C-conjugation odd color charge correlations in the light-front wave function of the proton at moderately small x

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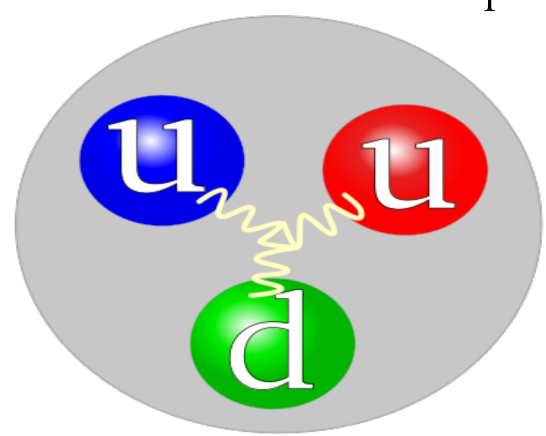
wikipedia



What's that ?

talk based on collaborations with
H. Mäntysaari, R. Paatelainen: 2103.11682
T. Stebel, V. Skokov: 2001.04516
G. Miller, R. Venugopalan: 1808.02501

Motivation :

* C-odd color charge fluctuations first appear at cubic order in $<\rho^a\rho^b\rho^c>$ correlator ($<\rho^a> \sim \mathrm{tr}\, t^a = 0$; $<\rho^a\rho^b> \sim \mathrm{tr}\, t^a t^b$ is C-even)

* related to non-Gaussian color charge fluctuations in the proton

* related to some of transverse spin physics (e.g. T-odd dipole gluon Sivers function)

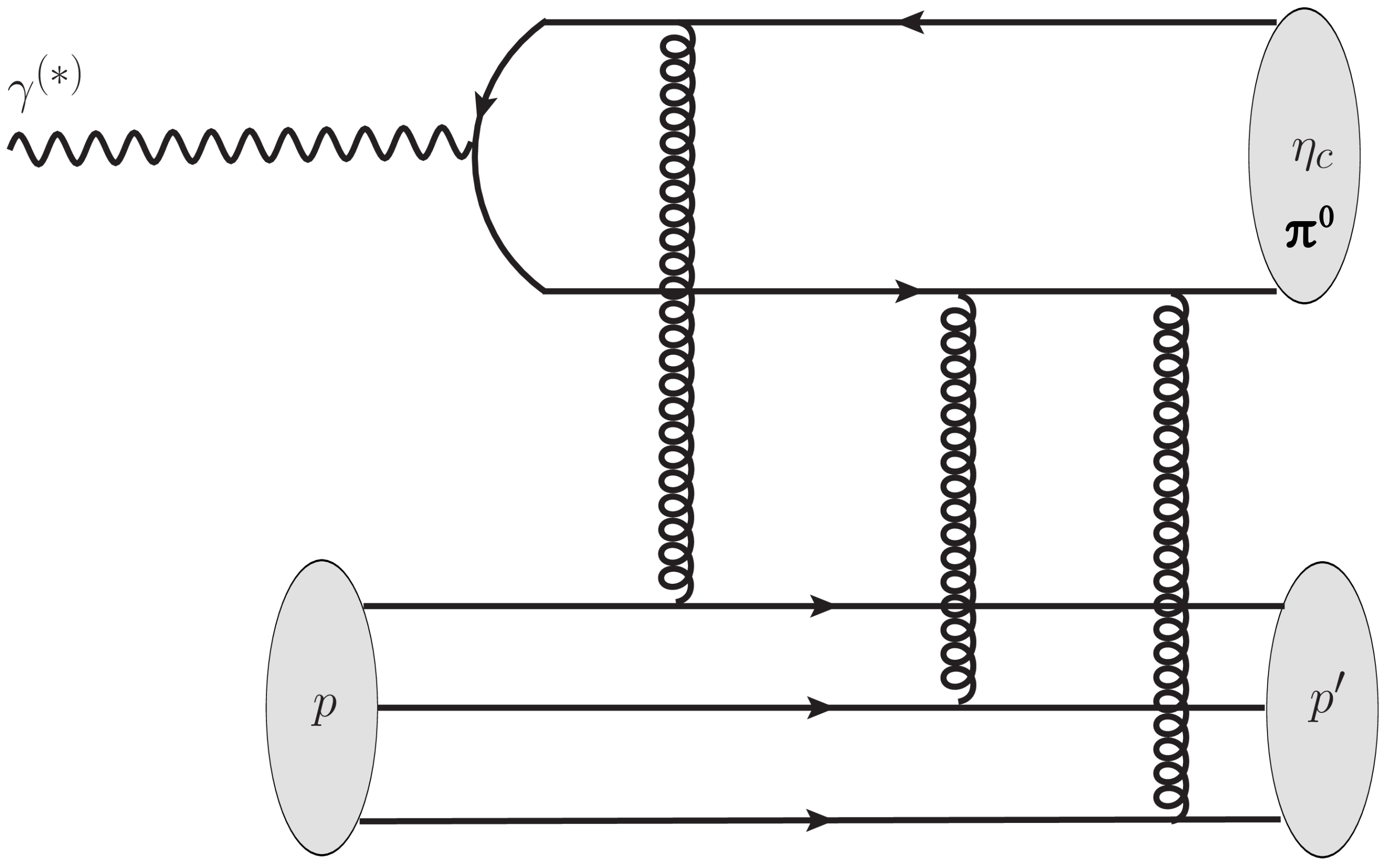
same root!

