Probing gluon saturation via diffractive jet production at the EIC

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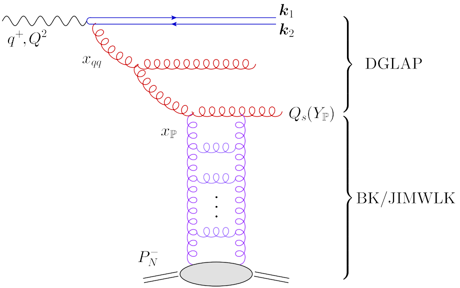
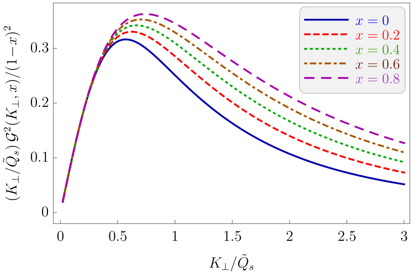
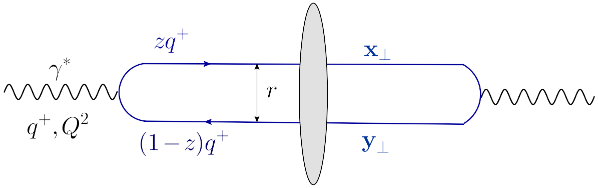
with A.H. Mueller, D.N. Triantafyllopoulos (2112.06353, PRL) and S.-Y. Wei (w.i.p.)





How to measure gluon saturation in DIS ?

- High energy/small Bjorken x: one can use the Colour Dipole Picture



$$\sigma_{\gamma^* p}(Q^2, x) = \int \mathrm{d}^2 r \int_0^1 \mathrm{d}z \left|\Psi_{\gamma^* \to q\bar{q}}(r, z; Q^2)\right|^2 \underbrace{\sigma_{\text{dipole}}(r, A, x)}_{2\pi R_A^2\, T(r,A,x)}$$

- Gluon saturation $\Longleftrightarrow$ Strong (multiple) scattering: $T \simeq 1$ ("black disk")
- Saturation requires large dipoles: $r \gtrsim 1/Q_s$ with $Q_s^2(A, x) \propto A^{1/3} x^{-\lambda}$
- The dipole size is limited by virtuality: $r \lesssim 1/\bar{Q}$ with $\bar{Q}^2 \equiv z(1-z)Q^2$
- In order to probe saturation, one needs $z(1-z)Q^2 \lesssim Q_s^2$

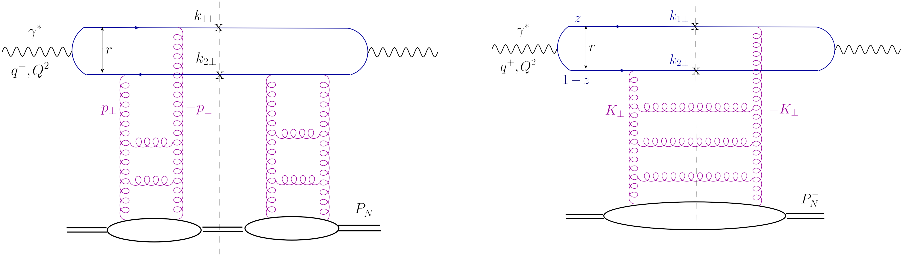
Saturation fits to DIS at HERA

- Excellent saturation/CGC fits at HERA: $x_{\rm Bj} \le 10^{-2}$, $Q^2 \le 50$ GeV^2
  - the pioneers: Golec-Biernat and Wüsthoff, 1999
  - many subsequent analyses, with more and more input from first principles *(BK/JIMWLK evolution, NLO effects, resummations...)*
  - Ducloué, E.I., Soyez and Triantafyllopoulos, 2019
  - Beuf, Hänninen, Lappi, Mäntysaari, 2020 ...
- Interesting qualitative predictions: geometric scaling, diffraction ...
- However, gluon saturation only marginally probed: $Q_s^2 \sim 1$ GeV^2
  - limited region in phase-space, non-perturbative contamination
- Can one measure saturation at high $Q^2 \gg Q_s^2$?
- Less inclusive observables: particle (jet/hadron) production, diffraction

Elastic scattering (LO Diffraction)

