Twist decomposition of non-linear effects in the Balitsky-Kovchegov evolution

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The goal and the outline

- The goal: we want to use information provided by the low-\(x\) BFKL/BK framework to input the collinear framework:

\[ K_T \rightarrow \text{collinear} \]

BFKL \(\rightarrow\) DGLAP + Higher Twists

BK \(\rightarrow\) GLR

- Outline: use the Mellin moment space conjugate to scale to isolate contributions of subsequent twist operators in BFKL / BK. Trace effects of nonlinearity at all twists
Puzzle: problems of DGLAP in precision HERA data at low $x$ and moderate $Q^2$

- The final, precise HERA data show problems of DGLAP fits of DIS and DDIS when $Q^2 < 5 \text{ GeV}^2$ data are included fits, problems at small $x$

- Explanations proposed:

  Expectations: $\text{twist 4} / \text{twist 2} \sim \left( \frac{Q_0^2}{Q^2} \right) \left( \frac{x_0}{x} \right)^\lambda$

- **Small $x$ resummation beyond DGLAP** [R. Ball et. al, 2017]
Possible Higher Twist effects in $F_L$

Somewhat surprising finding from DGLAP + Higher Twist fits: small corrections in $F_2$, significant and positive corrections in $F_L$

[arXiv:1707.05992]
Higher Twist effects from saturation models

- Golec-Biernat–Wüsthoff or Bartels–Golec-Biernat–Kowalski saturation models assume multiple independent (eikonal) scattering:
  \[ \sigma(x, Q^2) = \sigma_0 \left[ 1 - \exp \left( -\sigma_1(x, Q^2)/\sigma_0 \right) \right] \]

- HT contributions found to increase quickly with decreasing \( x \), as
  \[ \sigma^{(\text{twist}=2^n)} \sim \sigma_1^n \]

- This is, however a model that does not fully agree with QCD-based analyzes concerning the HT components.
QCD: HT from BFKL equation [LM, M. Sadzikowski, arXiv:1411.7774] — at LL(1/x) the HT contributions found to decrease with decreasing x in the asymptotic regime.

The reason: gluon Reggeization binds t-channel gluons that couple to color dipole produced by $\gamma^*$ into two Reggeized gluons, that span a single gluon ladder dominated by twist 2 contribution.

BFKL bootstrap — gluon Reggeization

**BFKL equation — two interacting Reggeized gluons**
The Higher Twist puzzle

- Saturation (eikonal) models: small HT effects in $F_2$, large negative HT corrections in $F_L$
The Higher Twist puzzle

- **Saturation (eikonal)**: models: small HT effects in $F_2$, large negative HT corrections in $F_L$

- **BFKL**: small HT twist corections in $F_2$, small negative HT corrections in $F_L$

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The Higher Twist puzzle

- **Saturation (eikonal) models:** small HT effects in $F_2$, large negative HT corrections in $F_L$

- **BFKL:** small HT twist corections in $F_2$, small negative HT corrections in $F_L$

- **DGLAP + HT fits to HERA data:** small HT twist corections in $F_2$, large positive, up to $\sim 50\%$ positive corrections in $F_L$ at small $x$ and moderate $Q^2$
The Higher Twist puzzle

- Saturation (eikonal) models: small HT effects in $F_2$, large negative HT corrections in $F_L$

- BFKL: small HT twist corrections in $F_2$, small negative HT corrections in $F_L$

- DGLAP + HT fits to HERA data: small HT twist corrections in $F_2$, large positive, up to $\sim 50\%$ positive corrections in $F_L$ at small $x$ and moderate $Q^2$

A possible solution: coupling of two BFKL ladders to a splitted $\gamma^*$ dipole:

To be consistent, one should include the BFKL evolution both in the dipole wave function and in the ladders. At LL($1/x$): one iteration of nonlinearity in the BK equation
Triple Pomeron Vertex allows for a transition from single BFKL ladder to two and more ladders, that carry the HT contributions with the strongest enhancement due to evolution.