* couple to C-odd ggg exchange (Odderon) which appears in a variety of processes

Processes involving the “hard” Odderon :

kinematic regime: hard ggg exchange at $x \sim 0.01 - 0.1$

* exclusive production of pseudo-scalar mesons in $\gamma^* p \longrightarrow M_{PS}\, p$, large $|t|$
  (and high Q^2 in case of π^0 production)

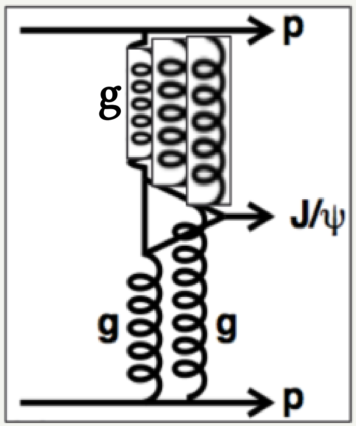


Czyzewski, Kwiecinski, L. Motyka, M. Sadzikowski, PLB 398 (1997);
Engel, Ivanov, Kirschner, Szymanowski, Eur.Phys.J.C4 (1998);
Kilian, Nachtmann, Eur.Phys.J.C5 (1998); A.D, T. Stebel, Phys.Rev.D 99 (2019)

* exclusive production of vector mesons in pp $\longrightarrow M_V\,p$, large $|t|$

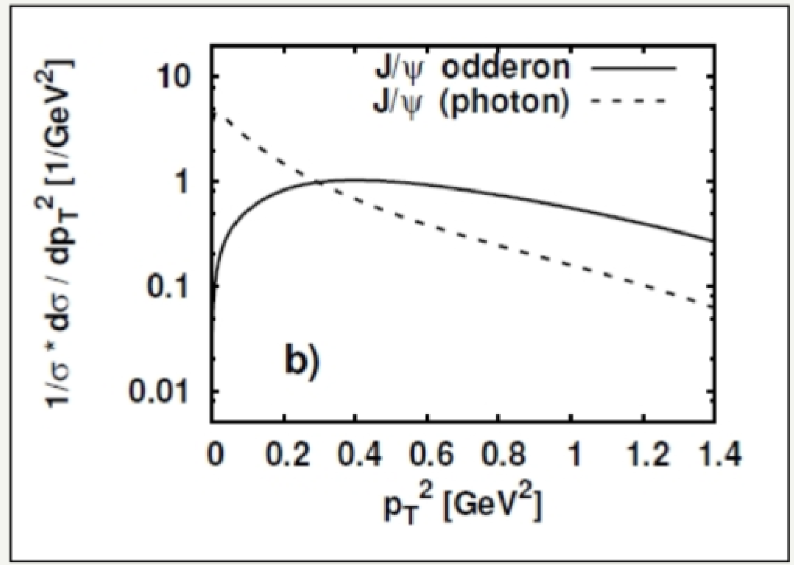
Odderon

Visible in heavy V.M. at high p_T^2?



$$\frac{d\sigma}{dt} \sim e^{bt}$$

Photoproduction: b~6 GeV^{-2}
Proton dissociation b~1 GeV^{-2}
Odderon b small



Bzdak, Motyka, Szymanowski, Cudell
PRD 75 (2007) 094023

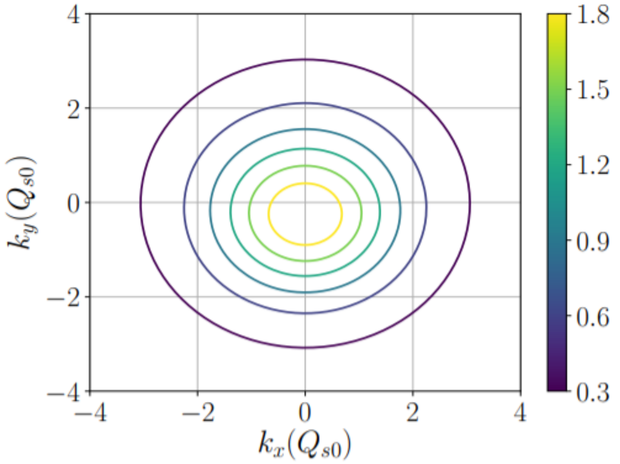
$d\sigma^{corr}/dy$	J/ψ	
	odderon	photon
Tevatron	0.3–1.3–5 nb	0.8–5–9 nb
LHC	0.3–0.9–4 nb	2.4–15–27 nb

Given incoherent backgrounds it is likely difficult to attribute a tail to odderon but the comparison of spectra in e-p, pA, AA to pp may be sufficient.

