



NATIONAL CENTRE
FOR NUCLEAR RESEARCH
ŚWIERK

DIS dijet production beyond eikonal accuracy

Arantxa Tymowska

for DIS2022, Santiago de Compostela, Spain.

03/05/2022

Work in progress in collaboration with Tolga Altinoluk, Guillaume Beuf and Alina Czajka

Color Glass Condensate (I)

- Regge-Gribov limit: $x \rightarrow 0$
- At small $x \rightarrow$ saturation
- In this regime we use the **Color Glass Condensate** to describe a scattering process

The interaction between projectile and the target: each parton coming from the projectile picks up a Wilson line during the interaction

$$\mathcal{U}_R(x) = \mathcal{P}_+ \exp \left[ig \int dx^+ T_R^a A_a^-(x^+, x_\perp) \right]$$

Color Glass Condensate (II)

CGC formalism used for dilute-dense scattering so we apply Semi-classical approximation

- Dense target: classical background field $A_a^\mu(x)$
- Dilute projectile: virtual photon treated in perturbation theory

Color Glass Condensate (II)

CGC formalism used for dilute-dense scattering so we apply Semi-classical approximation

- Dense target: classical background field $A_a^\mu(x)$
- Dilute projectile: virtual photon treated in perturbation theory

We also adopt Eikonal approximation which amounts to taking the high energy limit $s \rightarrow \infty$. Beyond eikonal limit give corrections of order $1/s$. We can obtain this limit boosting the target in following way:

$$A_a^\mu(x) \rightarrow \begin{cases} \gamma_t A_a^-(\gamma_t x^+, \frac{1}{\gamma_t} x^-, x_\perp) \\ \frac{1}{\gamma_t} A_a^+(\gamma_t x^+, \frac{1}{\gamma_t} x^-, x_\perp) \\ A_a^i(\gamma_t x^+, \frac{1}{\gamma_t} x^-, x_\perp) \end{cases}$$

Eikonal approximation

The Eikonal approximation is given by:

- $A_a^\mu(x) \propto \delta(x^+)$

Eikonal approximation

The Eikonal approximation is given by:

- $A_a^\mu(x) \propto \delta(x^+)$
- $A_a^\mu(x) \simeq \delta^{\mu-} A_a^-(x)$

Eikonal approximation

The Eikonal approximation is given by:

- $A_a^\mu(x) \propto \delta(x^+)$
- $A_a^\mu(x) \simeq \delta^{\mu-} A_a^-(x)$
- $A_a^\mu(x) \simeq A_a^\mu(x^+, \vec{x})$

Eikonal approximation

The Eikonal approximation is given by:

- $A_a^\mu(x) \propto \delta(x^+)$
- $A_a^\mu(x) \simeq \delta^{\mu-} A_a^-(x)$
- $A_a^\mu(x) \simeq A_a^\mu(x^+, \vec{x})$

In order to get the full next-to eikonal corrections we include:

- Target with finite width: transverse motion of the parton within the medium

Eikonal approximation

The Eikonal approximation is given by:

- $A_a^\mu(x) \propto \delta(x^+)$
- $A_a^\mu(x) \simeq \delta^{\mu-} A_a^-(x)$
- $A_a^\mu(x) \simeq A_a^\mu(x^+, \vec{x})$

In order to get the full next-to eikonal corrections we include:

- Target with finite width: transverse motion of the parton within the medium
- Interactions with the perpendicular component of the field
[T. Altinoluk, G. Beuf, A. Czajka, A. Tymowska \(2021\) \[arXiv:2012.03886\]](#) see also
[G.A. Chirilli \[arXiv:1807.11435\]](#), [\[arxiv:2101.12744\]](#)

Eikonal approximation

The Eikonal approximation is given by:

