

Jet anisotropy and jet shape in ep collisions

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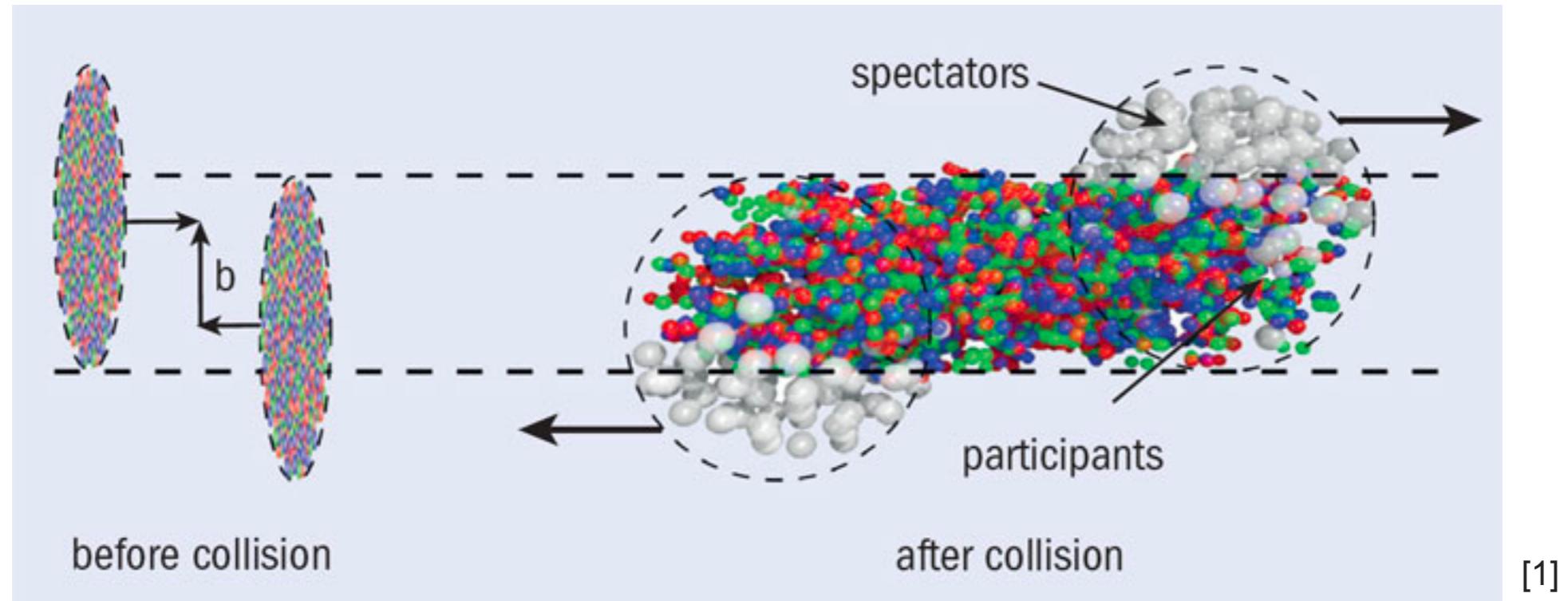


R. Esha, Z.-B. Kang, K. Lee,
D. Shao and FZ,
arXiv:2205.XXXX

May 4th, 2022

Motivation

- In heavy-ion collisions,



$$E \frac{d^3N}{d^3\mathbf{p}} = \frac{1}{2\pi} \frac{d^2N}{p_t dp_t dy} \left(1 + 2 \sum_{n=1}^{\infty} v_n \cos [n(\varphi - \Psi_{RP})] \right)$$

$$v_n(p_t, y) = \langle \cos[n(\varphi - \Psi_{RP})] \rangle$$

v_1 : directed flow

v_2 : elliptic flow

v_3 : triangular flow

v_4 : quadrupole flow

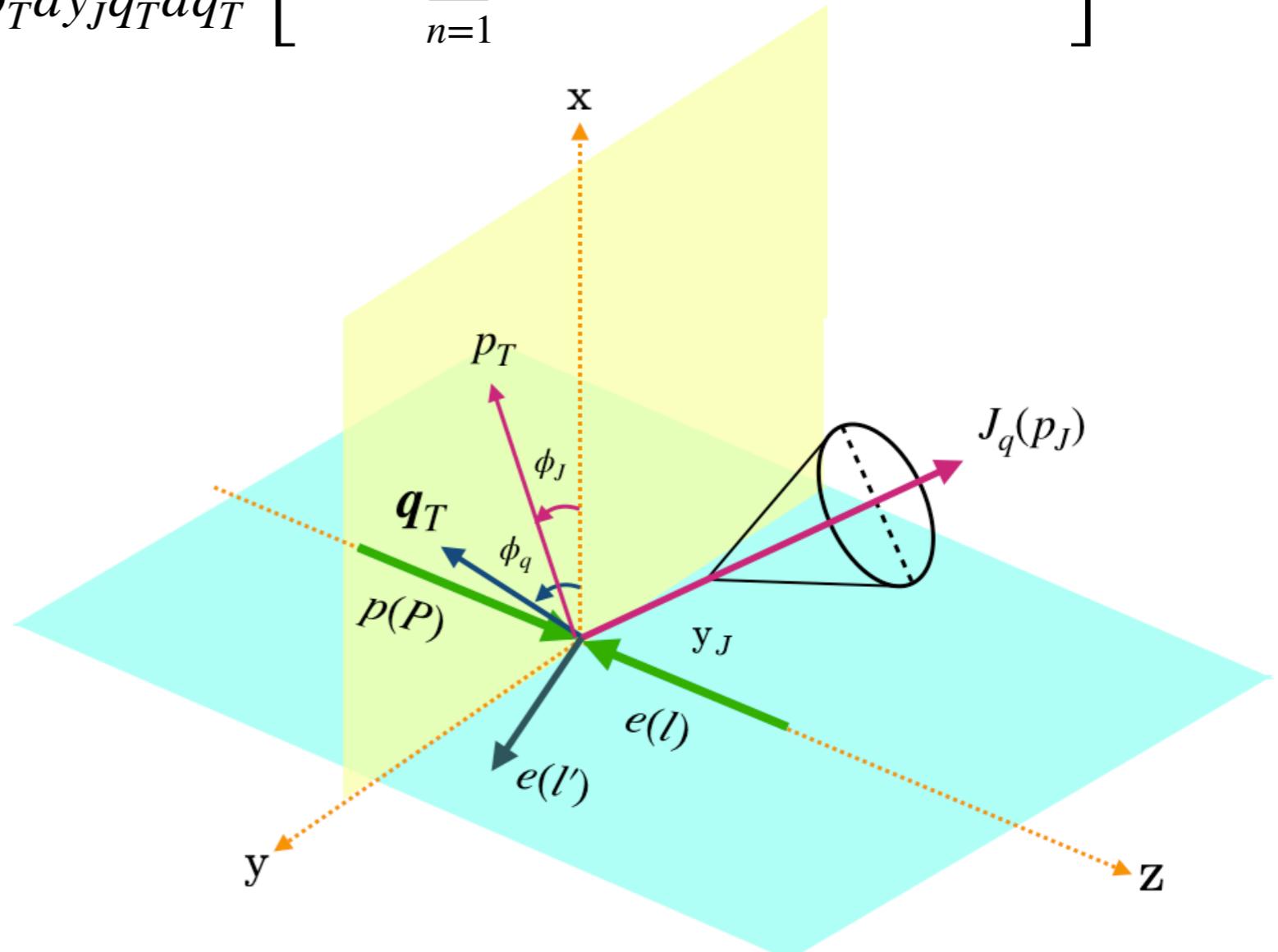
[1] Snellings, Raimond. "Elliptic flow: a brief review." *New Journal of Physics* 13.5 (2011): 055008.

