

Exclusive quarkonium production at next-to-leading order in the Color Glass Condensate framework

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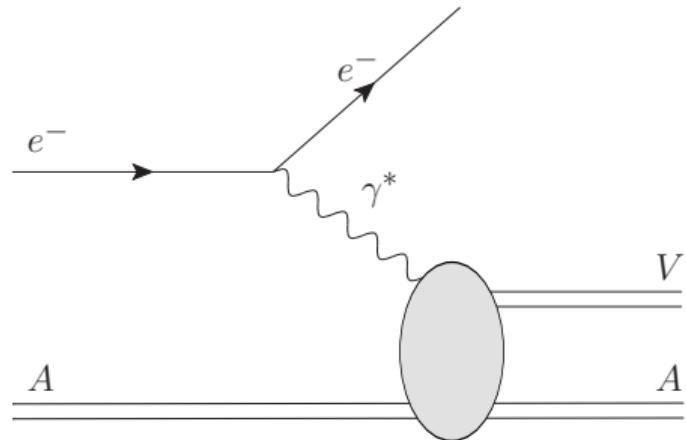


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DIS 2022

Exclusive vector meson production in deep inelastic scattering

- $\gamma_\lambda^* + A \rightarrow V_\lambda + A$
- Requires an exchange of at least two gluons
 - ⇒ Highly sensitive to the gluon structure of the nucleus
- Momentum transfer Δ can be measured:
 - Conjugate to the impact parameter \mathbf{b}
 - ⇒ Access to the spatial distribution of gluons
- Heavy vector mesons (quarkonia) $V = J/\psi, \Upsilon \dots$
- $\lambda = \text{Longitudinal, Transverse}$



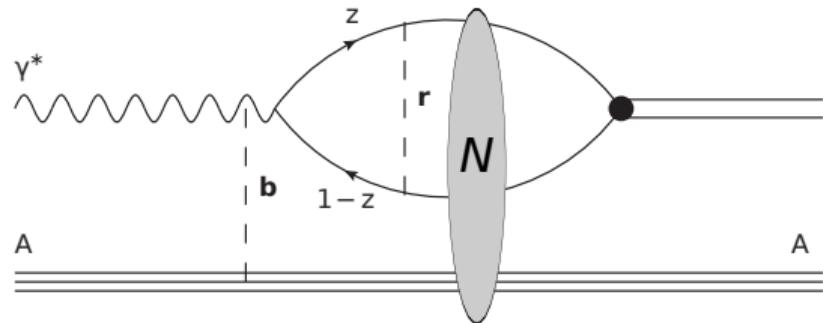
Quarkonium production at the leading order in the dipole picture

- Factorization in the high-energy limit:

Invariant amplitude for exclusive quarkonium production

$$-i\mathcal{A}^\lambda = 2 \int d^2\mathbf{b} d^2\mathbf{r} \frac{dz}{4\pi} e^{-i(\mathbf{b} + (\frac{1}{2} - z)\mathbf{r}) \cdot \Delta} \Psi_{\gamma^*}^{q\bar{q}}(\mathbf{r}, z) \mathcal{N}(\mathbf{r}, \mathbf{b}, Y) \Psi_V^{q\bar{q}*}(\mathbf{r}, z)$$

- $\Psi_{\gamma^*}^{q\bar{q}}$: Photon light-front wave function
- \mathcal{N} : Dipole-target scattering amplitude
- $\Psi_V^{q\bar{q}}$: Quarkonium light-front wave function



Exclusive quarkonium production at NLO

Invariant amplitude for exclusive quarkonium production

$$-i\mathcal{A}_{t=0} = 2 \int d^2\mathbf{x}_0 d^2\mathbf{x}_1 \int \frac{dz_0 dz_1}{(4\pi)} \delta(z_0 + z_1 - 1) \Psi_{\gamma^*}^{q\bar{q}} N_{01} \Psi_V^{q\bar{q}*}$$
$$+ 2 \int d^2\mathbf{x}_0 d^2\mathbf{x}_1 d^2\mathbf{x}_2 \int \frac{dz_0 dz_1 dz_2}{(4\pi)^2} \delta(z_0 + z_1 + z_2 - 1) \Psi_{\gamma^*}^{q\bar{q}g} N_{012} \Psi_V^{q\bar{q}g*}$$

