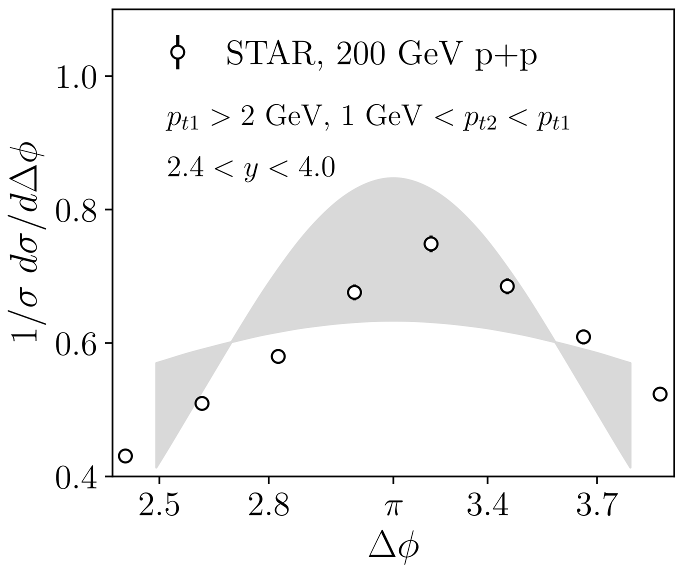
Back to comparisons with data

Giacalone, CM, Matas and Wei, in preparation

Low-pt di-hadrons

the width of the away-side peak is well described
however, we lose control on the normalization





this is because we are very sensitive to non-perturbative physics
this was to be expected since the Sudakov logarithms are not really large

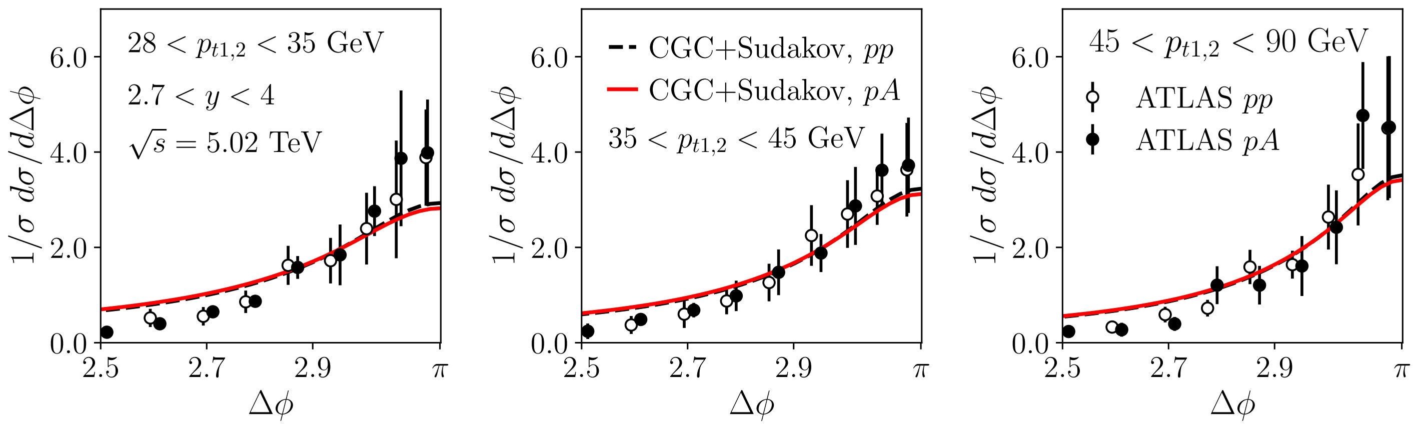
with such hadron p_T's (1-3 GeV), it is better to stick to the original CGC formulation

