Exclusive quarkonium production at next-to-leading order in the Color Glass Condensate framework

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Exclusive vector meson production in deep inelastic scattering

- \( \gamma^*_\lambda + A \rightarrow V_\lambda + A \)

- Requires an exchange of at least two gluons
  \( \Rightarrow \) Highly sensitive to the gluon structure of the nucleus

- Momentum transfer \( \Delta \) can be measured:
  - Conjugate to the impact parameter \( b \)
  \( \Rightarrow \) Access to the spatial distribution of gluons

- Heavy vector mesons (quarkonia) \( V = J/\psi, \Upsilon \ldots \)

- \( \lambda = \) Longitudinal, Transverse
Quarkonium production at the leading order in the dipole picture

- Factorization in the high-energy limit:

\[ -iA^\lambda = 2 \int d^2b d^2r \frac{dz}{4\pi} e^{-i(b + (\frac{1}{2} - z)r)} \cdot \Delta \psi^{q\bar{q}}(r, z) N(r, b, Y) \psi^{q\bar{q}}(r, z) \]

- \( \psi^{q\bar{q}} \): Photon light-front wave function
- \( N \): Dipole-target scattering amplitude
- \( \psi^V \): Quarkonium light-front wave function
Exclusive quarkonium production at NLO

Invariant amplitude for exclusive quarkonium production

\[ -i A_{t=0} = 2 \int d^2x_0 \, d^2x_1 \int \frac{dz_0 \, dz_1}{(4\pi)^2} \delta(z_0 + z_1 - 1) \psi_{q\bar{q}} N_{01} \psi_{\gamma^* V}^{q\bar{q}*} \]

\[ + 2 \int d^2x_0 \, d^2x_1 \, d^2x_2 \int \frac{dz_0 \, dz_1 \, dz_2}{(4\pi)^2} \delta(z_0 + z_1 + z_2 - 1) \psi_{\gamma^* g} N_{012} \psi_{V}^{q\bar{q}g*} \]

- Also contribution from the $q\bar{q}g$ state
- Need the wave functions for $q\bar{q}$ state at NLO
  - Photon: Beuf, Lappi, Paatelainen, 2103.14549, 2112.03158, 2204.02486
  - Quarkonium: Nonperturbative
- Consider only $t = -\Delta^2 = 0$ case:
  - No need to model $b$-dependence of $N_{01}$
Nonrelativistic expansion for quarkonium

- Non-Relativistic QCD (NRQCD): Parametrically \( v \sim \alpha_s(vM_V) > \alpha_s(M_V) \)

\[ \Rightarrow \text{Expansion in } v \text{ and } \alpha_s: 1 > \alpha_s > v^2 > \ldots \]

Nonrelativistic expansion

\[ \psi^n_V = \sum_{m,k} C^k_{n\leftarrow m} \int_0^1 \frac{dz'}{4\pi} \left( \frac{1}{m_q} \nabla \right)^k \phi^m(r = 0, z') \]

- \( \phi^m = \text{leading-order wave function for Fock state } m \)
- \( \alpha_s \text{ corrections included in } C^k_{n\leftarrow m} \)
- Relativistic corrections go as \( v^k \) in the index \( k \)
NLO calculation in the nonrelativistic limit

- Nonrelativistic limit: Leading-order wave function $\phi^{q\bar{q}}(\vec{k}) \sim (2\pi)^3 \delta^3(\vec{k})$

- Quarkonium Fock states: $|V\rangle = \psi^{q\bar{q}}_V |q\bar{q}\rangle + \psi^{q\bar{g}}_V |q\bar{g}\rangle + \text{higher orders in NRQCD}$

- Only include the leading terms $O(\nu^0)$ in heavy quark velocity:

\[
\psi^{q\bar{q}}_V = C^0_{q\bar{q}\leftarrow q\bar{q}} \int_0^1 \frac{dz'}{4\pi} \phi^{q\bar{q}}(r = 0, z') \quad \psi^{q\bar{g}}_V = C^0_{q\bar{g}\leftarrow q\bar{q}} \int_0^1 \frac{dz'}{4\pi} \phi^{q\bar{g}}(r = 0, z')
\]