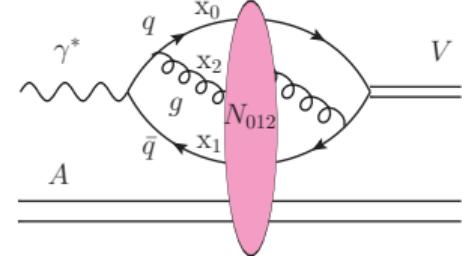
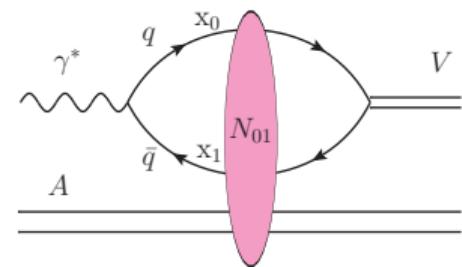
- Also contribution from the $q\bar{q}g$ state
- Need the wave functions for $q\bar{q}$ state at NLO

Photon: [Beuf, Lappi, Paatelainen, 2103.14549](#), [2112.03158](#), [2204.02486](#)

Quarkonium: Nonperturbative

- Consider only $t = -\Delta^2 = 0$ case:

No need to model \mathbf{b} -dependence of N_{01}



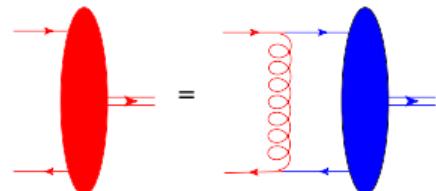
Nonrelativistic expansion for quarkonium

- Non-Relativistic QCD (NRQCD): Parametrically $v \sim \alpha_s(vM_V) > \alpha_s(M_V)$
⇒ Expansion in v and α_s : $1 > \alpha_s > v^2 > \dots$

Nonrelativistic expansion Escobedo, Lappi, 1911.01136

$$\Psi_V^n = \sum_{m,k} \underbrace{C_{n \leftarrow m}^k}_{\text{perturbative corrections}} \underbrace{\int_0^1 \frac{dz'}{4\pi} \left(\frac{1}{m_q} \nabla \right)^k \phi^m(\mathbf{r} = 0, z')}_{\text{nonperturbative constant}}$$

- ϕ^m = leading-order wave function for Fock state m
- α_s corrections included in $C_{n \leftarrow m}^k$
- Relativistic corrections go as v^k in the index k

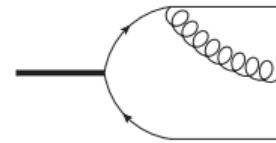
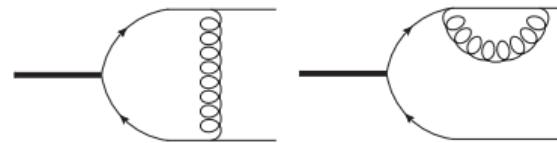


Escobedo, Lappi, 1911.01136

NLO calculation in the nonrelativistic limit

- Nonrelativistic limit: Leading-order wave function $\phi^{q\bar{q}}(\vec{k}) \sim (2\pi)^3 \delta^3(\vec{k})$
- Quarkonium Fock states: $|V\rangle = \Psi_V^{q\bar{q}} |q\bar{q}\rangle + \Psi_V^{q\bar{q}g} |q\bar{q}g\rangle + \text{higher orders in NRQCD}$
- Only include the leading terms $\mathcal{O}(v^0)$ in heavy quark velocity:

$$\Psi_V^{q\bar{q}} = C_{q\bar{q} \leftarrow q\bar{q}}^0 \int_0^1 \frac{dz'}{4\pi} \phi^{q\bar{q}}(\mathbf{r} = 0, z') \quad \Psi_V^{q\bar{q}g} = C_{q\bar{q}g \leftarrow q\bar{q}}^0 \int_0^1 \frac{dz'}{4\pi} \phi^{q\bar{q}}(\mathbf{r} = 0, z')$$



- $C_{q\bar{q} \leftarrow q\bar{q}}^0, C_{q\bar{q}g \leftarrow q\bar{q}}^0$ at $\mathcal{O}(\alpha_s)$: Escobedo, Lappi, 1911.01136

Cancellation of divergences

- UV divergences between the $q\bar{q}$ and $q\bar{q}g$ parts of the calculation cancel
 - IR divergences cancel when we take into account:
 - ➊ Renormalization of the leading-order wave function $\phi^{q\bar{q}}(\vec{r} = 0)$
 - Can be related to the dimensionally regularized wave function [Escobedo, Lappi, 1911.01136](#)
$$\int \frac{dz'}{4\pi} \phi^{q\bar{q}} = \int \frac{dz'}{4\pi} \phi_{\text{DR}}^{q\bar{q}} \times \left[1 - \frac{\alpha_s C_F}{2\pi} \frac{1}{\alpha} \right], \alpha = \text{gluon IR cutoff}$$
 - ➋ The rapidity dependence of the dipole amplitude which can be described in terms of the Balitsky-Kovchegov equation
- ⇒ The total production amplitude is finite and can be numerically evaluated
- Longitudinal NLO production: [Mäntysaari, J.P, 2104.02349](#)
 - Transverse NLO production: [Mäntysaari, J.P, 2204.14031](#)