Ronan McNulty, CFNS Workshop: Target fragmentation and diffraction physics
with novel processes: Ultraperipheral, electron-ion, and hadron collisions, Feb. 9 – 11, 2022
https://indico.bnl.gov/event/14009/

* dipole Gluon Sivers function for transv. pol. proton $f_{1T}^{\perp g}(x, k_T^2)$
 – unknown <u>normalization & sign</u> due to initial condition

Yao, Hagiwara, Hatta: 1812.03959;
Boer, Echevarria, Mulders, Zhou, PRL 116 (2016)
Boer, Cotogno, van Daal, Mulders, Signori, Zhou, JHEP 10 (2016)
Boussarie, Hatta, Szymanowski, Wallon, PRL 124 (2020)
Kovchegov, Santiago, 2108.03667



* group theory bounds:
Lappi, Ramnath, Rummukainen, Weigert, PRD 94 (2016)

* Boer, Hagiwara, Zhou, Zhou: Energy evolution of T-odd gluon TMDs at small x, arXiv:2203.00267
"the spin dependent odderon computed from the MV model is very small and would not lead to any measurable effects. [...] In the following numerical estimations, we use the diquark model"

The proton Fock state description on the light front

The proton on the light front (valence quark Fock state; L.C. time $x^+ = 0$)

$$
\begin{aligned}
|P\rangle \;=\; & \frac{1}{\sqrt{6}} \int \frac{dx_1 dx_2 dx_3}{\sqrt{x_1 x_2 x_3}} \delta(1 - x_1 - x_2 - x_3) \int \frac{d^2k_1 d^2k_2 d^2k_3}{(16\pi^3)^3} 16\pi^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \\
& \times \psi(k_1, k_2, k_3) \sum_{i_1, i_2, i_3} \epsilon_{i_1 i_2 i_3} |p_1, i_1; p_2, i_2; p_3, i_3\rangle
\end{aligned}
$$

+ higher Fock states

P. Lepage & Brodsky, 1979 - ....
Brodsky, Pauli, Pinsky, PR (1998)

* start from effective, non-perturbative 3q Fock space amplitude to describe "large"-x structure of the proton

* add corrections (such as |qqqg>) as needed

$\longrightarrow$ Evaluate color charge $\rho^a = g \int dx^- \, \overline{q} \gamma^+ t^a q$ correlators explicitly !

<$\rho^a\rho^b\rho^c$> correlator (C odd part, LO)

does not vanish (color charge fluct. not Gaussian) :

$$\left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\,\right\rangle_{K_\perp}\Big|_{\mathcal{C}=-} \equiv \frac{g^3}{4}\,d^{abc}\,G_3^-(\vec{q}_1,\vec{q}_2,\vec{q}_3)$$

$$G_3^-(\vec{q}_1,\vec{q}_2,\vec{q}_3) = \int [dx_i][dp_i]$$

1-body, positive

$$\Big[\psi^*(\vec{p}_1+(1-x_1)\vec{K}_\perp,\vec{p}_2-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp)$$

$$-\psi^*(\vec{p}_1-\vec{q}_1-x_1\vec{K}_\perp,\vec{p}_2+\vec{q}_1+(1-x_2)\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp)$$

2-body, negative

$$-\psi^*(\vec{p}_1+\vec{q}_2+(1-x_1)\vec{K}_\perp,\vec{p}_2-\vec{q}_2-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp)$$

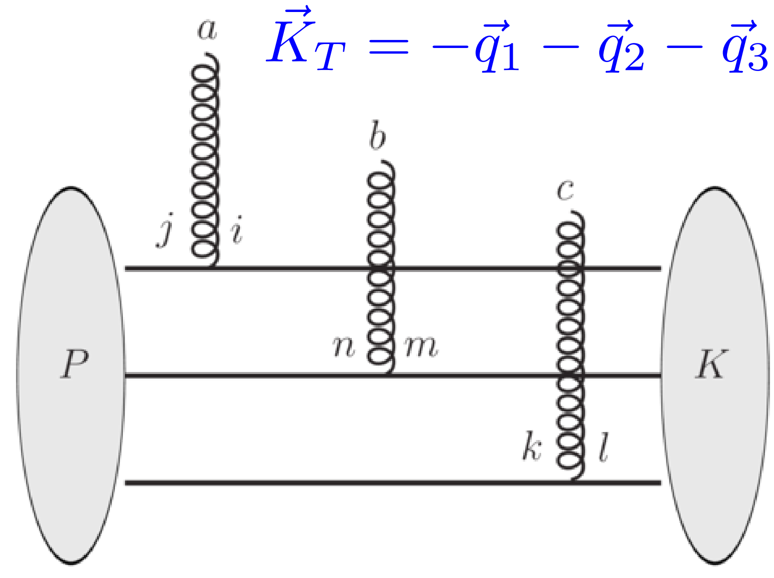
$$-\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_2-x_1\vec{K}_\perp,\vec{p}_2+\vec{q}_1+\vec{q}_2+(1-x_2)\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp)$$

