Emergence of Pion and proton parton distributions

K. Raya & J. Rodríguez-Quintero

FÍSICA MATEMÁTICAS COMPUTACIÓN

In collaboration with: D. Binosi, Z-F. Cui, M. Ding, J.M. Morgado, J. Papavassiliou, C.D. Roberts, S. Schmidt.

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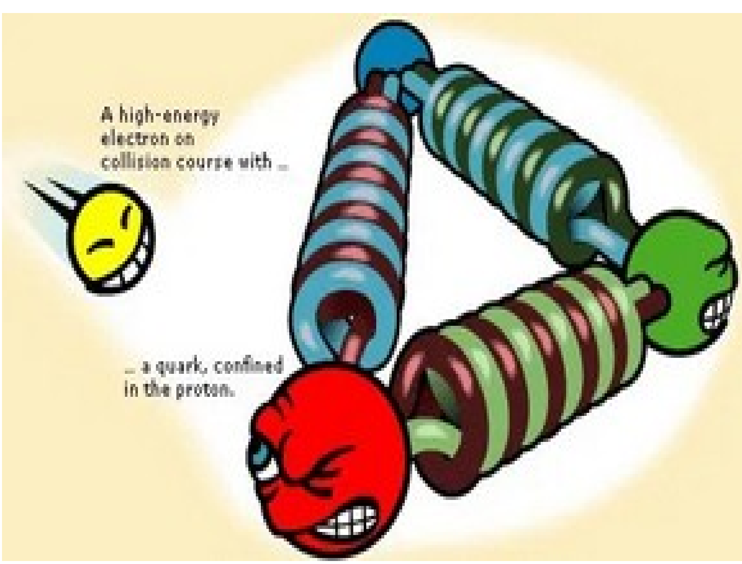
QCD: Basic Facts

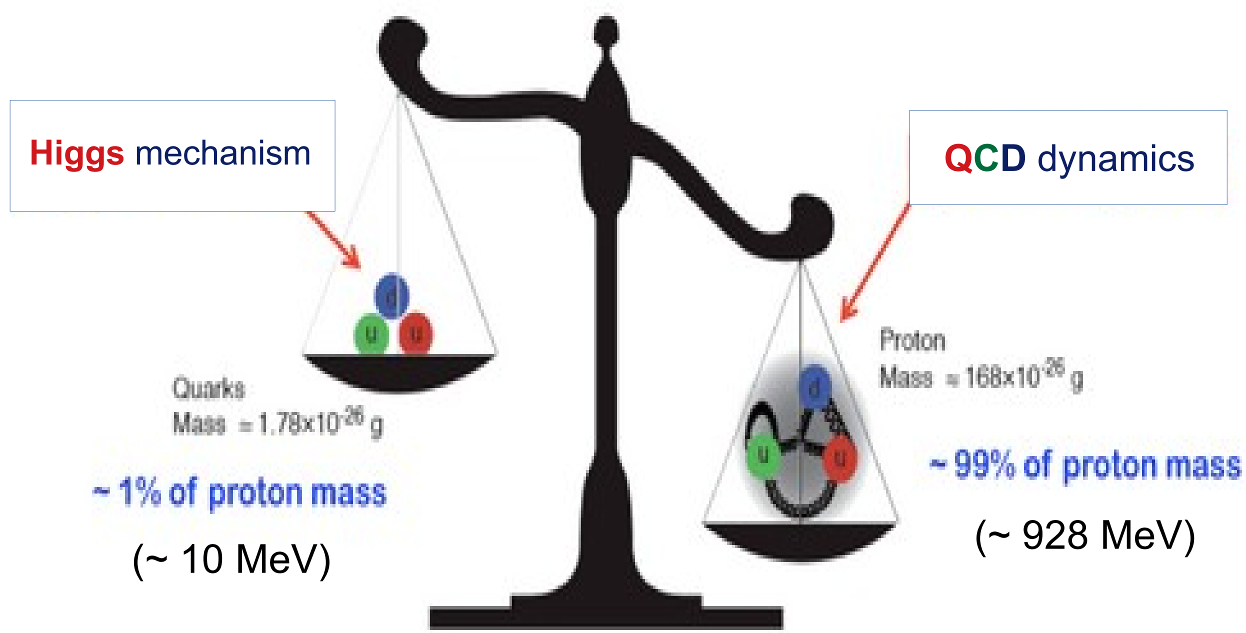
- **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\tfrac{1}{2}\lambda^a A^a_\mu,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

- Quarks and gluons not *isolated* in nature.
  - Formation of colorless bound states: "**Hadrons**"
  - **1-fm scale** size of hadrons?

- Emergence of hadron masses (EHM) from QCD **dynamics**





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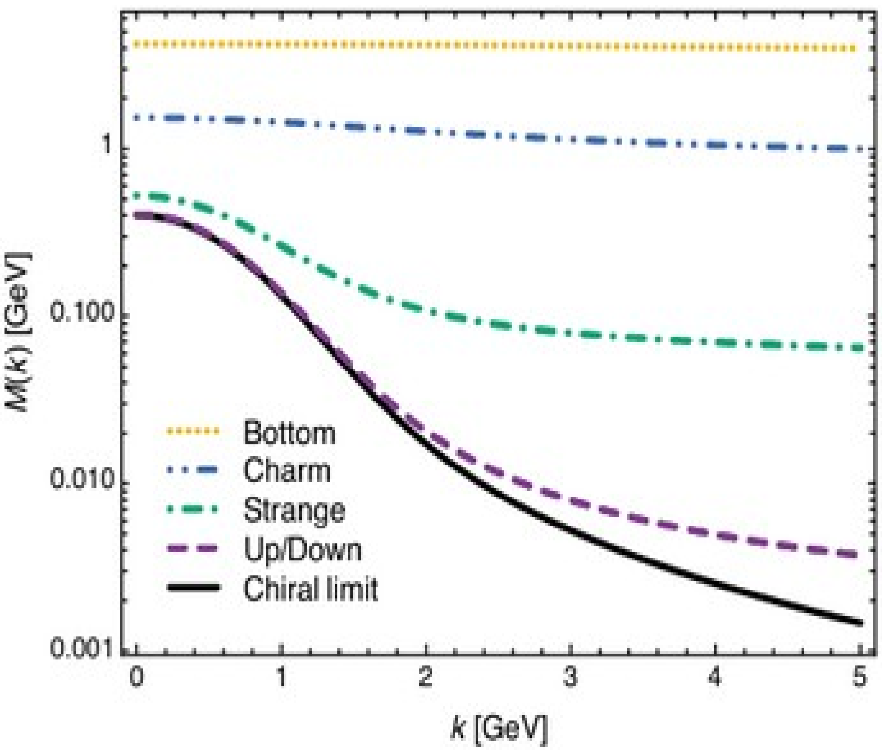
Can we trace them down to fundamental d.o.f ?

- Emergence of hadron masses (EHM) from QCD **dynamics**

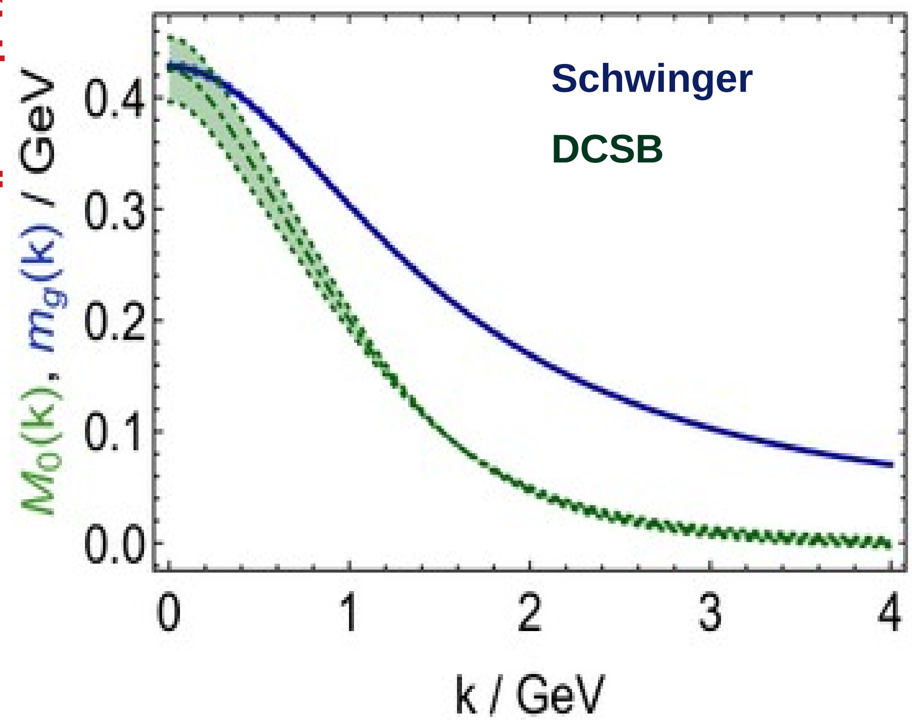
Dynamical masses

(**D**ynamical **C**hiral **S**ymmetry **B**reaking)



$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathrm{M_f(p^2)})$$

"Higgs" masses

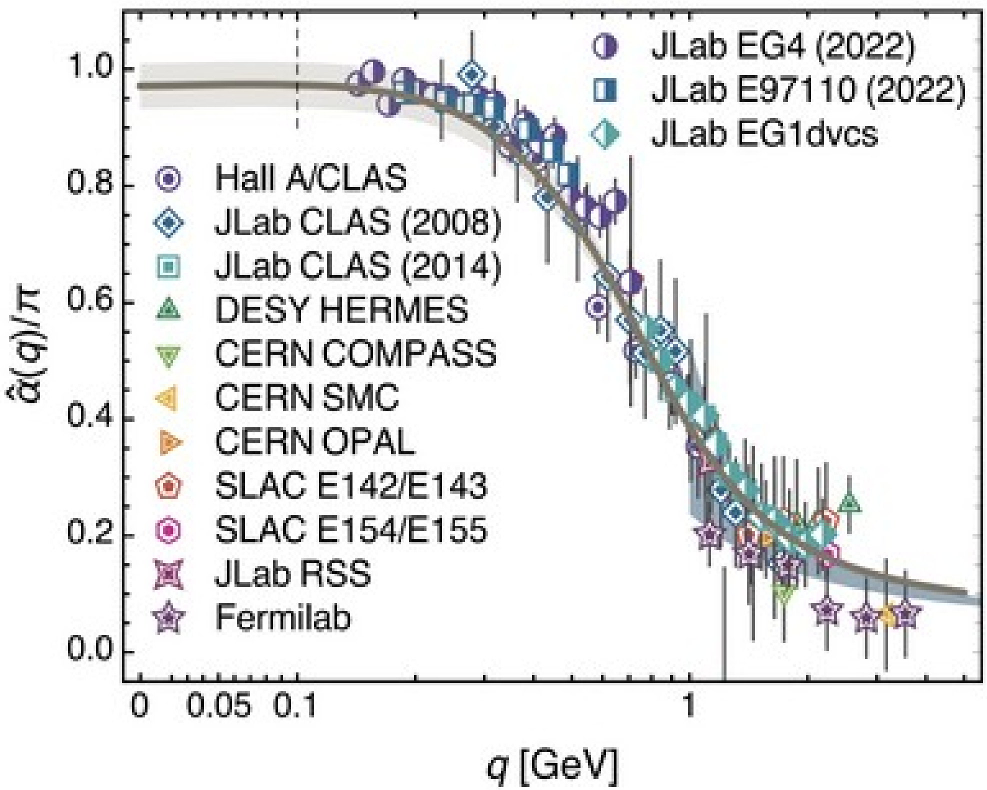


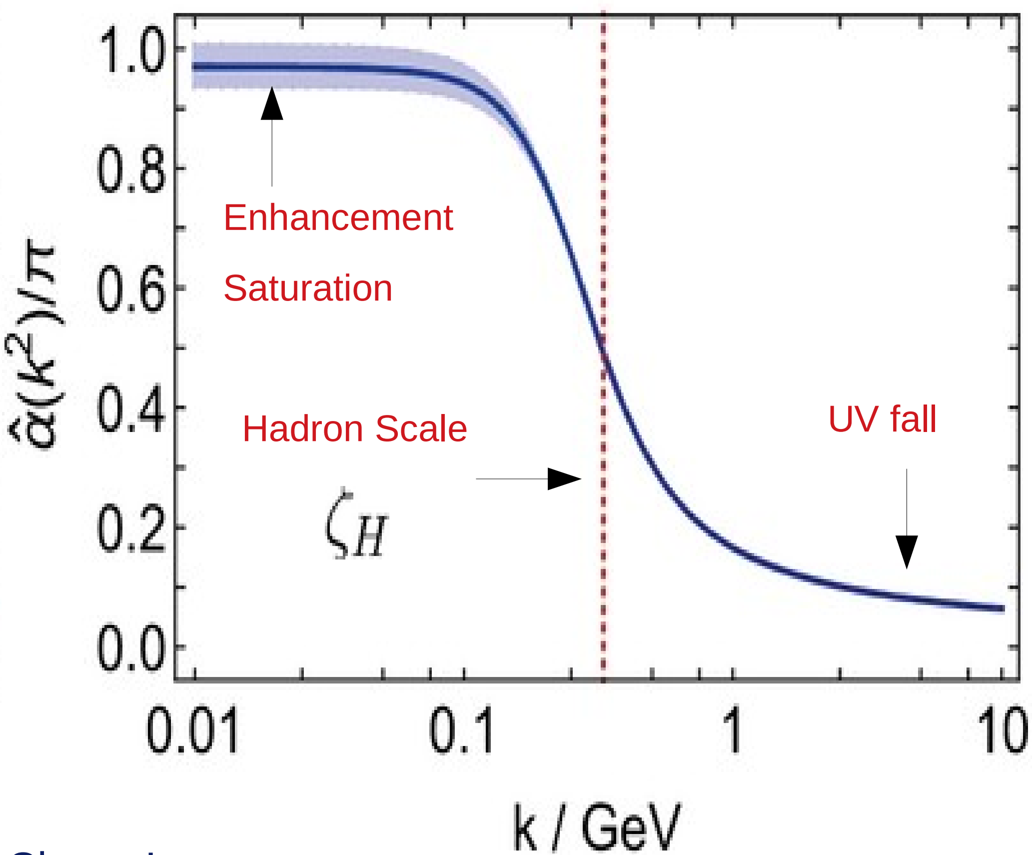
Gluon and quark *running masses*

QCD: Basic Facts

- **Confinement** and the **EHM** are tightly connected with **QCD's running coupling**.

(figure: D. Binosi's courtesy!)





Modern picture of **QCD** coupling. 'Effective Charge'

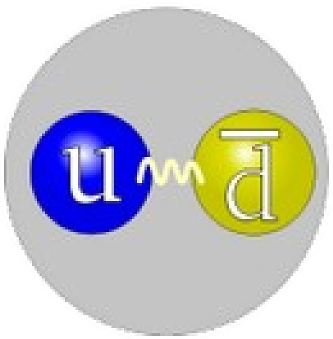
Combined continuum + QCD lattice analysis

ζ_H : Fully **dressed valence** quarks express all hadron's properties

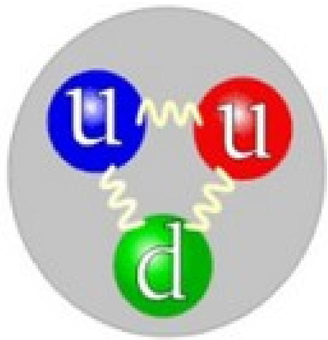
Why bother about pions?

- **Pions** and **kaons** emerge as (pseudo)-**Goldstone** bosons of **DCSB**.

(besides being ‘simple’ bound states)



$m_\pi \approx 0.140 \text{ GeV}$



$m_p \approx 0.940 \text{ GeV}$

- Their study is crucial to understand the **EHM** and the ***hadron structure***:

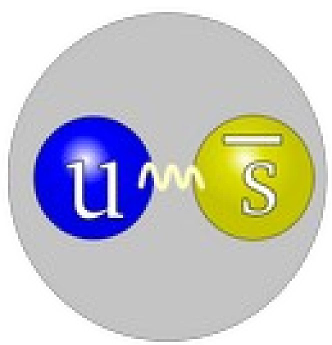
  - Dominated by **QCD** dynamics

    Simultaneously explains the mass of the proton and the *masslessness* of the pion

‘Higgs’ masses

$m_{u/d} \approx 0.004 \text{ GeV}$

$m_s \approx 0.095 \text{ GeV}$

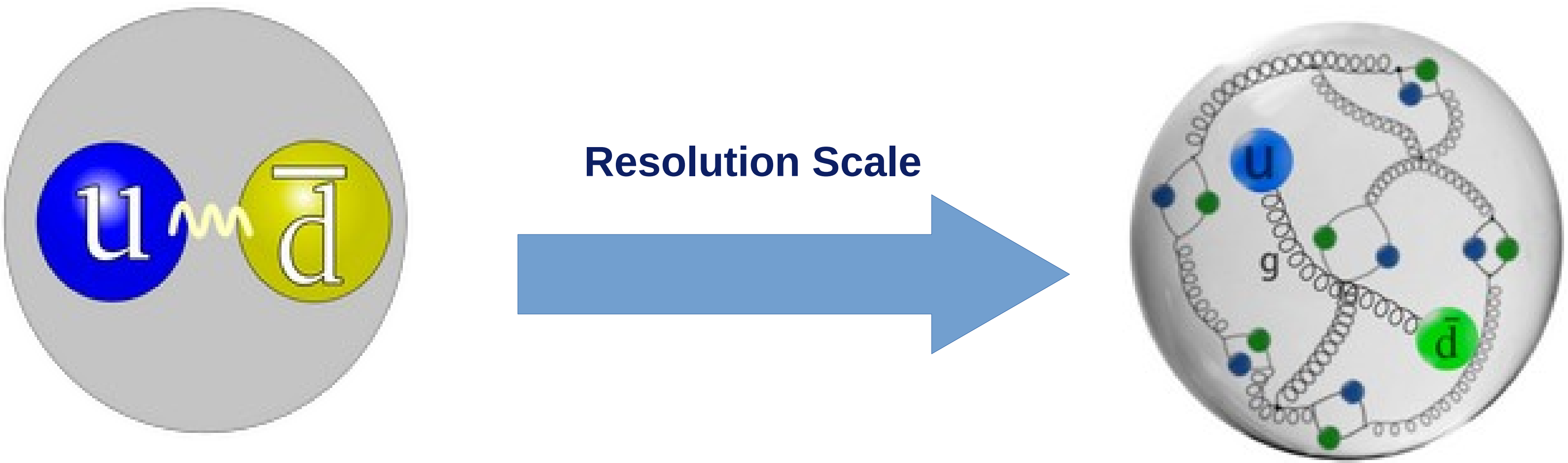


$m_K \approx 0.490 \text{ GeV}$

  - Interplay between **Higgs** and **strong** mass generating mechanisms.