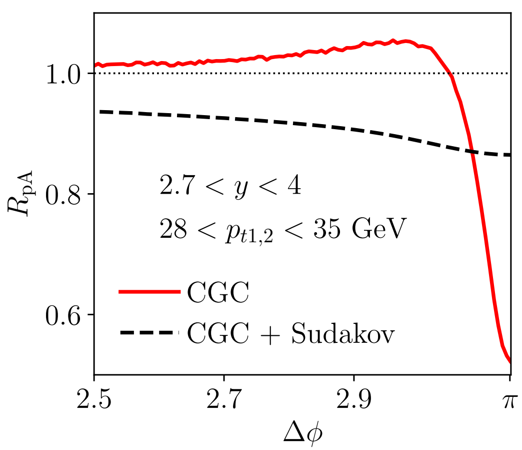
CM (2007), Albacete and CM (2010), Lappi and Mantysaari (2013)

LHC forward di-jets

ATLAS measured the di-jet correlation function at forward rapidities

ATLAS Collaboration (2019)





there is almost no difference between p+p and p+A

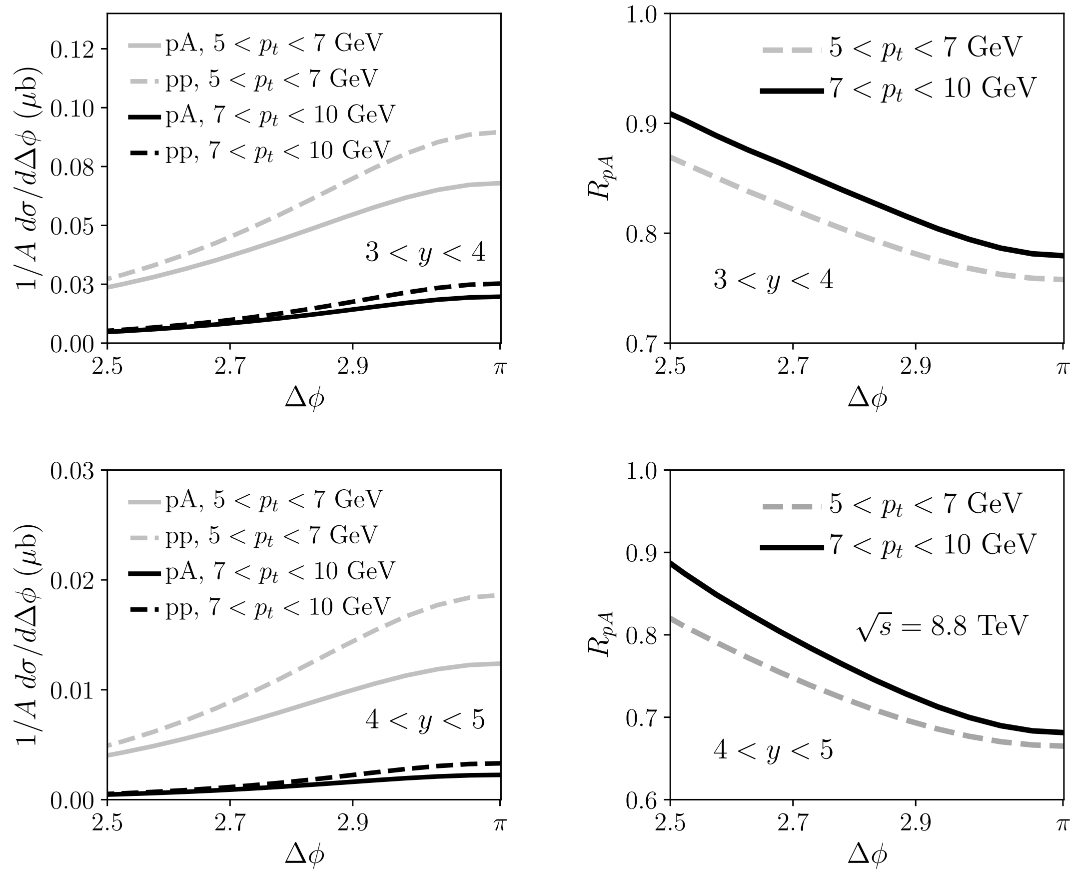
the Sudakov resummation dominates and R_{pA} stays rather close to unity

similar conclusions obtained with the MC model

van Hameren, Kotko, Kutak and Sapeta (2019)