- Elastic scattering is more sensitive to gluon saturation

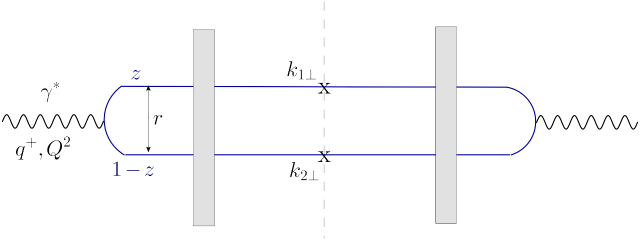
$$\sigma_{el} \propto T^2 \longleftrightarrow \sigma_{tot} \propto 2T$$



- Colourless exchange: 2-gluon ladder, (BFKL) Pomeron, rapidity gap $Y_{\mathbb{P}}$
- At weak scattering $T \ll 1$, elastic scattering is strongly suppressed
- Diffraction controlled by the black disk limit $(T \sim 1)$ even when $Q^2 \gg Q_s^2$

Dijet production in the correlation limit

- Two relatively hard jets (or hadrons) which are nearly back to back



$$k_{1\perp} \simeq k_{2\perp} \sim Q \;\gg\; K_\perp \equiv |\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}| \sim Q_s$$

- Momentum imbalance fixed by (multiple) scattering/gluon saturation
- Measure saturation from azimuthal correlations *(Marquet, 2007)*

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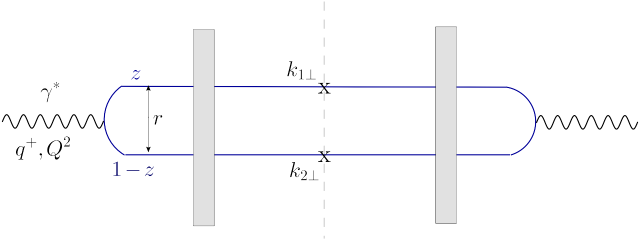

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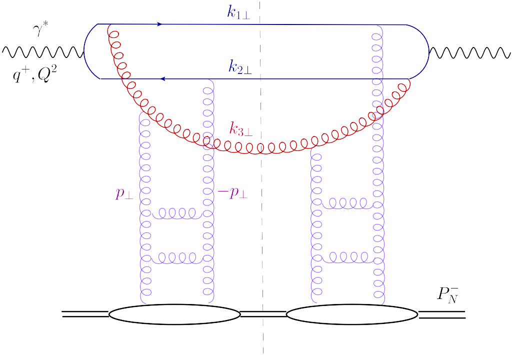
- Momentum imbalance fixed by (multiple) scattering/gluon saturation
- Measure saturation from azimuthal correlations *(Marquet, 2007)*
- Additional broadening due to final-state radiation (Sudakov effect) *(Mueller, Xiao and Yuan, 2013)*
- The two effects are difficult to disentangle in practice ☹

2+1 jets in hard DIS diffraction

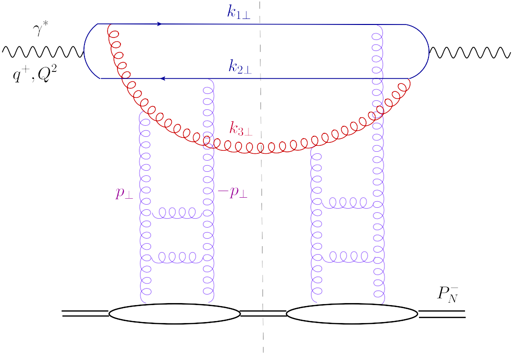
- What is the analog of the "correlation limit" for diffractive dijets ?
- 2+1 jet production: $k_{1\perp},\, k_{2\perp} \sim Q \;\gg\; k_{3\perp} \sim Q_s$
  - a value $k_{3\perp} \sim Q_s$ is naturally selected by the elastic scattering



- The longitudinal momentum is soft as well: $k_3^+ = \xi q^+$ with $\xi \lesssim \frac{Q_s^2}{Q^2} \ll 1$
  - time ordering: $k_3^+/k_{3\perp}^2 \lesssim q^+/Q^2$

2+1 jets in hard DIS diffraction (2)

- Coherent diffraction: elastic scattering, the target is not broken
  - total momentum transfer is negligible: $|\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3| \sim \Lambda_{\rm QCD}$