3-body, positive

$$+2\,\psi^*(\vec{p}_1-\vec{q}_1-x_1\vec{K}_\perp,\vec{p}_2+\vec{q}_1+\vec{q}_2+(1-x_2)\vec{K}_\perp,\vec{p}_3-\vec{q}_2-x_3\vec{K}_\perp)$$

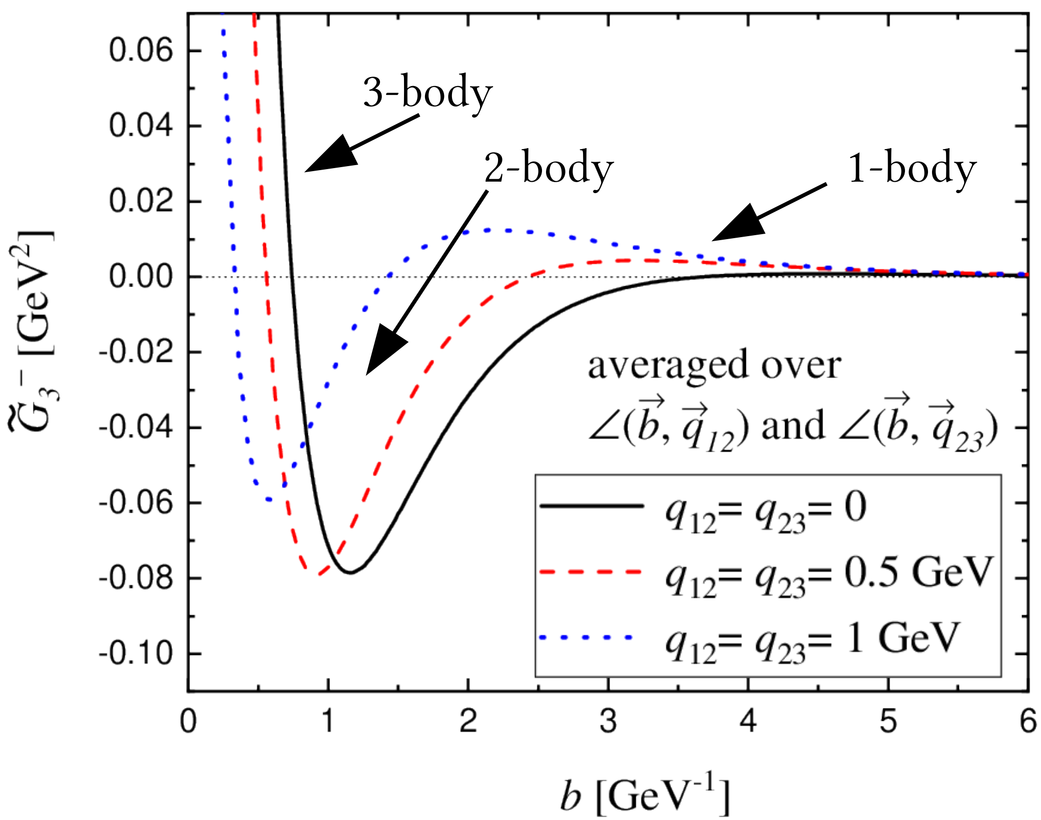
$$\Big]\,\psi(\vec{p}_1,\vec{p}_2,\vec{p}_3)$$

$$\vec{K}_T = -\vec{q}_1-\vec{q}_2-\vec{q}_3$$



* 1-, 2- and 3-body matrix elements, sum vanishes when either $q_i \longrightarrow 0$ (Ward identities)