Motivation

- Back-to-back electron-jet production from ep collision,

$$e(l) + p(P) \rightarrow e(l') + J_q(p_J) + X$$

$$\frac{d\sigma}{d^2\mathbf{p}_T dy_J d\phi_J d^2\mathbf{q}_T} = \frac{d\sigma}{2\pi d^2\mathbf{p}_T dy_J q_T dq_T} \left[1 + 2 \sum_{n=1}^{\infty} v_n(p_T, y_T) \cos(n(\phi_q - \phi_J)) \right]$$



Motivation

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$$e(l) + p(P) \rightarrow e(l') + J_q(p_J) + X$$

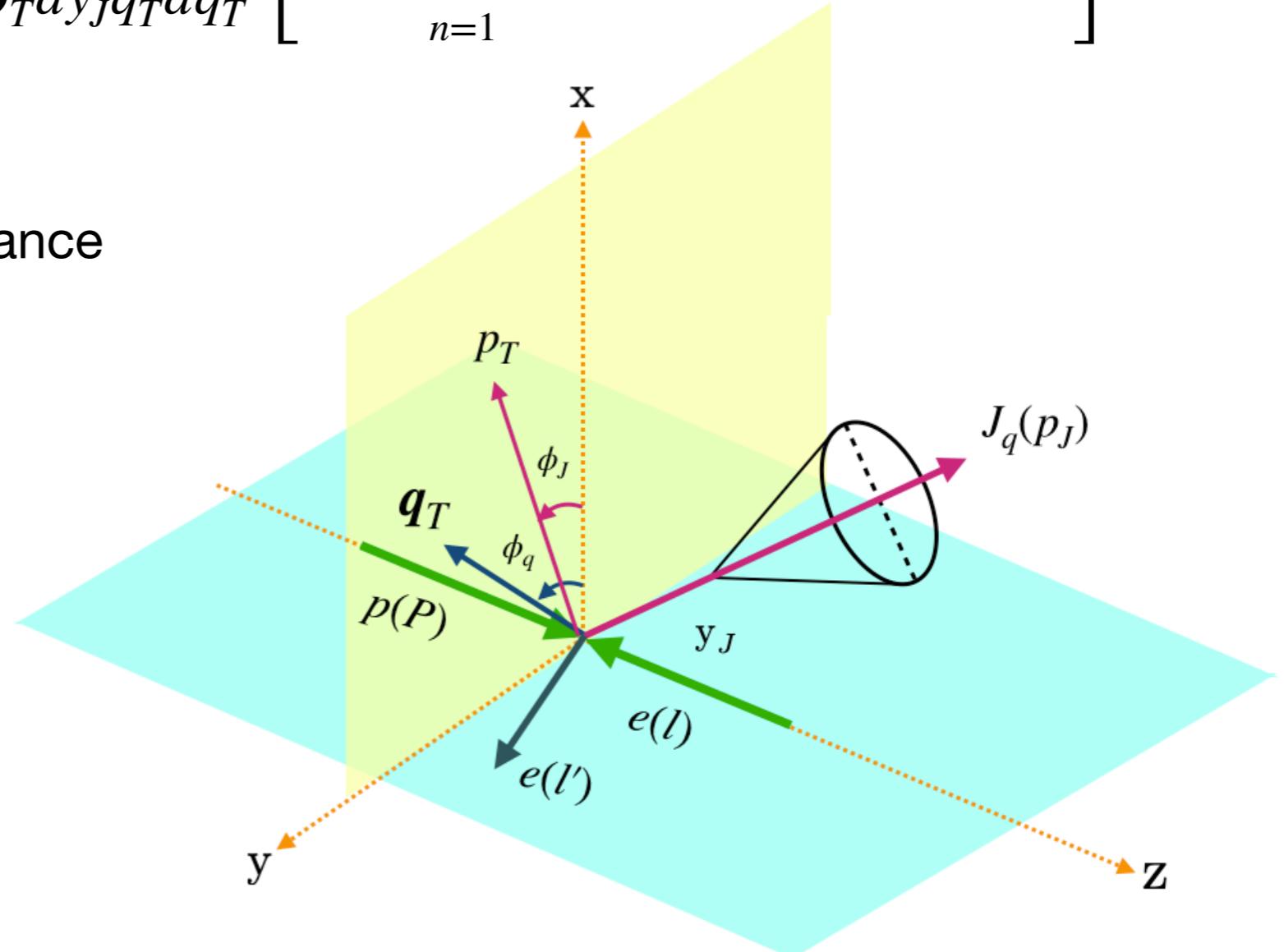
$$\frac{d\sigma}{d^2\mathbf{p}_T dy_J d\phi_J d^2\mathbf{q}_T} = \frac{d\sigma}{2\pi d^2\mathbf{p}_T dy_J q_T dq_T} \left[1 + 2 \sum_{n=1}^{\infty} v_n(p_T, y_T) \cos(n(\phi_q - \phi_J)) \right]$$

\mathbf{q}_T : transverse momentum imbalance

$$\mathbf{q}_T = \mathbf{l}'_T + \mathbf{p}_{JT}$$

p_T : jet transverse momentum

y_J : jet rapidity



Motivation

- Back-to-back electron-jet production from ep collision,

$$e(l) + p(P) \rightarrow e(l') + J_q(p_J) + X$$

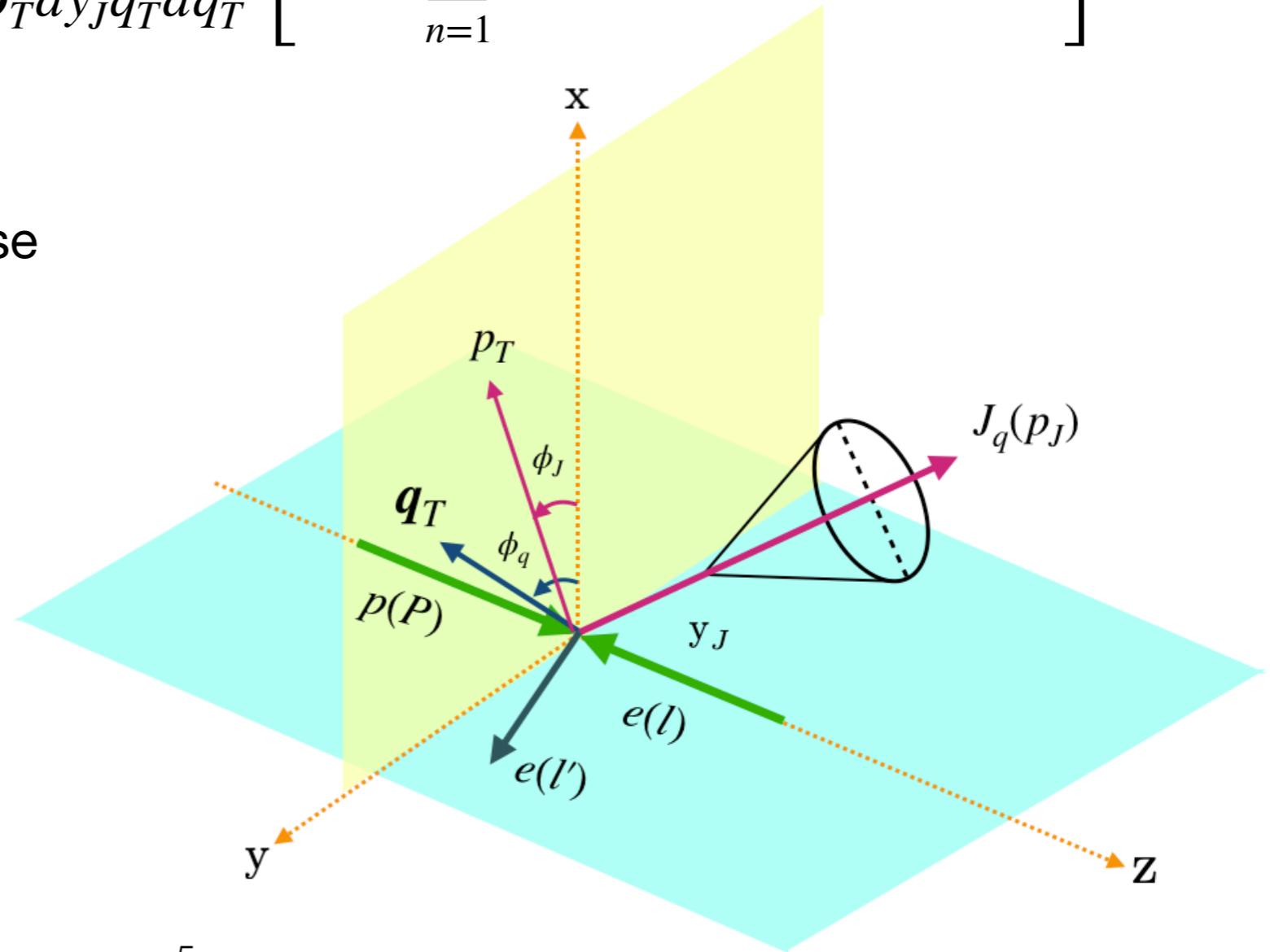
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ϕ_q : azimuthal angle of transverse momentum imbalance