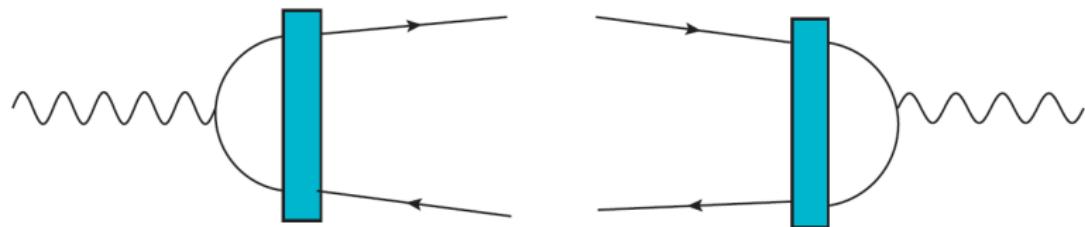
- $A_a^\mu(x) \propto \delta(x^+)$
- $A_a^\mu(x) \simeq \delta^{\mu-} A_a^-(x)$
- $A_a^\mu(x) \simeq A_a^\mu(x^+, \vec{x})$

In order to get the full next-to eikonal corrections we include:

- Target with finite width: transverse motion of the parton within the medium
- Interactions with the perpendicular component of the field
T. Altinoluk, G. Beuf, A. Czajka, A. Tymowska (2021) [[arXiv:2012.03886](#)] see also
G.A. Chirilli [[arXiv:1807.11435](#)], [[arxiv:2101.12744](#)]
- Taking into account x^- -dependence
T. Altinoluk, G. Beuf (2021) [[arXiv:2109.01620](#)]

Inclusive DIS dijet production

We want to calculate DIS dijet production at next-to eikonal order for the inclusive case

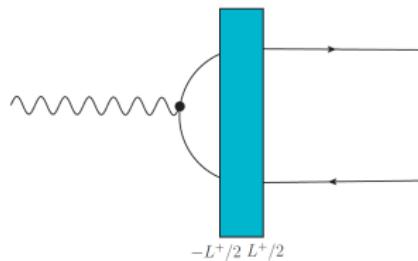


The process will be one of the focus for future EIC experiments.

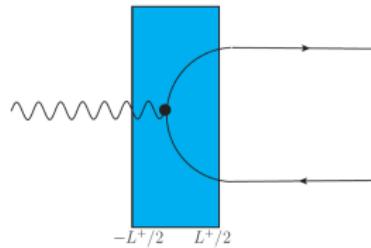
Lower energies at EIC compared to LHC \rightarrow NEik corrections

Diagrams for DIS dijet production at NEik

We need contributions from photon splitting into quark-antiquark pair
First diagrams contributes at both eikonal and next-to eikonal order



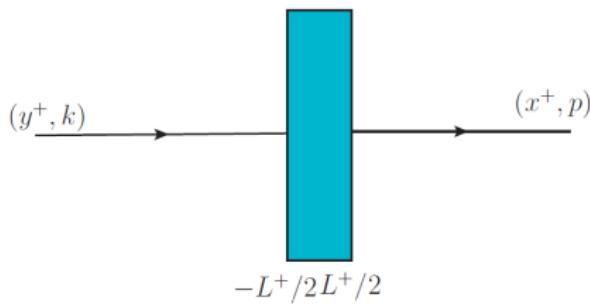
Second diagram contributes only at next-to eikonal order and vanishes when taking the longitudinal polarization of the photon.



Quark propagator at NEik order

The quark propagator through the whole medium is given by:

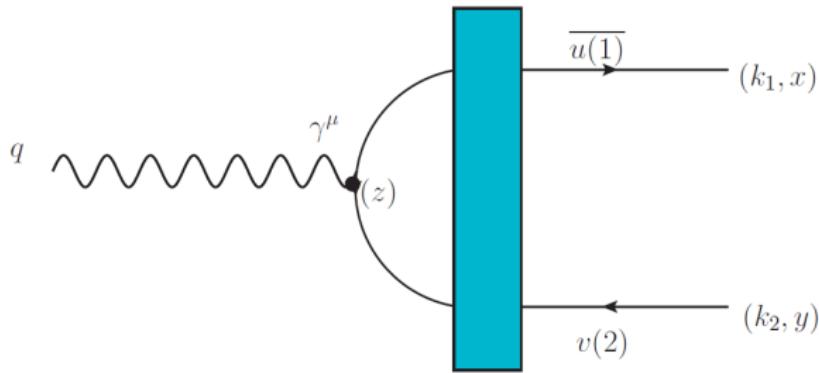
$$S_F(x, y) = \int \frac{d^3 p}{(2\pi)^3} \int \frac{d^3 k}{(2\pi)^3} \theta(p^+) \theta(k^+) e^{-ix \cdot p} e^{iy \cdot k} \int dz^- e^{iz^- (p^+ - k^+)} \int d^2 z e^{-iz \cdot (p - k)} \times \frac{(\not{p} + m)}{2p^+} \gamma^+ \left\{ \mathcal{U}_F\left(\frac{L^+}{2}, -\frac{L^+}{2}; z, z^-\right) \right. \\ - \frac{(p^j + k^j)}{2(p^+ + k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\left(\frac{L^+}{2}, z^+; z, z^-\right) \overleftrightarrow{\mathcal{D}_{z^+}} \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; z, z^-\right) \right] \\ - \frac{i}{(p^+ + k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \left[\mathcal{U}_F\left(\frac{L^+}{2}, z^+; z, z^-\right) \overleftarrow{\mathcal{D}_{z^+}} \overrightarrow{\mathcal{D}_{z^+}} \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; z, z^-\right) \right] \\ \left. + \frac{[\gamma^i, \gamma^j]}{4(p^+ + k^+)} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \mathcal{U}_F\left(\frac{L^+}{2}, z^+; z, z^-\right) g t \cdot \mathcal{F}_{ij}(z) \mathcal{U}_F\left(z^+, -\frac{L^+}{2}; z, z^-\right) \right\} \frac{(\not{k} + m)}{2k^+}$$



LSZ reduction formula

S -matrix element for the virtual photon splitting into a quark-antiquark pair is

$$S_{q_1 \bar{q}_2 \leftarrow \gamma^*} = -iee_f \frac{Q}{q^+} g_\mu^+ \lim_{x^+, y^+ \rightarrow \infty} \int d^2 x \int dx^- \int d^2 y \int dy^- \int d^4 z \\ \times e^{-iq \cdot z} e^{ik_1 \cdot x} e^{ik_2 \cdot y} \bar{u}(1) \gamma^+ S_F(x, z)_{\beta\alpha}(-\gamma^\mu) S_F(z, y)_{\alpha\delta} \gamma^+ v(2)$$



Amplitudes

We can write the amplitude divided in three contributions.

First the generalized Eikonal contribution where we don't have momentum conservation compared to eikonal because of common b^- in the Wilson lines

$$\begin{aligned} \mathcal{S}_{q_1 \bar{q}_2 \leftarrow \gamma^*}^{L-Eik} = & -\frac{ee_f Q}{2\pi} \delta_{h_2, -h_1} \theta(q^+ + k_1^+ - k_2^+) \theta(q^+ - k_1^+ + k_2^+) \int db^- e^{ib^-(k_1^+ + k_2^+ - q^+)} \\ & \times \int_{v,w} e^{-iv \cdot k_1} e^{-iw \cdot k_2} \frac{\sqrt{k_1^+ k_2^+}}{(q^+)^2} (q^+ + k_1^+ - k_2^+) (q^+ - k_1^+ + k_2^+) K_0(\hat{Q}|v-w|) (\mathcal{U}_F(v, b^-) \mathcal{U}_F^\dagger(w, b^-) - 1) \end{aligned}$$

and two next-to eikonal contributions where we divide them in helicity-dependent and helicity-independent. The helicity-dependent term is:

$$\begin{aligned} i\mathcal{M}_{q_1 \bar{q}_2 \leftarrow \gamma^*}^{L-NEik-Hdep} = & \frac{iee_f Q}{2\pi} \delta_{h_2, -h_1} \theta(k_1^+) \theta(k_2^+) \times \int_{v,w} e^{-iv \cdot k_1} e^{-iw \cdot k_2} \frac{2\sqrt{k_1^+ k_2^+}}{(q^+)^2} \left\{ K_0(\bar{Q}|v-w|) \right. \\ & \times \left[k_2^+ \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \mathcal{U}_F\left(\frac{L^+}{2}, v^+; v\right) \left(\textcolor{red}{h_1} \epsilon^{ij} g t \cdot \mathcal{F}_{ij}(v) \right) \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; v\right) \mathcal{U}_F^\dagger(w) \right. \\ & \left. + k_1^+ \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dw^+ \mathcal{U}_F(v) \mathcal{U}_F^\dagger\left(w^+, -\frac{L^+}{2}; w\right) \left(-\textcolor{red}{h_2} \epsilon^{ij} g t \cdot \mathcal{F}_{ij}(w) \right) \mathcal{U}_F\left(\frac{L^+}{2}, w^+; w\right) \right] \right\} \end{aligned}$$

$$\hat{Q} = \sqrt{m^2 + Q^2} \frac{(q^+ + k_1^+ - k_2^+) (q^+ - k_1^+ + k_2^+)}{4(q^+)^2}$$

$$\bar{Q} = \sqrt{m^2 + Q^2} \frac{k_1^+ k_2^+}{(q^+)^2}$$

Amplitudes

And the helicity-independent term is

$$\begin{aligned} i\mathcal{M}_{q_1\bar{q}_2 \leftarrow \gamma^*}^{\text{L-NEik-Hind}} &= \frac{ieef Q}{2\pi} \delta_{h_2, -h_1} \theta(k_1^+) \theta(k_2^+) \int_{v,w} e^{-iv \cdot k_1} e^{-iw \cdot k_2} \frac{2\sqrt{k_1^+ k_2^+ k_1^- k_2^-}}{(q^+)^2} \times \left\{ K_0(\bar{Q}|v-w|) \right. \\ &\times \left(+ \frac{L^+}{2} \left(\frac{1}{k_1^+} \Delta_{v^j} + \frac{1}{k_2^+} \Delta_{w^j} \right) \mathcal{U}_F(v) \mathcal{U}_F^\dagger(w) - \frac{1}{k_1^+} \mathcal{U}_F^{(2)}(v) \mathcal{U}_F^\dagger(w) - \frac{1}{k_2^+} \mathcal{U}_F(v) \mathcal{U}_F^{(2)}(w) \right) \\ &+ \left(-K_0(\bar{Q}|v-w|) \left(\frac{1}{k_1^+} - \frac{1}{k_2^+} \right) + 2(k_2^+ - k_1^+) \frac{Q^2}{(q^+)^2} |v-w| K_1(\bar{Q}|v-w|) \right) \\ &\times \left. \left(\mathcal{U}_F(w, b^-) \overleftrightarrow{\partial}_- \mathcal{U}_F^\dagger(v, b^-) \right) \Big|_{b^- = 0} \right\} \end{aligned}$$

$$\mathcal{U}_F^{(2)}(v) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \mathcal{U}_F\left(\frac{L^+}{2}, v^+; v\right) \overrightarrow{\mathcal{D}_{v^j}} \overrightarrow{\mathcal{D}_{v^j}} \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; v\right)$$

We see we have some new next-to eikonal corrections that come from the x^- dependence that were not considered before

Cross Section

We have the cross section for the generalized Eikonal part:

$$\frac{d\sigma_{\gamma_L^* \rightarrow q\bar{q}}^{gEik}}{dP.S.(1,2)} = \frac{1}{2q^+} \int db^- e^{ib^-(k_1^+ + k_2^+ - q^+)} \left\langle M^\dagger\left(\frac{b^-}{2}\right) M\left(-\frac{b^-}{2}\right) \right\rangle$$