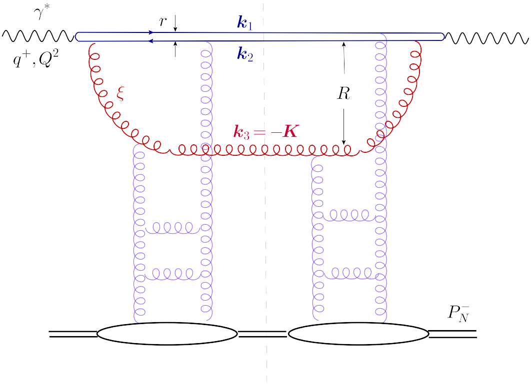


- The gluon jet needs not be measured ... yet, it plays an essential role:
  - it controls the imbalance between the hard jets: $K_\perp \equiv |\boldsymbol{k}_1 + \boldsymbol{k}_2| \simeq k_{3\perp}$
  - it allows for strong scattering (saturation) even at high Q^2

The gluon dipole picture

- A simpler, effective, picture for the colour flow *(Wüsthoff, 97)*

$R \sim 1/k_{3\perp} \sim 1/Q_s \gg r \sim 1/Q \implies$ effective gg dipole

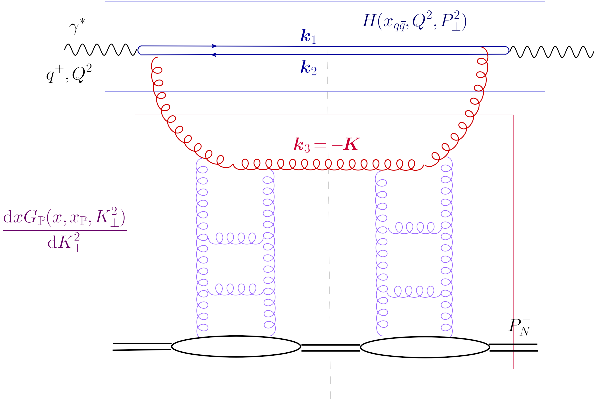


- A gg dipole scatters twice as strong as a $q\bar{q}$ one: $Q_s^2(gg) = (9/4)Q_s^2(q\bar{q})$
- The saturation momentum is evaluated at the rapidity gap: $Q_s^2(Y_{\mathbb{P}})$

TMD factorisation for diffractive dijets

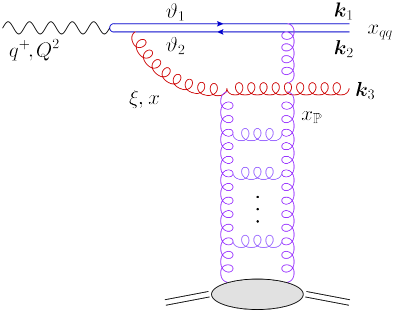
(E.I., A.H. Mueller, D.N. Triantafyllopoulos, arXiv:2112.06353)

- So far, the soft gluon was seen as part of the projectile wavefunction
- It can be viewed as a part of target wavefunction as well



- Hard dijet cross-section $=$ Hard factor $\times$ UGD of the Pomeron
- The hard factor: the same as for inclusive dijets *(cf. talk by Bowen Xiao)*

Rapidity scales



$$\Delta Y = \ln \frac{1}{x}$$

$$Y_{\mathbb{P}} = \ln \frac{1}{x_{\mathbb{P}}}$$

$$x \equiv \frac{x_{q\bar{q}}}{x_{\mathbb{P}}} = \frac{Q^2 + M^2_{q\bar{q}}}{Q^2 + M^2_{q\bar{q}g}}$$

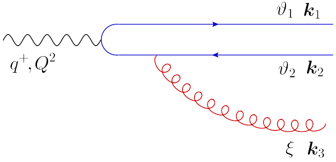
$$M^2_{q\bar{q}} = \frac{P^2_{\perp}}{\vartheta_1 \vartheta_2} \sim Q^2$$

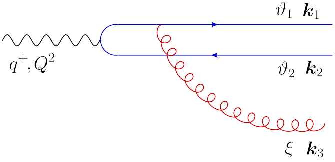
$$M^2_{q\bar{q}g} = M^2_{q\bar{q}} + \frac{k^2_{3\perp}}{\xi}$$