Parton distributions: energy scales

ζ_H $\qquad\qquad\qquad\qquad$ $\zeta > \zeta_H$



- **Fully-dressed valence quarks**

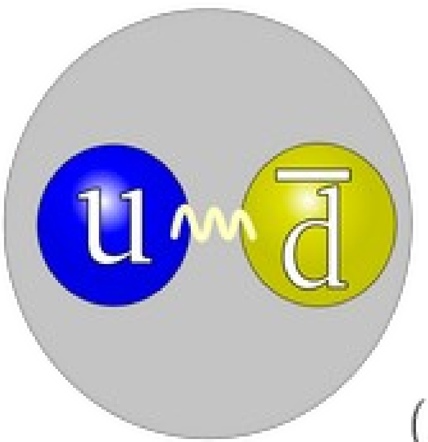
(quasiparticles)

- **Unveiling of glue and sea d.o.f.**

(partons)

Parton distributions: energy scales

ζ_H

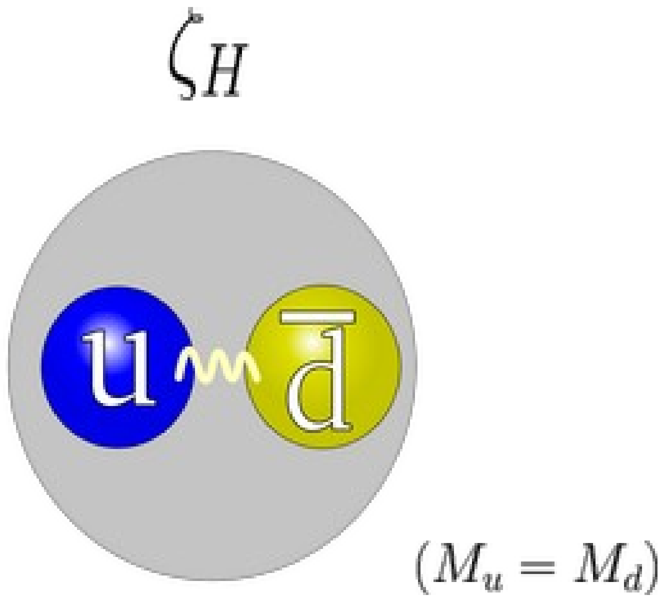


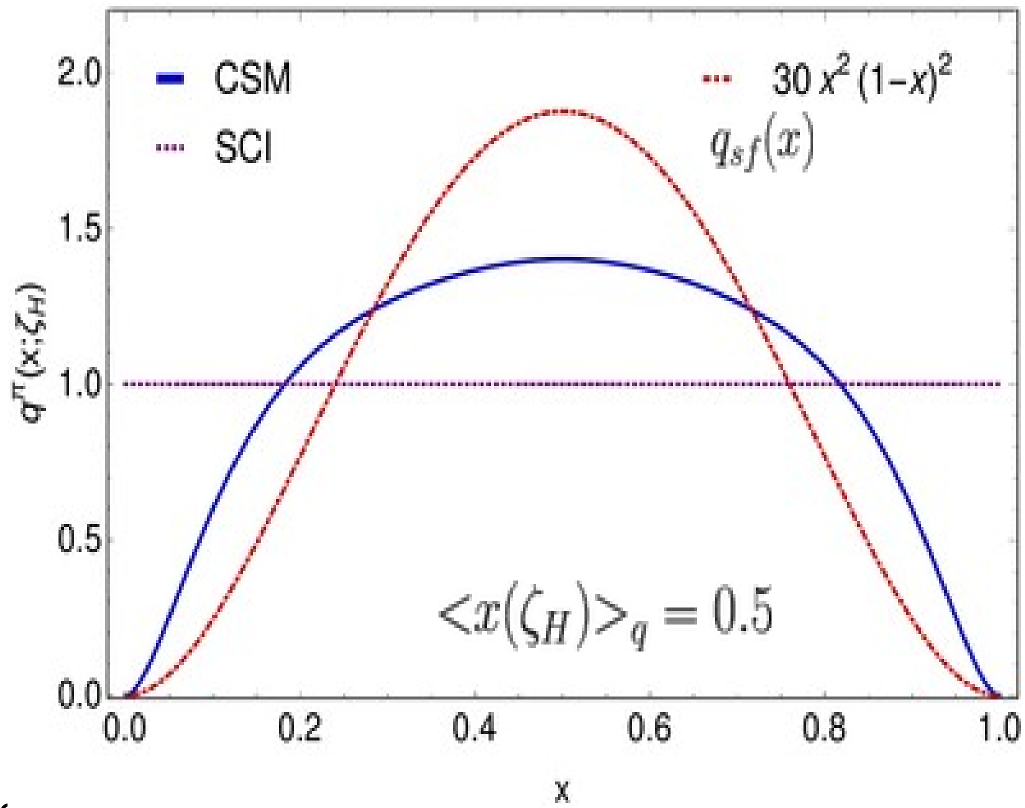
$(M_u = M_d)$

- **Fully-dressed valence quarks**
  - At this scale, **all properties** of the hadron are contained within their valence quarks.
  - **QCD constraints** are defined from here (e.g. large-x behavior of the PDF)

$$u^{\pi}(x;\zeta) \overset{x\simeq 1}{\sim} (1-x)^{\beta = 2+\gamma(\zeta)}$$

Parton distributions: energy scales





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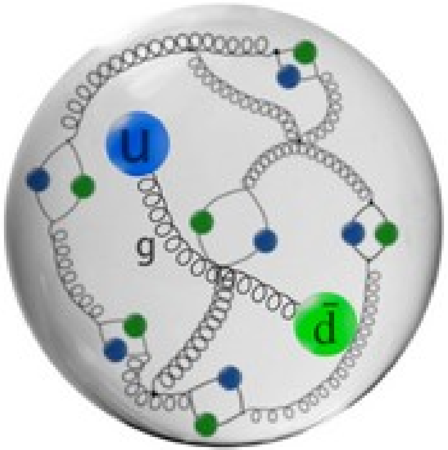
- **CSM** results produce:
  - **EHM-induced** dilated distributions
  - Soft end-point behavior

Cui:2020tdf

Parton distributions: energy scales

$$\zeta > \zeta_H$$



- **Unveiling of glue and sea d.o.f.**

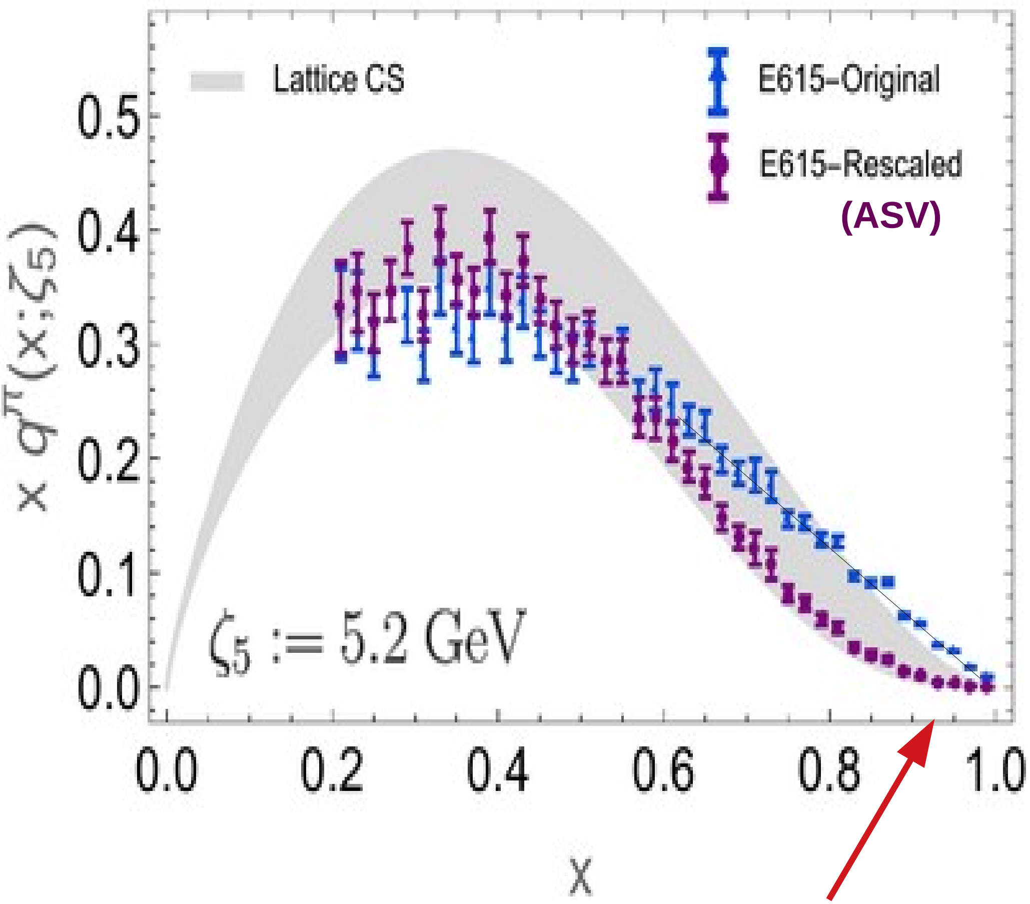
➢ **Experimental** data is given **here**.

➢ The interpretation of parton distributions from cross sections demands **special care**.

➢ In addition, the synergy with **lattice QCD** and phenomenological approaches is welcome.

Parton distributions: energy scales

Conway:1989fs Aicher:2010cb Sufian:2020vzb



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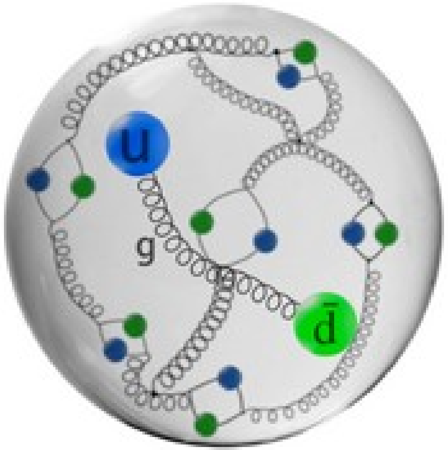

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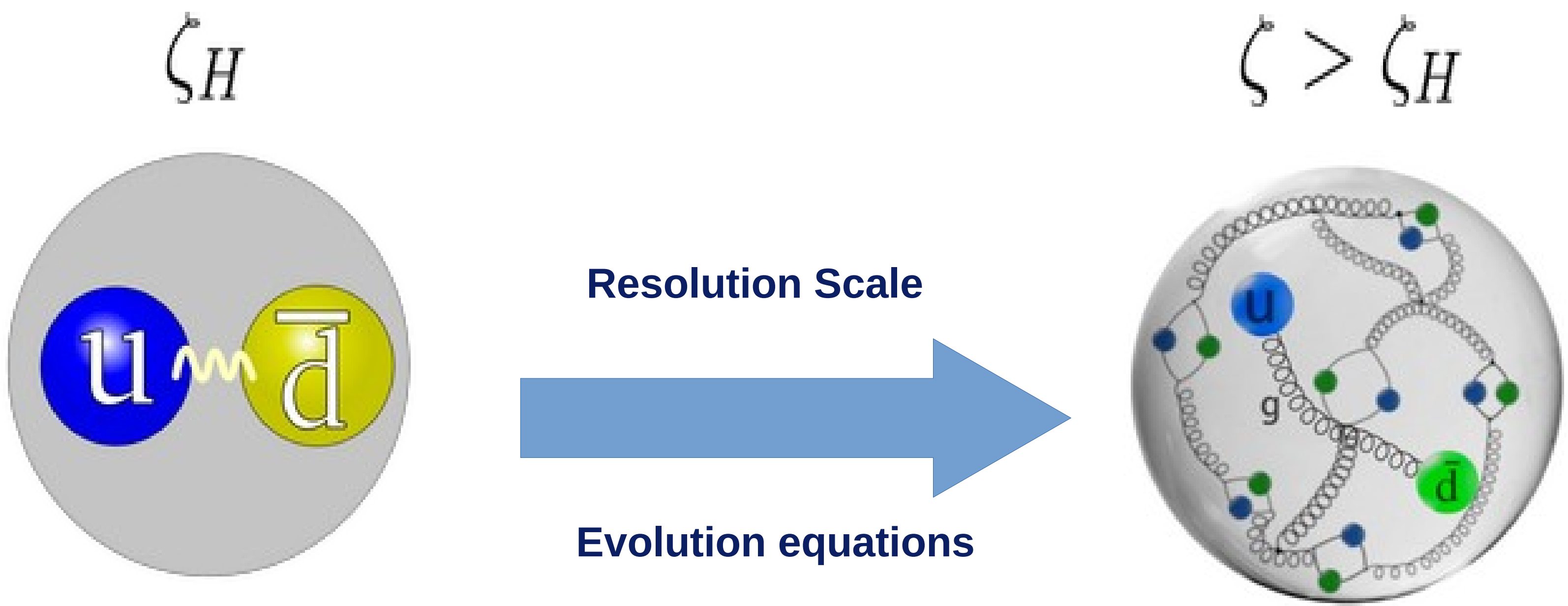

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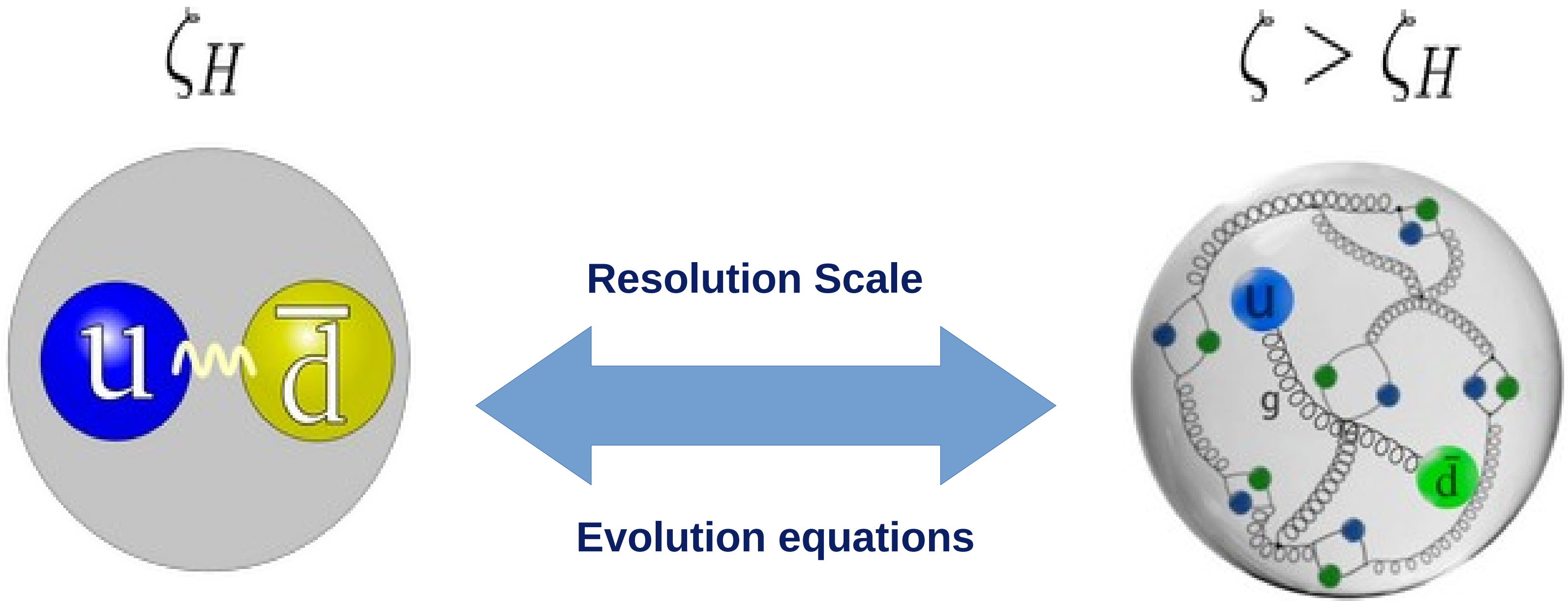

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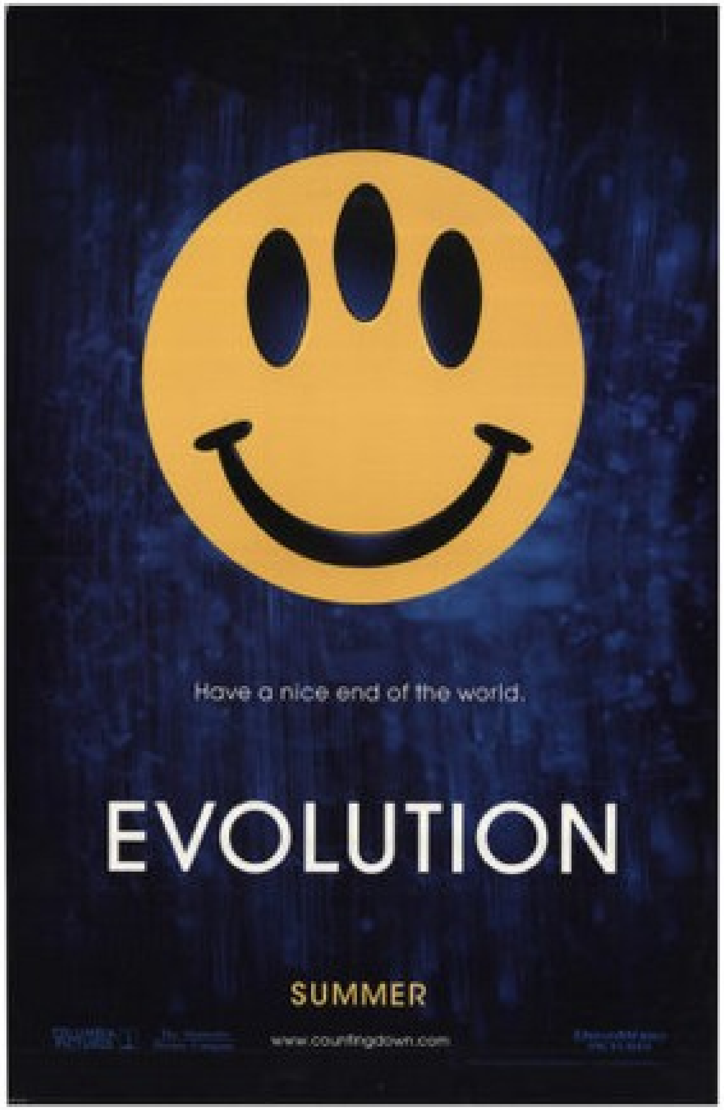
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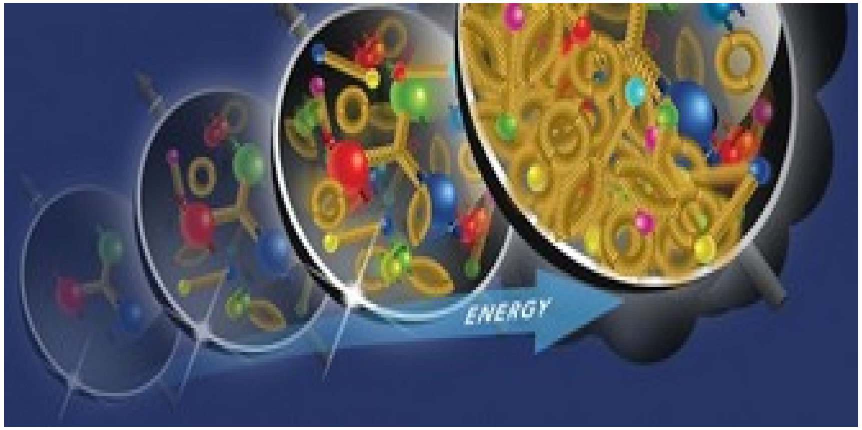
DGLAP: All orders evolution