Final expression (transverse production)

$$-i\mathcal{A}^T = \int \frac{dz'}{4\pi} \phi_{\text{DR}}^{q\bar{q}} \times \sqrt{\frac{N_c}{2}} \frac{ee_f m_q}{\pi} 2 \int d^2 \mathbf{x}_{01} \int d^2 \mathbf{b} \left\{ \mathcal{K}_{q\bar{q}}^{\text{LO}}(Y_0) + \frac{\alpha_s C_F}{2\pi} \mathcal{K}_{q\bar{q},\Psi}^{\text{NLO}}(Y_{\text{dip}}) + \frac{\alpha_s C_F}{2\pi} \int d^2 \mathbf{x}_{20} \int_{z_{\min}}^{1/2} dz_2 \mathcal{K}_{q\bar{q}g}(Y_{q\bar{q}g}) \right\}$$

where $\mathcal{K}_{q\bar{q}}^{\text{LO}}(Y_0) = K_0(\zeta) N_{01}(Y_0)$, $\zeta = |\mathbf{x}_{01}| \sqrt{\frac{1}{4} Q^2 + m_q^2}$,

$$\mathcal{K}_{q\bar{q},\Psi}^{\text{NLO}} = \left[I_{\mathcal{VMS}}^T \left(\frac{1}{2}, \mathbf{x}_{01} \right) + \mathcal{K}^T + K_0(\zeta) \left(-\Omega_{\mathcal{V}}^T \left(\gamma; \frac{1}{2} \right) + L \left(\gamma; \frac{1}{2} \right) - \frac{\pi^2}{3} + \frac{5}{2} - 3 \ln \left(\frac{m_q |\mathbf{x}_{01}|}{2} \right) - 3\gamma_E \right) \right] N_{01}$$

and

$$\begin{aligned} \mathcal{K}_{q\bar{q}g}(Y_{q\bar{q}g}) = & 32\pi m_q \left\{ K_1(2m_q z_2 |\mathbf{x}_{20}|) \frac{i \mathbf{x}_{20}^i}{|\mathbf{x}_{20}|} \left[-\mathcal{I}_{(j)}^i ((1-z_2)^2 + z_2^2) - z_2^2 (2z_2 - 1) \hat{\mathcal{I}}_{(j)}^i + \mathcal{I}_{(k)}^i \frac{1}{2z_2 + 1} + \hat{\mathcal{I}}_{(k)}^i \frac{z_2^2 (2z_2 - 1)^2}{(2z_2 + 1)^2} \right] N_{012} \right. \\ & + \frac{z_2}{m_q} K_0(2m_q z_2 |\mathbf{x}_{20}|) \left[\frac{-1 + 2z_2}{2} \mathcal{I}_{(j)}^{ii} + \frac{1 + 2z_2}{2} \mathcal{I}_{(k)}^{ii} - 2(1 - 2z_2) z_2 \mathcal{J}_{(l)} - 4m_q^2 z_2^2 \mathcal{I}_{(j)}^i \right] N_{012} \\ & \left. - ((1-z_2)^2 + z_2^2) \frac{1}{8\pi^2 m_q z_2 |\mathbf{x}_{20}|^2} K_0(\zeta) e^{-\mathbf{x}_{20}^2 / (\mathbf{x}_{10}^2 e^{\gamma_E})} N_{01} \right\}. \end{aligned}$$

Including relativistic corrections at LO

- Keep terms up to v^2 in the nonrelativistic expansion
- Nonperturbative constants related to NRQCD matrix elements [Lappi, Mäntysaari, JP, 2006.02830](#)
- Relativistic corrections $v^2\alpha_s^0$ to the amplitude:

$$-iA_{\text{rel}}^T = \sqrt{\frac{N_c}{2}} \frac{ee_f m_q}{\pi} 2 \int d^2 \mathbf{x}_{01} N_{01}(Y_{\text{dip}}) \frac{1}{\sqrt{m_q}} \frac{\nabla^2 \phi_{\text{RF}}(0)}{6m_q^2} \left[\frac{1}{2} m_q^2 \mathbf{x}_{01}^2 K_0(\zeta) - \frac{\mathbf{x}_{01}^2 Q^2}{8\zeta} K_1(\zeta) - \frac{1}{2} \zeta K_1(\zeta) \right]$$