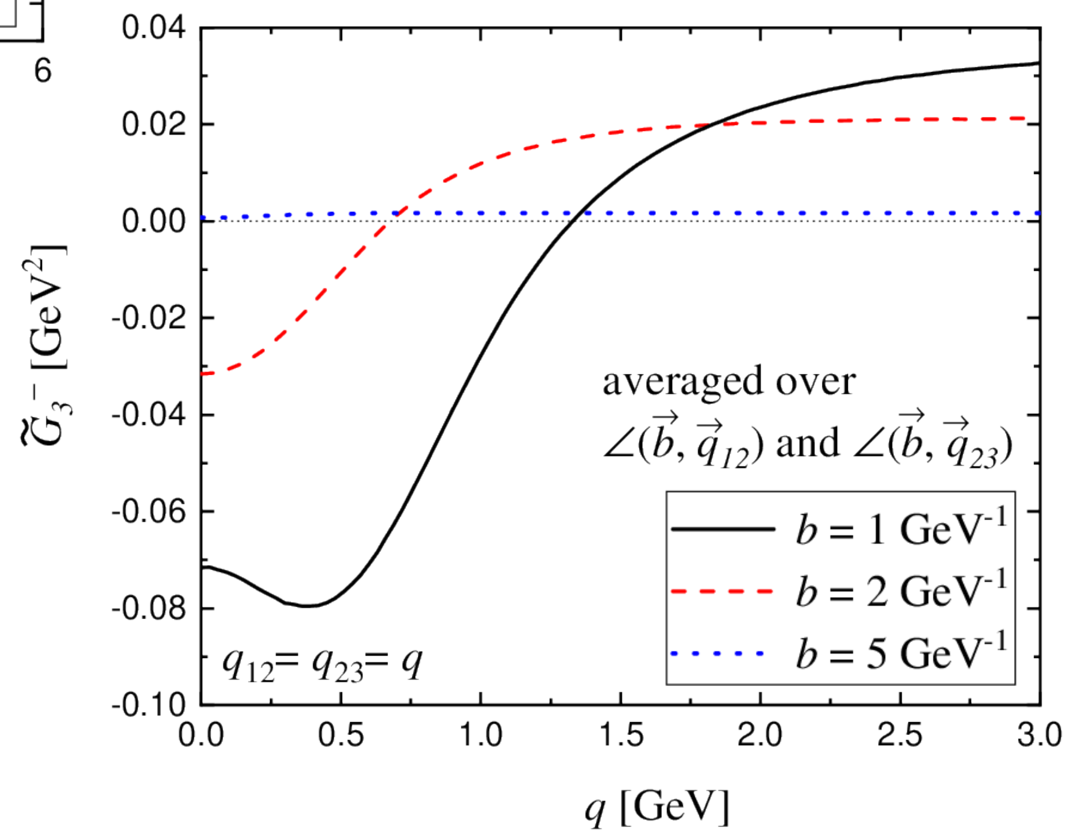
* "3-body" diagrams not (power-) suppressed when $\vec{q}_1 \sim \vec{q}_2 \sim \vec{q}_3 \sim -\vec{K}_T/3 \gg \Lambda_{\rm QCD}$ but actually dominant !



- very non-trivial structure; non-monotonic, sign changes etc
- diverges for b $\longrightarrow$ 0 due to contribution from high K_T

$$\left(\int \frac{\mathrm{d}^2 K_T}{K_T^2} \; e^{-i\vec{b}\cdot\vec{K}_T} \right)$$

A.D., T. Stebel, V. Skokov, arXiv:2001.04516



<ρ^{a}ρ^{b}ρ^c> correlator at LO (C even part)

$$\left\langle \rho^a(\vec{q}_1)\,\rho^b(\vec{q}_2)\,\rho^c(\vec{q}_3)\,\right\rangle_{K_\perp}\Big|_{\mathcal{C}=+} \equiv \frac{g^3}{4}\,if^{abc}\,G_3^+(\vec{q}_1,\vec{q}_2,\vec{q}_3)$$

$$\begin{aligned}
G_3^+(\vec{q}_1,\vec{q}_2,\vec{q}_3) &= \int[dx_i]\int[d^2p_i] \\
&\Big[\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_2-\vec{q}_3-x_1\vec{K}_\perp,\vec{p}_2-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\
&-\psi^*(\vec{p}_1-\vec{q}_2-\vec{q}_3-x_1\vec{K}_\perp,\vec{p}_2-\vec{q}_1-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\
&+\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_3-x_1\vec{K}_\perp,\vec{p}_2-\vec{q}_2-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\
&-\psi^*(\vec{p}_1-\vec{q}_1-\vec{q}_2-x_1\vec{K}_\perp,\vec{p}_2-\vec{q}_3-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\
&\Big]\,\psi(\vec{p}_1,\vec{p}_2,\vec{p}_3) \\
&= G_2(\vec{q}_1+\vec{q}_2,\vec{q}_3)-G_2(\vec{q}_1+\vec{q}_3,\vec{q}_2)+G_2(\vec{q}_1,\vec{q}_2+\vec{q}_3)
\end{aligned}$$