Raya:2021zrz

Cui:2020tdf

$$\left\{\zeta^2\frac{d}{d\zeta^2}\int_0^1 dy\delta(y-x) - \frac{\alpha(\zeta^2)}{4\pi}\int_x^1\frac{dy}{y}\begin{pmatrix} P_{qq}^{\rm NS}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^{\rm S}\left(\frac{\mathbf{x}}{\mathbf{y}}\right)\end{pmatrix}\right\}\begin{pmatrix} H_\pi^{\rm NS,+}(y,t;\zeta) \\ \mathbf{H}_\pi^{\rm S}(y,t;\zeta)\end{pmatrix} = 0$$

DGLAP leading-order evolution equations



DGLAP: All orders evolution

Assumption: define an **effective** charge such that

Raya:2021zrz

Cui:2020tdf

Starting from fully-dressed **quasiparticles**, at ζ_H

Sea and **Gluon** content unveils, as prescribed by **QCD**

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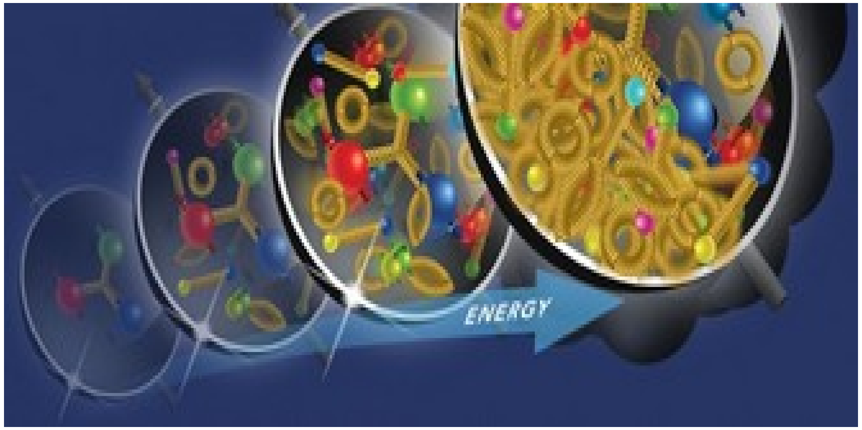

- **Not** the **LO** QCD coupling but an **effective** one.
- Making this equation **exact**.
- Connecting with the **hadron scale**, at which the **fully-dressed** valence-**quarks** express **all** of the hadron's properties.

(thus carrying all the momentum)

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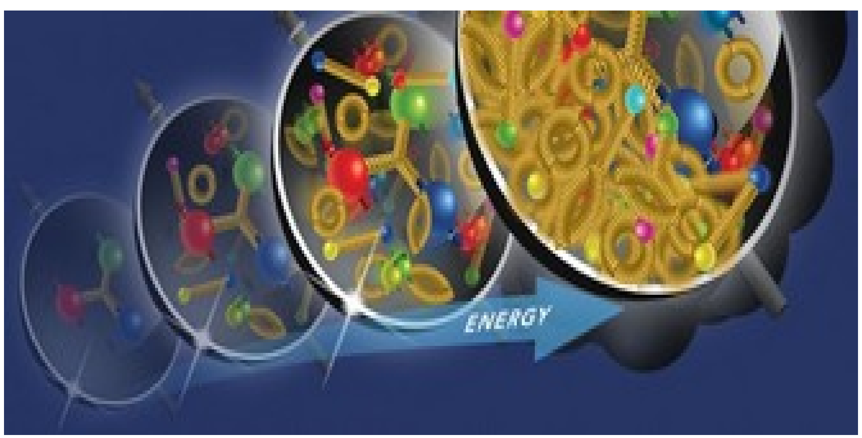
DGLAP ~~leading order~~ evolution equations

$$\gamma_{AB}^{(n)} = -\int_0^1 dx\, x^n P_{AB}^C(x)$$



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Cui:2020tdf

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This ratio encodes the information of the charge

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This ratio encodes the information of the charge and use isospin symmetry

$$\langle x(\zeta_H)\rangle_u = \langle x(\zeta_H)\rangle_{\bar d} = 1/2$$

Direct connection bridging from hadron to experimental scale: only one input is needed to evolve "all" the Mellin moments up and reconstruct the PDF.

DGLAP: All orders evolution

Cui:2020tdf

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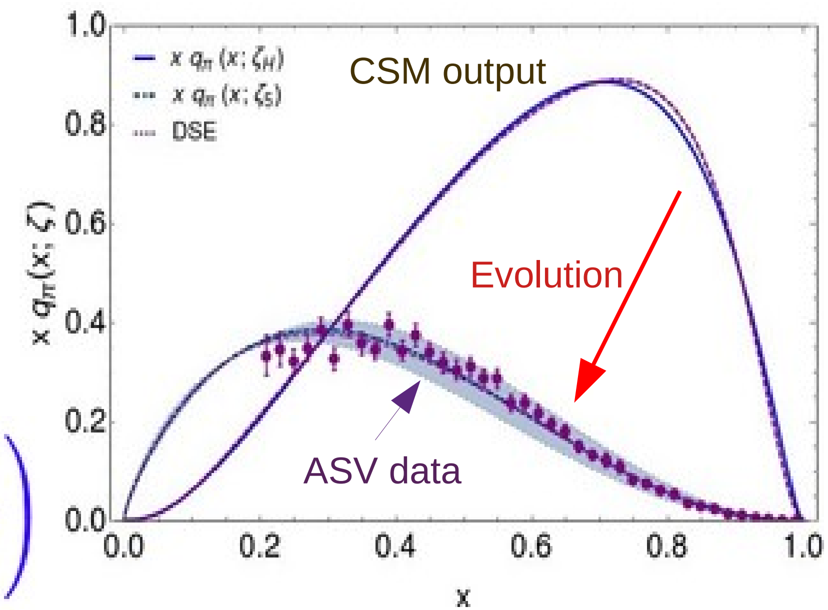
Reconstruction after evolving a CSM PDF



DGLAP: All orders evolution

Implication 2: glue and sea-quark distributions (n_f=4)

$$
\begin{aligned}
\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi} S(\zeta_H,\zeta_f)\right), \qquad q = u, \bar{d}; \\
\langle x(\zeta_f)\rangle_{\rm sea} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - (\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}}), \\
&= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u \\
\langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1 - \langle 2x(\zeta_f)\rangle_u^{7/4}\right);
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- Obtained from valence-quark inputs

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$$\boxed{\langle 2x(\zeta_f)\rangle_q + \langle x(\zeta_f)\rangle_{\rm sea} + \langle x(\zeta_f)\rangle_g = 1}$$

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$\zeta_f/\zeta_H \to \infty$

Asymptotic (massless) limit is manifestly in agreement with textbook results: G. Altarelli, Phys. Rep. 81, 1 (1982)

DGLAP: All orders evolution

Implication 2: glue and sea-quark distributions (n_f=4)

$$
\begin{aligned}
\langle 2x(\zeta_f)\rangle_q &= \exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_f)\right), \qquad q=u,\bar{d};\\
\langle x(\zeta_f)\rangle_{\rm sea} &= \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - (\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}}),\\
&= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u\\
\langle x(\zeta_f)\rangle_g &= \frac{4}{7}\left(1-\langle 2x(\zeta_f)\rangle_u^{7/4}\right);
\end{aligned}
$$

Obtained from valence-quark inputs

Momentum sum rule:

$$\boxed{\langle 2x(\zeta_f)\rangle_q + \langle x(\zeta_f)\rangle_{\rm sea} + \langle x(\zeta_f)\rangle_g = 1}$$

$\zeta_f/\zeta_H \to \infty$ Asymptotic (massless) limit is manifestly in agreement with textbook results: G. Altarelli, Phys. Rep. 81, 1 (1982)

R.S. Sufian et al., arXiv:2001.04960

ζ_5	$\langle 2x\rangle_q^\pi$	$\langle x\rangle_g^\pi$	$\langle x\rangle_{\rm sea}^\pi$
Ref.[55]	0.412(36)	0.449(19)	0.138(17)
Herein	$\boxed{0.40(4)}$	0.45(2)	0.14(2)

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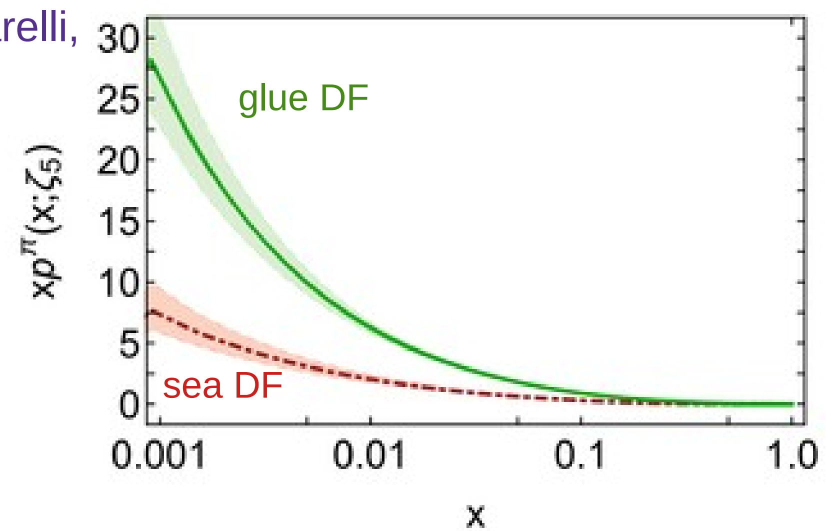
Herein	0.40(4)	0.45(2)	0.14(2)

Compute all the moments and reconstruct:



DGLAP: All orders evolution

Implication 3: recursion of Mellin moments

$$\langle x^{2n+1}\rangle^{\zeta}_{u_\pi} = \frac{(\langle 2x\rangle^{\zeta}_{u_\pi})^{\gamma_0^{2n+1}/\gamma_0^1}}{2(n+1)} \times \sum_{j=0,1,\ldots}^{2n} (-)^j \binom{2(n+1)}{j} \langle x^j\rangle^{\zeta}_{u_\pi} (\langle 2x\rangle^{\zeta}_{u_\pi})^{-\gamma_0^j/\gamma_0^1} .$$

- Since **isospin symmetry** limit implies:

$$q(x;\zeta_H) = q(1-x;\zeta_H)$$

- **Odd** moments can be expressed in terms of previous **even** moments.
- Thus arriving at the recurrence **relation** on the left which is satisfied if, and only if, the source distribution is related by evolution to a symmetric one at the initial scale .

DGLAP: All orders evolution

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Reported lattice moments agree very well with the recursion formula

n	$\langle x^n\rangle^{\zeta_5}_{u_\pi}$ Ref. [99]	Eq. (17)
1	0.230(3)(7)	0.230
2	0.087(5)(8)	0.087
3	0.041(5)(9)	0.041
4	0.023(5)(6)	0.023
5	0.014(4)(5)	0.015
6	0.009(3)(3)	0.009
7		0.0078

[99] C. Alexandrou et al., PRD104(2021)054504

DGLAP: All orders evolution

Implication 3: recursion of Mellin moments

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Reported lattice moments agree very well with the recursion formula and so also does and estimate for the 7-th moment from lattice reconstruction.

Moments from global fits can be also compared to the estimated from recursion !

n	$\langle x^n\rangle^{\zeta_5}_{u_\pi}$ Ref. [99]	Eq. (17)
1	0.230(3)(7)	0.230
2	0.087(5)(8)	0.087
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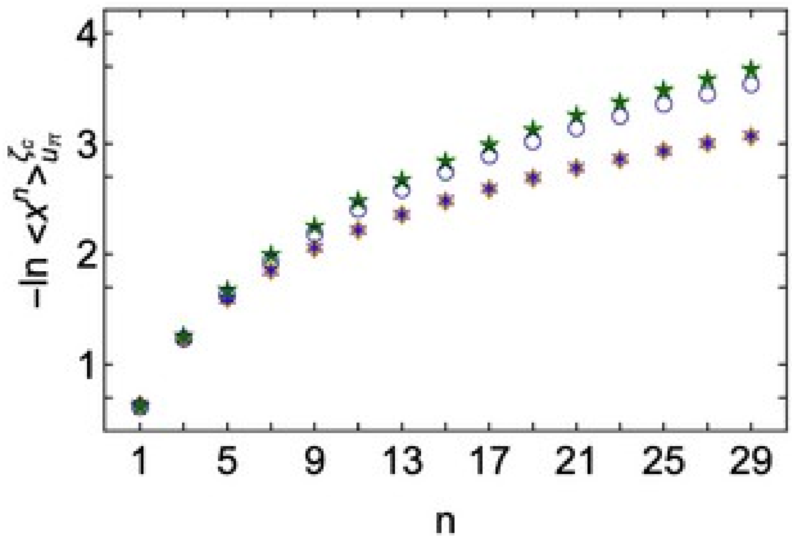
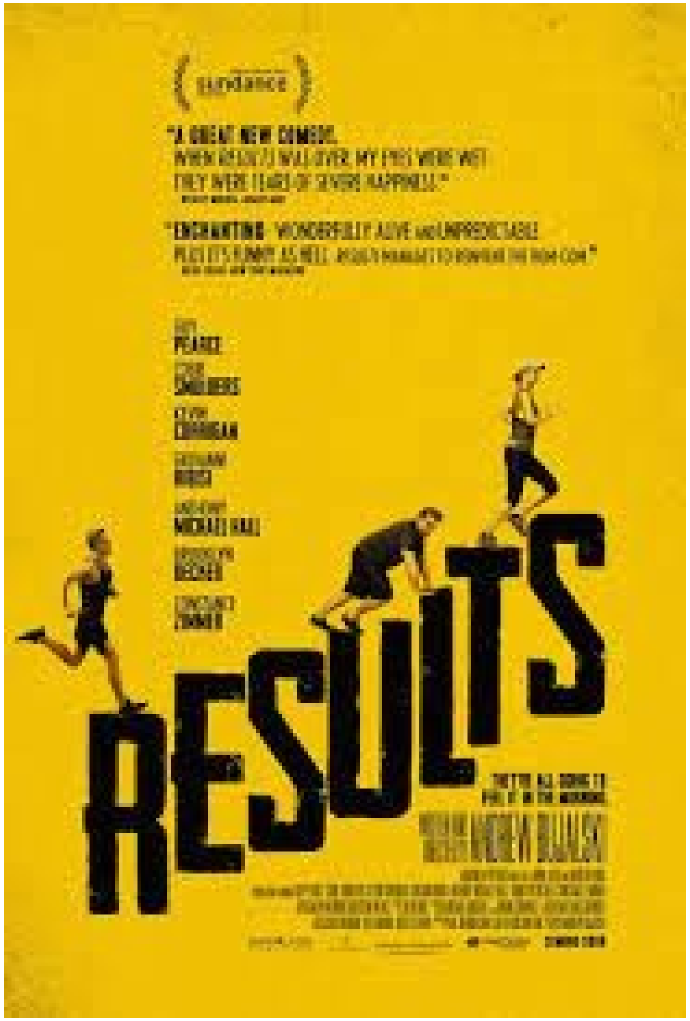
[99] C. Alexandrou et al., PRD104(2021)054504

Moments computed from: P. Barry et al., PRL127(2021)232001




sundance
"A GREAT NEW COMEDY.
WHEN RESULTS WAS OVER, MY EYES WERE WET
THEY WERE TEARS OF SEVERE HAPPINESS."
"ENCHANTING. WONDERFULLY ALIVE AND UNPREDICTABLE.
PLUS IT'S FUNNY AS HELL. RESULTS MANAGES TO REINVENT THE ROM-COM."
PEARCE
SMULDERS
CORRIGAN
RESULTS
ANDREW BUJALSKI

Pion PDF: from CSM (DSEs) to the experiment

Symmetry-preserving DSE computation of the valence-quark PDF:
[L. Chang et al., Phys.Lett.B737(2014)23]
[M. Ding et al., Phys.Rev.D101(2020)054014