This mechanism should provide the most reliable estimate of HT effects in the proton structure in the LL\((1/x)\) approximation in QCD.
Higher Twists from LL Baltisky–Kovchegov (BK) equation

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\[ \gamma^*(Q^2) \]

\[ \text{Triple Pomeron (BK) vertex — transition to 4 Reggeized gluons} \]

**Note however:** the BK Triple Pomeron Vertex vanishes at LL\((Q^2)\)

[J. Bartels, K. Kutak, 2007]
Calcutational method

- The HT determinations are performed in the space of Mellin moments conjugate to the hard scale.

- The total cross sections related to $F_2$ and $F_L$ structure functions may be decomposed into twist components by isolating the singularities in the Mellin plane, that at small $\alpha_s$ lead to terms with the canonical $Q^2$ scaling, $(Q^2)^{-n}$.

- For GBW saturation model one finds simple poles in the Mellin plane for integer $n$.

- For LL BFKL equation essential singularities appear at integer $n$.

- For LL BK equation there are multiple ladders, that lead to multiple Mellin variables, and convolutions of the Mellin integrals. The convoluted expressions have essential singularities in all Mellin variables.

- Present results on HT from BK: one Triple Pomeron Vertex included, leading to two independent Mellin variables. Enough to estimate the leading twist 4 contribution.
The basic object: impact parameter density of dipole scattering amplitude $N(y, r)$, which is related to unintegrated and collinear gluon densities by linear integral transformations. It is convenient to use $\phi(y, k^2)$ instead

$$\phi(y, k^2) = \int \frac{d^2r}{2\pi} e^{-ik \cdot r} \frac{N(y, r)}{r^2}.$$ 

The Balitsky–Kovchegov equation for $\phi(y, k^2)$

$$\frac{\partial \phi(y, k^2)}{\partial y} = \bar{\alpha}_s \int_0^\infty dq^2 q^2 \left\{ \frac{q^2 \phi(y, q^2) - k^2 \phi(y, k^2)}{|q^2 - k^2|} + \frac{k^2 \phi(y, k^2)}{\sqrt{4q^4 + k^2}} \right\} - \bar{\alpha}_s \phi^2(y, k^2)$$

The solution in terms of power series of in nonlinearity (for a large nucleus, this is an expansion in $\zeta = A^{1/3}$)

[Yu. Kovchegov, 1999]:

$$\phi = \zeta \left( \phi_0 + \zeta \phi_1 + \zeta^2 \phi_2 + \ldots \right) = \zeta \sum_{n=0}^\infty \zeta^n \phi_n$$

The order $n$ is equal to number of Triple Pomeron Vertices. For $n = 0$ the linear — i.e. the BFKL equation is reproduced, for $n = 1$ the $1 \to 2$ BFKL ladders transition is described.
The iterative solution

We solve the equation directly in the Mellin space:

\[
\frac{\partial \tilde{\phi}_0(y, \gamma)}{\partial y} = \bar{\alpha}_s \chi(\gamma) \tilde{\phi}_0(y, \gamma)
\]

\[
\frac{\partial \tilde{\phi}_1(y, \gamma)}{\partial y} = \bar{\alpha}_s \chi(\gamma) \tilde{\phi}_1(y, \gamma)
\]

\[-2\pi i \bar{\alpha}_s \int_{c_1-i\infty}^{c_1+i\infty} \frac{d\gamma_1}{2\pi i} \int_{c_2-i\infty}^{c_2+i\infty} \frac{d\gamma_2}{2\pi i} \delta(\gamma - \gamma_1 - \gamma_2) \tilde{\phi}_0(y, \gamma_1) \tilde{\phi}_0(y, \gamma_2)\]