- $x_{\mathbb{P}}$: fraction of the target P^-_N transferred to $q\bar{q}g$ system (by the Pomeron)
- $x_{q\bar{q}} \sim x_{\rm Bj}$: the respective quantity for the hard $q\bar{q}$ dijets
- $x \equiv x_{q\bar{q}}/x_{\mathbb{P}}$: gluon splitting fraction w.r.t. the Pomeron (a.k.a. β)
- The most interesting situation: $\xi \sim \frac{Q_s^2}{Q^2} \ll 1 \implies M^2_{q\bar{q}g} \sim Q^2$ & $x \sim 1$
  - $\Delta Y \ll Y_{\mathbb{P}} \Longrightarrow$ maximal value for the gap $Y_{\mathbb{P}}$, hence for $Q_s^2(Y_{\mathbb{P}})$

Gluon dipole wavefunction (1)

- Two diagrams: gluon emission by the quark and by the antiquark





$$\mathcal{A}^{lj} = \left[\frac{k_1^l\left(k_3^j + \frac{\xi}{1-\vartheta_1}k_1^j\right)}{k_{1\perp}^2 + \bar{Q}^2} + \frac{k_2^l\left(k_3^j + \frac{\xi}{1-\vartheta_2}k_2^j\right)}{k_{2\perp}^2 + \bar{Q}^2}\right]\frac{1}{k_{3\perp}^2 + \mathcal{M}^2}$$

- $\mathcal{M}^2 \equiv \xi\left(\frac{k_{1\perp}^2}{\vartheta_1} + \frac{k_{2\perp}^2}{\vartheta_2} + Q^2\right)$: effective virtuality of the gluon emission

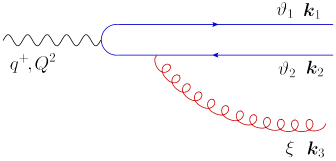
(Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans, 2016; G. Beuf, '17;

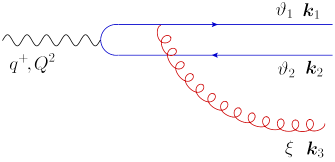
Caucal, Salazar, Venugopalan, 2021; Taels, Altinoluk, Beuf, Marquet, 2022; Y. Mulian and E.I., to appear...)

- Both the gluon vertex and the energy denominators violate factorisation
  - mixing of hard ($k_{1\perp}$, $k_{2\perp}$, ϑ_1, ϑ_2) and soft ($k_{3\perp}$, ξ) variables

Gluon dipole wavefunction (2)

- Two diagrams: gluon emission by the quark and by the antiquark



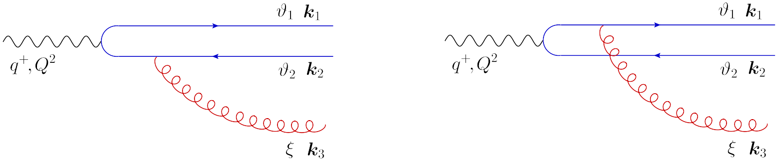


$$\mathcal{A}^{lj} = \left[\frac{k_1^l\left(k_3^j + \frac{\xi}{1-\vartheta_1}k_1^j\right)}{k_{1\perp}^2 + \bar{Q}^2} + \frac{k_2^l\left(k_3^j + \frac{\xi}{1-\vartheta_2}k_2^j\right)}{k_{2\perp}^2 + \bar{Q}^2}\right]\frac{1}{k_{3\perp}^2 + \mathcal{M}^2}$$

- Change variables: $\boldsymbol{k}_1,\, \boldsymbol{k}_2 \;\longrightarrow\; \boldsymbol{P} \equiv \vartheta_2\boldsymbol{k}_1 - \vartheta_1\boldsymbol{k}_2, \quad \boldsymbol{K} \equiv \boldsymbol{k}_1 + \boldsymbol{k}_2 = -\boldsymbol{k}_3$
- Expand to first order in $K_\perp/P_\perp \ll 1$ and $\xi \sim \frac{Q_s^2}{Q^2} \ll 1$
  - zeroth order terms cancel in the sum
- Add contributions from graphs with instantaneous quark propagators

Gluon dipole wavefunction (3)

- The final result is remarkably simple:



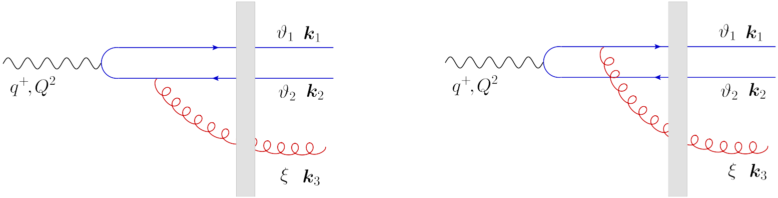
$$\mathcal{A}^{lj} = \underbrace{\frac{1}{P_\perp^2 + \bar{Q}^2}\left(\delta^{li} - \frac{2P^l P^i}{P_\perp^2 + \bar{Q}^2}\right)}_{\text{hard factor}} \underbrace{\frac{K_\perp^i K_\perp^j - (\delta^{ij}/2)K_\perp^2}{K_\perp^2 + \mathcal{M}^2}}_{\text{gluon dipole wavefunction}}$$

$$\mathcal{M}^2 = \xi\left(\frac{P_\perp^2}{\vartheta_1\vartheta_2} + Q^2\right) = \frac{x}{1-x}K_\perp^2$$

- Similar expression proposed by Wüsthoff in 1997
  - gluon contribution to the diffractive structure function
- Factorisation holds only when using the target rapidity: x rather than ξ

Gluon dipole wavefunction (4)

- Adding the scattering is simple: multiply with the gg dipole amplitude



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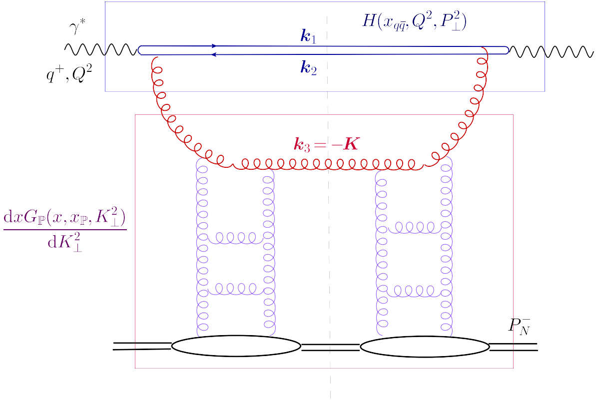
- A Bessel transform of the gluon dipole scattering amplitude:

$$\mathcal{G}(x, K_\perp, Y_{\mathbb{P}}) = \mathcal{M}^2 \int_0^\infty \mathrm{d}R\, R\, \mathrm{J}_2(K_\perp R)\mathrm{K}_2(\mathcal{M}R)\, \mathcal{T}_{gg}(R, Y_{\mathbb{P}})$$

  - $\mathrm{J}_2(K_\perp R)$ reflects the tensor structure; $\mathrm{K}_2(\mathcal{M}R)$: the virtuality

TMD factorisation for diffractive dijets

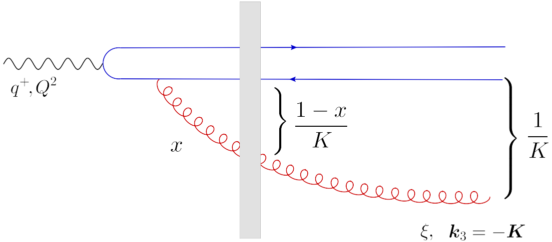
- We have proven collinear (TMD) factorisation, with an explicit result for the Pomeron gluon distribution obtained from first principles



$$\frac{\mathrm{d}\, xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} \propto \left[\mathcal{G}(x, K_\perp, Y_{\mathbb{P}})\right]^2$$

$$\frac{\mathrm{d}\, xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} \simeq (1-x)\begin{cases} 1, & K_\perp \lesssim \tilde{Q}_s(x) \\ \dfrac{\tilde{Q}_s^4(x)}{K_\perp^4}, & K_\perp \gg \tilde{Q}_s(x) \end{cases}$$

- Effective (x-dependent) saturation momentum: $\tilde{Q}_s^2(x, Y_{\mathbb{P}}) = (1-x)Q_s^2(Y_{\mathbb{P}})$
  - the gluon dipole size R is limited by the virtuality: $R \lesssim 1/\mathcal{M}$



- Very fast decrease $\sim 1/K_\perp^4$ at large gluon momenta $K_\perp \gg \tilde{Q}_s(x)$
  - the cross-section is controlled by the Black Disk Limit

