Twist decomposition of non-linear effects in the Balitsky-Kovchegov evolution

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The goal and the outline

- The goal: we want to use information provided by the low-x BFKL/BK framework to input the collinear framework:

  $K_T \rightarrow$ collinear

  BFKL $\rightarrow$ DGLAP + Higher Twists

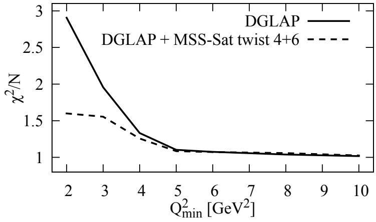
  BK $\rightarrow$ GLR

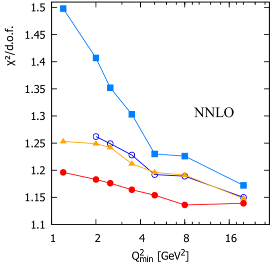
- Outline: use the Mellin moment space conjugate to scale to isolate contributions of subsequent twist operators in BFKL / BK. Trace effects of nonlinearity at all twists

Puzzle: problems of DGLAP in precision HERA data at low x and moderate Q^2

- The final, precise HERA data show problems of DGLAP fits of DIS and DDIS when $Q^2 < 5$ GeV^2 data are included fits, problems at small x
- Explanations proposed:

1. Higher Twist corrections [LM, M. Sadzikowski, W. Słomiński, 2012, L. Harland–Lang et al., 2016; I. Abt et al. 2016; LM, M. Sadzikowski, W. Słomiński, K. Wichman, 2017]
   Expectations: twist 4 / twist 2 $\sim (Q_0^2/Q^2)(x_0/x)^\lambda$

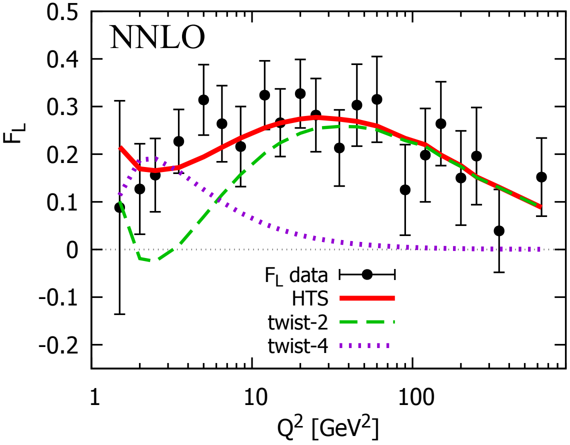




2. Small x resummation beyond DGLAP [R. Ball et. al, 2017]

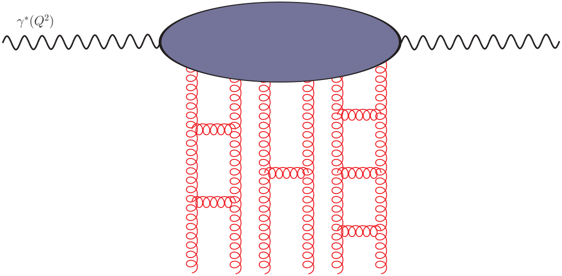
Possible Higher Twist effects in F_L

Somewhat surprising finding from DGLAP + Higher Twist fits: small corrections in F_2, significant and **positive** corrections in F_L [arXiv:1707.05992]



Higher Twist effects from saturation models

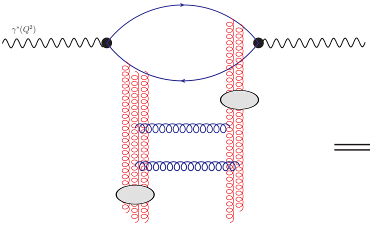
- Golec-Biernat–Wüsthoff or Bartels–Golec-Biernat–Kowalski saturation models assume multiple independent (eikonal) scattering: $\sigma(x, Q^2) = \sigma_0 \left[1 - \exp(-\sigma_1(x, Q^2)/\sigma_0)\right]$
- HT contributions found to increase quickly with decreasing x, as $\sigma^{(\text{twist}=2n)} \sim \sigma_1^n$
- This is, however a model that does not fully agree with QCD-based analyzes concerning the HT components.