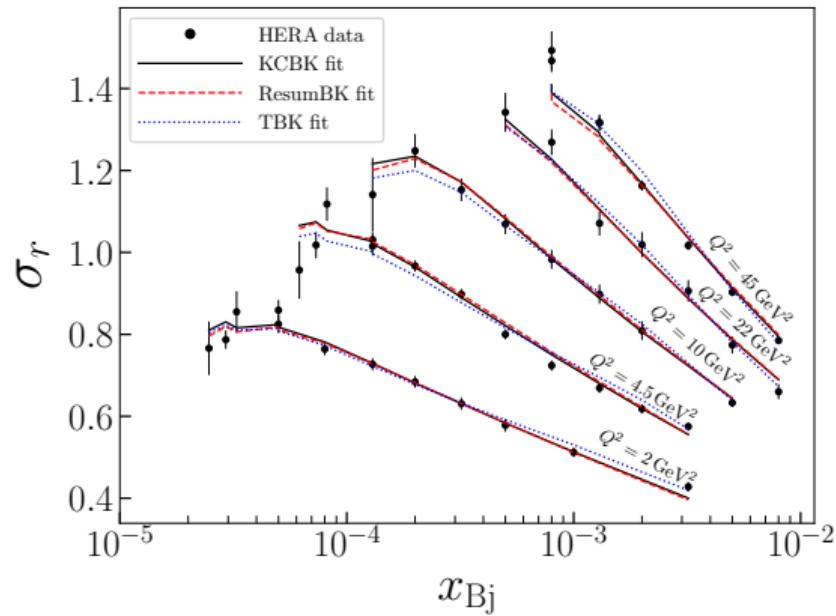
where the derivative of the *rest-frame* wave function ϕ_{RF} can be written as

$$\nabla^2 \phi_{\text{RF}}(0) = -\langle \vec{q}^2 \rangle_V \frac{1}{\sqrt{2N_c}} \sqrt{\langle \mathcal{O}_1 \rangle_V}$$

\Rightarrow first higher-order corrections $v^0\alpha_s$ and $v^2\alpha_s^0$ included in the production amplitude

Initial condition fit for the dipole amplitude at NLO

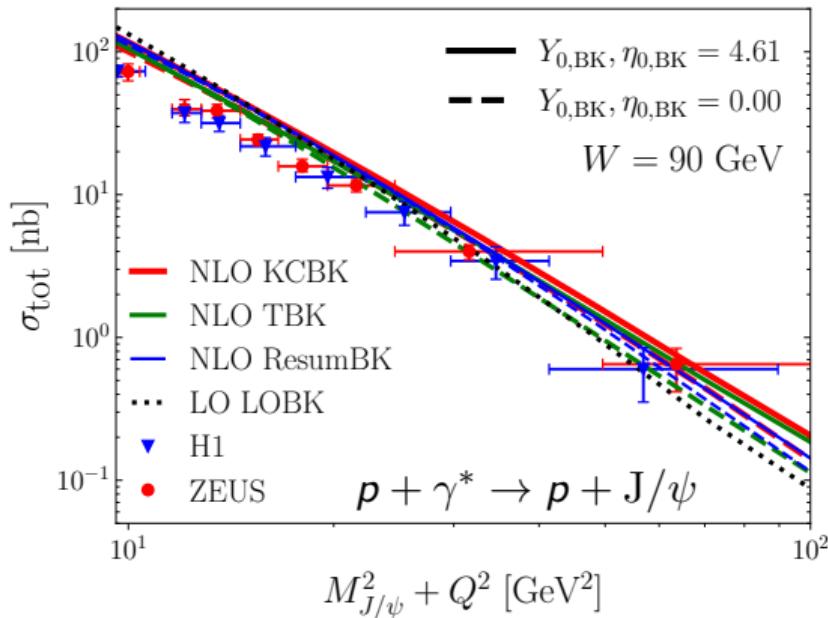
- Fitted to the HERA structure function data
- LO fit: [Lappi, Mäntysaari, 1309.6963](#)
- NLO calculation: needs an NLO fit
- NLO BK: numerically heavy
 - Use different approximations:
KCBK, ResumBK, TBK
 - Two starting points for the BK evolution:
 $Y_{0,\text{BK}} = 0.00$ and $Y_{0,\text{BK}} = 4.61$
- $3 \times 2 = 6$ different NLO dipole amplitude fits
- Note: only massless quarks in this fit