The Pomeron UGD

$$\frac{\mathrm{d}\, xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} \simeq (1-x) \begin{cases} 1, & K_\perp \lesssim \tilde{Q}_s(x) \\ \dfrac{\tilde{Q}_s^4(x)}{K_\perp^4}, & K_\perp \gg \tilde{Q}_s(x) \end{cases}$$

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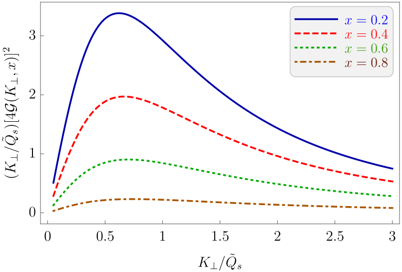
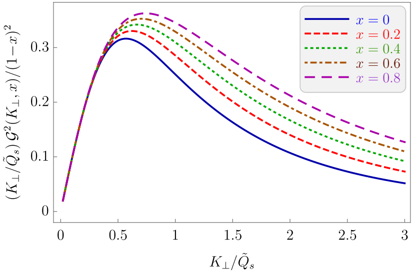




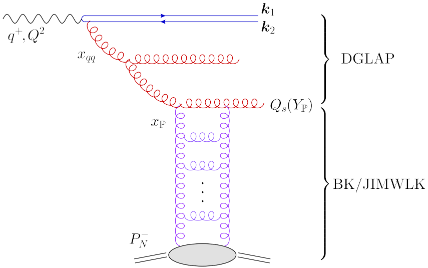
- Very fast decrease $\sim 1/K_\perp^4$ at large gluon momenta $K_\perp \gg \tilde{Q}_s(x)$
- Geometric scaling after dividing through $1-x$: a function of $K_\perp/\tilde{Q}_s(x)$

The integrated gluon distribution of the Pomeron

- Sensitivity to saturation persists after integrating out the $K_\perp$–distribution
  - the integral is rapidly converging and effectively cut off at $K_\perp \sim \tilde{Q}_s(x)$

$$xG_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2) \;\propto\; (1-x)^2\, Q_s^2(Y_{\mathbb{P}})$$

- Why is this interesting ?
  - eliminates the Sudakov effect (final state radiation)
  - introduces the DGLAP evolution: powers of $\alpha_s \ln \frac{P_\perp^2}{Q_s^2}$



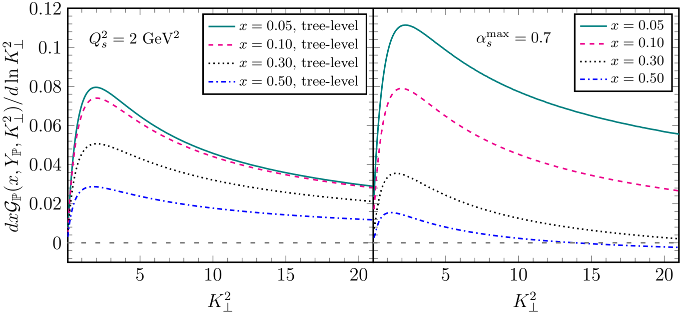
- BK evolution of the Pomeron
- DGLAP evolution with initial condition provided by saturation

- A unique situation where DGLAP and BK/JIMWLK can be interconnected

Recent results including DGLAP

(E.I., A.H. Mueller, D.N. Triantafyllopoulos, S.-Y. Wei, in preparation)

- Initial condition taken from the MV model (adding BK in project)
- The unintegrated gluon distribution (multiplied by $K_\perp^2$)



- left plot: the initial condition (MV)
- right plot: DGLAP evolution preserves the sensitivity to saturation
- a pronounced peak at $K_\perp^2 \simeq (1-x)Q_s^2$

Conclusions

- A new, hard, process to study gluon saturation in DIS
- Strongly sensitive to saturation since relying on elastic scattering
- TMD factorisation for diffractive jets emerging from the Color Dipole Picture
- The experimental measurement of the gluon-gluon dipole would be a bonus !
- Saturation remains important if the dijet imbalance is not measured
  - initial condition for DGLAP evolution emerging from first principles
- Open problems: adding BK (easy), including Sudakov (difficult), phenomenology, feasibility at the EIC, LHeC ...
- Similar processes might be interesting in ultraperipheral pA and AA