ϕ_J : azimuthal angle of jet transverse momentum

v_n : anisotropic Fourier coefficients

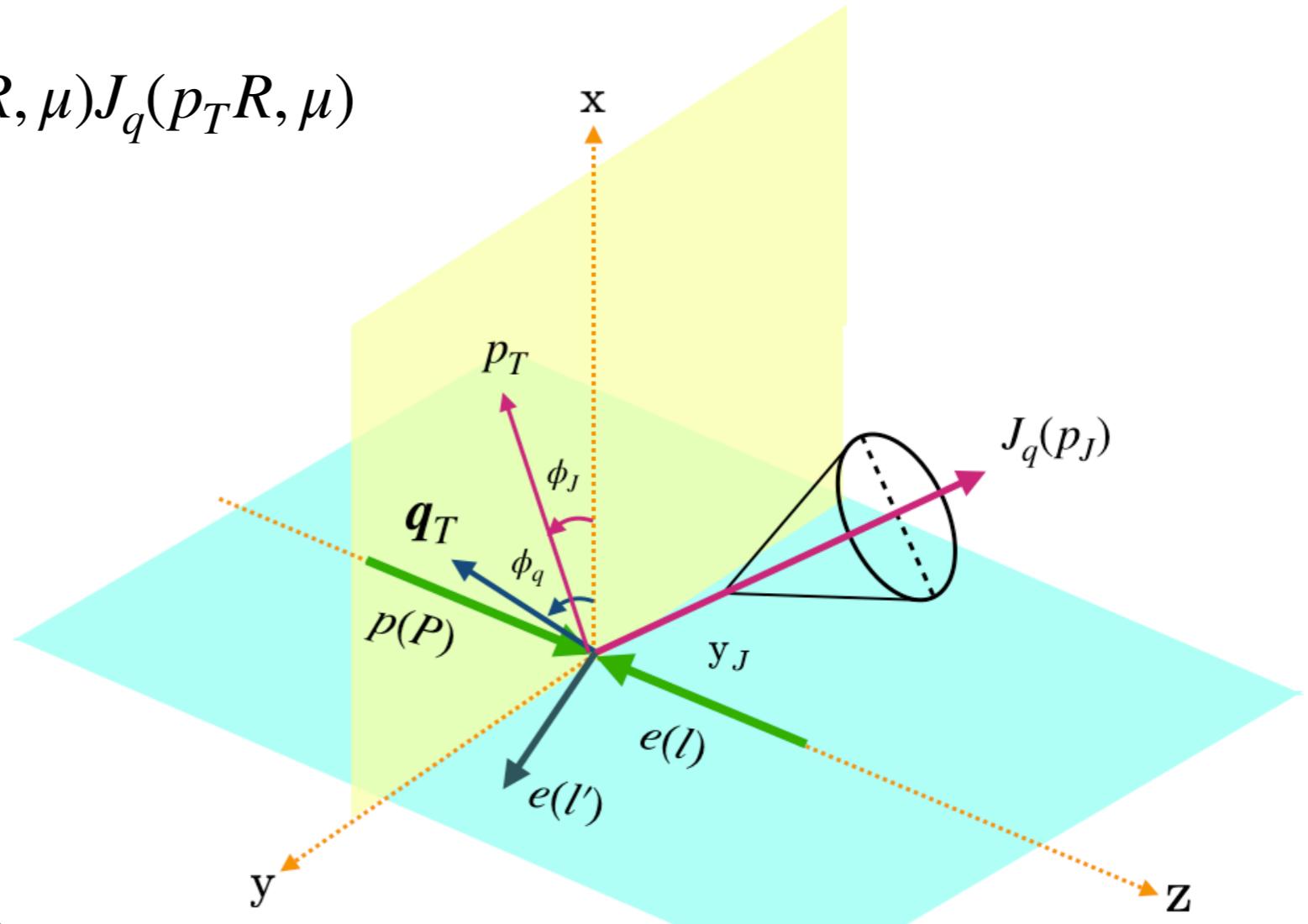
$$\sim \langle \cos(n(\phi_q - \phi_J)) \rangle$$



Theoretical framework

- At small $|q_T|$ limit, TMD factorization gives ^[2]

$$\frac{d\sigma^{e+p \rightarrow e+\text{jet}+X}}{d^2 p_T dy_J d\mathbf{q}_T} = \hat{\sigma}_0 \sum_q e_q^2 H(Q, \mu) \int \frac{d^2 \mathbf{b}}{(2\pi)^2} \exp(-i\mathbf{b} \cdot \mathbf{q}_T) x \tilde{f}_1(x, \mathbf{b}, \mu) \\ \times S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu) J_q(p_T R, \mu)$$



[2] Arratia, Miguel, et al. "Jet-based measurements of Sivers and Collins asymmetries at the future electron-ion collider." *Physical Review D* 102.7 (2020): 074015.

Jet anisotropy

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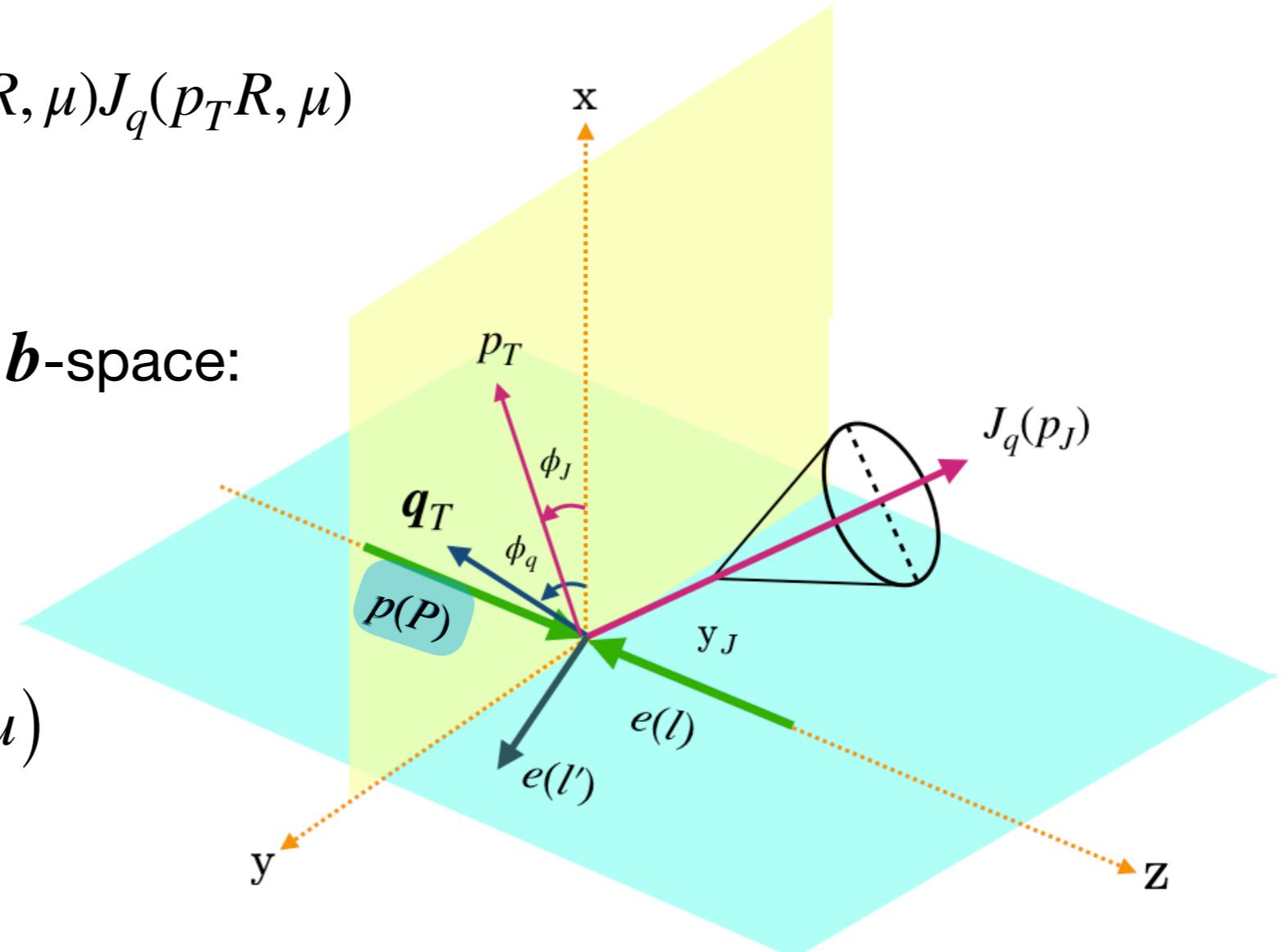
$$\frac{d\sigma^{e+p \rightarrow e+\text{jet}+X}}{d^2 p_T dy_J d\mathbf{q}_T} = \hat{\sigma}_0 \sum_q e_q^2 H(Q, \mu) \int \frac{d^2 \mathbf{b}}{(2\pi)^2} \exp(-i\mathbf{b} \cdot \mathbf{q}_T) x \tilde{f}_1(x, \mathbf{b}, \mu)$$