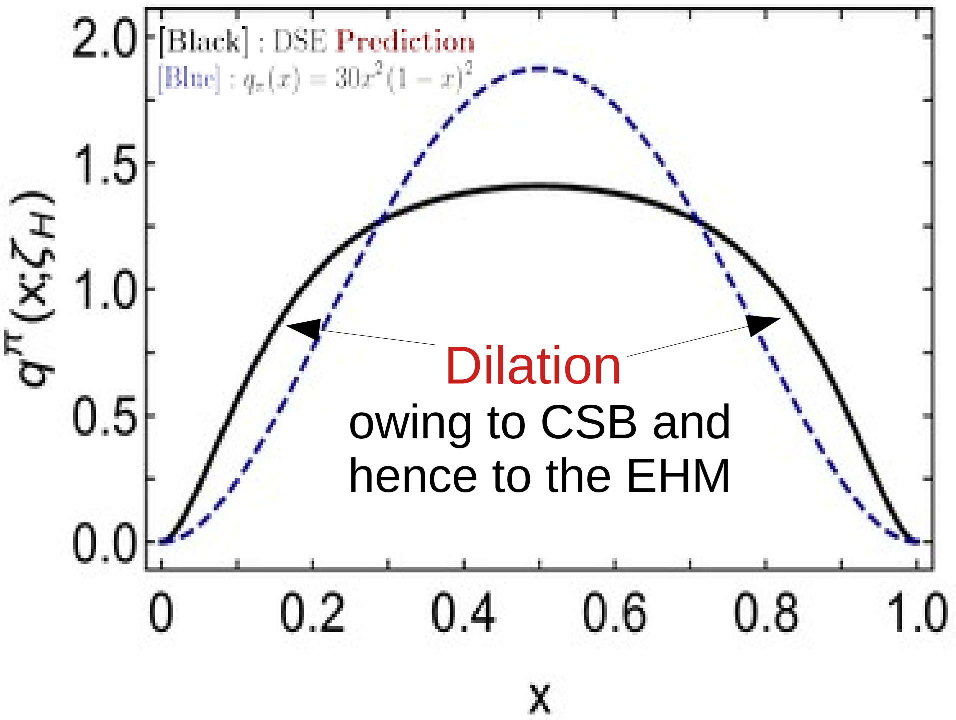
$$q^\pi(x;\zeta) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta) \Gamma_\pi^P(k_{\bar\eta\eta};\zeta)\, S(k_{\bar\eta};\zeta) \times \{n\cdot \frac{\partial}{\partial k_\eta}\left[\Gamma_\pi^{-P}(k_{\eta\bar\eta};\zeta) S(k_\eta;\zeta)\right]\}\,.$$

$$q_{\mathbb{O}}^\pi(x;\zeta_H) = 213.32\, x^2(1-x)^2 \times [1 - 2.9342\sqrt{x(1-x)} + 2.2911\, x(1-x)]$$

$$q(x;\zeta) \underset{x\to 1}{\sim} (1-x)^{\beta(\zeta)}\,(1+\mathcal{O}(1-x))$$

$$\beta(\zeta_H) = 2$$

Farrar, Jackson, Phys.Rev.Lett 35(1975)1416
Berger, Brodsky, Phys.Rev.Lett 42(1979)940



- The EHM-triggered broadening shortens the extent of the domain of convexity lying on the neighborhood of the endpoints, induced too by the QCD dynamics

- It cannot however spoil the asymptotic QCD behaviour at large-x (and, owing to isospin symmetry, at low-x)

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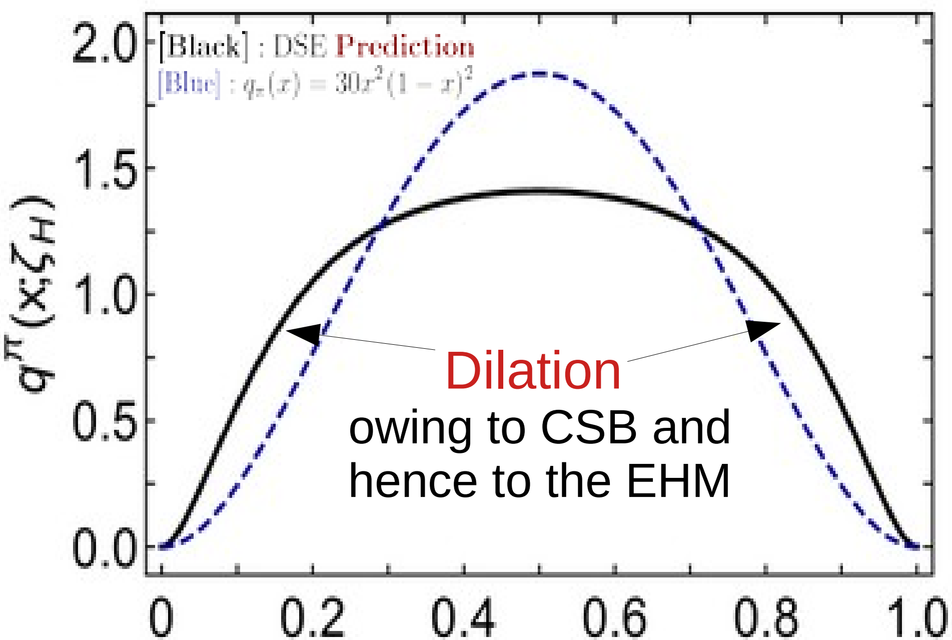

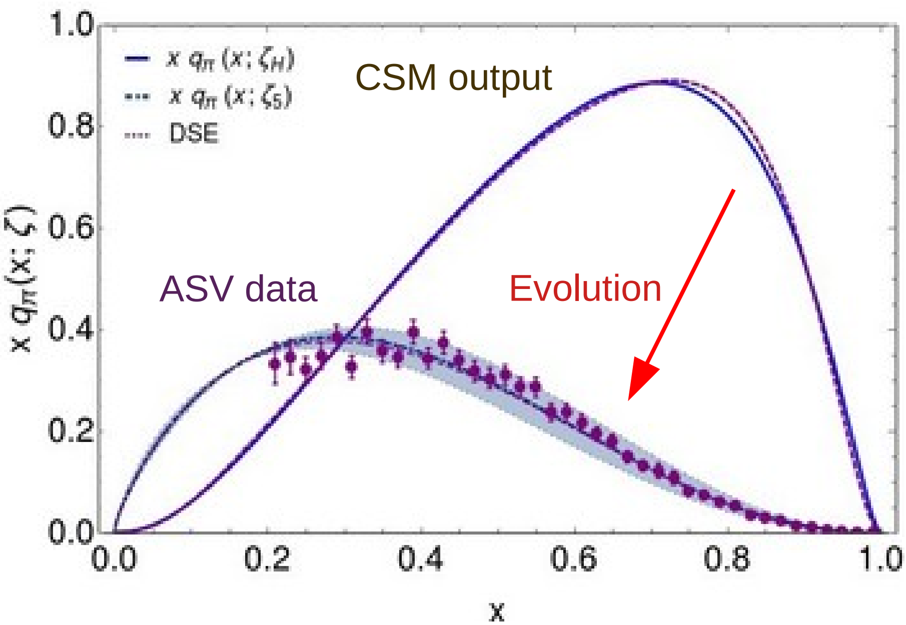

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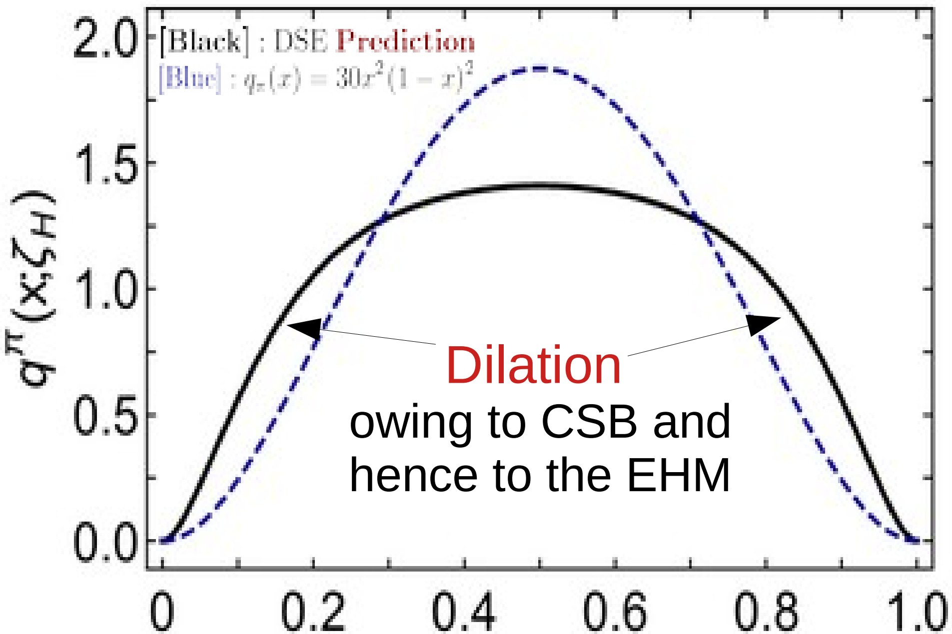
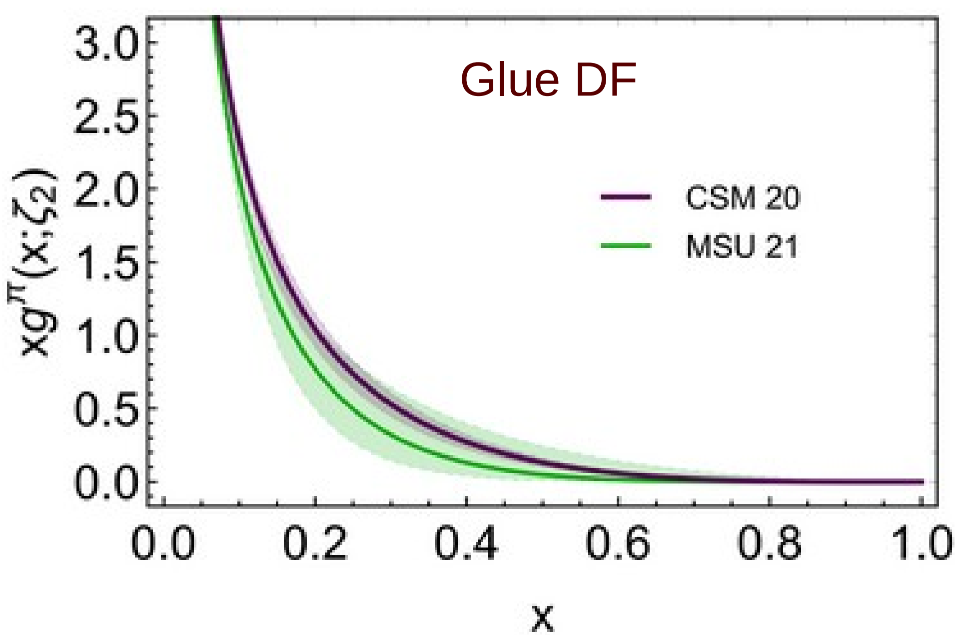
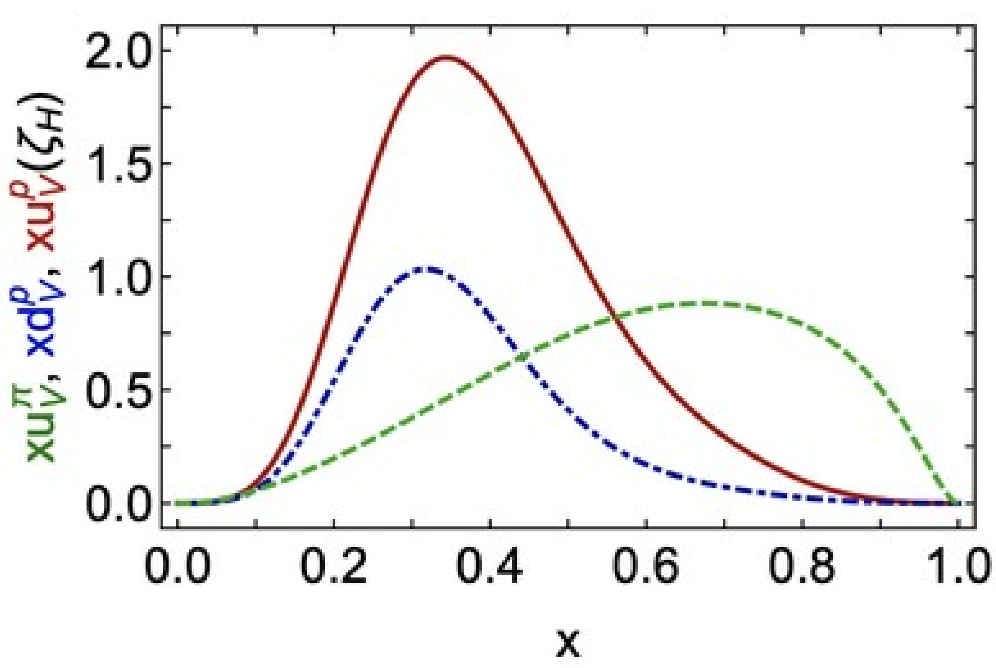




Proton PDF: from CSM (DSEs) to the experiment

An analogous symmetry-preserving DSE computation of the valence-quark PDFs within a proton, based on diquark-quark approach:
[L. Chang et al., Phys.Lett.B, arXiv:2201.07870]



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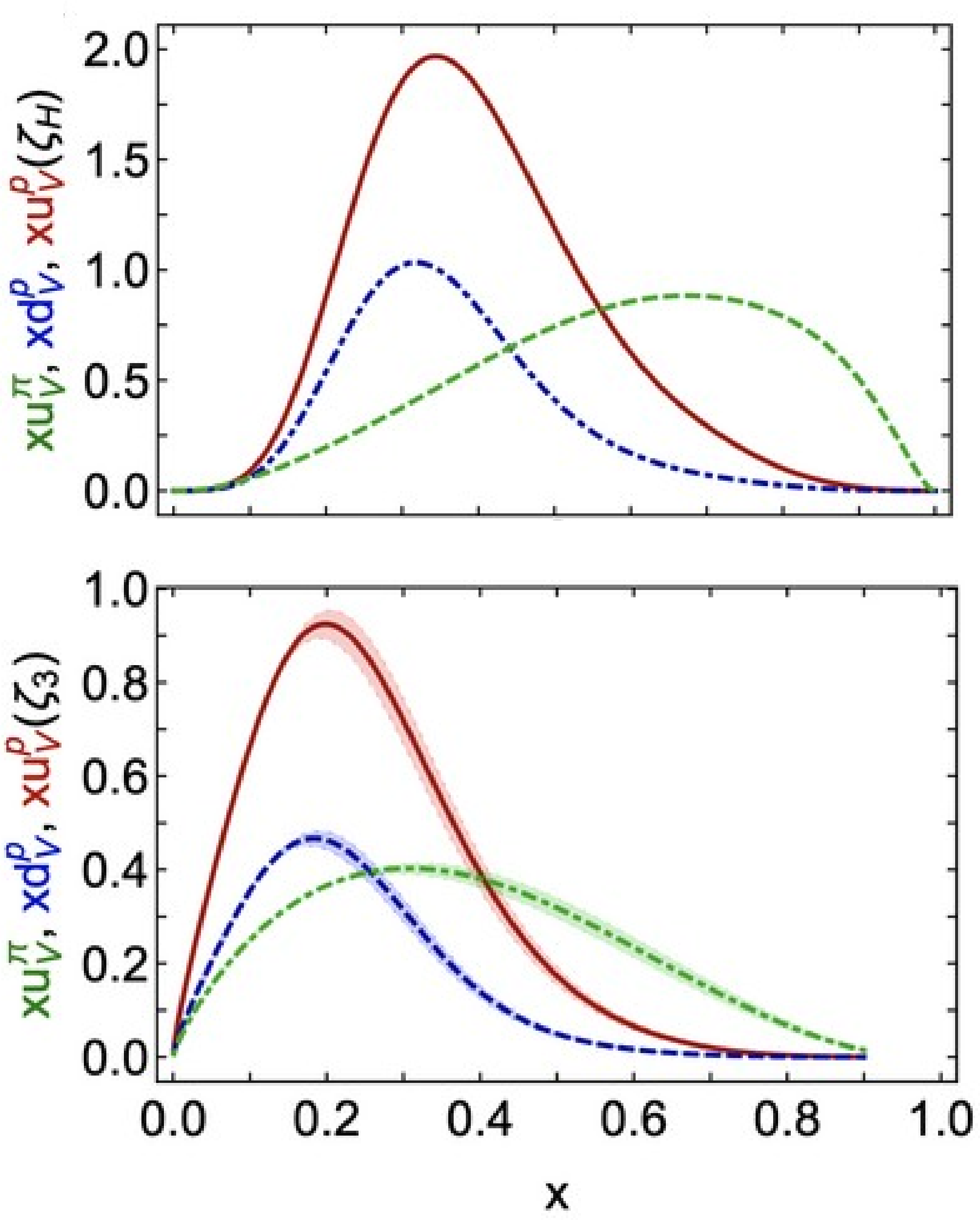
And analogous evolution approach:

$$\zeta^2 \frac{d}{d\zeta^2}\langle x^n\rangle^{\zeta}_{q_H} = -\frac{\alpha_{1\ell}(\zeta^2)}{4\pi}\gamma^n_{qq}\langle x^n\rangle^{\zeta}_{q_H}$$

$$\zeta^2 \frac{d}{d\zeta^2}\langle x^n\rangle^{\zeta}_{\Sigma^q_H} = -\frac{\alpha_{1\ell}(\zeta^2)}{4\pi}\left[\gamma^n_{qq}\langle x^n\rangle^{\zeta}_{\Sigma^q_H} + 2\mathcal{P}^{\zeta}_{qg}\gamma^n_{qg}\langle x^n\rangle^{\zeta}_{g_H}\right]$$

$$\zeta^2 \frac{d}{d\zeta^2}\langle x^n\rangle^{\zeta}_{g_H} = -\frac{\alpha_{1\ell}(\zeta^2)}{4\pi}\left[\sum_q \gamma^n_{gq}\langle x^n\rangle^{\zeta}_{\Sigma^q_H} + \gamma^n_{gg}\langle x^n\rangle^{\zeta}_{g_H}\right]$$