- $C^0_{q\bar{q}\leftarrow q\bar{q}}, C^0_{q\bar{g}\leftarrow q\bar{q}}$ at $O(\alpha_s)$: Escobedo, Lappi, 1911.01136
Cancellation of divergences

- UV divergences between the $q\bar{q}$ and $q\bar{q}g$ parts of the calculation cancel

- IR divergences cancel when we take into account:
  1. Renormalization of the leading-order wave function $\phi^{q\bar{q}}(\vec{r'} = 0)$
     - Can be related to the dimensionally regularized wave function Escobedo, Lappi, 1911.01136
     \[
     \int \frac{dz'}{4\pi} \phi^{q\bar{q}} = \int \frac{dz'}{4\pi} \phi_{DR}^{q\bar{q}} \times \left[ \frac{1}{2\pi} \frac{1}{\alpha} \right], \alpha = \text{gluon IR cutoff}
     \]
  2. The rapidity dependence of the dipole amplitude which can be described in terms of the Balitsky-Kovchegov equation

$\Rightarrow$ The total production amplitude is finite and can be numerically evaluated

- Longitudinal NLO production: Mäntysaari, J.P, 2104.02349

- Transverse NLO production: Mäntysaari, J.P, 2204.14031
\[-i A^T = \int \frac{dz'}{4\pi} \phi_D^{q\bar{q}} \times \sqrt{\frac{N_c}{2} e_{ef} m_q} \cdot 2 \int d^2 x_{01} \int d^2 b \left\{ \mathcal{K}^{LO}_{qq} (Y_0) + \frac{\alpha_s C_F}{2\pi} \mathcal{K}^{NLO}_{qq,\psi} (Y_{\text{dip}}) + \frac{\alpha_s C_F}{2\pi} \int d^2 x_{20} \int_{z_{\text{min}}}^{1/2} dz_2 \mathcal{K}_{qqg} (Y_{qqg}) \right\} \]

where \( \mathcal{K}^{LO}_{qq} (Y_0) = K_0 (\zeta) N_{01} (Y_0) \), \( \zeta = |x_{01}| \sqrt{\frac{1}{4} Q^2 + m_q^2} \),

\[
\mathcal{K}^{NLO}_{qq,\psi} = \left[ I^T_{\text{V,MS}} \left( \frac{1}{2}, x_{01} \right) + K^T + K_0 (\zeta) \left( -\Omega^T \left( \gamma; \frac{1}{2} \right) + L \left( \gamma; \frac{1}{2} \right) - \frac{\pi^2}{3} + \frac{5}{2} - 3 \ln \left( \frac{m_q |x_{01}|}{2} \right) - 3 \gamma_E \right) \right] N_{01}
\]

and

\[
\mathcal{K}_{qqg} (Y_{qqg}) = 32\pi m_q \left\{ K_1 (2m_q z_2 |x_{20}|) \left[ i x_{20} \left( \mathcal{I}_{(j)}^i \right) \left( (1 - z_2)^2 + z_2^2 \right) - z_2^2 (2z_2 - 1) \hat{\mathcal{I}}_{(j)}^i + \frac{1}{2} \hat{\mathcal{I}}_{(k)}^i \right] \frac{z_2^2 (2z_2 - 1)^2}{(2z_2 + 1)^2} \right\} N_{012}
\]

\[
+ \frac{z_2}{m_q} K_0 (2m_q z_2 |x_{20}|) \left[ -\frac{1 + 2z_2 \mathcal{I}_{(j)}^i}{2} + \frac{1 + 2z_2 \mathcal{I}_{(k)}^i}{2} - 2(1 - 2z_2) z_2 \mathcal{J}_{(j)} - 4m_q^2 z_2^2 \mathcal{I}_{(j)} \right] N_{012}
\]

\[
- ( (1 - z_2)^2 + z_2^2 ) \frac{1}{8\pi^2 m_q z_2 |x_{20}|^2} K_0 (\zeta) e^{-x_{20}^2 / (x_{10}^2 e^{\gamma_E})} N_{01}
\].
Including relativistic corrections at LO