$$\times S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu) J_q(p_T R, \mu)$$

The unpolarized TMDPDF \tilde{f}_1 in \mathbf{b} -space:

$$\tilde{f}_1(x, \mathbf{b}, \mu)$$

$$= 2\pi \int dk_T k_T J_0(k_T b) f(x, k_T^2, \mu)$$



Jet anisotropy

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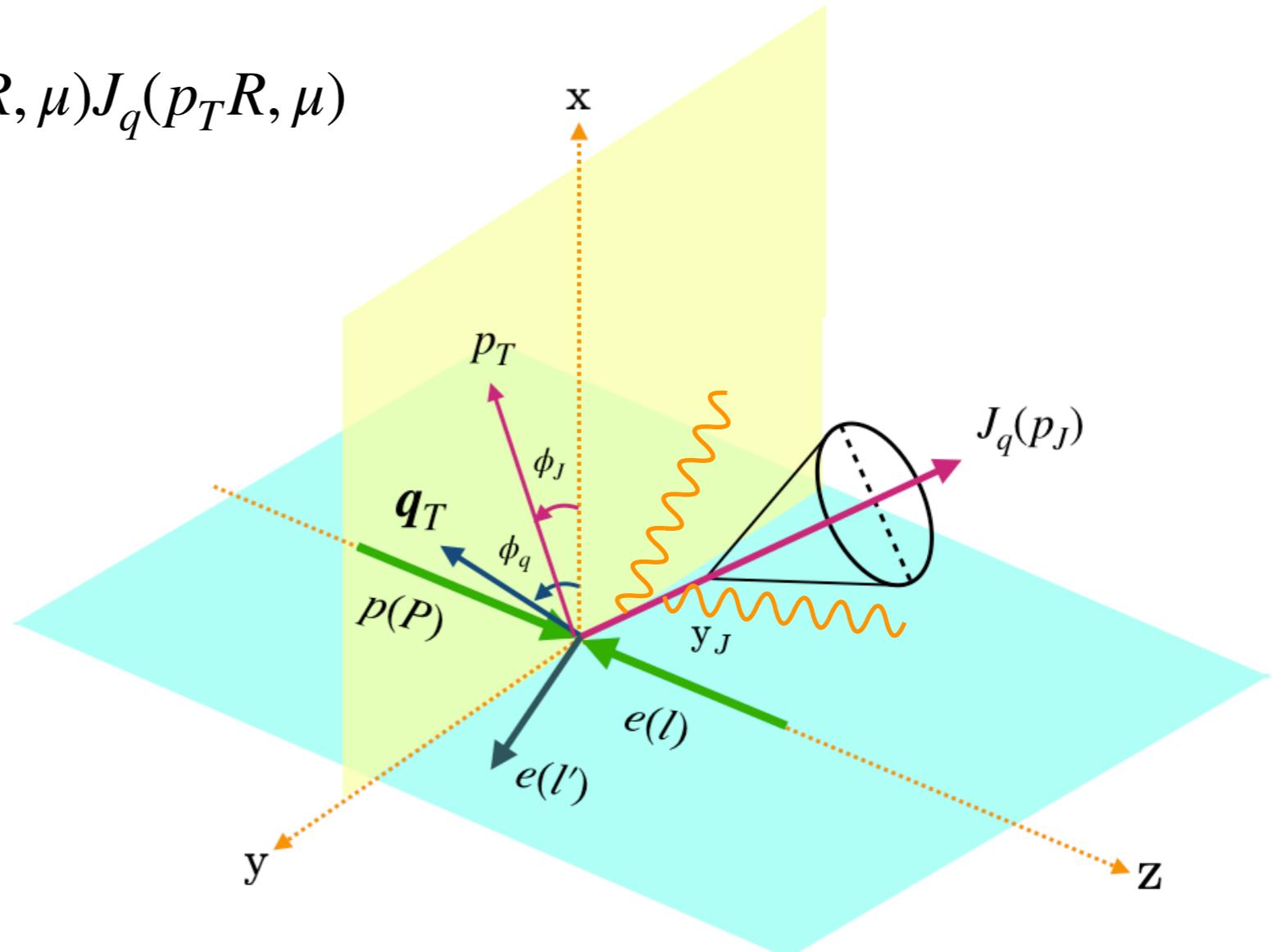
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$$\times S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu) J_q(p_T R, \mu)$$

The global soft function:

$$S_{\text{global}}(\mathbf{b}, \mu)$$

- Depends on the magnitude b and azimuthal angle ϕ_b of the vector \mathbf{b}



Jet anisotropy

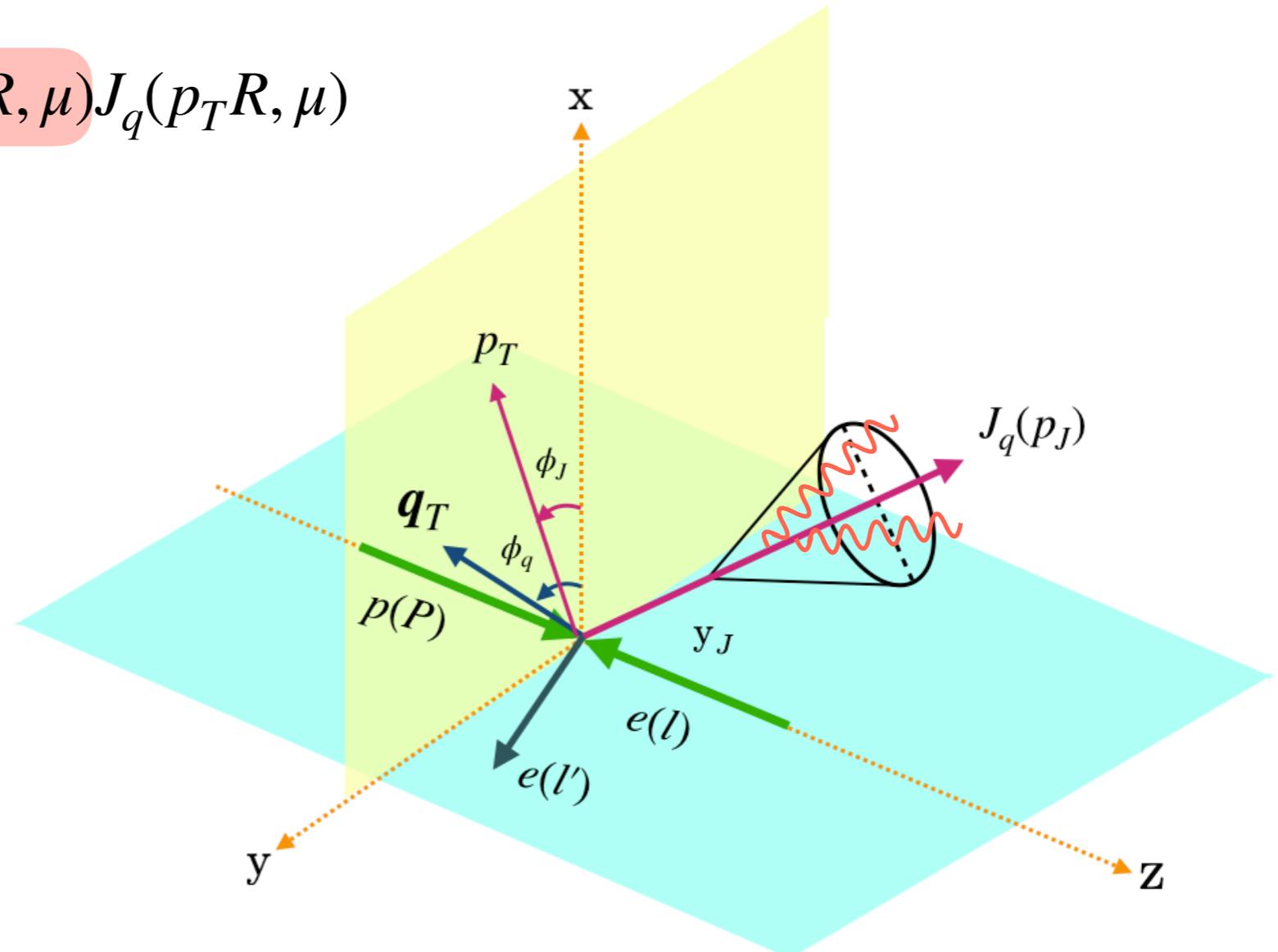
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The collinear-soft function:

$$S_{\text{cs}}(\mathbf{b}, R, \mu)$$

- Depends on the magnitude b and azimuthal angle ϕ_b of the vector \mathbf{b}
- Sensitive to the jet direction.



Jet anisotropy

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$$\times S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu) J_q(p_T R, \mu)$$

$$\phi_{bJ} = \phi_b - \phi_J$$

$$\phi_{qJ} = \phi_q - \phi_J$$



$$\frac{d^2 \mathbf{b}}{(2\pi)^2} \exp(-i\mathbf{b} \cdot \mathbf{q}_T) = \frac{bdb}{2\pi} \left\{ J_0(bq_T) \frac{d\phi_{bJ}}{2\pi} + 2 \sum_{n=1}^{\infty} (-i)^n J_n(bq_T) \cos(n\phi_{qJ}) \cos(n\phi_{bJ}) \frac{d\phi_{bJ}}{2\pi} \right\}$$

Jet anisotropy

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$$\times S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu) J_q(p_T R, \mu) \quad \downarrow \quad \begin{aligned} \phi_{bJ} &= \phi_b - \phi_J \\ \phi_{qJ} &= \phi_q - \phi_J \end{aligned}$$

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Average of the soft function:

$$\bar{S}_q(\mathbf{b}, R, \mu) = \int S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu) \frac{d\phi_{bJ}}{2\pi}$$

Jet anisotropy

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Average of the soft function:

$$\bar{S}_q(\mathbf{b}, R, \mu) = \int S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu) \frac{d\phi_{bJ}}{2\pi}$$

Average of the soft function weighted by $\cos(n\phi_{bJ})$:

$$S_q^{\langle \cos(n\phi_{qJ}) \rangle}(\mathbf{b}, R, \mu) = \int S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu) \cos(n\phi_{bJ}) \boxed{\frac{d\phi_{bJ}}{2\pi}}$$

Jet anisotropy

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$$\phi_{bJ} = \phi_b - \phi_J \\ \phi_{qJ} = \phi_q - \phi_J$$

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$$\times \left\{ J_0(bq_T) \bar{S}_q(\mathbf{b}, R, \mu) + 2 \sum_{n=1} J_n(bq_T) \left[\cos(n\phi_{qJ}) (-i)^n S_q^{\langle \cos(n\phi_{qJ}) \rangle}(\mathbf{b}, R, \mu) \right] \right\}$$

Jet anisotropy

- Azimuthal anisotropy of particle spectrum

$$v_n \sim \frac{\tilde{f}_1^{\text{TMD}}(x, \mathbf{b}, \mu) \otimes J_n(bq_T)(-i)^n S_q^{\langle \cos(n\phi_{qJ}) \rangle}(\mathbf{b}, R, \mu)}{\tilde{f}_1^{\text{TMD}}(x, \mathbf{b}, \mu) \otimes J_0(bq_T) \bar{S}_q(\mathbf{b}, R, \mu)}$$

⇒ Can be measured in experiments as the expectation of
the n -th order harmonics $v_n = \langle \cos(n\phi_{qJ}) \rangle$

$$\begin{aligned} \frac{d\sigma^{e+p \rightarrow e+\text{jet}+X}}{d^2 p_T dy_J d\mathbf{q}_T} &= \hat{\sigma}_0 \sum_q e_q^2 H(Q, \mu) \int \frac{bdb}{2\pi} x \tilde{f}_1(x, \mathbf{b}, \mu) J_q(p_T R, \mu) \\ &\times \left\{ J_0(bq_T) \bar{S}_q(\mathbf{b}, R, \mu) + 2 \sum_{n=1} \left[\cos(n\phi_{qJ}) (-i)^n S_q^{\langle \cos(n\phi_{qJ}) \rangle}(\mathbf{b}, R, \mu) \right] \right\} \end{aligned}$$

Jet anisotropy

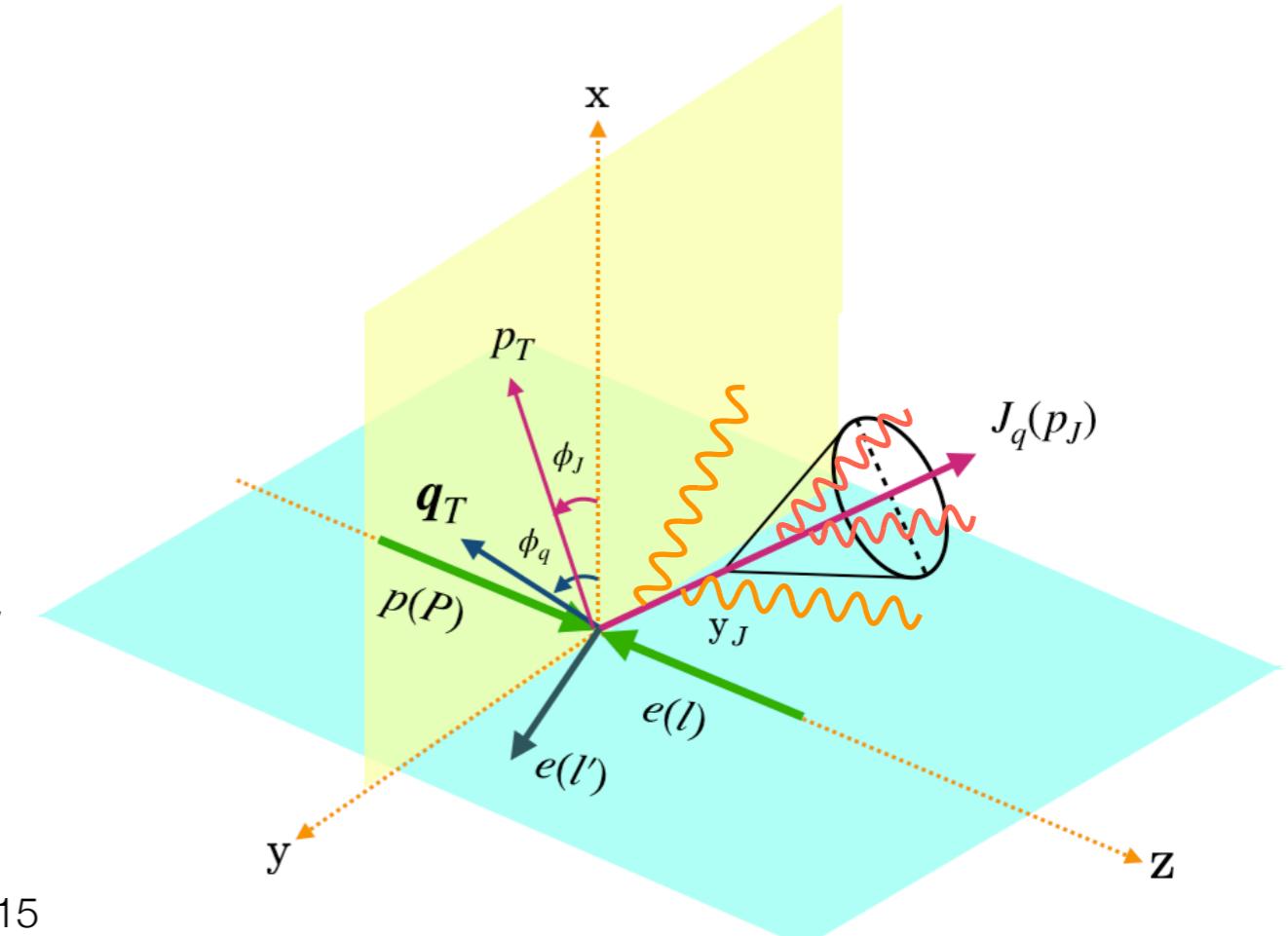
$$\frac{d\sigma}{d^2\mathbf{p}_T dy_J d\phi_J d^2\mathbf{q}_T} = \frac{d\sigma}{2\pi d^2\mathbf{p}_T dy_J q_T dq_T} \left[1 + 2 \sum_{n=1}^{\infty} v_n(p_T, y_T) \cos(n(\phi_q - \phi_J)) \right]$$