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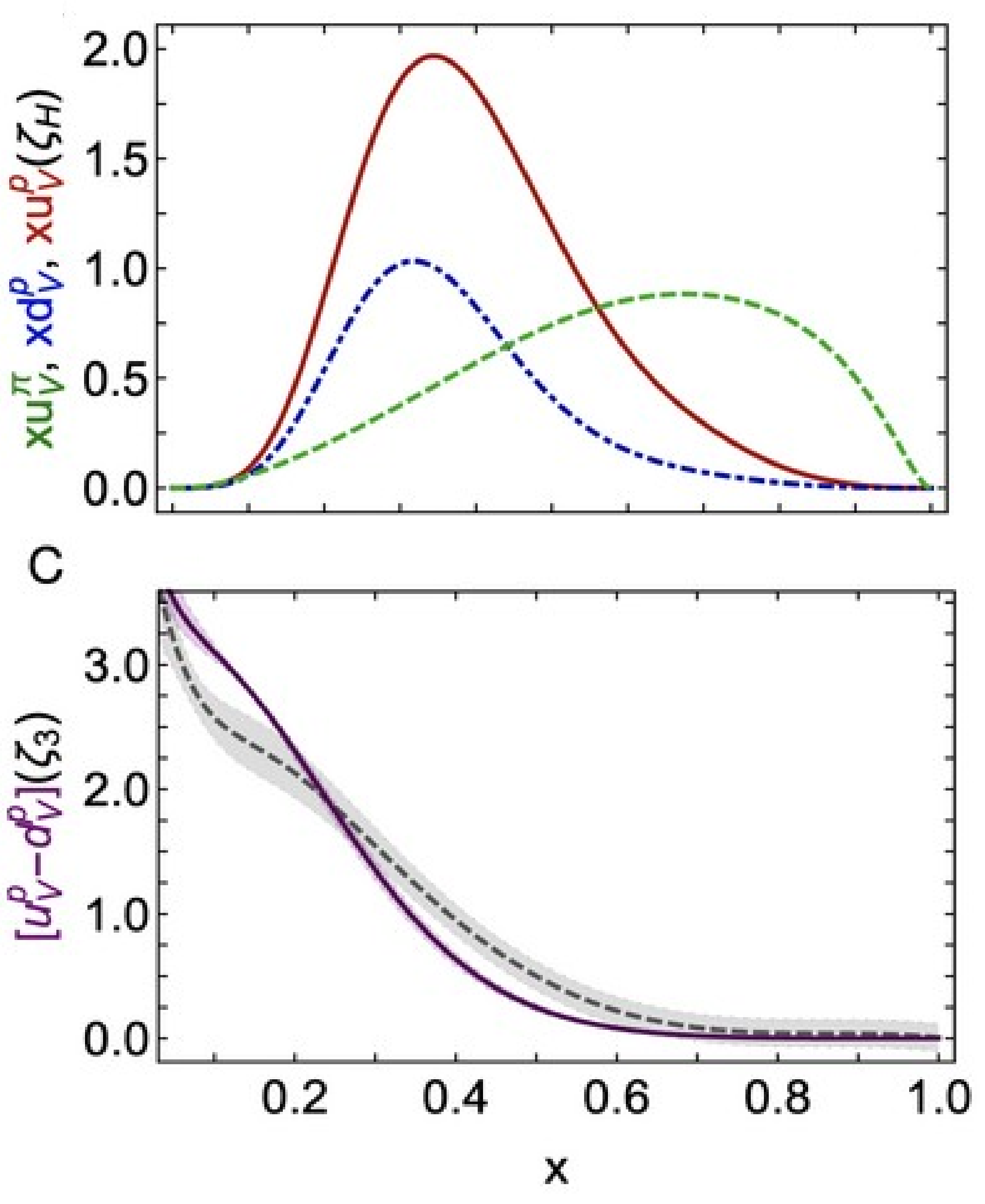

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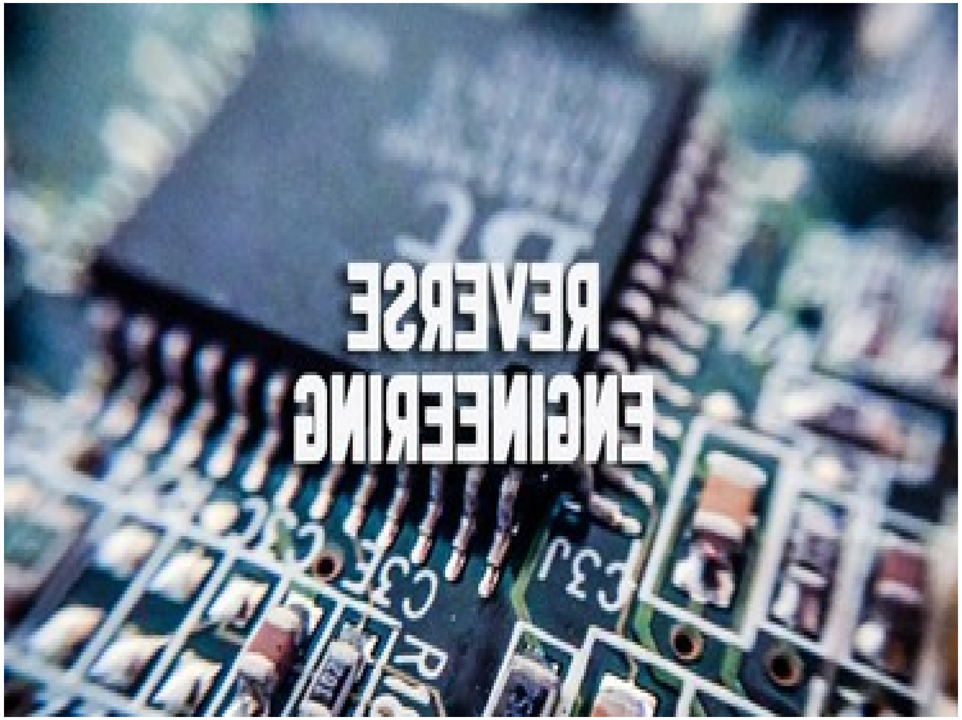
Producing an isovector distribution in fair agreement with lattice results
[H-W. Lin et al., arXiv:2011.14791]

Reverse engineering the PDF data



Pion PDF

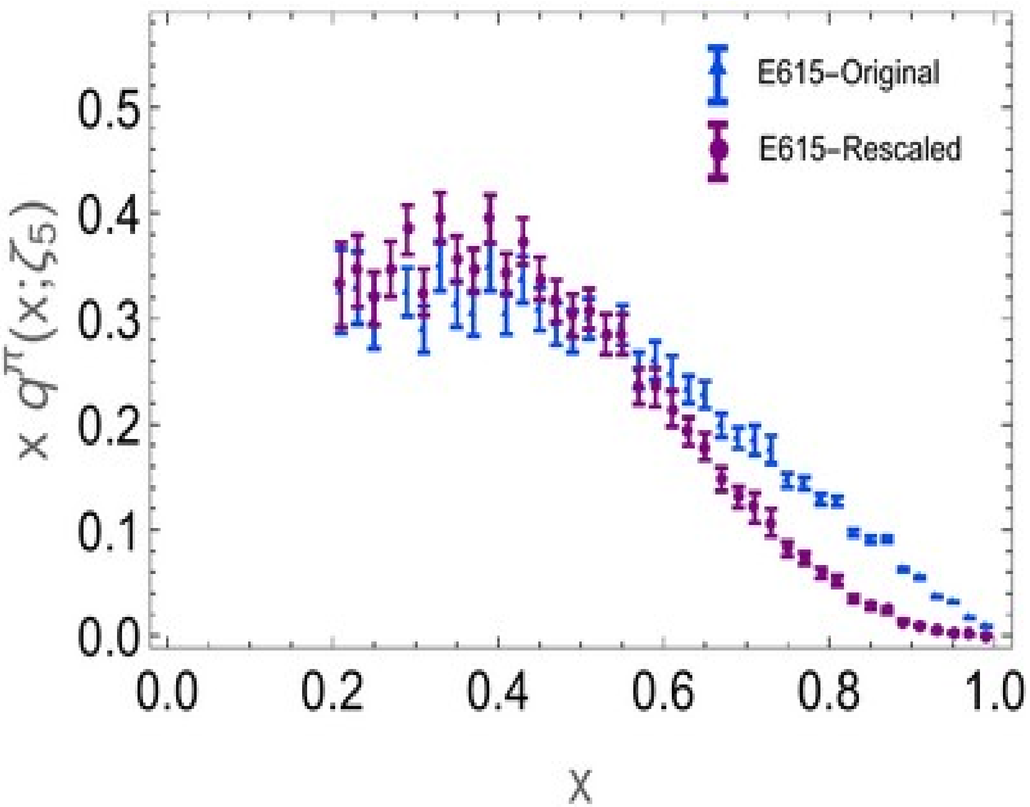
- Let us assume the data can be parameterized with a certain functional form, i.e.:

$$u^{\pi}(x;[\alpha_i];\zeta) = n_u^{\zeta} x^{\alpha_1^{\zeta}} (1-x)^{\alpha_2^{\zeta}} (1+\alpha_3^{\zeta} x^2)$$

Normalization

$$\{\alpha_i^{\zeta} | i = 1,2,3\}$$

Free parameters



- Then, we proceed as follows:

1) Determine the **best values α_i** via least-squares fit to the data.

2) Generate new **values α_i,** distributed randomly around the best fit.

3) Using the latter set, evaluate:

$$\chi^2 = \sum_{l=1}^{N} \frac{(u^{\pi}(x_l;[\alpha_i];\zeta_5) - u_j)^2}{\delta_l^2}$$

Data point with error

4) Accept a replica with probability:

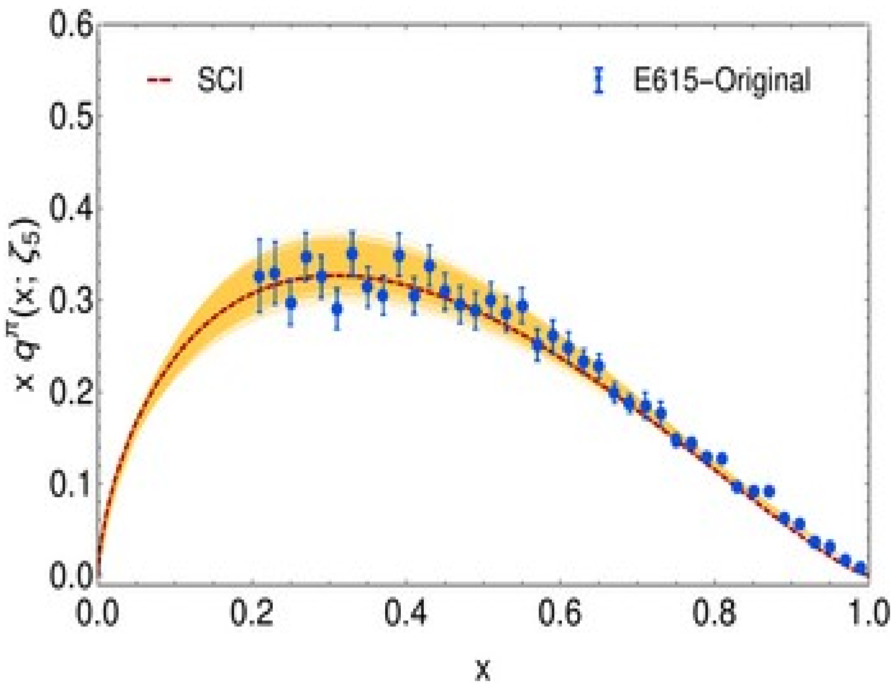
$$\mathcal{P} = \frac{P(\chi^2;d)}{P(\chi_0^2;d)}, \quad P(y;d) = \frac{(1/2)^{d/2}}{\Gamma(d/2)} y^{d/2-1} e^{-y/2}$$

5) Evolve back to ζ_H

Repeat (2-5).

Pion PDF: Original E615 Data

➢ Applying this algorithm to the original data yields:



(average)

Mean values (of moments) and errors, ζ_H

{{0.5, 2.52187 × 10^-17}, {0.331527, 0.00803273}, {0.247615, 0.0110893}, {0.19784, 0.0121977}, {0.165066, 0.0124911}, {0.141928, 0.0124198}, {0.124755, 0.0121811}, {0.111521, 0.0118683}, {0.101021, 0.0115275}, {0.0924926, 0.0111824}, {0.085431, 0.010845}, {0.0794897, 0.0105214}, {0.0744232, 0.0102142}, {0.0700521, 0.00992435}, {0.0662432, 0.00965182}}

(SCI)

Moments from SCI, ζ_H

{0.5, 0.332885, 0.249327, 0.199231, 0.165865, 0.142056, 0.124215, 0.11035, 0.0992657, 0.090203, 0.0826552, 0.0762721, 0.0708035, 0.0660661, 0.0619225}

✔ The produced moments are compatible with a **symmetric PDF** at the **hadronic scale**.

✗ But also exhibit agreement with the **SCI results**.

$$q_{\rm SCI}(x;\zeta_H) \approx 1$$

Thus, given the **QCD prescription**,

$$u^{\pi}(x;\zeta) \overset{x\simeq 1}{\approx} (1-x)^{\beta=2+\gamma(\zeta)}$$

We shall **discard** this for the upcoming construction of the valence quark GPD

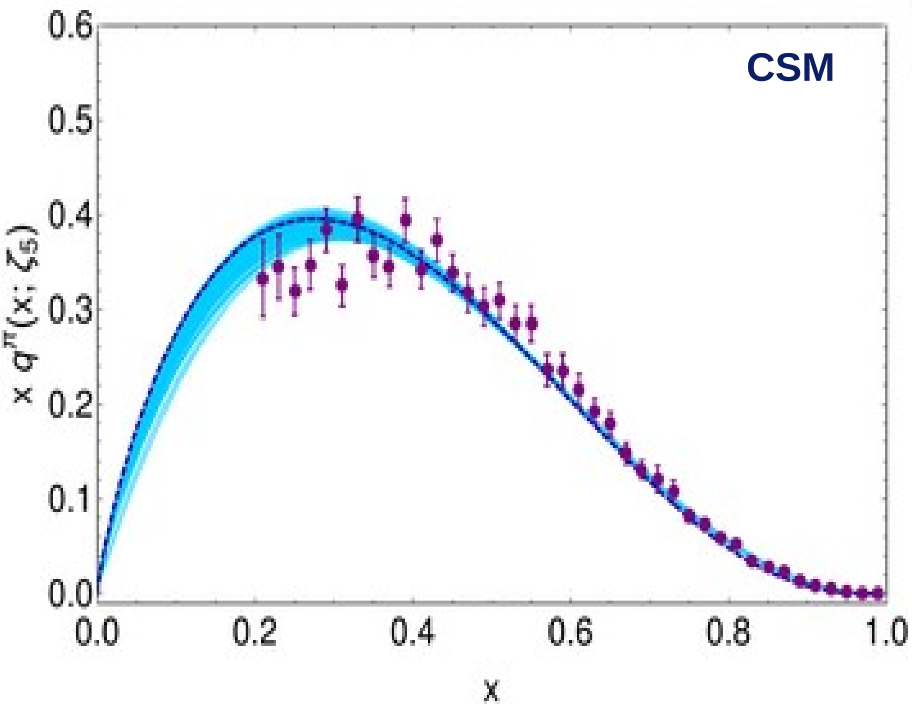
Pion PDF: ASV Data

➢ Applying this algorithm to the **ASV data** yields:

(average)

Mean values (of moments) and errors

{{0.5, 2.75144×10^{-17}}, {0.299833, 0.00647045}, {0.199987, 0.00735448}, {0.142895, 0.0068623},
{0.107274, 0.00608759}, {0.0835168, 0.00532834}, {0.0668711, 0.0046596},
{0.0547511, 0.00409028}, {0.0456496, 0.00361041}, {0.0386394, 0.00320609}}

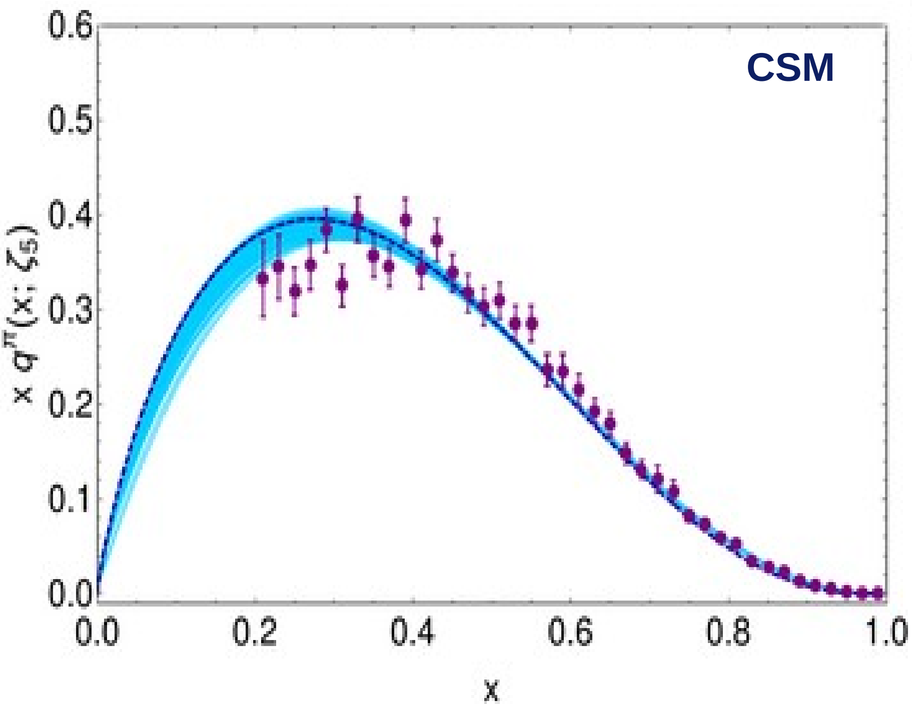


✔ The produced moments are compatible with a **symmetric PDF** at the **hadronic scale**.

✔ It seems it favors a **soft end-point** behavior… just like the **CSM result**.

Pion PDF: ASV Data

- Applying this algorithm to the **ASV data** yields:



Mean values (of moments) and errors

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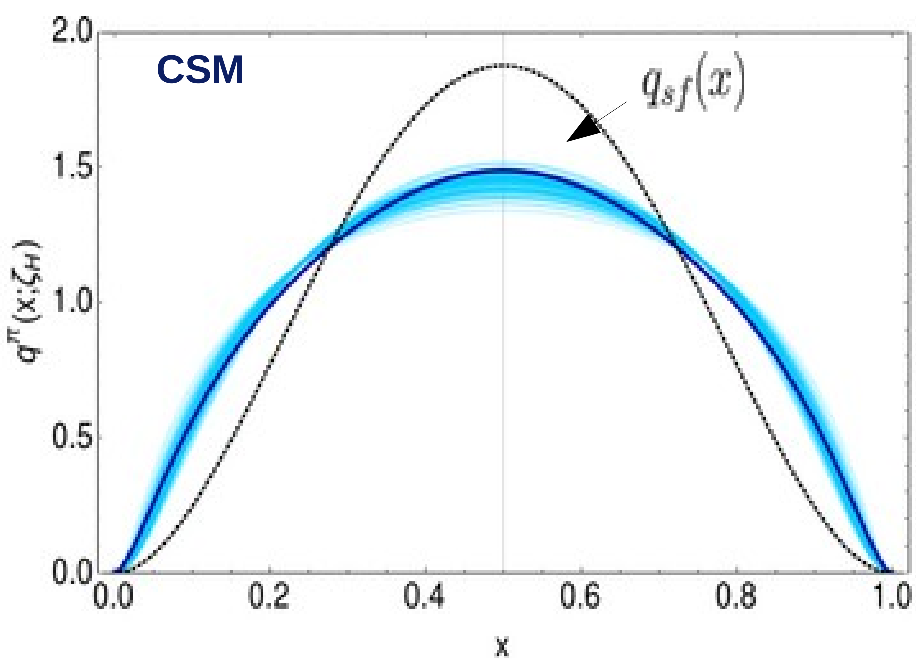
{0.0547511, 0.00409028}, {0.0456496, 0.00361041}, {0.0386394, 0.00320609}}

✔ Then, we can **reconstruct** the moments produced by each replica, using the **single-parameter Ansatz**:

$$u^{\pi}(x;\zeta_{\mathcal{H}}) = n_0 \ln(1 + x^2(1-x)^2/\rho^2)$$



✔ The produced moments are compatible with a **symmetric PDF** at the **hadronic scale**.