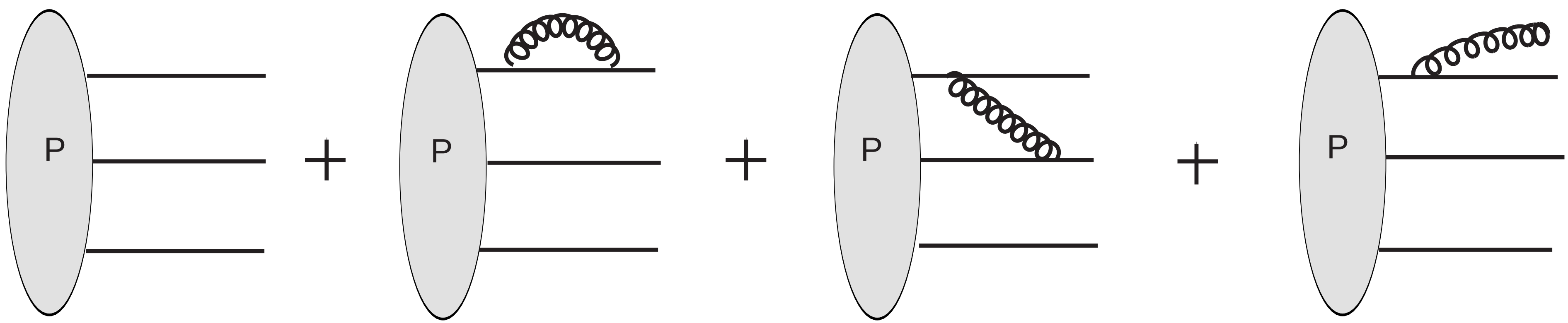
(like Reggeized 2-gluon exchange)

whereas

$$\begin{aligned}
G_3^-(\vec{q}_1,\vec{q}_2,\vec{q}_3) &= G_2(\vec{q}_1+\vec{q}_2,\vec{q}_3)+G_2(\vec{q}_1+\vec{q}_3,\vec{q}_2)+G_2(\vec{q}_2+\vec{q}_3,\vec{q}_1) \\
&-2\int[dx_i]\int[d^2p_i]\,\Big[\Psi^*(\vec{p}_1+(1-x_1)\vec{K}_\perp,\vec{p}_2-x_2\vec{K}_\perp,\vec{p}_3-x_3\vec{K}_\perp) \\
&\quad-\Psi^*(\vec{p}_1-\vec{q}_1-x_1\vec{K}_\perp,\vec{p}_2-\vec{q}_2-x_2\vec{K}_\perp,\vec{p}_3-\vec{q}_3-x_3\vec{K}_\perp)\Big]\,\Psi(\vec{p}_1,\vec{p}_2,\vec{p}_3)
\end{aligned}$$

Now to $|P\rangle \sim \psi_{qqq}|qqq\rangle + \psi_{qqqg}|qqqg\rangle$

computed in LF perturbation theory (in LC gauge),
1-gluon emission / exchange,
w/o employing small-x approximation

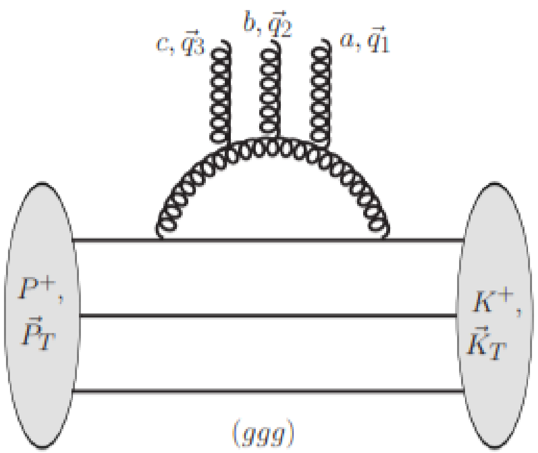


w/ R. Paatelainen & H. Mäntysaari

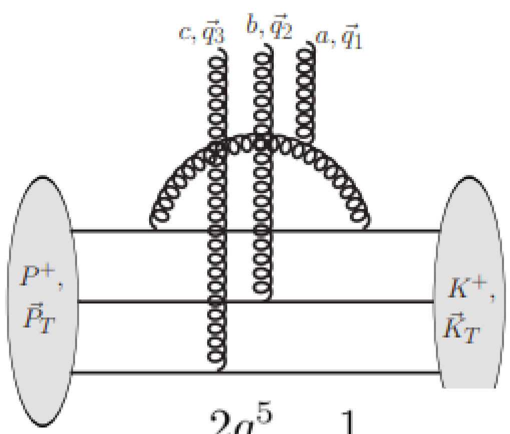
1st perturbative correction to <ρ^aρ^bρ^c> correlator

* C-odd contribution to dipole scattering amplitude, "initial condition"
  (small-x evol: BKP, Kovchegov, Szymanowski, Wallon, PLB 2004;
  Hatta, Iancu, Itakura, L. McLerran, NPA2005;
  Lappi, Ramnath, Rummukainen, Weigert, PRD 2016)

* Two sample diagrams (~100 more listed in arXiv:2106.12623)