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- | | |
|------------------------------------|-------------------------|
| $\langle \cos(\phi_{qJ}) \rangle$ | v_1 : directed flow |
| $\langle \cos(2\phi_{qJ}) \rangle$ | v_2 : elliptic flow |
| $\langle \cos(3\phi_{qJ}) \rangle$ | v_3 : triangular flow |
| $\langle \cos(4\phi_{qJ}) \rangle$ | v_4 : quadrupole flow |
| ... | |

...



Jet anisotropy

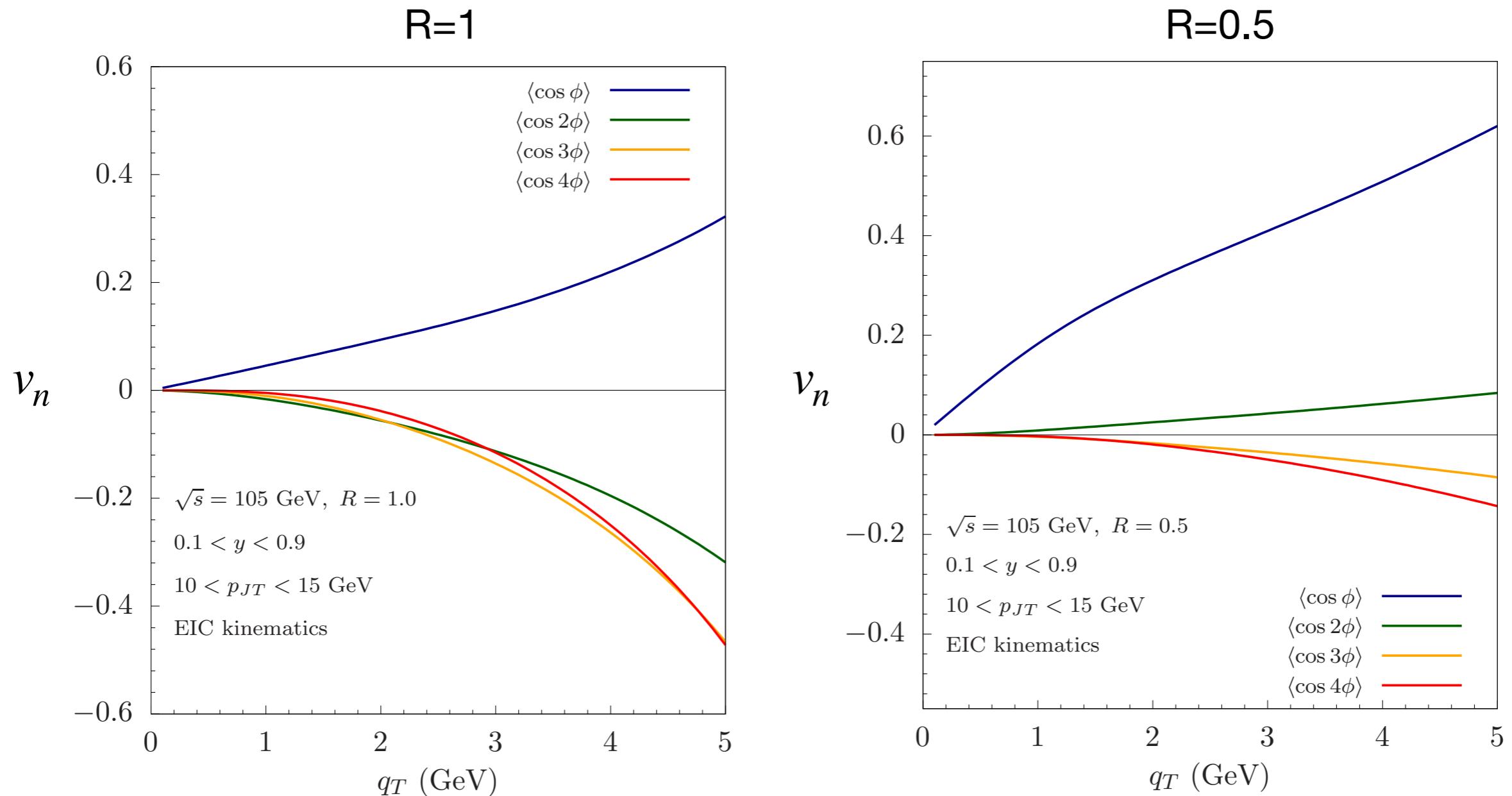
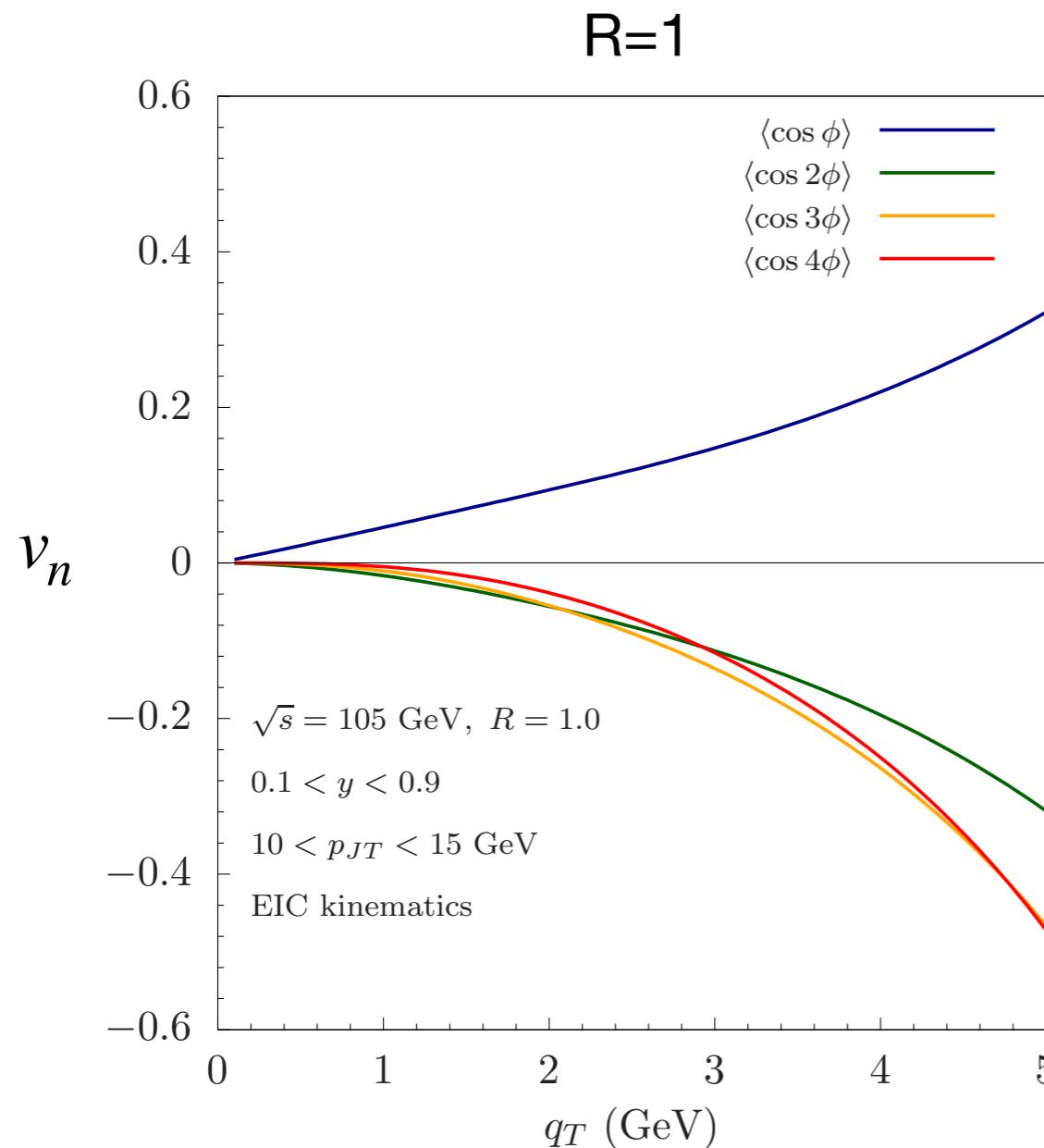
$$\bar{S}_q(\mathbf{b}, R, \mu) = 1 + \frac{\alpha_s}{2\pi} C_F \left[2y_J \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{\mu_b^2} \right) + \ln R^2 \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{\mu_b^2} \right) - \frac{\ln^2 R^2}{2} - \frac{\pi^2}{3} \right]$$