- Keep terms up to $v^2$ in the nonrelativistic expansion

- Nonperturbative constants related to NRQCD matrix elements \cite{Lappi, Mäntysaari, JP, 2006.02830}

- Relativistic corrections $v^2\alpha_s$ to the amplitude:

\[
-i A_{\text{rel}}^T = \sqrt{\frac{N_c}{2}} \frac{e ef m_q}{\pi} 2 \int d^2x_{01} N_{01}(Y_{\text{dip}}) \frac{1}{\sqrt{m_q}} \frac{\nabla^2 \phi_{\text{RF}}(0)}{6m_q^2} \left[ \frac{1}{2} m_q^2 x_{01}^2 K_0(\zeta) - \frac{x_{01}^2 Q^2}{8\zeta} K_1(\zeta) - \frac{1}{2} \zeta K_1(\zeta) \right]
\]

where the derivative of the rest-frame wave function $\phi_{\text{RF}}$ can be written as

\[
\nabla^2 \phi_{\text{RF}}(0) = -\langle \vec{q}^2 \rangle \sqrt{\frac{1}{2N_c}} \sqrt{\langle O_1 \rangle} v
\]

⇒ first higher-order corrections $v^0\alpha_s$ and $v^2\alpha_s$ included in the production amplitude
Initial condition fit for the dipole amplitude at NLO

- Fitted to the HERA structure function data
- LO fit: Lappi, Mäntysaari, 1309.6963
- NLO calculation: needs an NLO fit
- NLO BK: numerically heavy
  - Use different approximations: KCBK, ResumBK, TBK
  - Two starting points for the BK evolution:
    \( Y_{0,BK} = 0.00 \) and \( Y_{0,BK} = 4.61 \)
- \( 3 \times 2 = 6 \) different NLO dipole amplitude fits
- Note: only massless quarks in this fit

\( Q^2 = 2 \text{ GeV}^2 \)
\( Q^2 = 4.5 \text{ GeV}^2 \)
\( Q^2 = 10 \text{ GeV}^2 \)
\( Q^2 = 22 \text{ GeV}^2 \)
\( Q^2 = 45 \text{ GeV}^2 \)
Total $J/\psi$ production – dependence on the photon virtuality $Q^2$

- Nonrelativistic limit
- With $\nu^2$ relativistic corrections

- NLO corrections moderate, get a good description of HERA data
- Relativistic $\nu^2$ corrections important at low $Q^2$
Total $J/\psi$ production – dependence on the center-of-mass energy $W$

**Nonrelativistic limit**

- $Y_{0,BK}, \eta_{0,BK} = 4.61$
- $Y_{0,BK}, \eta_{0,BK} = 0.00$
- $Q^2 = 0.05 \text{ GeV}^2$

$\sigma_{\text{tot}} \text{ [nb]}$

- $p + \gamma^* \to p + J/\psi$
- $W \text{ [GeV]}$
- $Y_{0,BK}, \eta_{0,BK} = 4.61$
- $Y_{0,BK}, \eta_{0,BK} = 0.00$
- $Q^2 = 0.05 \text{ GeV}^2$

$\sigma_{\text{tot}} \text{ [nb]}$

- Good description of the energy dependence
- $Y_{0,BK} = 0.00$: unphysical results at low $W$
Longitudinal-to-transverse ratio for $J/\psi$ production

**Nonrelativistic limit**

$p + \gamma^* \rightarrow p + J/\psi$

\[ Q^2 \text{[GeV]} \]

\[ W = 90 \text{ GeV} \]

- $\sigma_L/\sigma_T$

**With $v^2$ relativistic corrections**

$p + \gamma^* \rightarrow p + J/\psi$

\[ Q^2 \text{[GeV]} \]

\[ W = 90 \text{ GeV} \]

- $\sigma_L/\sigma_T$

- NLO corrections moderate
Total $\Upsilon$ production – dependence on center-of-mass energy $W$