~ T^a_{bc}, C even



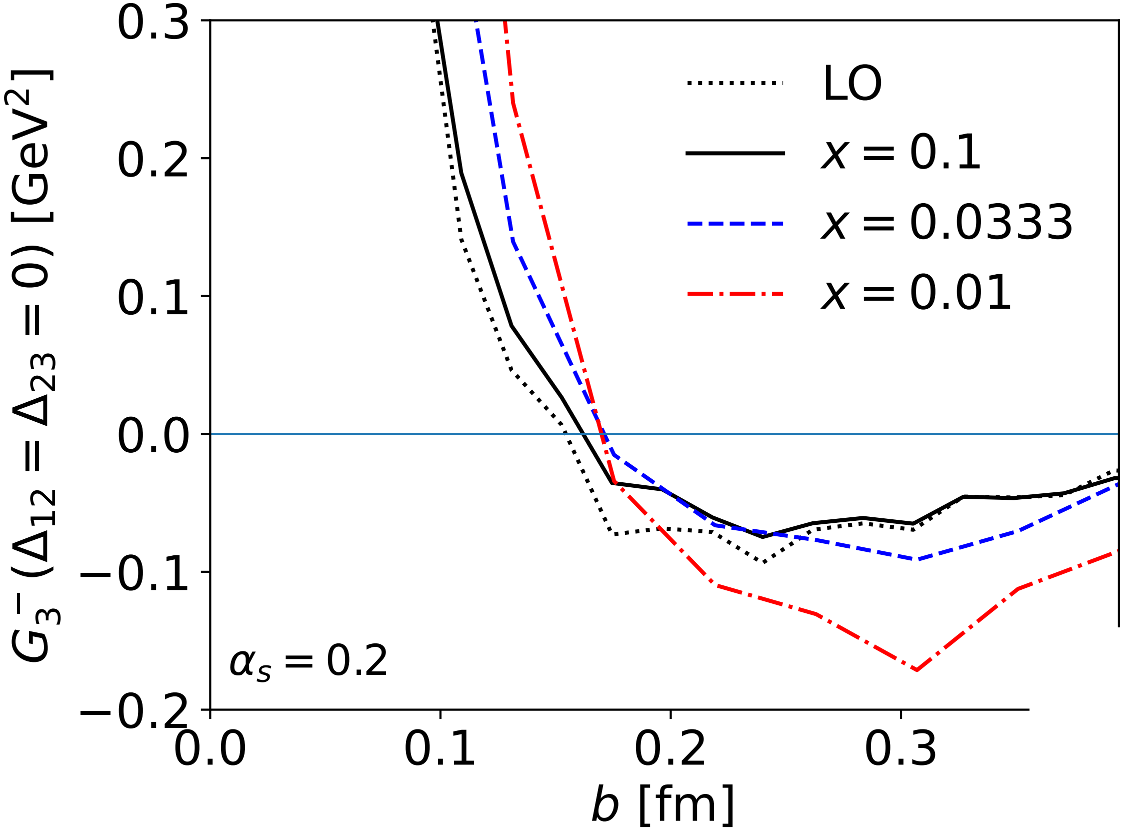
~d^{abc}, C odd
probes couple to higher-x quarks too

$$\frac{2g^5}{3\cdot 16\pi^3}\,\frac{1}{2}N_c\,\mathrm{tr}\left(t^at^bt^c+t^at^ct^b\right)\int[\mathrm{d}x_i]\int\left[\mathrm{d}^2k_i\right]\,\Psi_{qqq}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3)$$

$$\Psi^*_{qqq}(x_1,\vec{k}_1+x_1\vec{q}-\vec{q}_1;x_2,\vec{k}_2+x_2\vec{q}-\vec{q}_2;x_3,\vec{k}_3+x_3\vec{q}-\vec{q}_3)$$

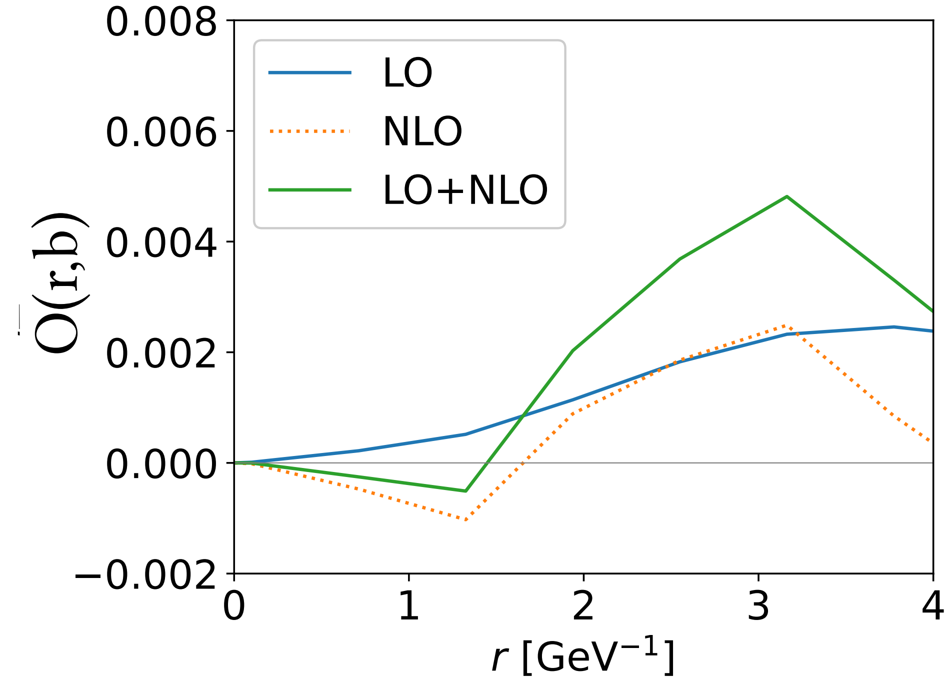
$$\frac{(2\pi)^{D-1}}{2p_1^+}\int\frac{\widetilde{\mathrm{d}k_g}}{2(p_1^+-k_g^+)}\langle S|\hat{\psi}_{q\to qg}(\vec{p}_1;\vec{p}_1-\vec{k}_g,\vec{k}_g)\,\hat{\psi}^*_{q\to qg}(\vec{p}_1-\vec{q}_1;\vec{p}_1-\vec{k}_g,\vec{k}_g-\vec{q}_1)|S\rangle$$