Directed flow	$v_1 \sim$	$(-i)S_q^{\langle \cos(\phi_{bJ}) \rangle}(\mathbf{b}, R, \mu) = \frac{\alpha_s}{\pi} C_F \left[\ln \frac{1}{R^2} + 2(\ln 4 - 1) \right]$	Positive
Elliptic flow	$v_2 \sim$	$(-i)^2 S_q^{\langle \cos(2\phi_{bJ}) \rangle}(\mathbf{b}, R, \mu) = \frac{\alpha_s}{2\pi} C_F \left[\ln \frac{1}{R^2} - 1 \right]$	
Triangular flow	$v_3 \sim$	$(-i)^3 S_q^{\langle \cos(3\phi_{bJ}) \rangle}(\mathbf{b}, R, \mu) = \frac{\alpha_s}{3\pi} C_F \left[\ln \frac{1}{R^2} + \frac{2(3 \ln 4 - 7)}{3} \right]$	
Quadrupole flow	$v_4 \sim$	$(-i)^4 S_q^{\langle \cos(4\phi_{bJ}) \rangle}(\mathbf{b}, R, \mu) = \frac{\alpha_s}{4\pi} C_F \left[\ln \frac{1}{R^2} - \frac{5}{2} \right]$	

Large R ($R = 1$) \Rightarrow Higher order flows are all negative

Small R ($R = 0.5$) \Rightarrow Higher order flow becomes positive ($v_2 > 0$)

EIC kinematics

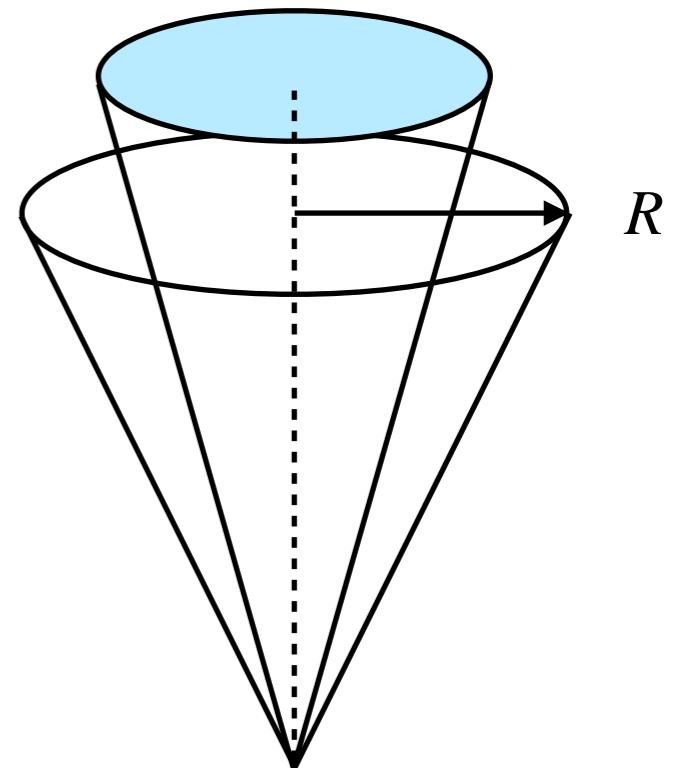


Jet shape

- Factorization formalism^[3]

$$\frac{d\sigma^{e+p \rightarrow e+\text{jet}+X}}{d^2 p_T dy_J d\mathbf{q}_T dz_r} = \hat{\sigma}_0 \sum_q e_q^2 \int \frac{d^2 \mathbf{b}}{(2\pi)^2} \exp(-i\mathbf{b} \cdot \mathbf{q}_T) x \tilde{f}_1(x, \mathbf{b}, \mu) S_q(\mathbf{b}, R, \mu) \hat{G}_q^{\text{jet}}(z_r, \omega_R, \mu)$$

$$S_q(\mathbf{b}, R, \mu) = S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu) \quad \psi(r)$$



[3] Kang, Zhong-Bo, Felix Ringer, and Wouter J. Waalewijn. "The energy distribution of subjets and the jet shape." *Journal of High Energy Physics* 2017.7 (2017): 1-39.

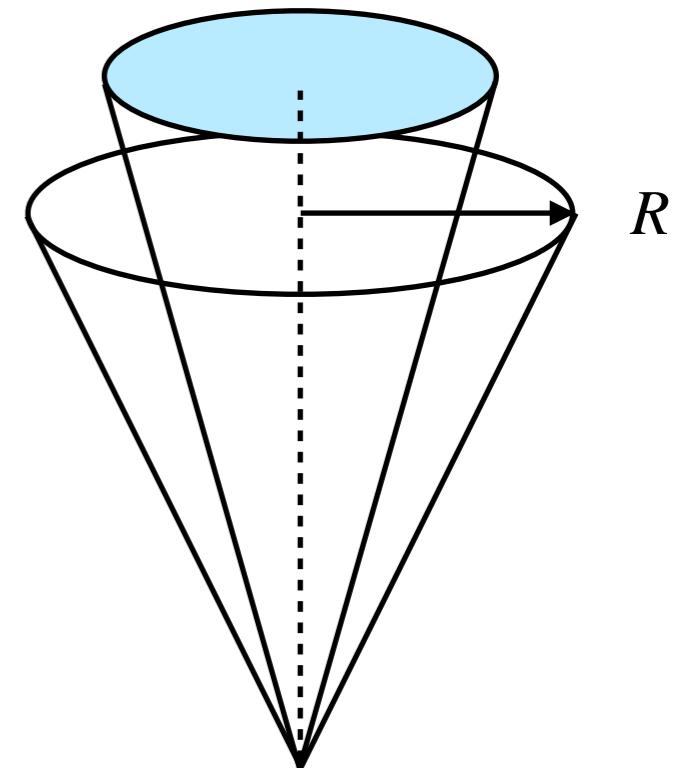
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subjet function (SJF)