where

$$\begin{aligned} \left\langle M^\dagger\left(\frac{b^-}{2}\right) M\left(-\frac{b^-}{2}\right) \right\rangle &= 2 \left(\frac{ee_F Q}{2\pi} \right)^2 \theta(q^+ + k_1^+ - k_2^+) \theta(q^+ - k_1^+ + k_2^+) \\ &\int_{v',w',v,w} e^{ik_1 \cdot (v' - v)} e^{ik_2 \cdot (w' - w)} \frac{k_1^+ k_2^+}{(q^+)^4} (q^+ + k_1^+ - k_2^+)^2 (q^+ - k_1^+ + k_2^+)^2 \\ &\times K_0(\hat{Q}|v' - w'|) K_0(\hat{Q}|v - w|) \left\langle \frac{1}{N_c} \text{Tr} \left[\left(U_F\left(w', \frac{b^-}{2}\right) U_F^\dagger\left(v', \frac{b^-}{2}\right) - 1 \right) \left(U_F\left(v, -\frac{b^-}{2}\right) U_F^\dagger\left(w, -\frac{b^-}{2}\right) - 1 \right) \right] \right\rangle \end{aligned}$$

- Generalized version of eikonal results which include non trivial longitudinal momentum exchange with the target due to the b^- dependence of the Wilson lines

Cross Section contribution at NEik (I)

$$\frac{d\sigma^{NEik}}{dP.S.(1,2)} = \frac{1}{2q^+} 2\pi \delta(k_1^+ + k_2^+ - q^+) \left[\left\langle M_{Eik}^\dagger M_{NEik} \right\rangle + \left\langle M_{NEik}^\dagger M_{Eik} \right\rangle \right]$$

$$= \frac{1}{2q^+} 2\pi \delta(k_1^+ + k_2^+ - q^+) \left[\left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{kinematic} + \left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{NEik-genuine} + \left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{x^--dep} + cc \right]$$

$$\left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{kinematic} = 16 \left(\frac{e e_f Q}{2\pi} \right)^2 \theta(k_1^+) \theta(k_2^+) \int_{v', w', v, w} e^{i(v' - v) \cdot k_1} e^{i(w' - w) \cdot k_2} \mathcal{H}_{kinematic}$$

$$\times \left(Q(w', v', v, w) - d(v, w) - d(v', w') + 1 \right)$$

$$\mathcal{H}_{kinematic} = -\frac{L^+}{2} \frac{(k_1^+ k_2^+)^3}{(q^+)^4} \left[\frac{1}{k_1^+} \left(-k_1^2 - 2i k_1^j \partial_{v^j} + \Delta_{v^j} \right) + \frac{1}{k_2^+} \left(-k_2^2 - 2i k_2^j \partial_{w^j} + \Delta_{w^j} \right) \right]$$

$$\times K_0(\bar{Q}|v' - w'|) K_0(\bar{Q}|v - w|)$$

$$d(v, w) = \left\langle \frac{1}{N_c} \text{Tr} \left[\mathcal{U}_F(v) \mathcal{U}_F^\dagger(w) \right] \right\rangle$$

$$Q(w', v', v, w) = \left\langle \frac{1}{N_c} \text{Tr} \left[\mathcal{U}_F(w') \mathcal{U}_F^\dagger(v') \mathcal{U}_F(v) \mathcal{U}_F^\dagger(w) \right] \right\rangle$$

Cross Section contribution at NEik(II)

The cross section for the Next-to-Eikonal contribution is:

$$\begin{aligned} \frac{d\sigma_{\gamma_L^* \rightarrow q\bar{q}}^{NEik}}{dP.S.(1,2)} &= \frac{1}{2q^+} 2\pi\delta(k_1^+ + k_2^+ - q^+) \left[\left\langle M_{Eik}^\dagger M_{NEik} \right\rangle + \left\langle M_{NEik}^\dagger M_{Eik} \right\rangle \right] \\ &= \frac{1}{2q^+} 2\pi\delta(k_1^+ + k_2^+ - q^+) \left[\left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{\text{kinematic}} + \left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{\text{NEik-genuine}} + \left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{x^- \text{-dep}} + cc \right] \end{aligned}$$

with

$$\left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{\text{NEik-genuine}} = 16 \left(\frac{ee_f Q}{2\pi} \right)^2 \theta(k_1^+) \theta(k_2^+) \int_{v', w', v, w} e^{i(v' - v) \cdot k_1} e^{i(w' - w) \cdot k_2} \mathcal{H}_{\text{NEik-genuine}}$$