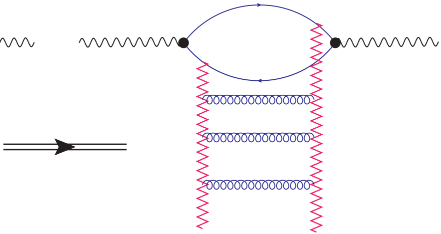
Independent multiple scattering
— eikonal approximation —
GBW saturation model

Higher Twist effects from LL BFKL equation

- QCD: HT from BFKL equation [LM, M. Sadzikowski, arXiv:1411.7774] — at LL($1/x$) the HT contributions found to decrease with decreasing x in the asymptotic regime.
- The reason: gluon Reggeization binds t-channel gluons that couple to color dipole produced by γ^* into two Reggeized gluons, that span a single gluon ladder dominated by twist 2 contribution.



BFKL bootstrap — gluon Reggeization



BFKL equation — two interacting Reggeized gluons

The Higher Twist puzzle

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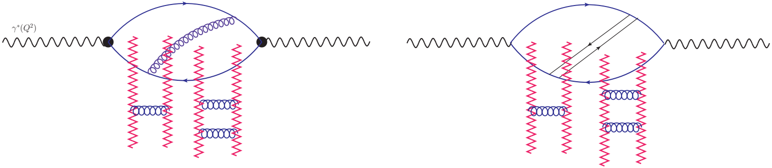
A possible solution: coupling of two BFKL ladders to a splitted γ^* dipole:

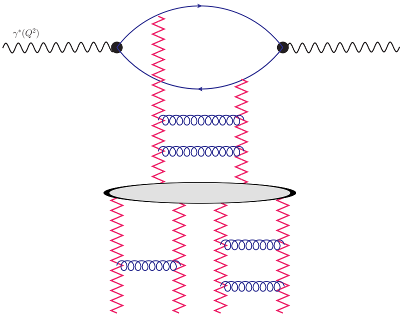


To be consistent, one should include the BFKL evolution both in the dipole wave function and in the ladders. At LL($1/x$): one iteration of nonlinearity in the BK equation

Higher Twists from LL Baltisky–Kovchegov (BK) equation

Triple Pomeron Vertex allows for a transition from single BFKL ladder to two and more ladders, that carry the HT contributions with the strongest enhancement due to evolution.

This mechanism should provide the most reliable estimate of HT effects in the proton structure in the LL($1/x$) approximation in QCD.



Triple Pomeron (BK) vertex — transition to 4 Reggeized gluons

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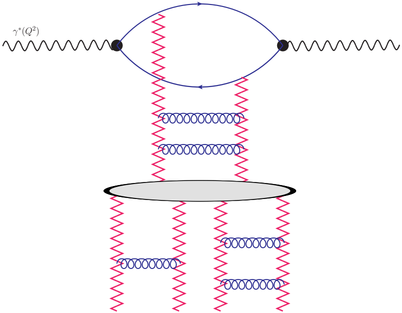


Triple Pomeron (BK) vertex —
transition to 4 Reggeized gluons

Note however: the BK Triple Pomeron Vertex vanishes at LL(Q^2)
[J. Bartels, K. Kutak, 2007]

Calculational method

- The HT determinations are performed in the space of Mellin moments conjugate to the hard scale.
- The total cross sections related to F_2 and F_L structure functions may be decomposed into twist components by isolating the singularities in the Mellin plane, that at small α_s lead to terms with the canonical Q^2 scalling, $(Q^2)^{-n}$.
- For GBW saturation model one finds simple poles in the Mellin plane for integer n.
- For LL BFKL equation essential singularities appear at integer n.
- For LL BK equation there are multiple ladders, that lead to multiple Mellin variables, and convolutions of the Mellin integrals. The convoluted expressions have essential singularities in all Mellin variables.
- Present results on HT from BK: one Triple Pomeron Vertex included, leading to two independent Mellin variables. Enough to estimate the leading twist 4 contribution.

Expansion of Balitsky–Kovchegov equation in nonlinearity

The basic object: impact parameter density of dipole scattering amplitude $N(y, \mathrm{r})$, which is related to unintegrated and collinear gluon densities by linear integral transformations. It is convenient to use $\phi(y, k^2)$ instead

$$\phi(y, k_\perp^2) = \int \frac{d^2 r}{2\pi} e^{-i\mathrm{k}_\perp \cdot \mathrm{r}} \frac{N(y, \mathrm{r})}{r^2}.$$