The solution is obtained iteratively,

\[
\phi_0(y, k^2_{\perp}) = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} \left( \frac{4k^2_{\perp}}{Q^2_0} \right)^{-\gamma} C_0(\gamma) \exp[\bar{\alpha}_s \chi(\gamma)y]
\]

\[
\tilde{\phi}_1(y, \gamma) = \int \frac{d\gamma_1}{2\pi i} \frac{d\gamma_2}{2\pi i} 2\pi i \delta(\gamma - \gamma_1 - \gamma_2) C_0(\gamma_1) C_0(\gamma_2) \exp(\bar{\alpha}_s y \chi(\gamma)) - \exp(\bar{\alpha}_s y \chi(\gamma_1) + \bar{\alpha}_s y \chi(\gamma_2)) \frac{\chi(\gamma_1) + \chi(\gamma_2) - \chi(\gamma)}{\chi(\gamma)}
\]
Mellin representation of $\gamma^*$ cross sections

The $\gamma_T^*$ and $\gamma_L^*$ cross sections may be also expanded

$$\sigma_{T,L}^{\gamma^* A} = \sum_{i=0}^{\infty} \sigma_{T,L}^{(i) \gamma^* A}$$

$$\sigma_{T,L}^{(i) \gamma^* A}(x, Q^2) = \sigma_0 \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} \left( \frac{Q_0^2}{Q^2} \right)^\gamma \tilde{H}_{T,L}(\gamma) \frac{\Gamma(1-\gamma)}{2^{2\gamma-1} \Gamma(\gamma)} \tilde{\phi}_i(y, \gamma)$$

The Mellin fundamental strip is located in $-3/4 < \Re c < 0$. Functions $\tilde{H}_{T,L}$ are Mellin transforms of the photon wave functions.

The linear (BFKL) component

$$\sigma_{T,L}^{(0) \gamma^* A} = \sigma_0 \int_{-1/2-i\infty}^{-1/2+i\infty} \frac{d\gamma}{2\pi i} \left( \frac{Q_0^2}{Q^2} \right)^\gamma \tilde{H}_{T,L}(\gamma) \Gamma(-\gamma) e^{\bar{\alpha}_s y \chi(\gamma)}$$

The first nonlinear correction:

$$\sigma_{T,L}^{(1) \gamma^* A} = \sigma_0 \int_c d\gamma \left( \frac{Q_0^2}{Q^2} \right)^\gamma \int_{c_1} d\gamma_1 \tilde{H}_{T,L}(\gamma) \frac{\Gamma(1-\gamma) B(\gamma_1, \gamma - \gamma_1)}{2 \gamma_1 (\gamma - \gamma_1)}$$

$$\times \frac{\exp(\bar{\alpha}_s y \chi(\gamma)) - \exp(\bar{\alpha}_s y \chi(\gamma_1) + \bar{\alpha}_s y \chi(\gamma - \gamma_1))}{-\chi(\gamma) + \chi(\gamma_1) + \chi(\gamma - \gamma_1)}$$

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The double Mellin form of $\gamma^*$ cross sections

- The cross sections

$$
\sigma_{T,L}^{(1)}{\gamma^* A} = \sigma_0 \int_c \frac{d\gamma}{2\pi i} \left( \frac{Q_0^2}{Q^2} \right)^\gamma \int_{c_1} \frac{d\gamma_1}{2\pi i} \tilde{H}_{T,L}(\gamma) \frac{\Gamma(1-\gamma)B(\gamma_1,\gamma-\gamma_1)}{2\gamma_1(\gamma-\gamma_1)} 
\times \frac{\exp(\tilde{\alpha_s}y\chi(\gamma)) - \exp(\tilde{\alpha_s}y\chi(\gamma_1) + \tilde{\alpha_s}y\chi(\gamma-\gamma_1))}{-\chi(\gamma) + \chi(\gamma_1) + \chi(\gamma-\gamma_1)}
$$

have isolated singularities in the double Mellin plane $(\gamma, \gamma_1)$ in the right half-planes.

- The first line contributes with poles while essential singularities appear in the second one.

- Singularities from $\Gamma(\gamma_i)$ and $\chi(\gamma_i)$ are found for integer $\gamma$ and $\gamma_1$, but also for integer $\gamma - \gamma_1$.