Finding the sweet spot

we tried to pinpoint the optimal hadron p_T range
where both the small-x and Sudakov logs matter



we identified the 5-10 GeV window

Conclusions

• we revisited forward di-hadron production in the CGC framework, focusing on nearly back-to-back hadrons

→ there, saturation effects are most relevant, as the di-hadron transverse momentum imbalance $|k_t|$ is of the order of the saturation scale Qs, or smaller

• we obtain a TMD-factorized expression

→ this is well adapted to further implement the soft-gluon resummation needed to get the correct width of the away-side peak

• if the hadron p_T's are too low, various non-perturbative effects hinder robust theory calculations

• if the hadron/jet p_T's are too big, large Sudakov logarithms dominate the small-x logs and hide the saturation effects

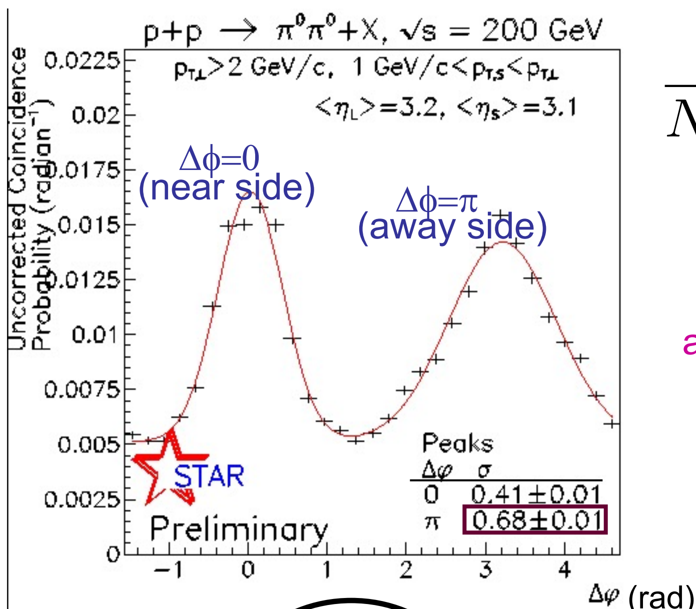
we hope to see at the LHC (LHCb, FOCAL) a confirmation of the saturation signal seen at RHIC: using intermediate p_T di-hadrons (5-10 GeV)