The Balitsky–Kovchegov equation for $\phi(y, k_\perp^2)$

$$\frac{\partial \phi(y, k_\perp^2)}{\partial y} = \bar{\alpha}_s \int_0^\infty \frac{dq_\perp^2}{q_\perp^2} \left\{ \frac{q_\perp^2 \phi(y, q_\perp^2) - k_\perp^2 \phi(y, k_\perp^2)}{|q_\perp^2 - k_\perp^2|} + \frac{k_\perp^2 \phi(y, k_\perp^2)}{\sqrt{4q_\perp^4 + k_\perp^2}} \right\} - \bar{\alpha}_s \phi^2(y, k_\perp^2)$$

The solutio n in terms of power series of in nonlinearity (for a large nucleus, this is an expansion in $\zeta = A^{1/3}$)
[Yu. Kovchegov, 1999]:

$$\phi \; = \; \zeta \left(\phi_0 + \zeta \phi_1 + \zeta^2 \phi_2 + \ldots \right) \; = \; \zeta \sum_{n=0}^{\infty} \zeta^n \phi_n$$

The order n is equal to number of Triple Pomeron Vertices. For $n = 0$ the linear — i.e. the BFKL equation is reproduced, for $n = 1$ the $1 \to 2$ BFKL ladders transition is described.

We solve the equation directly in the Mellin space:

$$\begin{aligned}
\frac{\partial\tilde{\phi}_0(y,\gamma)}{\partial y} &= \bar{\alpha}_s\chi(\gamma)\tilde{\phi}_0(y,\gamma) \\
\frac{\partial\tilde{\phi}_1(y,\gamma)}{\partial y} &= \bar{\alpha}_s\chi(\gamma)\tilde{\phi}_1(y,\gamma) \\
&\quad -2\pi i\,\bar{\alpha}_s\int_{c_1-i\infty}^{c_1+i\infty}\frac{d\gamma_1}{2\pi i}\int_{c_2-i\infty}^{c_2+i\infty}\frac{d\gamma_2}{2\pi i}\delta(\gamma-\gamma_1-\gamma_2)\tilde{\phi}_0(y,\gamma_1)\tilde{\phi}_0(y,\gamma_2)
\end{aligned}$$

The solution is obtained iteratively,

$$\phi_0(y,k_\perp^2) = \int_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}\left(\frac{4k_\perp^2}{Q_0^2}\right)^{-\gamma} C_0(\gamma)\exp[\bar{\alpha}_s\chi(\gamma)y]$$

$$\begin{aligned}
\tilde{\phi}_1(y,\gamma) &= \int\frac{d\gamma_1}{2\pi i}\frac{d\gamma_2}{2\pi i}2\pi i\delta(\gamma-\gamma_1-\gamma_2)C_0(\gamma_1)C_0(\gamma_2) \\
&\quad \frac{\exp\left(\bar{\alpha}_s y\chi(\gamma)\right)-\exp\left(\bar{\alpha}_s y\chi(\gamma_1)+\bar{\alpha}_s y\chi(\gamma_2)\right)}{\chi(\gamma_1)+\chi(\gamma_2)-\chi(\gamma)}
\end{aligned}$$

Mellin representation of γ^* cross sections

The γ_T^* and γ_L^* cross sections may be also expanded $\sigma_{T,L}^{\gamma^* A} = \sum_{i=0}^{\infty} \sigma_{T,L}^{(i)\,\gamma^* A}$

$$\sigma_{T,L}^{(i)\,\gamma^* A}(x, Q^2) = \sigma_0 \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{Q_0^2}{Q^2}\right)^{\gamma} \tilde{H}_{T,L}(\gamma) \frac{\Gamma(1-\gamma)}{2^{2\gamma-1}\Gamma(\gamma)} \tilde{\phi}_i(y,\gamma)$$

The Mellin fundamental strip is located in $-3/4 < \Re c < 0$. Functions $\tilde{H}_{T,L}$ are Mellin transforms of the photon wave functions.
The linear (BFKL) component

$$\sigma_{T,L}^{(0)\,\gamma^* A} = \sigma_0 \int_{-1/2-i\infty}^{-1/2+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{Q_0^2}{Q^2}\right)^{\gamma} \tilde{H}_{T,L}(\gamma)\Gamma(-\gamma) e^{\bar{\alpha}_s y \chi(\gamma)},$$

The first nonlinear correction:

$$\begin{aligned}
\sigma_{T,L}^{(1)\,\gamma^* A} &= \sigma_0 \int_c \frac{d\gamma}{2\pi i} \left(\frac{Q_0^2}{Q^2}\right)^{\gamma} \int_{c_1} \frac{d\gamma_1}{2\pi i} \tilde{H}_{T,L}(\gamma) \frac{\Gamma(1-\gamma)B(\gamma_1, \gamma-\gamma_1)}{2\gamma_1(\gamma-\gamma_1)} \\
&\times \frac{\exp(\bar{\alpha}_s y \chi(\gamma)) - \exp(\bar{\alpha}_s y \chi(\gamma_1) + \bar{\alpha}_s y \chi(\gamma-\gamma_1))}{-\chi(\gamma) + \chi(\gamma_1) + \chi(\gamma-\gamma_1)}
\end{aligned}$$