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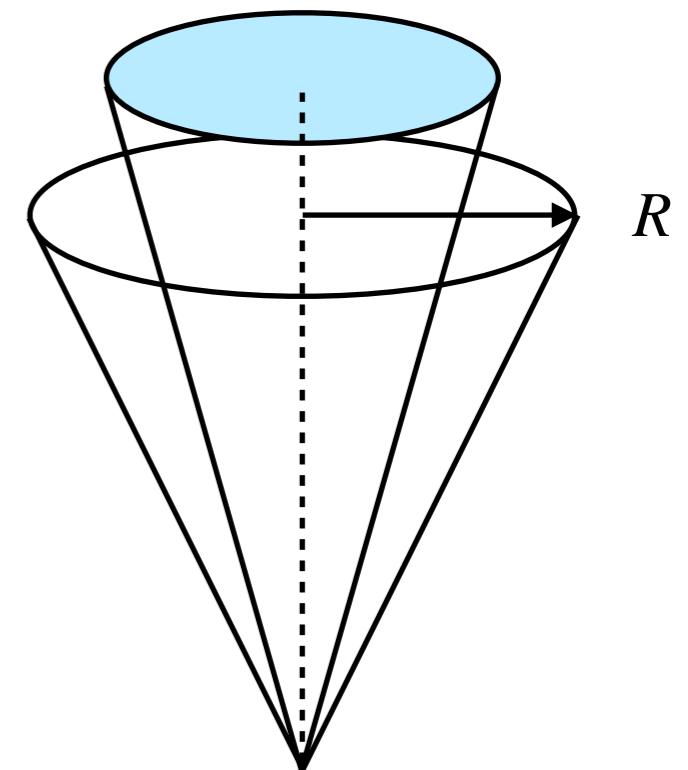
$$S_q(\mathbf{b}, R, \mu) = S_{\text{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu) \quad \psi(r)$$

Integrated jet shape:

$$\psi(r) = \int dz_r z_r \frac{d\sigma}{d^2 p_T dy_J d^2 \mathbf{q}_T dz_r} \Bigg/ \frac{d\sigma}{d^2 p_T dy_J d^2 \mathbf{q}_T}$$

Differential jet shape:

$$\rho(r) = \frac{d\psi(r)}{dr}$$



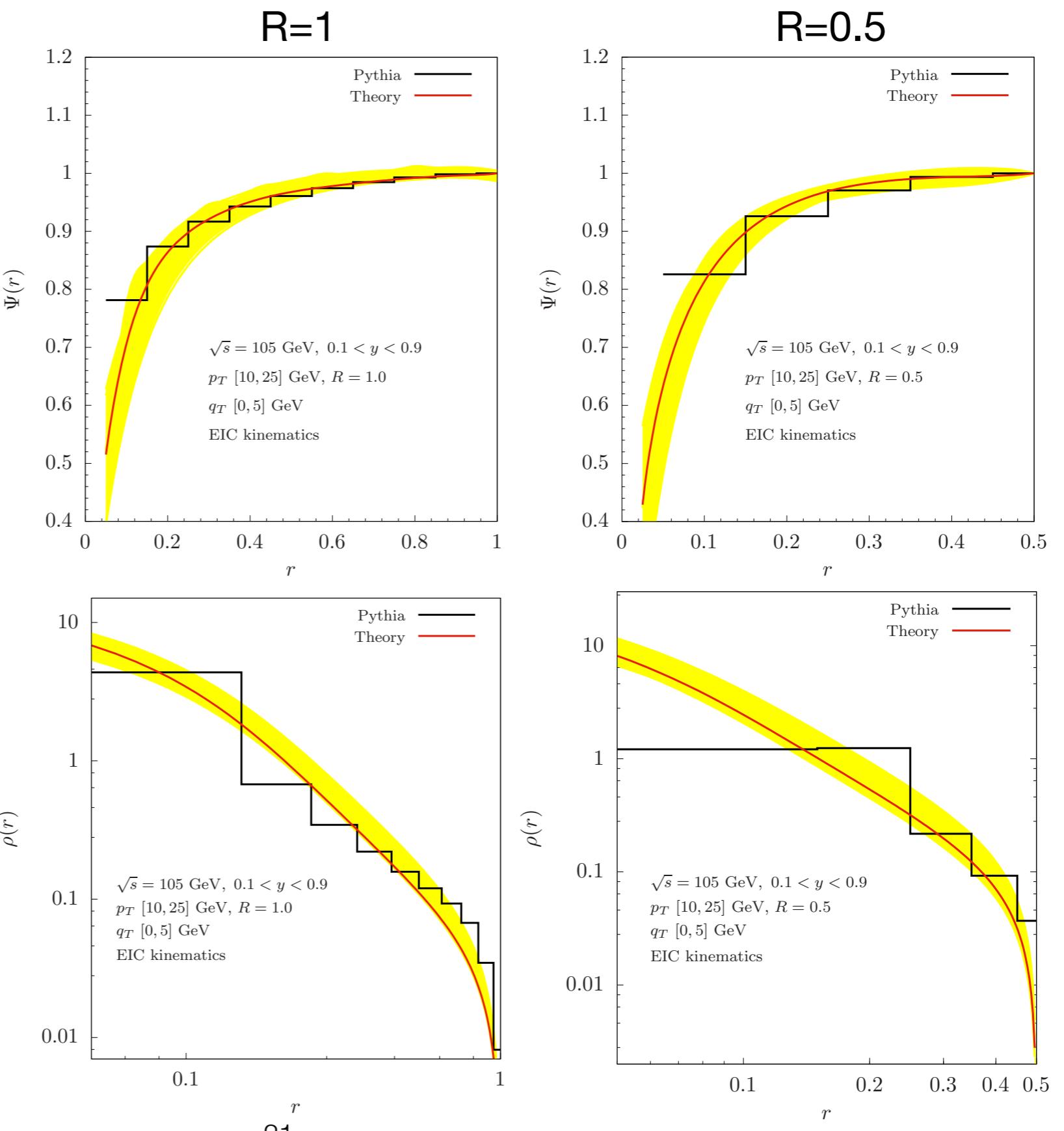
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Jet shape

EIC kinematics

Integrated jet shape:

$$\psi(r) = \frac{\int dz_r z_r \frac{d\sigma}{d^2 p_T dy_J d^2 q_T dz_r}}{\frac{d\sigma}{d^2 p_T dy_J d^2 q_T}}$$



Differential jet shape:

$$\rho(r) = \frac{d\psi(r)}{dr}$$

Summary

- We study back-to-back lepton-jet production in ep collisions. In this work, we study the azimuthal anisotropy for the azimuthal angle difference $\phi_{qJ} = \phi_q - \phi_J$
- We present the numerical results of such azimuthal anisotropy for EIC kinematics and find that the directed flow component related to $\cos(\phi_{qJ})$ azimuthal asymmetry is dominant.
- Consistent results for jet shape from theory and pythia simulations
- These are promising observables for studying lepton-jet correlations in future experiments.

Backup

Theoretical framework

- The global soft function: soft radiation that has no phase space restriction and does not resolve the jet cone.

$$S_{\text{global}}(\mathbf{b}, \mu, \nu) = 1 + \frac{\alpha_s}{2\pi} C_F \left\{ \left[-\frac{2}{\eta} + \ln \frac{\mu^2}{\nu^2} + 2y_J + 2\ln(-2i\cos(\phi_{bJ})) \right] \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{\mu_b^2} \right) + \frac{2}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu^2}{\mu_b^2} - \frac{\pi^2}{6} \right\}$$

$\mu_b = 2e^{-\gamma_E}/b$, $\phi_{bJ} \equiv \phi_b - \phi_J$ with ϕ_b and ϕ_J are the azimuthal angles of the vector \mathbf{b} and jet transverse momentum \mathbf{p}_T

Theoretical framework

- The global soft function: soft radiation that has no phase space restriction and does not resolve the jet cone.

$$S_{\text{global}}(\mathbf{b}, \mu, \nu) = 1 + \frac{\alpha_s}{2\pi} C_F \left\{ \left[-\frac{2}{\eta} + \ln \frac{\mu^2}{\nu^2} + 2y_J + 2 \ln(-2i \cos(\phi_{bJ})) \right] \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{\mu_b^2} \right) \right. \\ \left. + \frac{2}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{\mu^2}{\mu_b^2} - \frac{\pi^2}{6} \right\}$$

- The collinear-soft function: soft radiation which is only sensitive to the jet direction and resolves the jet cone

$$S_{cs}(\mathbf{b}, R, \mu) = 1 - \frac{\alpha_s}{2\pi} C_F \left[\frac{1}{\epsilon^2} + \frac{2}{\epsilon} \ln \left(\frac{-2i \cos(\phi_{bJ}) \mu}{\mu_b R} \right) + 2 \ln^2 \left(\frac{-2i \cos(\phi_{bJ}) \mu}{\mu_b R} \right) + \frac{\pi^2}{4} \right]$$

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