Back-up slides

Di-hadron angular correlations

comparisons between d+Au → h_1 h_2 X (or p+Au → h_1 h_2 X) and p+p → h_1 h_2 X

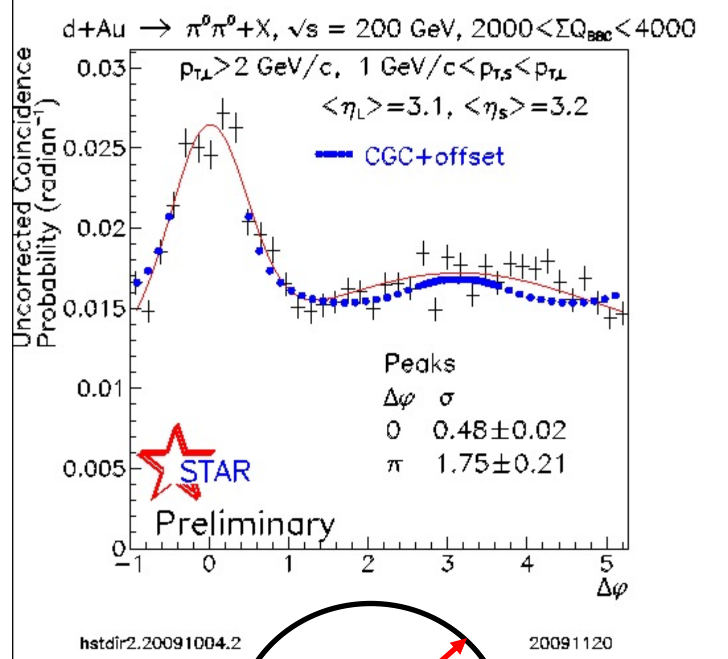
p+p collisions

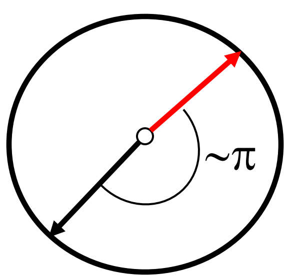


$$\frac{1}{N_{trig}}\frac{dN_{pair}}{d\Delta\varphi}$$

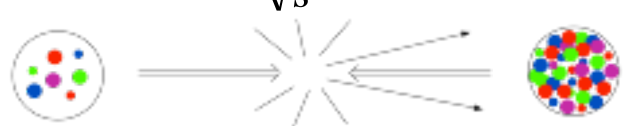
Albacete
and CM (2010)

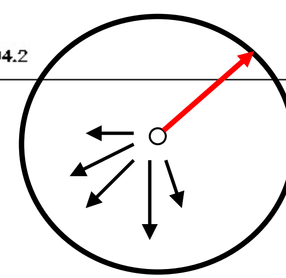
central d+Au collisions





$$x_A = \frac{k_1\, e^{-y_1} + k_2\, e^{-y_2}}{\sqrt{s}} << 1$$





however, when $y_1 \sim y_2 \sim 0$ (and therefore $x_A \sim 0.03$),
the p+p and d+Au curves are almost identical

Nearly back-to-back di-hadrons

- The full CGC formula is notoriously difficult to deal with

CM (2007)

presented at Quark Matter 10 years ago, no complete implementation, but several approximations studied

Albacete, CM (2010) Stasto, Xiao and Yuan (2012) Lappi and Mantysaari (2013)

- instead, we shall focus on a restricted kinematic window where saturation effects are most important: the vicinity of $\Delta\Phi = \pi$

this is where the k_t of the small-x_2 gluon in the target is the smallest

- there, one can take the limit $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$ and simplify the full formula

in practice, we keep only the leading $1/|p_{ti}|$ power, but we still have all orders in $(Q_s/k_t)^n$

the result is a Transverse Momentum Dependent (TMD) factorization formula

- only valid in asymmetric situations

Collins and Qiu (2007), Xiao and Yuan (2010)

does not apply with TMD parton densities for both colliding projectiles

The back-to-back regime

- this TMD factorization formula for $x_2 \ll x_1 \sim 1$ can be derived in two ways:

from the generic TMD factorization framework (valid up to power corrections): by taking the small-x limit

Bomhof, Mulders and Pijlman (2006)

Kotko, Kutak, CM, Petreska, Sapeta and van Hameren (2015)

from the CGC framework (valid at small-x): by extracting the leading power

Dominguez, CM, Xiao and Yuan (2011)

CM, Petreska, Roiesnel (2016)

- at small x, the TMD gluon distributions can be written as:

(showing here the $qg^* \to qg$ channel TMDs only)

$$U_{\mathbf{x}} = \mathcal{P} \exp\left[ig \int_{-\infty}^{\infty} dx^+ A_a^-(x^+, \mathbf{x}) t^a \right]$$

$$\mathcal{F}_{qg}^{(1)}(x_2, |k_t|) = \frac{4}{g^2} \int \frac{d^2x d^2y}{(2\pi)^3}\, e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})} \left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{y}})(\partial_i U_{\mathbf{x}}^\dagger) \right] \right\rangle_{x_2}$$

$$\mathcal{F}_{qg}^{(2)}(x_2, |k_t|) = -\frac{4}{g^2} \int \frac{d^2x d^2y}{(2\pi)^3}\, e^{-ik_t\cdot(\mathbf{x}-\mathbf{y})} \frac{1}{N_c} \left\langle \mathrm{Tr}\left[(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger \right] \mathrm{Tr}\left[U_{\mathbf{y}} U_{\mathbf{x}}^\dagger \right] \right\rangle_{x_2}$$

these Wilson line correlators also emerge directly in CGC calculations when $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$

x evolution of the gluon TMDs

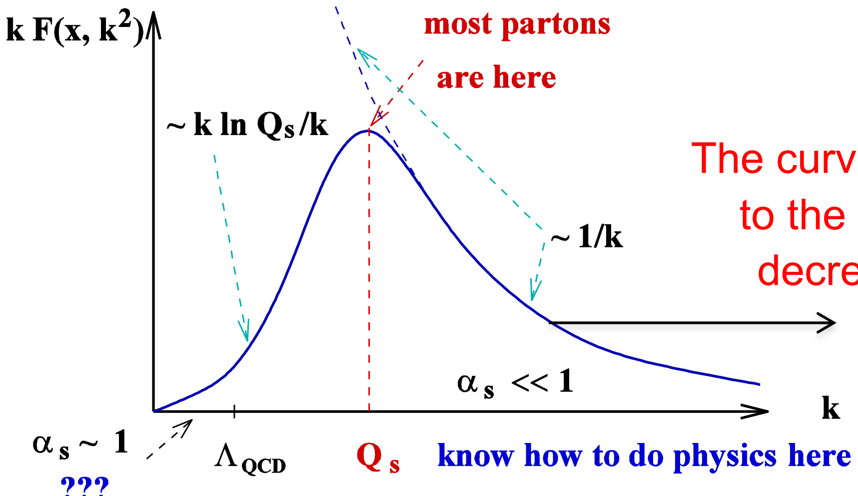
the evolution of Wilson line correlators with decreasing x can be computed from the so-called JIMWLK equation

$$\frac{d}{d\ln(1/x_2)}\langle O\rangle_{x_2} = \langle H_{JIMWLK}\ O\rangle_{x_2}$$

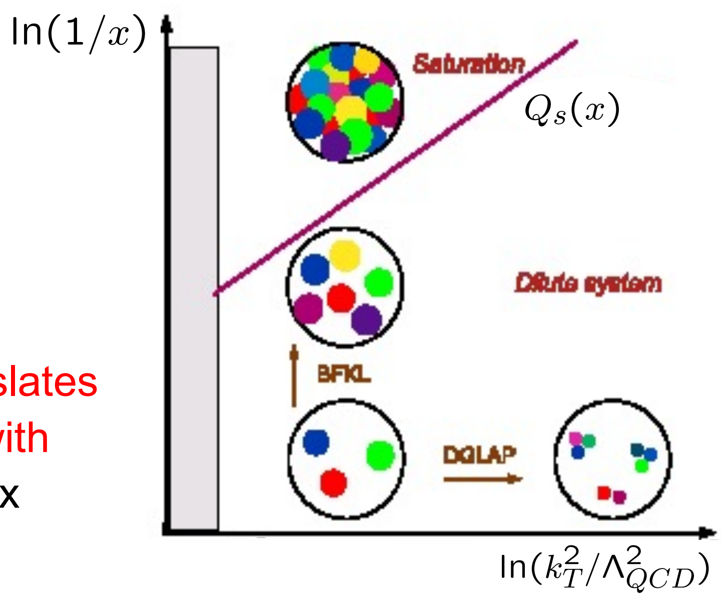
Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

a functional RG equation that resums the leading logarithms in $y = \ln(1/x_2)$

- qualitative solutions for the gluon TMDs:



The curve translates to the right with decreasing x



the distribution of partons as a function of x and k_T

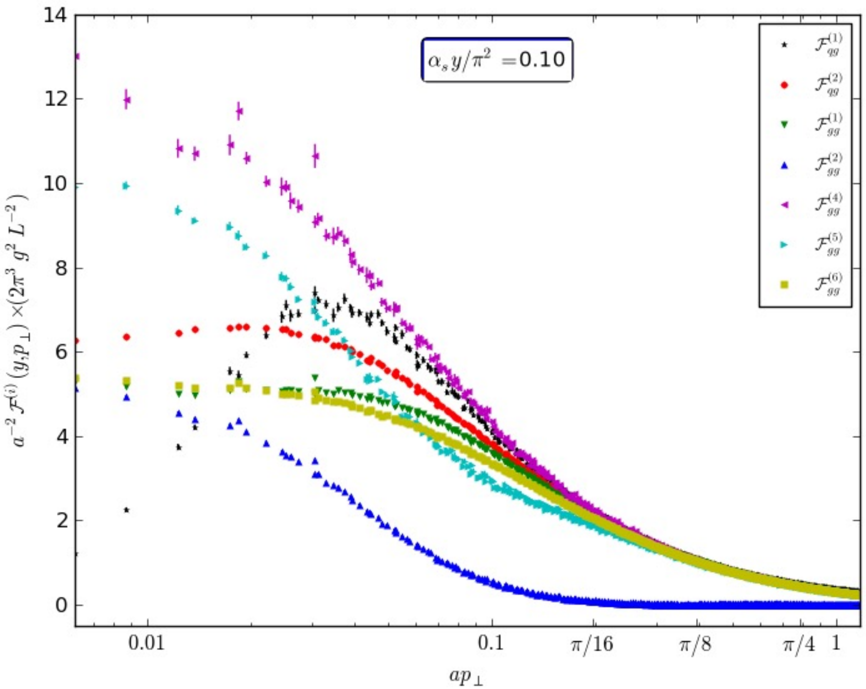
JIMWLK numerical results

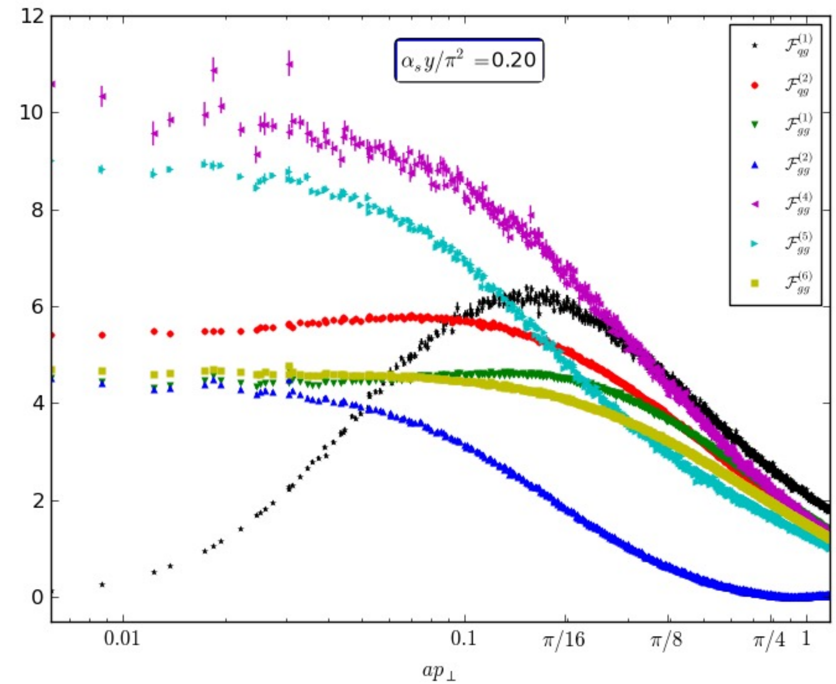
using a code written by Claude Roiesnel

initial condition at y=0 : McLerran-Venugopalan model

evolution: JIMWLK at leading log

CM, Petreska, Roiesnel (2016)