✔ It seems it favors a **soft end-point** behavior… just like the **CSM result**.

Pion PDF: Lattice Data

- We can follow an analogous procedure to infer, based upon **lattice data**, how the **hadronic scale PDF** should look like.

- Let us consider the list of **lattice QCD** moments:

Joo:2019bzr Sufian:2019bol Alexandrou:2021mmi

n	[61]	[62]	[63]
1	0.254(03)	0.18(3)	0.23(3)(7)
2	0.094(12)	0.064(10)	0.087(05)(08)
3	0.057(04)	0.030(05)	0.041(05)(09)
4			0.023(05)(06)
5			0.014(04)(05)
6			0.009(03)(03)

- Those verify the recurrence relation, thus being compatible with a **symmetric PDF** at ζ_H

- While also falling within the **physical bounds**.

$$\frac{1}{2^n} \leq \langle x^n \rangle^{\zeta}_{u_\pi} \left(\langle 2x \rangle^{\zeta}_{u_\pi}\right)^{-\gamma_0^n/\gamma_0^1} \leq \frac{1}{1+n}$$

Produced by

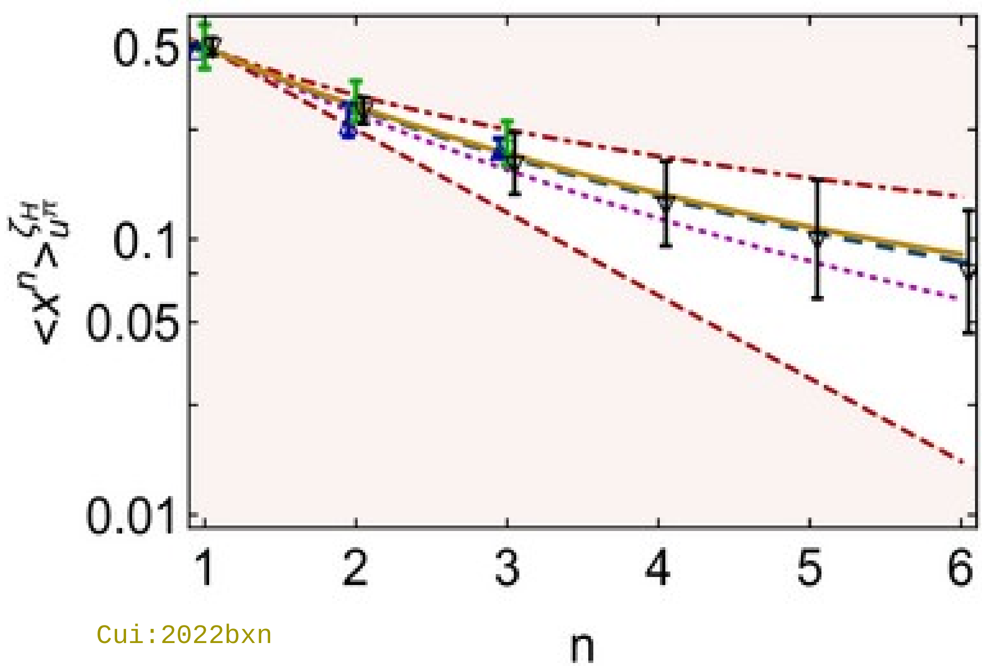
$$q(x;\zeta_H) = \delta(x - 1/2)$$

(infinitely heavy valence quarks)

Produced by

$$q(x;\zeta_H) = 1$$

(massless SCI case)



Cui:2022bxn

Pion PDF: recapitulation

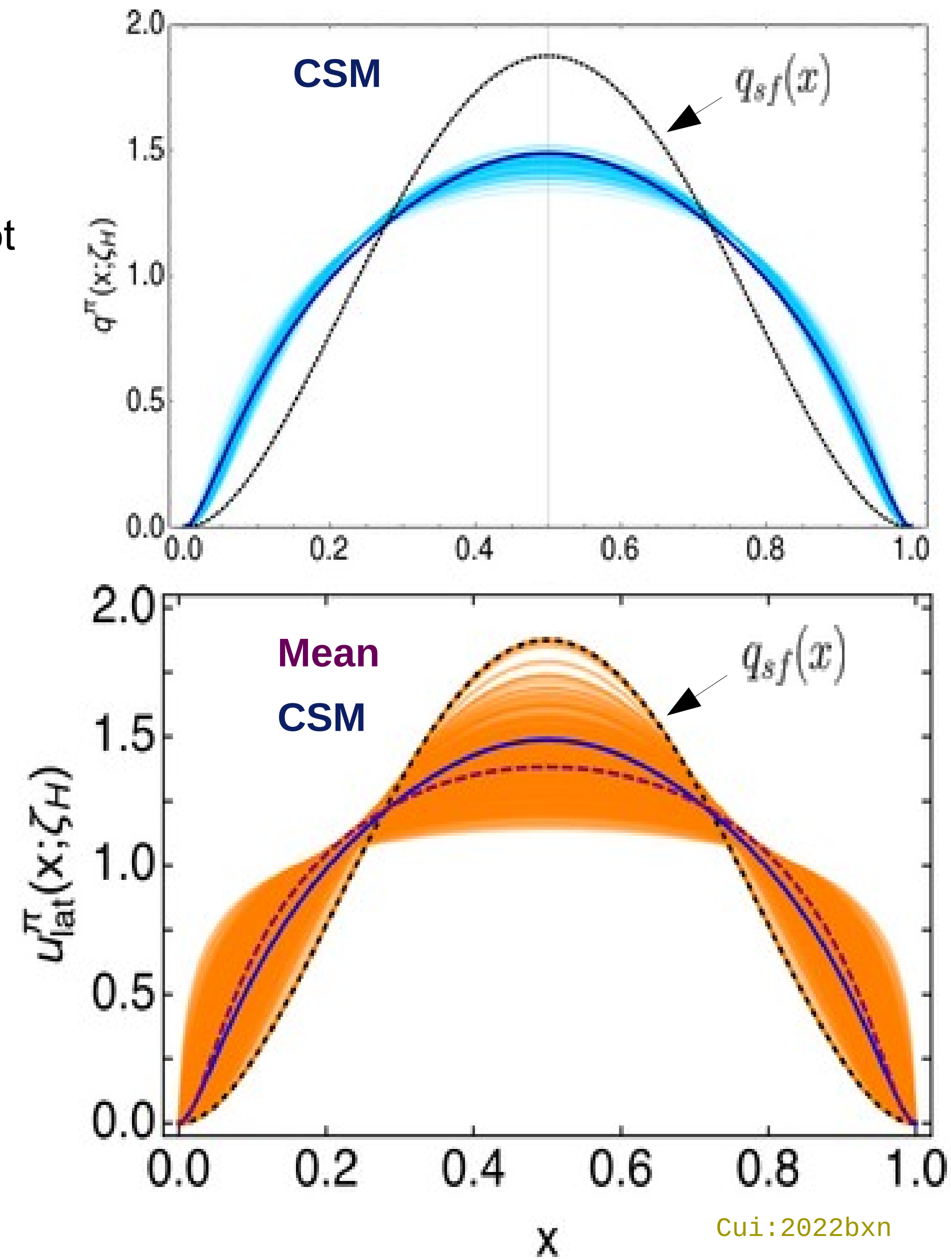
- The (**original**) **experimental** data yield a hadronic scale **PDF** compatible with **SCI results**.

  ➔Thus should be disfavored since it does not produce the expected large-x behavior.

- Both (**ASV**) **experimental** and **lattice** data yield hadronic scale **PDFs** exhibiting soft end-point behavior and **EHM-induced broadening**.

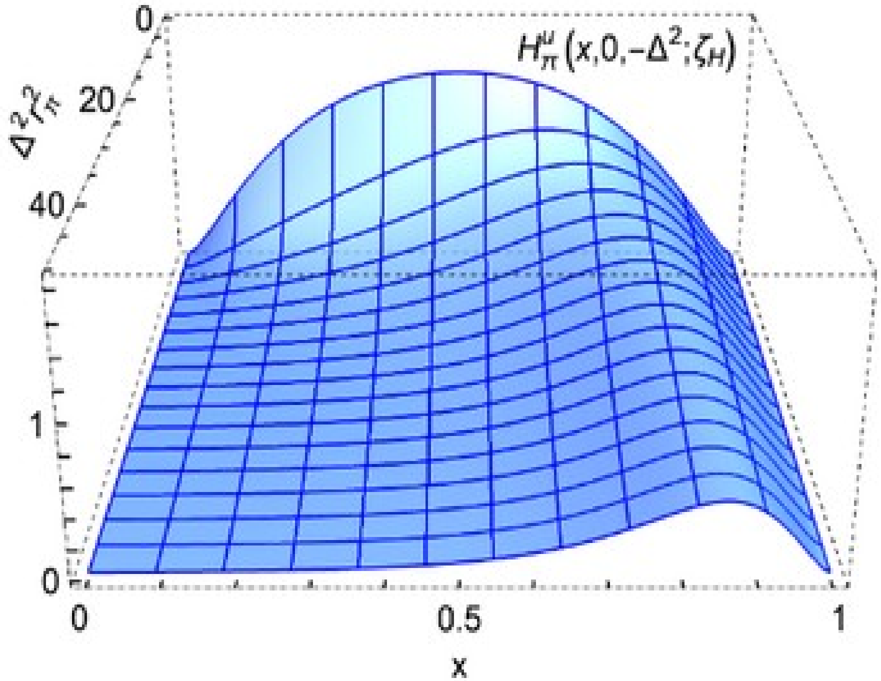
- The results are **compatible**, although current precision of the lattice moments still leaves us with a somewhat **wide band** of **uncertainty**.

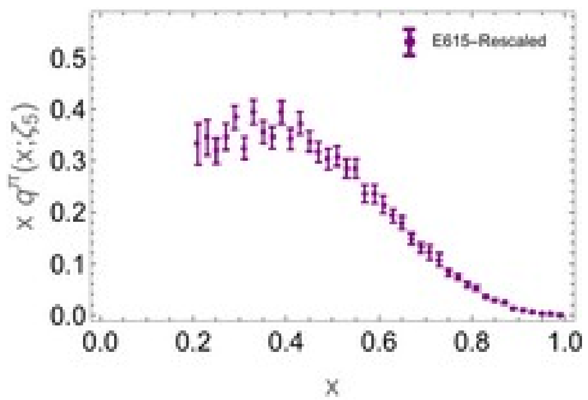
- Thus we focus on the **ASV** data for the rest of the discussion.

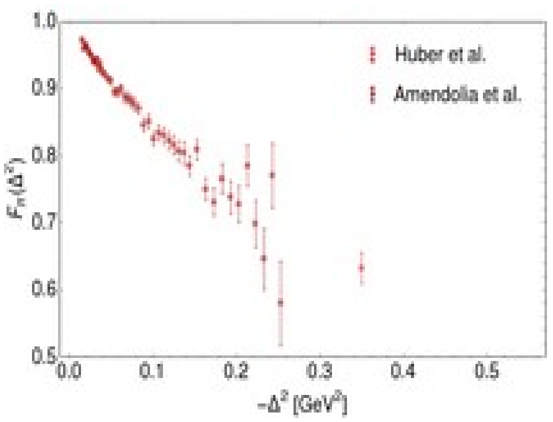


Cui:2022bxn

GPDs from PDFs and form factors







Light-front wave functions

- Many **distributions** are related via the leading-twist light-front wave function (**LFWF**), e.g.:

Distribution amplitudes

$$f_{\mathsf{P}}\varphi^{u}_{\mathsf{P}}(x,\zeta_{\mathcal{H}}) = \int \frac{dk_\perp^2}{16\pi^3}\psi^{u}_{\mathsf{P}}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)$$

Distribution functions

$$u^{\mathsf{P}}(x;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\left|\psi^{u}_{\mathsf{P}}\left(x,k_\perp^2;\zeta_{\mathcal{H}}\right)\right|^2$$

- In the **DGLAP** kinematic domain, this is also the case of the valence-quark **GPD**:

$$H^{u}_{\mathsf{P}}(x,\xi,t;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\psi^{u*}_{\mathsf{P}}\left(x_-,k_{\perp-}^2;\zeta_{\mathcal{H}}\right)\psi^{u}_{\mathsf{P}}\left(x_+,k_{\perp+}^2;\zeta_{\mathcal{H}}\right)$$

$$x_\mp = (x\mp\xi)/(1\mp\xi)\,,$$
$$k_{\perp\mp} = k_\perp \pm (\Delta_\perp/2)(1-x)/(1\mp\xi)$$

$$\psi^{u}_{\mathsf{P}}(x,k_\perp^2;\zeta)$$

"One ring to rule them all"

LFWF: Factorized models

Raya:2021zrz

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- If the **x-k** dependence is factorized, then:

$$\psi^{\uparrow\downarrow}_{\mathbf{P}_u}(x,k_\perp^2;\zeta_H) = \tilde{\psi}^{\mathbf{u}}_{\mathbf{P}_u}(k_\perp^2)\left[u^{\mathbf{P}}(x;\zeta_H)\right]^{1/2}$$

➔ The **x-dependence** of the LFWF lies within the **PDF** or, equivalently, the **PDA**:

$$u^{\mathbf{P}}(x;\zeta_H) = [\varphi^u_{\mathbf{P}}(x;\zeta_H)]^2 \Big/ \int_0^1 dx[\varphi^u_{\mathbf{P}}(x;\zeta_H)]^2$$

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- Our experience with **CSM** have revealed correlations proportional to

$$M_{\mathbf{P}}^2,\ M_h^2 - M_q^2$$

- So it should be a very good **Ansatz** for the **pion**, and fairly good for the **kaon**.

LFWF: Factorized models

Raya:2021zrz

- Starting with a **factorized LFWF**, $\psi^{\uparrow\downarrow}_{\mathbf{P}_u}(x, k_\perp^2; \zeta_H) = \tilde{\psi}^{\mathbf{u}}_{\mathbf{P}_u}(k_\perp^2)\,[u^{\mathbf{P}}(x;\zeta_H)]^{1/2}$

This one shall be obtained as in the first part of the talk

- The overlap representation for the **GPD** entails:

$$H^u_{\mathsf{P}}(x,\xi,t;\zeta_{\mathcal{H}}) = \int \frac{d^2k_\perp}{16\pi^3}\psi^{u*}_{\mathsf{P}}\left(x_-,k^2_{\perp-};\zeta_{\mathcal{H}}\right)\psi^u_{\mathsf{P}}\left(x_+,k^2_{\perp+};\zeta_{\mathcal{H}}\right)$$

$$= \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

Heaviside Theta

This dictates the off-forward behavior of the GPD

- Where $z = s_\perp^2 = -t(1-x)^2/(1-\xi^2)^2$ and:

… will be driven by the electromagnetic form factor

$$\boxed{\Phi^u_{\mathbf{P}}(z;\zeta_H) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\tilde{\psi}^{u*}_{\mathbf{P}}\left(\mathbf{k}_\perp^2;\zeta_H\right)\tilde{\psi}^u_{\mathbf{P}}\left(\left(\mathbf{k}_\perp-\mathbf{s}_\perp\right)^2;\zeta_H\right)}$$

The GPD **model**

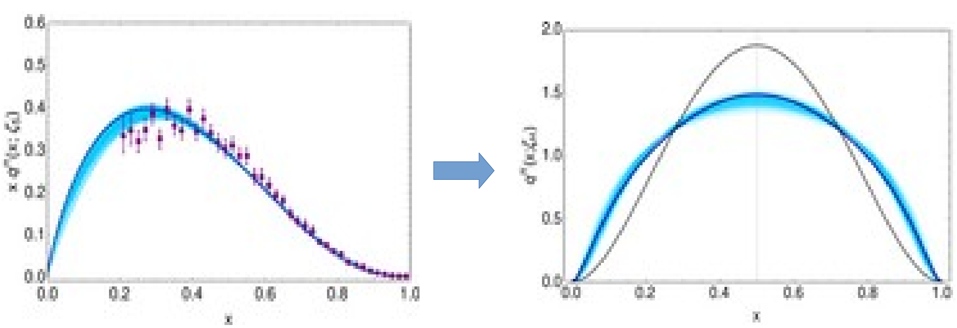
Raya:2021zrz

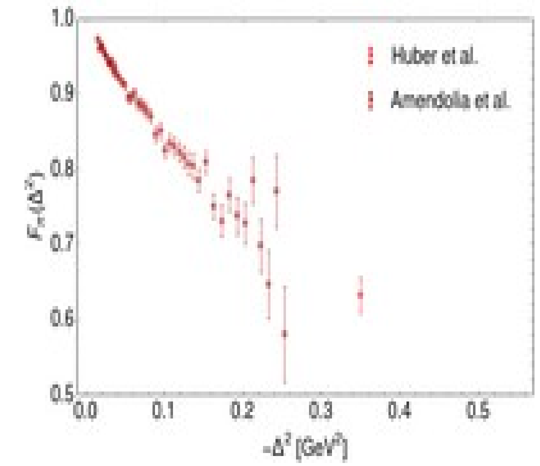
➢ The factorized **LFWF** motivates the following **GPD** model:

$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathrm{P}}(x_-;\zeta_H)u^{\mathrm{P}}(x_+;\zeta_H)}\Phi_{\mathrm{P}}(z;\zeta_H)$$

- The **PDF** might be inferred from **data**, as described before.
- Thus, **parameterized** by:

$$u^{\pi}(x;\zeta_{\mathcal{H}}) = n_0 \ln(1 + x^2(1-x)^2/\rho^2)$$





- The **GPD** connects **Φ(z)** with the **EFF** via**:**

$$F_{\pi}(t) = \int_0^1 dx\, u^{\pi}(x;\zeta_H)\Phi_{\pi}(z;\zeta_H)$$

- A useful **parametrization** is:

$$\Phi_{\pi}(z;\zeta_H) = \frac{1 + (b_1 - 1)r_{\pi}^2/(6<x^2>)z}{1 + b_1 r_{\pi}^2/(6<x^2>)z + b_2 z^2}$$

- Where $\mathbf{r_{\pi}}$ is taken from **PDG** and $\mathbf{b_{1,2}}$ are parameters to be fitted to the experimental data.