$$\times \left[\frac{1}{k_1^+} \left(Q^{(2)}(w', v', v_*, w) - d^{(2)}(v_*, w) \right) + \frac{1}{k_2^+} \left(Q^{(2)}(w', v', v, w_*) - d^{(2)}(v, w_*) \right) \right]$$

$$\mathcal{H}_{\text{NEik-genuine}} = \frac{(k_1^+ k_2^+)^3}{(q^+)^4} K_0(\bar{Q}|v' - w'|) K_0(\bar{Q}|v - w|)$$

$$d^{(2)}(v_*, w) = \left\langle \frac{1}{N_c} \text{Tr} \left[\mathcal{U}_F^{(2)}(v) \mathcal{U}_F^\dagger(w) \right] \right\rangle$$

$$Q^{(2)}(w'_*, v', v, w) = \left\langle \frac{1}{N_c} \text{Tr} \left[\mathcal{U}_F^{(2)}(w') \mathcal{U}_F^\dagger(v') \mathcal{U}_F(v) \mathcal{U}_F^\dagger(w) \right] \right\rangle$$

Cross Section contribution at NEik (III)

$$\frac{d\sigma_{\gamma_L^* \rightarrow q\bar{q}}^{NEik}}{dP.S.(1,2)} = \frac{1}{2q^+} 2\pi\delta(k_1^+ + k_2^+ - q^+) \left[\left\langle M_{Eik}^\dagger M_{NEik} \right\rangle + \left\langle M_{NEik}^\dagger M_{Eik} \right\rangle \right]$$

$$= \frac{1}{2q^+} 2\pi\delta(k_1^+ + k_2^+ - q^+) \left[+ \left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{\text{kinematic}} + \left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{\text{NEik-genuine}} + \left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{x^- \text{-dep}} + cc \right]$$

$$\left\langle M_{Eik}^\dagger M_{NEik} \right\rangle_{x^- \text{-dep}} = 16 \left(\frac{ee_f Q}{2\pi} \right)^2 \theta(k_1^+) \theta(k_2^+) \int_{v', w', v, w} e^{i(v' - v) \cdot k_1} e^{i(w' - w) \cdot k_2} \mathcal{H}_{x^-}$$

$$\times \left(Q^{(1)}(w', v', v_*, w_*) - d^{(1)}(v_*, w_*) \right)$$

$$\mathcal{H}_{x^-} = \frac{(k_1^+ k_2^+)^3}{(q^+)^4} (k_2^+ - k_1^+) K_0(\bar{Q}|v' - w'|) \left(\frac{K_0(\bar{Q}|v - w|)}{k_1^+ k_2^+} + \frac{2Q^2|v - w|}{(q^+)^2} K_1(\bar{Q}|v - w|) \right)$$

$$d^{(1)}(v_*, w_*) = \left\langle \frac{1}{N_c} \text{Tr} \left[\left(\mathcal{U}_F(v, b^-) \overleftrightarrow{\partial}_- \mathcal{U}_F^\dagger(w, b^-) \right) \Big|_{b^- = 0} \right] \right\rangle$$

$$Q^{(1)}(w'_*, v'_*, v, w) = \left\langle \frac{1}{N_c} \text{Tr} \left[\left(\mathcal{U}_F(w', b^-) \overleftrightarrow{\partial}_- \mathcal{U}_F^\dagger(v', b^-) \right) \Big|_{b^- = 0} \mathcal{U}_F(v) \mathcal{U}_F^\dagger(w) \right] \right\rangle$$

Outlook

- We computed the cross section for the case of photon longitudinal polarization for DIS dijet production at full NEik order from the gluon background field
- Next-to eikonal corrections include:
 - Relaxing the shockwave approximation → transverse motion through the target
 - Including interactions with transverse component of the background field
 - Taking into account z^- -dependence → effects of longitudinal momentum exchange with the target
- Computation for the transverse polarization of the photon (in progress)
- NEik effect from the quark background field not yet included

Thank you for your attention