saturation effects impact the various gluon TMDs in very different ways

The Gaussian truncation

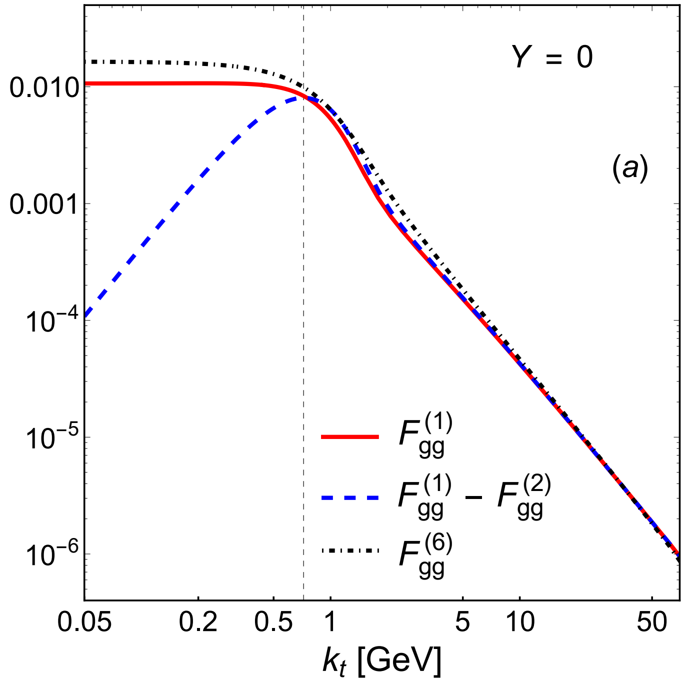
- this approximation allows to express any Wilson-line correlator in terms of the solution to a simpler equation: the Balitsky-Kovchegov equation

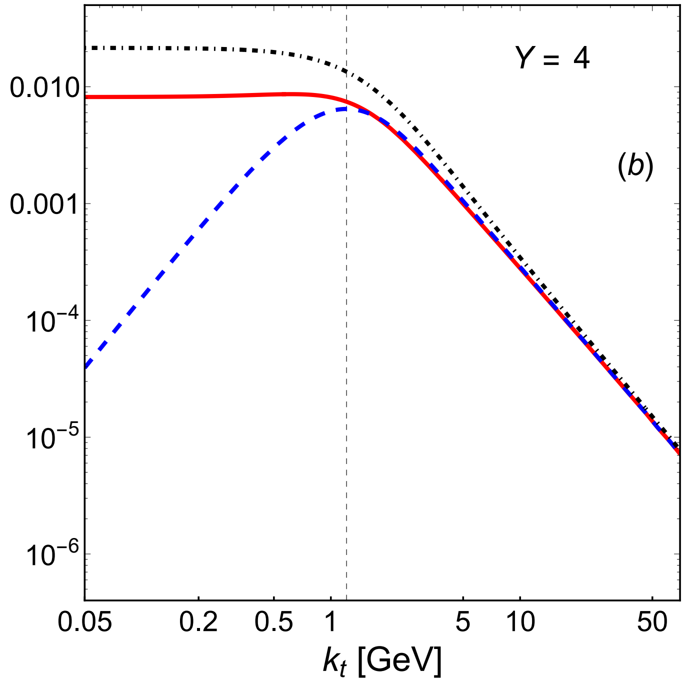
in addition, running-coupling corrections can be implemented

- some numerical results:

McLerran-Venugopalan initial condition

after some evolution: $Y=\ln(1/x_2)$





we use those gluon distributions to compute the forward di-hadron cross section