The GPD **model**

Raya:2021zrz

- We have a **3-parameter** model for the **GPD**: $\{\rho, b_1, b_2\}$

$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

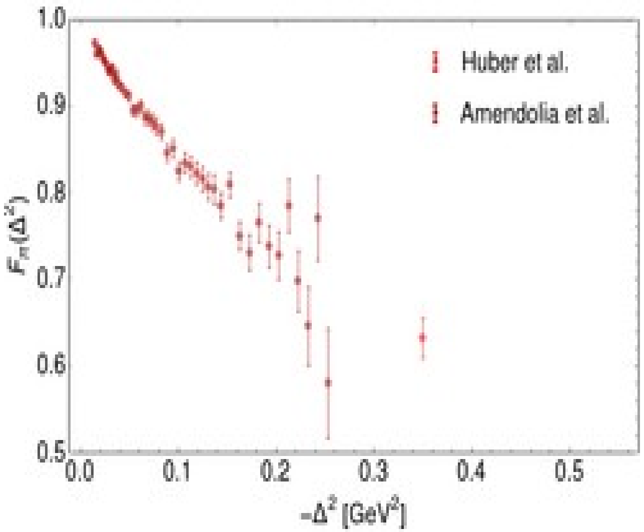
$$u^{\pi}(x;\zeta_{\mathcal{H}}) = n_0 \ln(1+x^2(1-x)^2/\rho^2) \qquad \Phi_\pi(z;\zeta_H) = \frac{1+(b_1-1)r_\pi^2/(6<x^2>)z}{1+b_1 r_\pi^2/(6<x^2>)z + b_2 z^2}$$

- The **strategy** is as follows:

1) Following the described procedure for the **PDF**, generate a replica "*i*", storing the value ρ_i, and its probability of acceptance $P(\rho_i)$.

2) Using such **replica**, integrate the **GPD** (for ξ=0) using random values of $b_{1,2}$ and varying randomly $\mathbf{r}_\pi$ within the range 0.659 +/- 0.005 fm (in agreement with its **PDG** value).

3) Compute the χ^2_i by comparing with the **EFF** experimental data [Amendolia:1984nz, JeffersonLab:2008jve].



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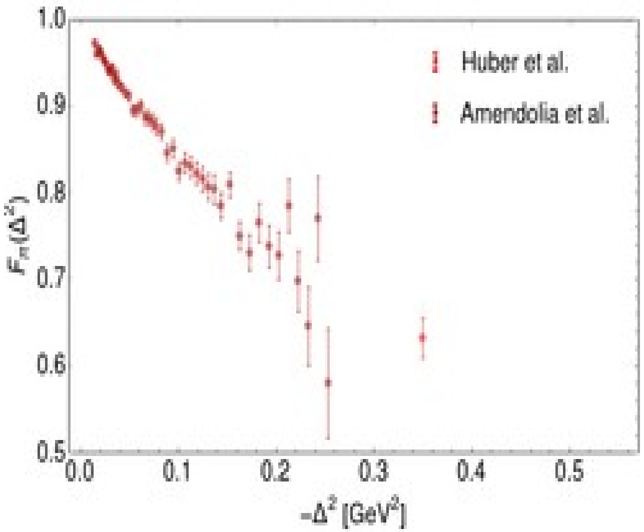
- The **strategy** is as follows:

4) Use χ^2_i to **calculate** $P(\{b_1^i, b_2^i\}|\rho_i)$

Subsequently, accept the set of parameters with probability:

$$P(\{\rho_i, b_1^{(i)}, b_2^{(i)}\}) = P(\{b_1^{(i)}, b_2^{(i)}\}|\rho_i)P(\rho_i)$$

Repeat.



Numerical **Results**

$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_{\mathcal{H}}) = \Theta(x_-)\sqrt{u^{\mathbf{P}}(x_-;\zeta_H)u^{\mathbf{P}}(x_+;\zeta_H)}\Phi_{\mathbf{P}}(z;\zeta_H)$$

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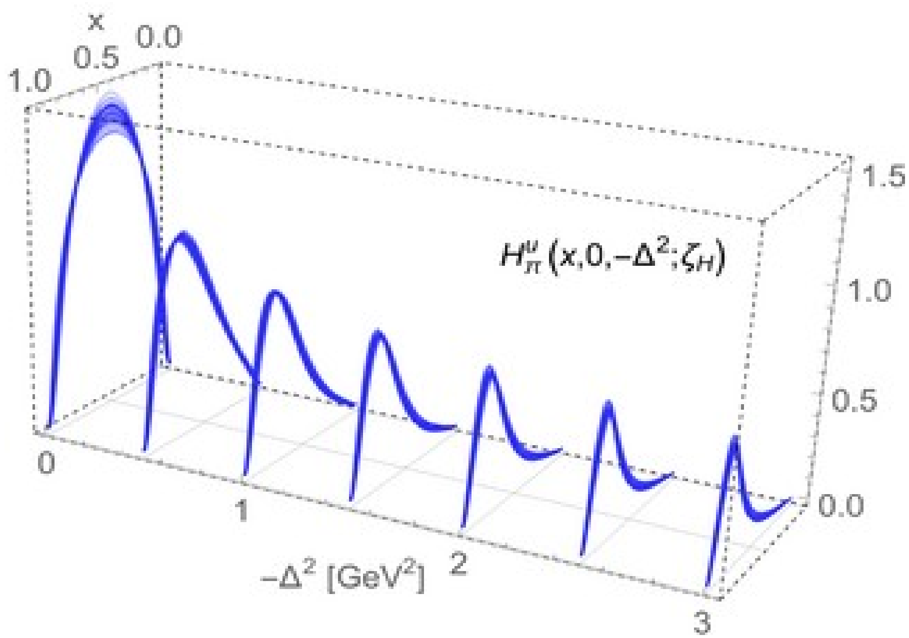
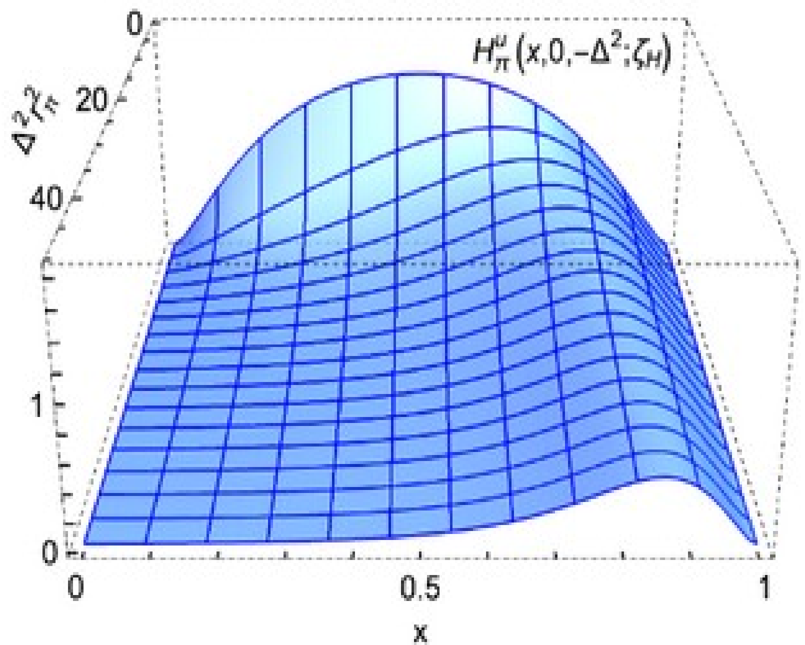
- Combining **pion PDF** data (**ASV**) and **pion EFF** data, one arrives at:

$$\rho = 0.07 \pm 0.03,\ b_1 = 0.46 \pm 0.40,\ b_2 = 18.67 \pm 4.38$$

(with proper mass units)





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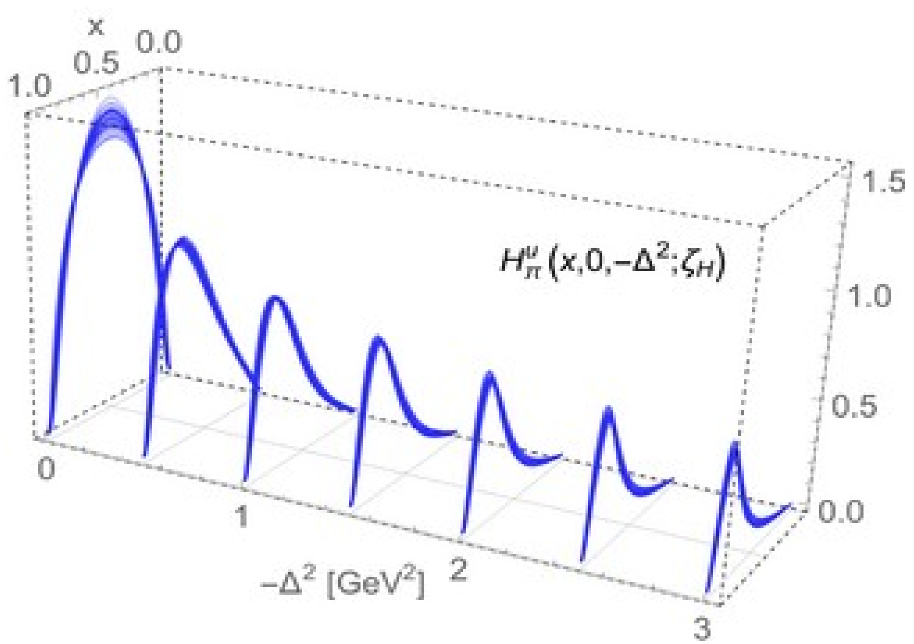

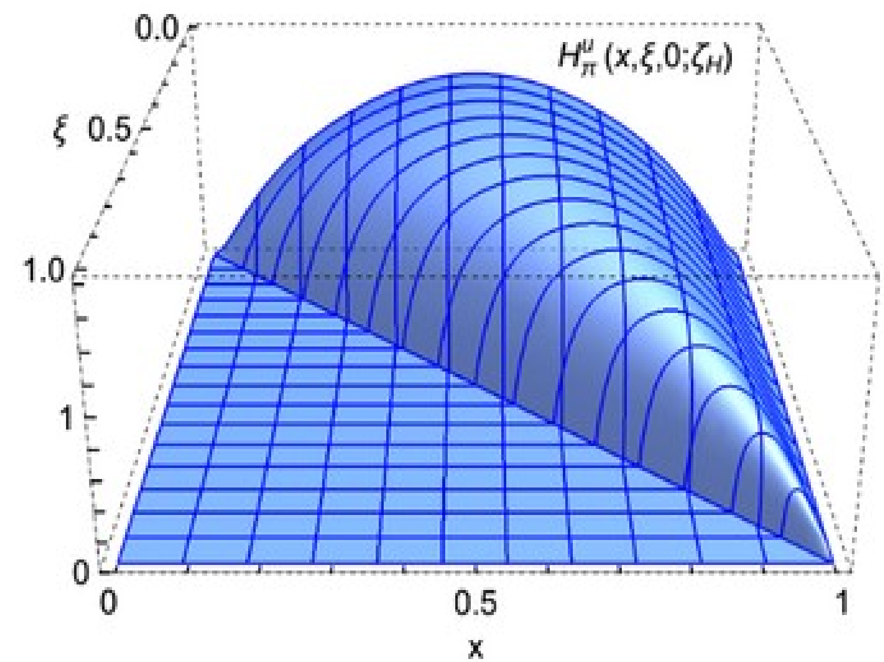

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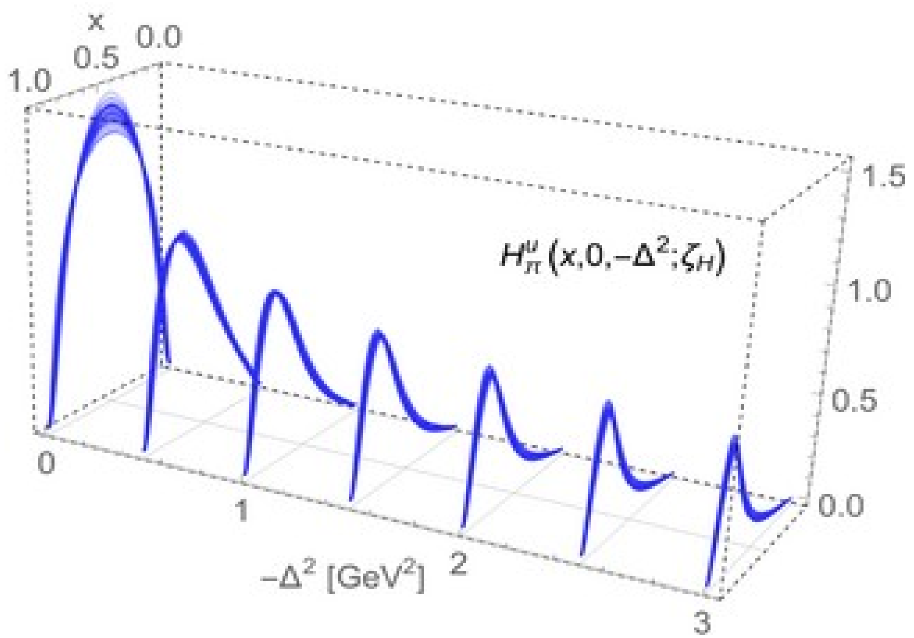