The double Mellin form of γ^* cross sections

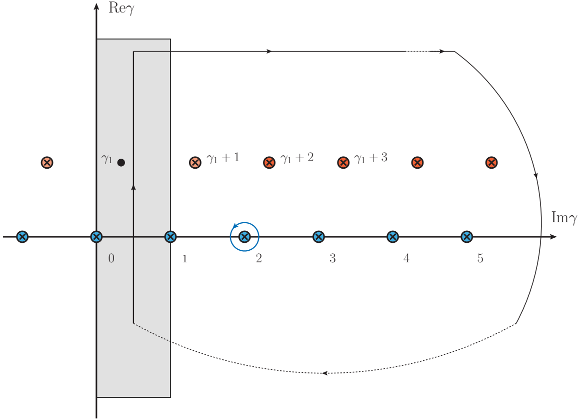
- The cross sections

$$\sigma^{(1)\,\gamma^* A}_{T,L} = \sigma_0 \int_c \frac{d\gamma}{2\pi i}\left(\frac{Q_0^2}{Q^2}\right)^{\gamma}\int_{c_1}\frac{d\gamma_1}{2\pi i}\tilde{H}_{T,L}(\gamma)\frac{\Gamma(1-\gamma)B(\gamma_1,\gamma-\gamma_1)}{2\gamma_1(\gamma-\gamma_1)}$$
$$\times \quad \frac{\exp(\bar{\alpha}_s y\chi(\gamma)) - \exp(\bar{\alpha}_s y\chi(\gamma_1)+\bar{\alpha}_s y\chi(\gamma-\gamma_1))}{-\chi(\gamma)+\chi(\gamma_1)+\chi(\gamma-\gamma_1)}$$

  have isolated singularities in the double Mellin plane (γ, γ_1) in the right half-planes
- The first line contributes with poles while essential singularities appear in the second one
- Singularities from $\Gamma(\gamma_i)$ and $\chi(\gamma_i)$ are found for integer γ and γ_1, but also for integer $\gamma - \gamma_1$
- Recall that the eigenvalue of the BFKL kernel $\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$ has simple poles for all integer values of γ. In our convention the twist poles correspond to singularities in positive integer values of γ.

Twist decomposition of γ^* cross sections

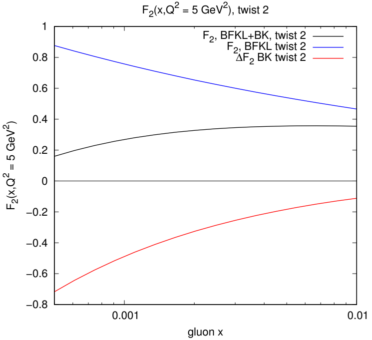
- Decomposition strategy: represent the cross sections as sums of contributions coming from singularities in γ at fixed γ_1

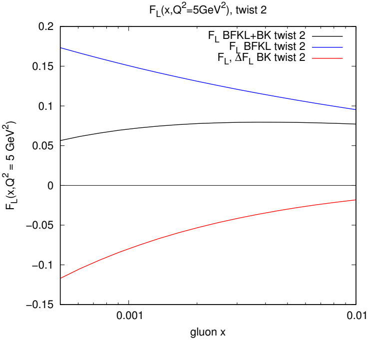


- The integrations around γ-singularities performed numerically. The resulting expression has terms with γ_1 dependence factored out from Q^2 dependence and terms of the form of $\int \frac{d\gamma_1}{2\pi i}(Q_0^2/Q^2)^{\gamma_1} G(\gamma_1)$ where $G(\gamma_1)$ has singularities at integer γ_1