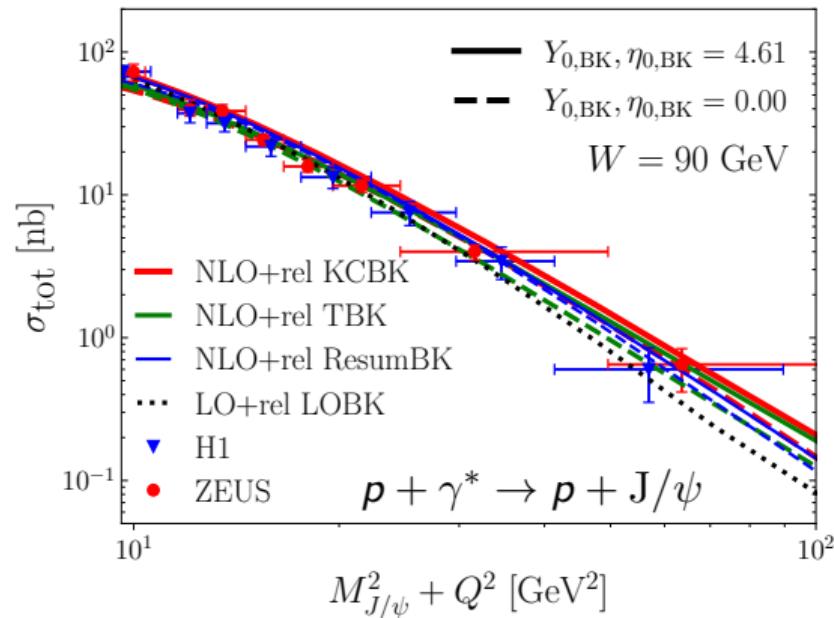
[Hänninen et al., 2007.01645](#)

Total J/ψ production – dependence on the photon virtuality Q^2

Nonrelativistic limit



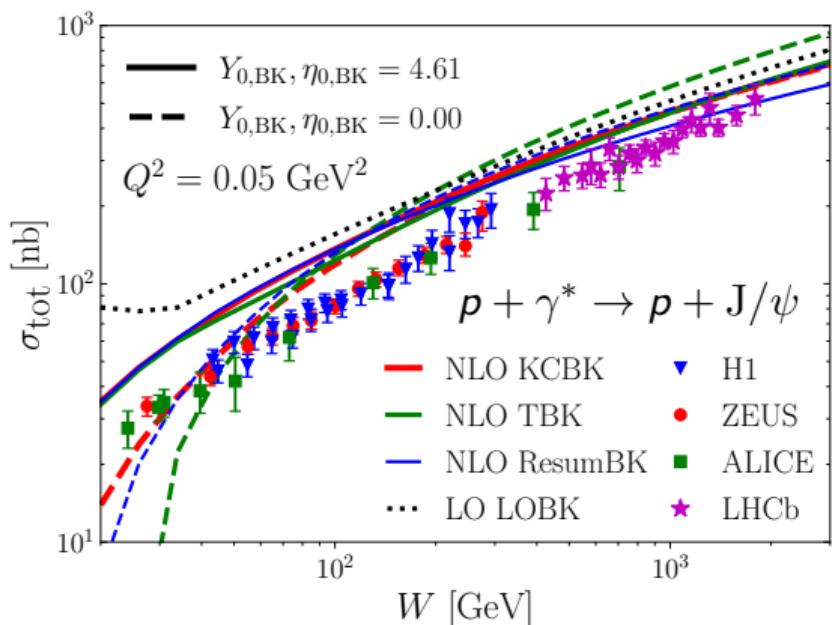
With v^2 relativistic corrections



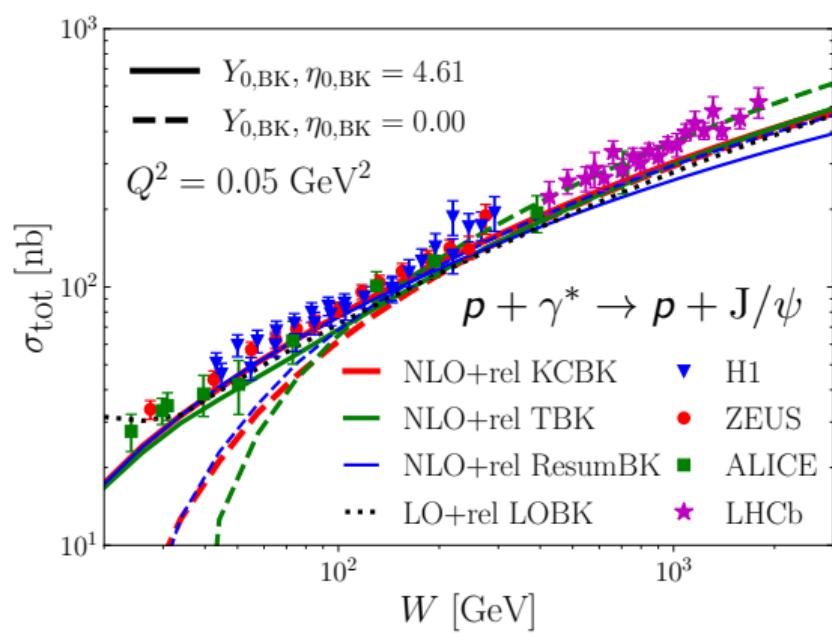
- NLO corrections moderate, get a good description of HERA data
- Relativistic v^2 corrections important at low Q^2