- Recall that the eigenvalue of the BFKL kernel $\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma)$ has simple poles for all integer values of $\gamma$. In our convention the twist poles correspond to singularities in positive integer values of $\gamma$. 
Twist decomposition of $\gamma^*$ cross sections

- Decomposition strategy: represent the cross sections as sums of contributions coming from singularities in $\gamma$ at fixed $\gamma_1$

- The integrations around $\gamma$-singularities performed numerically. The resulting expression has terms with $\gamma_1$ dependence factored out from $Q^2$ dependence and terms of the form of $\int \frac{d\gamma_1}{2\pi i} \left( \frac{Q_0^2}{Q^2} \right)^{\gamma_1} G(\gamma_1)$ where $G(\gamma_1)$ has singularities at integer $\gamma_1$
The leading twist shadowing in BK in proton $F_2$ (left) and $F_L$ (right)

$Q^2 = 5 \text{ GeV}^2$, twist 2 only

The curves: **BFKL + BK**, **BFKL**, **BK correction**

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**Note:** the calculations are performed for the BFKL term adjusted to data. The plots show the strength of the BK effects at twist 2 assuming a single iteration of the Triple Pomeron Vertex.
$Q^2 = 5 \text{ GeV}^2$, the ratio of higher twist contributions to the BFKL+BK twist-2 contribution

The curves: $\text{BFKL + BK}$, $\text{BFKL}$, $\text{BK correction}$

The BFKL/BK higher twist corrections are found to be a small ($F_2$) or moderate ($F_L$) fraction of the leading twist contributions.
Non-linear effects in collinear evolution

BK equation for unintegrated gluon density $f(x, k^2)$ (with kernels $K_1$ and $K_2$)

$$\frac{xdf(x, k^2)}{dx} = \alpha_s K_1[f](x, k^2) - \alpha_s^2 K_2[f \otimes f](x, k^2)$$

may be transformed at DLA to GLR-like equation for $G(x, Q^2) \equiv xg(x, Q^2)$

$$\frac{\mu^2 dG(x, \mu^2)}{d\mu^2} = \alpha_s P_1[G](x, \mu^2) - \frac{\alpha_s^2}{\mu^2 R^2} P_2[G \otimes G](x, \mu^2)$$

The nonlinear term $\sim Q_s^2 / \mu^2$ affects evolution between $Q_0^2$ and $Q^2$ mostly for $\mu^2 \sim Q_0^2$

Represent $G(x, \mu^2)$ as $G(x, \mu^2) = h(x, \mu^2) G_{\text{linear}}(x, \mu^2)$, solve non-linear equation. Neglecting $x$-dependence:

$$\frac{h(Q^2) - 1}{h(Q^2)} = -c \left[ \frac{G_{\text{linear}}(Q_0^2)}{R^2 Q_0^2} - \frac{G_{\text{linear}}(Q^2)}{R^2 Q^2} \right] \equiv \frac{\delta G}{G_{\text{linear}}}$$

At $Q^2 \to \infty$: $\delta G / G_{\text{linear}} \sim -c [ G_{\text{linear}}(Q_0^2) / Q_0^2 ]$ — does not vanish.

The effects on the shape of $G(Q^2)$ are, however, strongest for $Q^2 \to Q_0^2$. 

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The three evolution regimes

$log(Q^2)$

Probe : $Q^2$

Input : $Q^0$

Probe : $Q^2$

DGLAP

"GLR"

Non − DGLAP

Nonlinear corrections

$Q_{sat}^2$

$log(1/x)$

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Conclusions and outlook

- We have performed the twist decomposition of the proton structure functions at small $x$ from the LL($1/x$) Balitsky–Kovchegov equation, assuming a single iteration of the Triple Pomeron Vertex.

- The non-linear (saturation) corrections enter strongly at twist 2, the unitarization is driven by the twist 2 contributions.

- The BK introduces small higher twist corrections in $F_2$ and moderate corrections in $F_L$.

- We have obtained a GLR-like equation from BK for the gluon density. The non-linear corrections in the evolution may affect precision DIS data.
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THANKS!