$Q^2 = 5$ GeV^2, twist 2 only

The curves: **BFKL + BK**, **BFKL**, **BK correction**

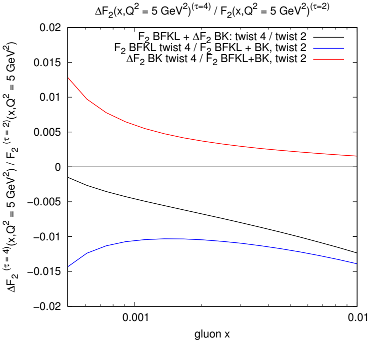


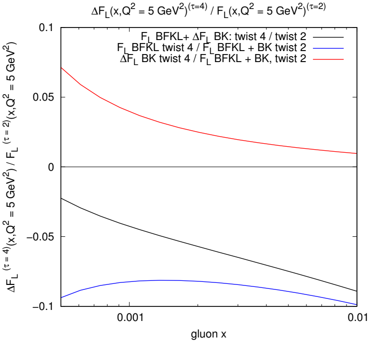


Note: the calculations are performed for the BFKL term adjusted to data. The plots show the strength of the BK effects at twist 2 assuming a single iteration of the Triple Pomeron Vertex.

$Q^2 = 5$ GeV^2, the ratio of higher twist contributions to the BFKL+BK twist-2 contribution

The curves: **BFKL + BK**, **BFKL**, **BK correction**





The BFKL/BK higher twist corrections are found to be a small (F_2) or moderate (F_L) fraction of the leading twist contributions.

Non-linear effects in collinear evolution

BK equation for unintegrated gluon density $f(x,k^2)$ (with kernels K_1 and K_2)

$$\frac{xdf(x,k^2)}{dx} = \alpha_s K_1[f](x,k^2) - \alpha_s^2 K_2[f \otimes f](x,k^2)$$

may be transformed at DLA to GLR-like equation for $G(x,Q^2) \equiv xg(x,Q^2)$

$$\frac{\mu^2\, dG(x,\mu^2)}{d\mu^2} = \alpha_s P_1[G](x,\mu^2) - \frac{\alpha_s^2}{\mu^2 R^2} P_2[G \otimes G](x,\mu^2)$$

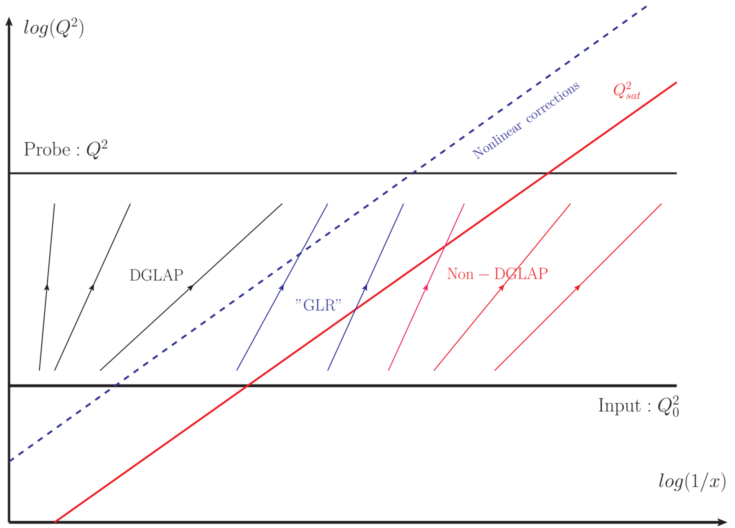
The nonlinear term $\sim Q_s^2/\mu^2$ affects evolution between Q_0^2 and Q^2 mostly for $\mu^2 \sim Q_0^2$

Represent $G(x,\mu^2)$ as $G(x,\mu^2) = h(x,\mu^2) G_{\mathrm{linear}}(x,\mu^2)$, solve non-linear equation. Neglecting x-dependence:

$$\frac{h(Q^2)-1}{h(Q^2)} = -c\left[\frac{G_{\mathrm{linear}}(Q_0^2)}{R^2 Q_0^2} - \frac{G_{\mathrm{linear}}(Q^2)}{R^2 Q^2}\right] \equiv \frac{\delta G}{G_{\mathrm{linear}}}$$

At $Q^2 \to \infty$: $\delta G/G_{\mathrm{linear}} \sim -c[G_{\mathrm{linear}}(Q_0^2)/Q_0^2]$ — does not vanish.
The effects on the shape of $G(Q^2)$ are, however, strongest for $Q^2 \to Q_0^2$.

The three evolution regimes



Conclusions and outlook

- We have performed the twist decomposition of the proton structure functions at small x from the LL$(1/x)$ Balitsky–Kovchegov equation, assuming a single iteration of the Triple Pomeron Vertex
- The non-linear (saturation) corrections enter strongly at twist 2, the unitarization is driven by the twist 2 contributions
- The BK introduces small higher twist corrections in F_2 and moderate corrections in F_L
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THANKS!