Total J/ψ production – dependence on the center-of-mass energy W

Nonrelativistic limit



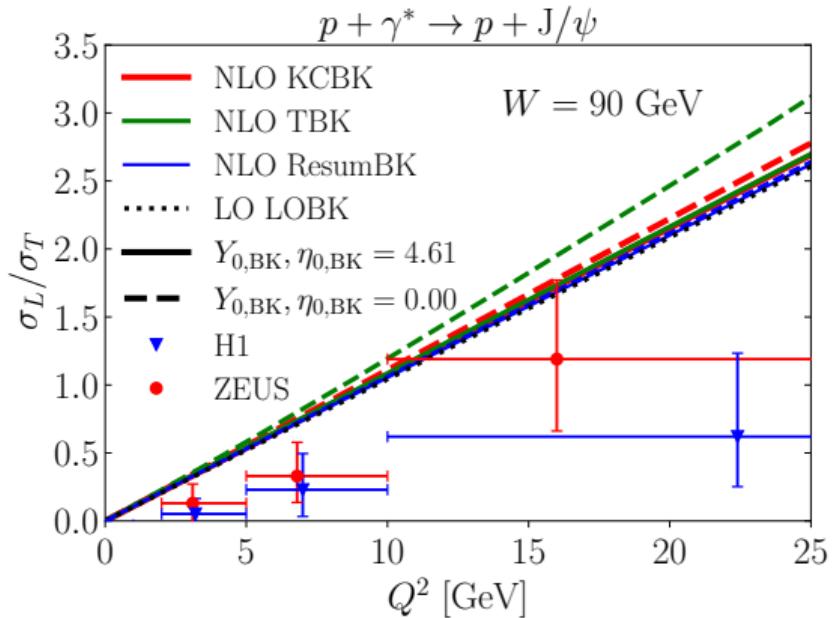
With v^2 relativistic corrections



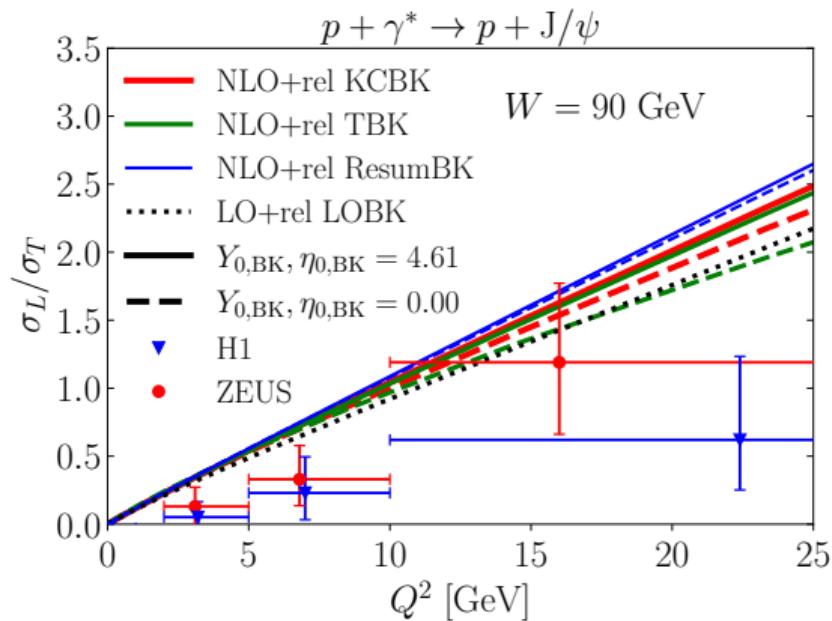
- Good description of the energy dependence
- $Y_{0,\text{BK}} = 0.00$: unphysical results at low W