Some numerical results :



* at x=0.1 there is basically no contribution from |qqqg>

* substantial effect at x~10^{-2}
→ resummation ?



Summary

* computed correlator $<\rho^a \rho^b \rho^c>$ of three color charge operators in a model proton (with "reasonable" quark x, $\mathbf{k}_T$ distributions, color and momentum correlations at x > 0.1)
+ first perturbative (1-gluon emission / exchange) correction, numerically small at x~0.1, increases towards lower x

* quantum correlations in the LFwf lead to a very intricate structure as a function of **b**, **r**, and their relative azimuth;   and x

* if $<\rho^3>\neq 0$ is seen $\longrightarrow$ evidence for non-Gaussian color charge fluctuations in the proton at sub-femtometer scales !

* Outlook: Im S, evolution to smaller x, T-odd gluon TMDs, ...

Thank you !

Backup Slides

NLO BK: evolution in terms of target rapidity (i.e. in x) :

B. Ducloué et al: 1902.06637

$$\begin{aligned}
\frac{\partial \bar{S}_{\boldsymbol{xy}}(\eta)}{\partial \eta} = & \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\,\Theta(\eta-\delta_{\boldsymbol{xyz}})\left[\bar{S}_{\boldsymbol{xz}}(\eta-\delta_{\boldsymbol{xz};r})\bar{S}_{\boldsymbol{zy}}(\eta-\delta_{\boldsymbol{zy};r})-\bar{S}_{\boldsymbol{xy}}(\eta)\right] \\
& - \frac{\bar{\alpha}_s^2}{4\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \ln\frac{(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2} \ln\frac{(\boldsymbol{y}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}\left[\bar{S}_{\boldsymbol{xz}}(\eta)\bar{S}_{\boldsymbol{zy}}(\eta)-\bar{S}_{\boldsymbol{xy}}(\eta)\right] \\
& + \frac{\bar{\alpha}_s^2}{2\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{z}\,\mathrm{d}^2\boldsymbol{u}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{u})^2(\boldsymbol{u}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\left[\ln\frac{(\boldsymbol{u}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}+\delta_{\boldsymbol{uy};r}\right]\bar{S}_{\boldsymbol{xu}}(\eta)\left[\bar{S}_{\boldsymbol{uz}}(\eta)\bar{S}_{\boldsymbol{zy}}(\eta)-\bar{S}_{\boldsymbol{uy}}(\eta)\right] \\
& + \bar{\alpha}_s^2 \times \text{“regular”},
\end{aligned} \tag{6.4}$$

non-local in rapidity, involves S at rapidities $\eta < \eta_0 = \log 1/x_0$!

Model LFwf for the proton (Brodsky & Schlumpf, PLB 329, 1994)

$$\begin{aligned}\psi_{\rm H.O.}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3) &= N_{\rm H.O.}\exp(-\mathcal{M}^2/2\beta^2)\ , \\ \psi_{\rm Power}(x_1,\vec{k}_1;x_2,\vec{k}_2;x_3,\vec{k}_3) &= N_{\rm Power}(1+\mathcal{M}^2/\beta^2)^{-p}\ .\end{aligned}$$

$$\mathcal{M}^2 = \sum_{i=1}^{3} \frac{\vec{k}^2_{\perp i} + m^2}{x_i}$$

m = 0.26 GeV, β = 0.55 for H.O. wf

m = 0.263, β = 0.607, p = 3.5 for PWR wf

With these parameters they fit:

- proton radius $R^2 = -6\ \frac{dF_1(Q^2)}{dQ^2}\Big|_{Q^2=0} = (0.76\ \mathrm{fm})^2$
- proton / neutron magnetic moments $1 + F_2(Q^2 \to 0) = 2.81\ /\ -1.66$
- axial vector coupling g_A = 1.25