- Relativistic effects small
  - This is expected for $\Upsilon$
- Good agreement with the data

$$\sigma_{\text{tot}} \text{ [nb]} \quad Q^2 = 0.05 \text{ GeV}^2$$

$p + \gamma^* \rightarrow p + \Upsilon$

$W$ [GeV]

$W$ [GeV]
Summary

- We have calculated both longitudinal and transverse quarkonium production at NLO
  \[ \Rightarrow \text{Comparisons with data possible} \]
- Both NLO and \( v^2 \) relativistic corrections numerically important
  \[ \Rightarrow \text{Generally good agreement with the data when both are included} \]
- Can distinguish between different NLO dipole fits
  \[ \Rightarrow \text{At small } W: \text{Dipoles with } Y_{0,BK} = 0.00 \text{ give unphysical results} \]
- Future: Use NLO dipole fits with \textit{massive} quarks
- NLO light vector meson production at large \( Q^2 \): Mäntysaari, JP, 2203.16911
- Important developments: precise measurements expected at ultra-peripheral collisions at the LHC and the future Electron-Ion Collider
Backup
The rapidity in the dipole amplitude is defined as $Y = \log z_2 + \log(\frac{q^+}{P^+})$

- $q^+, P^+$ = longitudinal momenta of the virtual photon and the target

Eikonal approximation: the invariant mass of the $q\bar{q}g$ system has to satisfy $M_{q\bar{q}g}^2 \ll W^2$

$\Rightarrow z_2 > z_{\text{min}} = \frac{P^+}{q^+} = \frac{Q_0^2}{W^2 + Q_0^2 - m_N^2}, Q_0^2 = \text{transverse momentum scale of the target}$

In total, we have three different rapidities in the expression:

- $Y_0 = \text{the initial rapidity}$
- $Y_{q\bar{q}g} = Y_0 + \log \frac{z_2}{z_{\text{min}}}, \text{evolution rapidity in the real contribution}$
- $Y_{\text{dip}} = Y_0 + \log \frac{1}{2z_{\text{min}}}, \text{evolution rapidity in the virtual contribution}$

The amount of evolution in rapidity: $\log 1/2z_{\text{min}} \approx \log \frac{W^2}{2Q_0^2}$

Following Beuf et al. 2007.01645, we choose $Q_0^2 = 1 \text{GeV}^2$ and $Y_0 = 0$
The leading order LO($Y_{\text{dip}}$) result includes the resummation of the large logs $\sim \alpha_s \log 1/x$

NLO = LO($Y_0$) + NLO$_{\text{dip}}$ + NLO$_{q\bar{q}g}$

Here the same dipole amplitude used for both LO and NLO

$\Rightarrow$ NLO − LO($Y_{\text{dip}}$) tells about the largeness of the NLO correction terms
• These results valid at $t = 0$

• Need the $t$-integrated cross section for comparisons with experimental data

• Use the experimental parametrization for $t$ dependence:

\[
\frac{d\sigma}{dt} = e^{-b|t|} \times \frac{d\sigma}{dt} (t = 0)
\]

• $b \approx$ transverse size of the target-meson system

• $b$ taken from a fit to experimental data

ZEUS collaboration, hep-ex/0201043
Calculated in Mäntysaari, J.P, 2203.16911, in the limit $Q^2 \gg M_V^2$

Excellent agreement with the data

$\sigma_L(\gamma^* + p \rightarrow \rho + p)$ [nb]

- $Y_{0,BK}, \eta_{0,BK} = 4.61$
- $Y_{0,BK}, \eta_{0,BK} = 0.00$

$Q^2 = 35.6$ GeV$^2$

$W = 75$ GeV

$\sigma_L(\gamma^* + p \rightarrow \rho + p)$ [nb]

- $Y_{0,BK}, \eta_{0,BK} = 4.61$
- $Y_{0,BK}, \eta_{0,BK} = 0.00$

$W = 75$ GeV

$M_{\rho}^2 + Q^2$ [GeV$^2$]