Longitudinal-to-transverse ratio for J/ψ production

Nonrelativistic limit



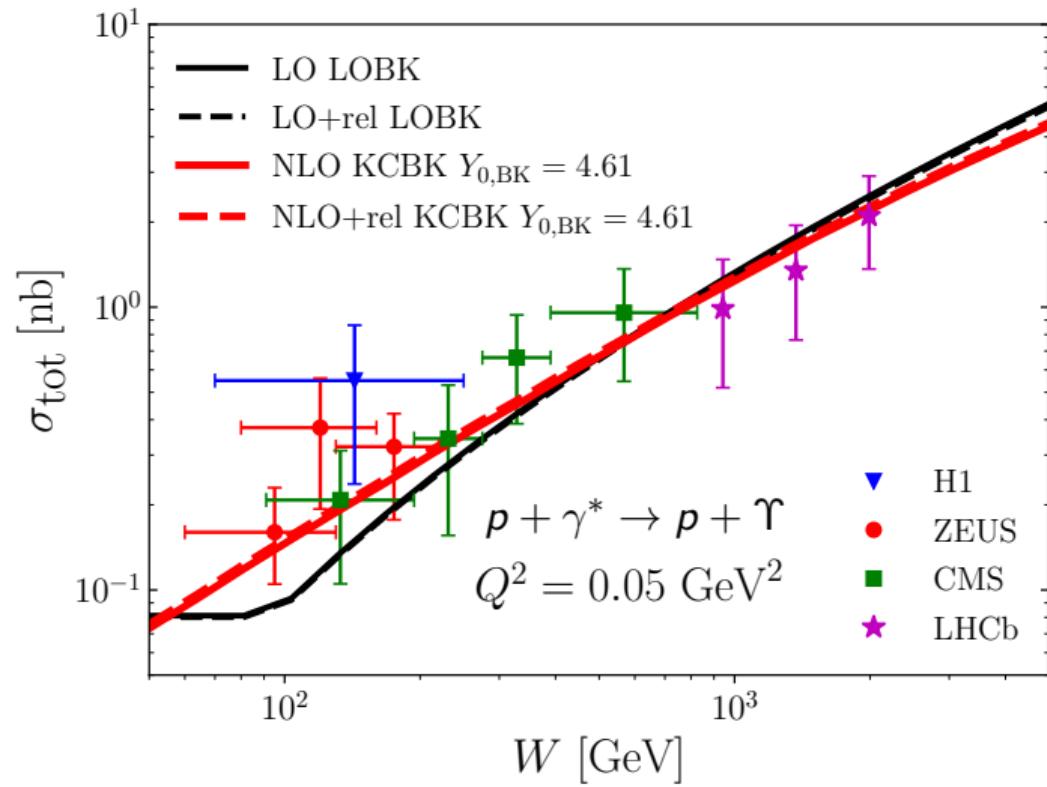
With v^2 relativistic corrections



- NLO corrections moderate

Total Υ production – dependence on center-of-mass energy W

- Relativistic effects small
 - This is expected for Υ
- Good agreement with the data



Summary

- We have calculated both longitudinal and transverse quarkonium production at NLO
⇒ Comparisons with data possible
- Both NLO and v^2 relativistic corrections numerically important
 - Generally good agreement with the data when both are included
- Can distinguish between different NLO dipole fits
 - At small W : Dipoles with $Y_{0,BK} = 0.00$ give unphysical results
- Future: Use NLO dipole fits with *massive* quarks
- NLO light vector meson production at large Q^2 : [Mäntysaari, JP, 2203.16911](#)
- Important developments: precise measurements expected at ultra-peripheral collisions at the LHC and the future Electron-Ion Collider

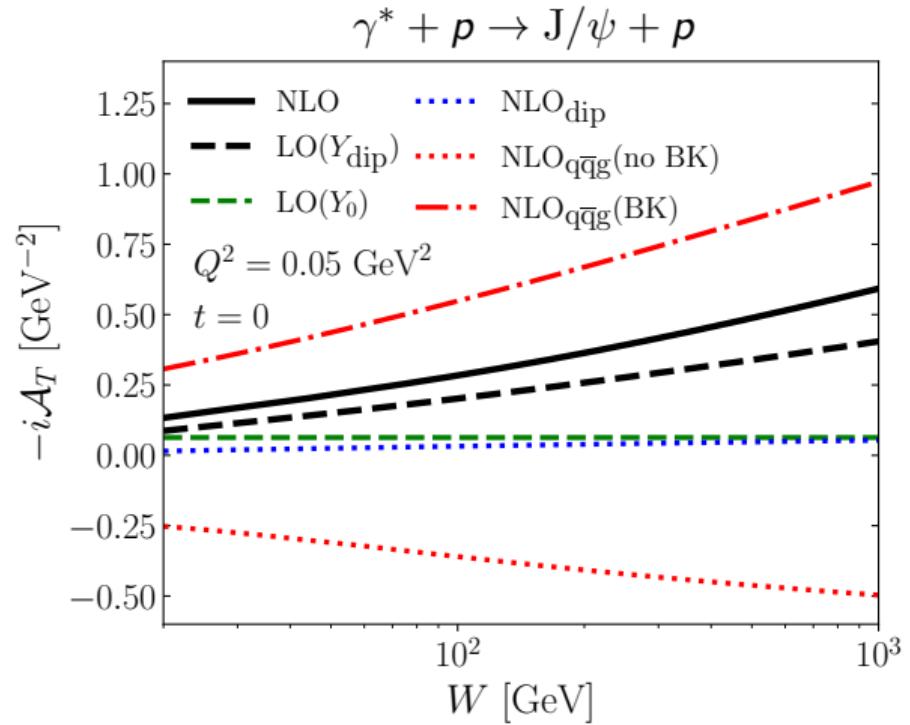
Backup

Backup - Rapidity

- The rapidity in the dipole amplitude is defined as $Y = \log z_2 + \log(q^+/P^+)$
 - q^+, P^+ = longitudinal momenta of the virtual photon and the target
- Eikonal approximation: the invariant mass of the $q\bar{q}g$ system has to satisfy $M_{q\bar{q}g}^2 \ll W^2$
 $\Rightarrow z_2 > z_{\min} = \frac{P^+}{q^+} = \frac{Q_0^2}{W^2 + Q^2 - m_N^2}$, Q_0^2 = transverse momentum scale of the target
- In total, we have three different rapidities in the expression:
 - Y_0 = the initial rapidity
 - $Y_{q\bar{q}g} = Y_0 + \log \frac{z_2}{z_{\min}}$, evolution rapidity in the real contribution
 - $Y_{\text{dip}} = Y_0 + \log \frac{1}{2z_{\min}}$, evolution rapidity in the virtual contribution
- The amount of evolution in rapidity: $\log 1/2z_{\min} \approx \log W^2/2Q_0^2$
- Following [Beuf et al. 2007.01645](#), we choose $Q_0^2 = 1 \text{ GeV}^2$ and $Y_0 = 0$

Backup – Decomposition of the production amplitude

- The leading order $\text{LO}(Y_{\text{dip}})$ result includes the resummation of the large logs $\sim \alpha_s \log 1/x$
- $\text{NLO} = \text{LO}(Y_0) + \text{NLO}_{\text{dip}} + \text{NLO}_{q\bar{q}}$
- Here the same dipole amplitude used for both LO and NLO
 $\Rightarrow \text{NLO} - \text{LO}(Y_{\text{dip}})$ tells about the largeness of the NLO correction terms

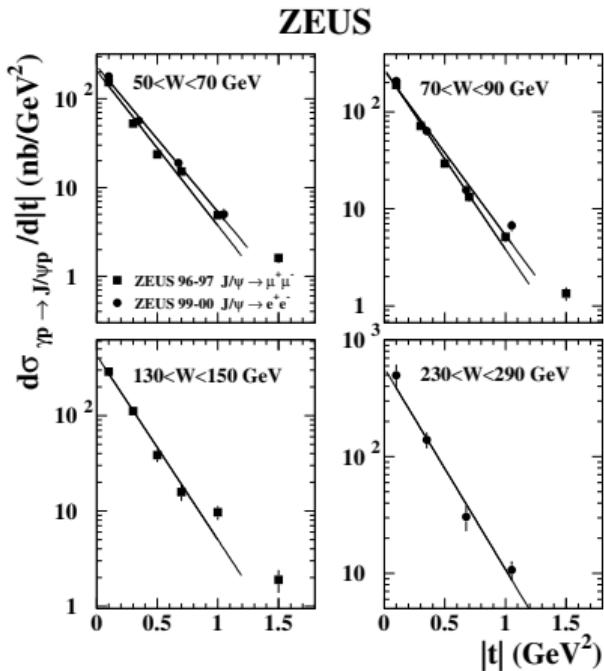


Backup – Performing the t integral

- These results valid at $t = 0$
- Need the t -integrated cross section for comparisons with experimental data
- Use the experimental parametrization for t dependence:

$$\frac{d\sigma}{dt} = e^{-b|t|} \times \frac{d\sigma}{dt}(t = 0)$$

- $b \approx$ transverse size of the target-meson system
- b taken from a fit to experimental data



ZEUS collaboration, [hep-ex/0201043](https://arxiv.org/abs/hep-ex/0201043)

Backup – Exclusive light vector meson production at NLO

- Calculated in [Mäntysaari, J.P, 2203.16911](#), in the limit $Q^2 \gg M_V^2$
- Excellent agreement with the data