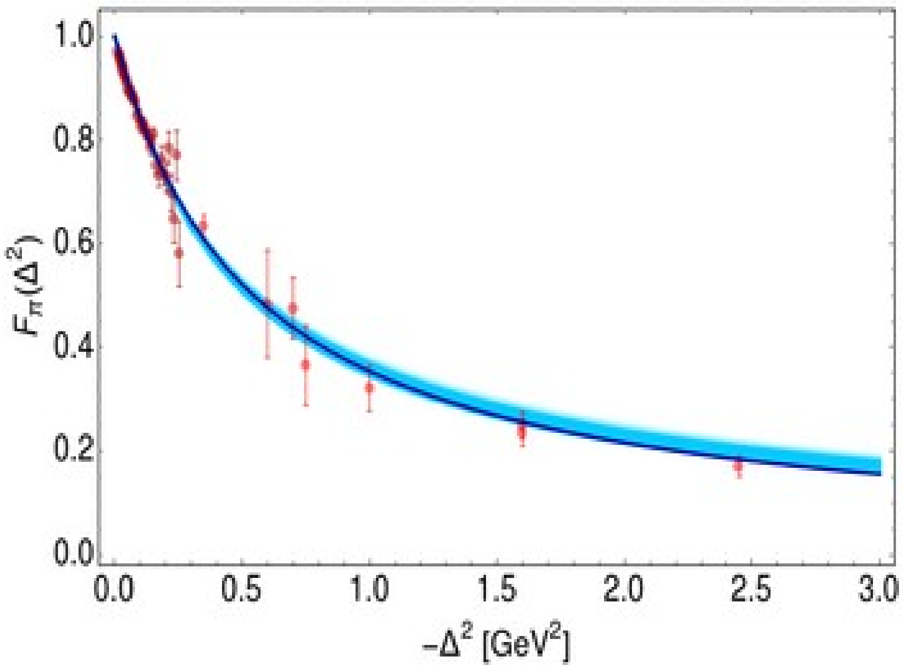

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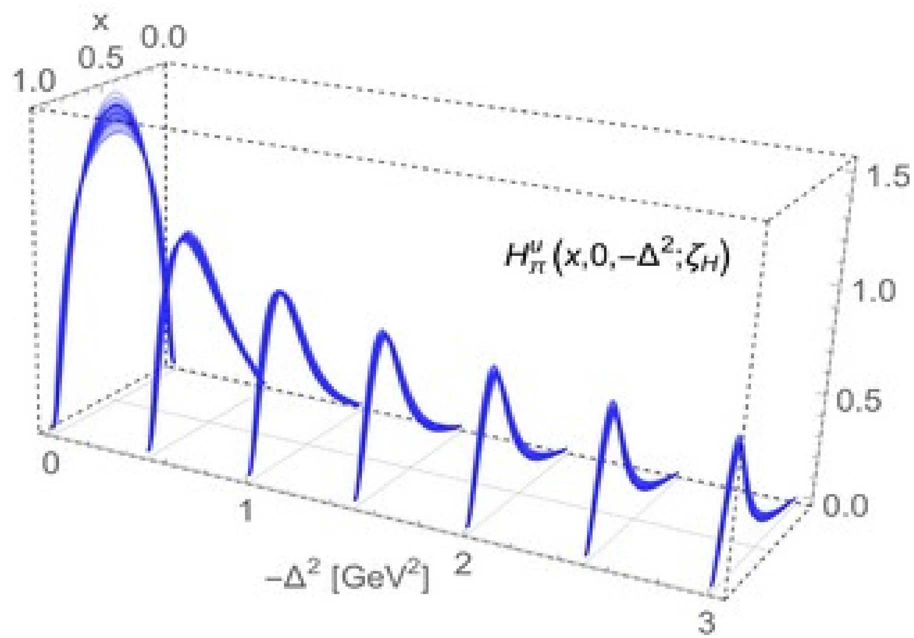
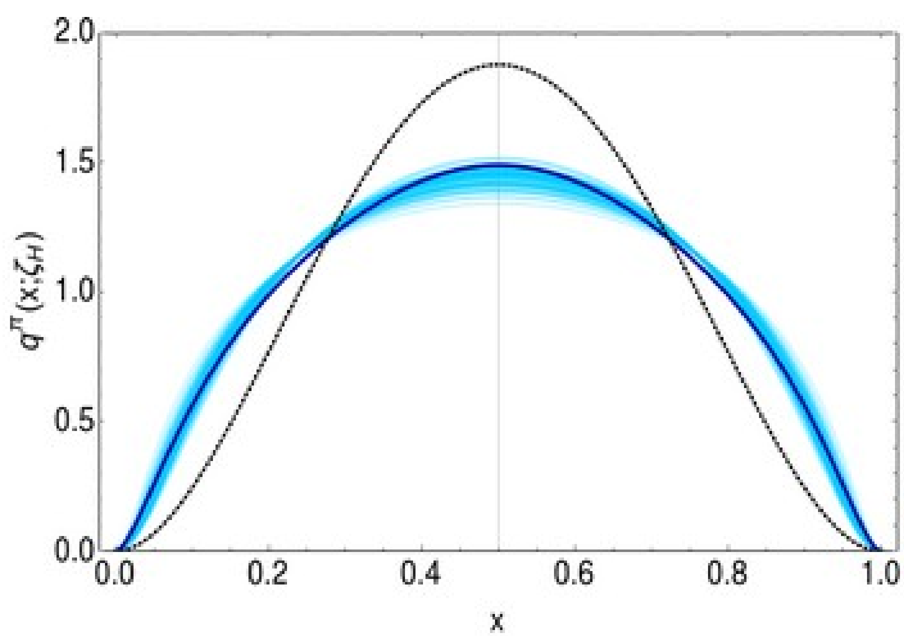
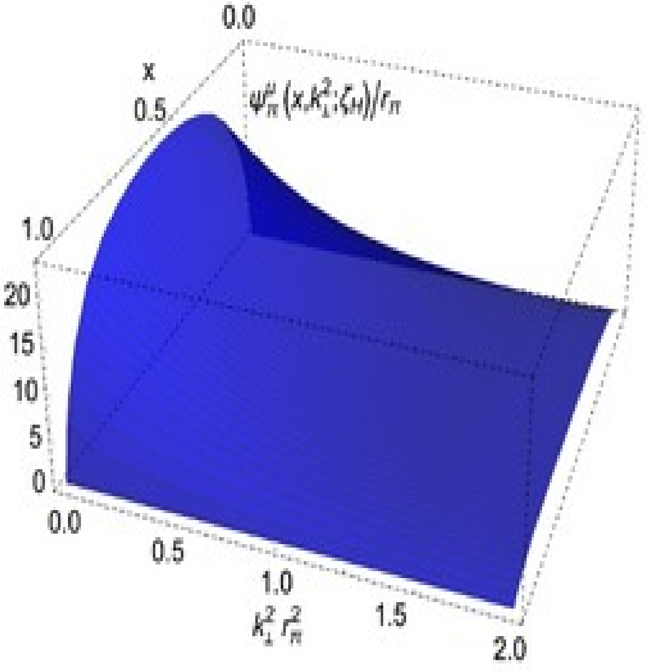
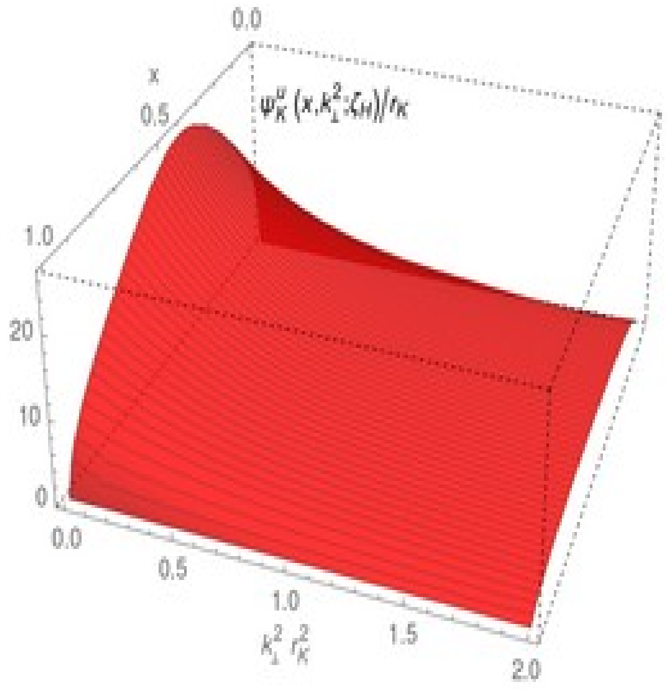
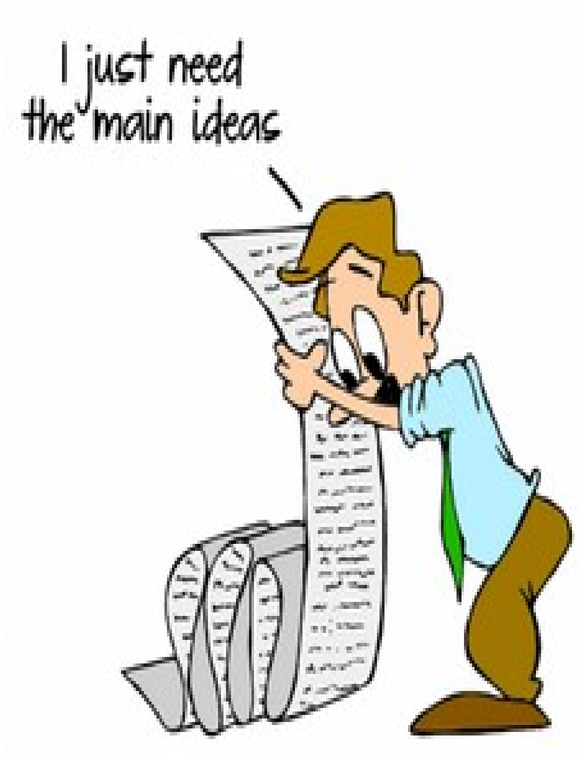




Summary and Scope

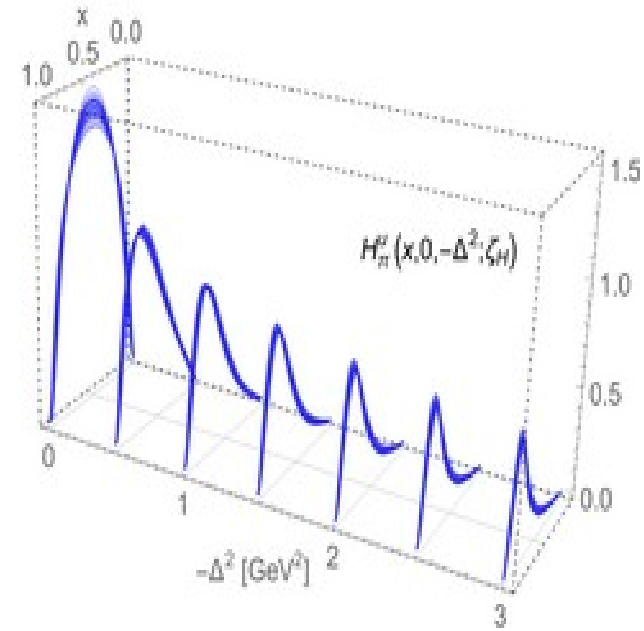


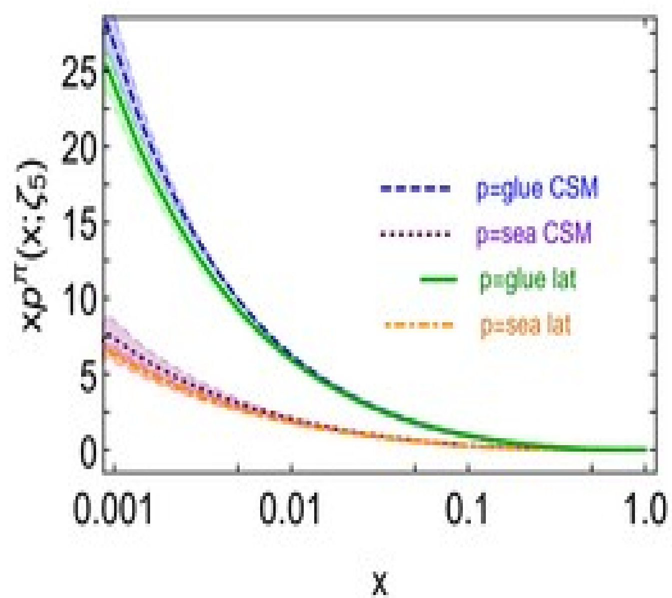




Summary and Scope

- We have derived, and tested, some key implications stemming from the evolution from a **hadronic scale** with the assumed **all orders** scheme.
- The experimental and lattice data of the **pion PDF** is **evolved**, downwards, toward the hadronic scale following the **all orders** evolution scheme.
- **Lattice QCD** and the **ASV** analysis favor the **CSM** results, but other sets of data could be used, if required.
- Contrasting with empirical information on the **EFF**, a **GPD** can be delivered and, at the end of the day, <u>is fully described by **only 3** parameters.</u>
- We can also evolve back to produce gluon and sea content!!





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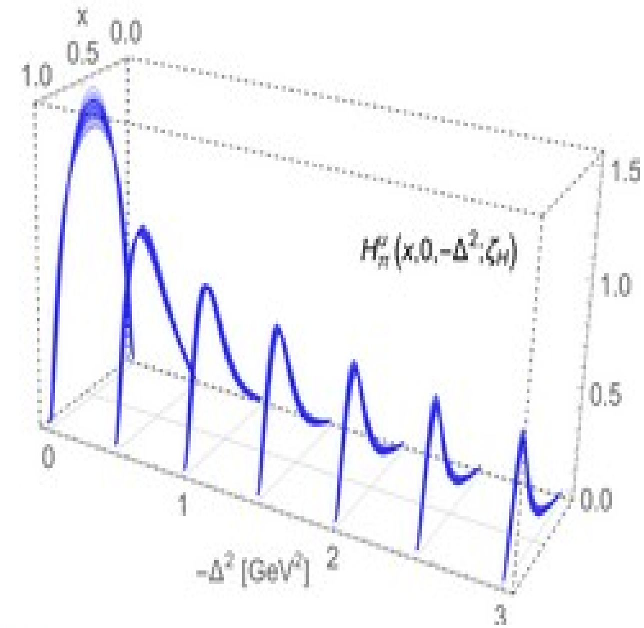
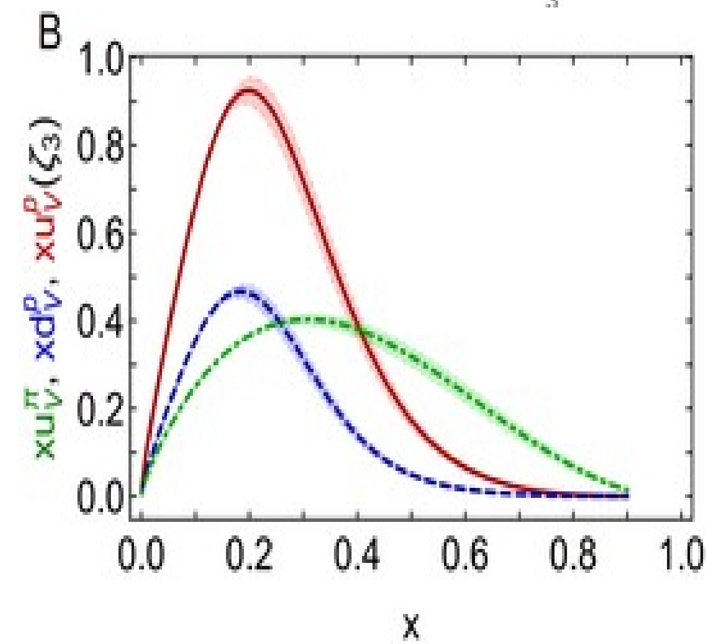
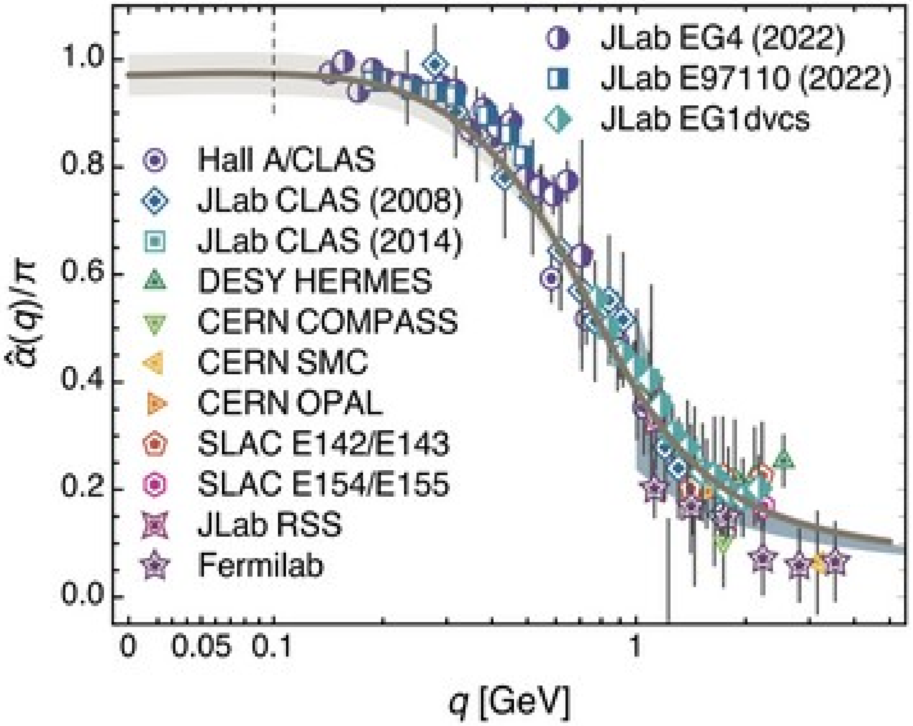
- We can also evolve back to produce gluon and sea content!!
- The robustness of the approach is to be tested in the case of proton PDFs.





Backslides

QCD effective charge



Modern continuum & lattice QCD analysis in the gauge sector delivers an analogue "Gell-Mann-Low" running charge, from which one obtains a process-independent, parameter-free prediction for the low-momentum saturation

- No landau pole
- Below a given mass scale, the interaction become scale-independent and QCD practically conformal again (as in the lagrangian).

Then, we define:

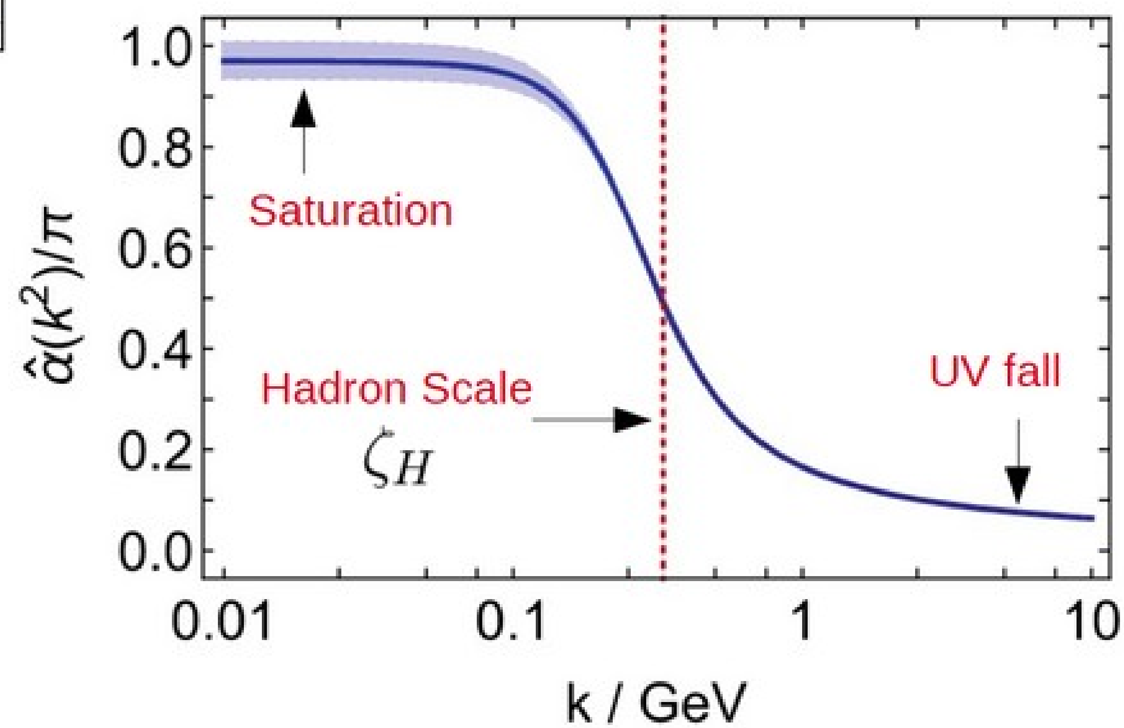
$$\alpha(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{M}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]}\,;\ \alpha(0) = 0.97(4)$$

where

$$\mathcal{M}(k^2 = \Lambda^2_{\rm QCD}) := m_G = 0.331(2)\ \text{GeV}$$

defines the screening mass and an associated wavelength, such that larger gluon modes decouple.

Then, we identify: $\zeta_H := m_G(1 \pm 0.1)$



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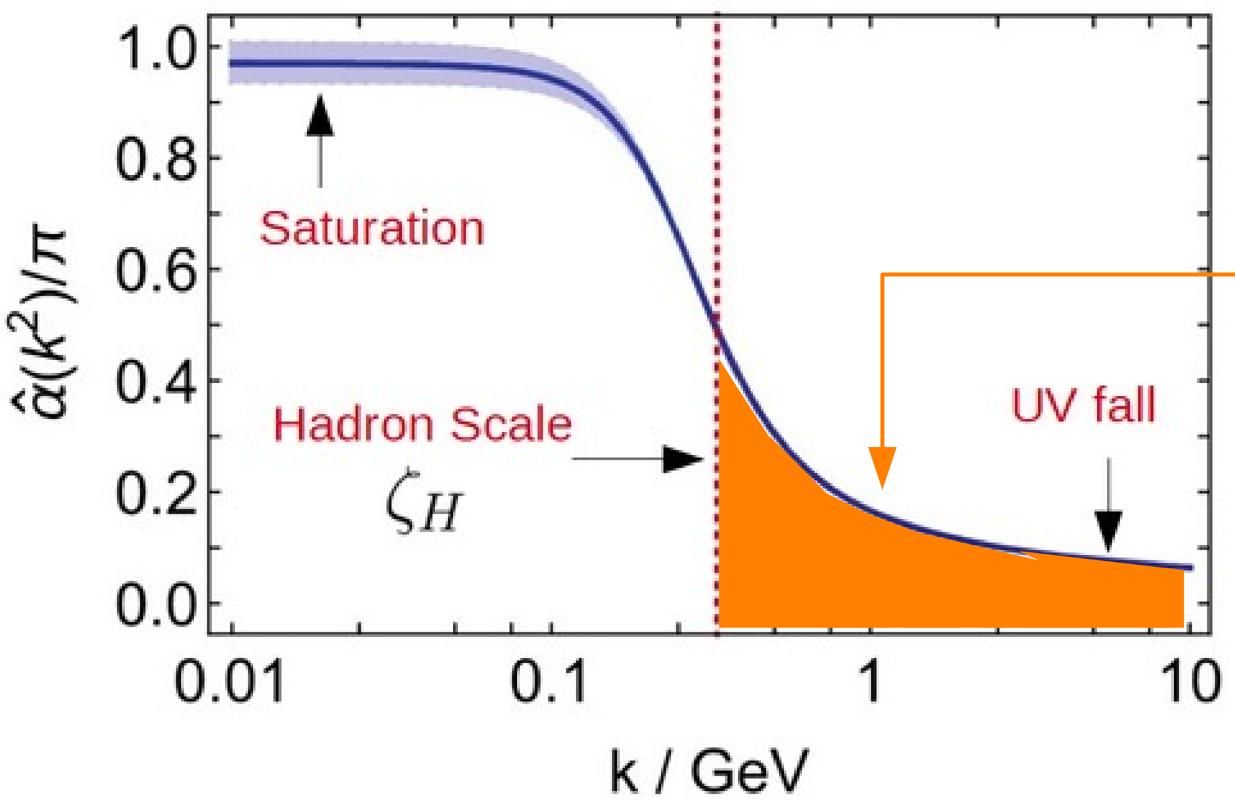
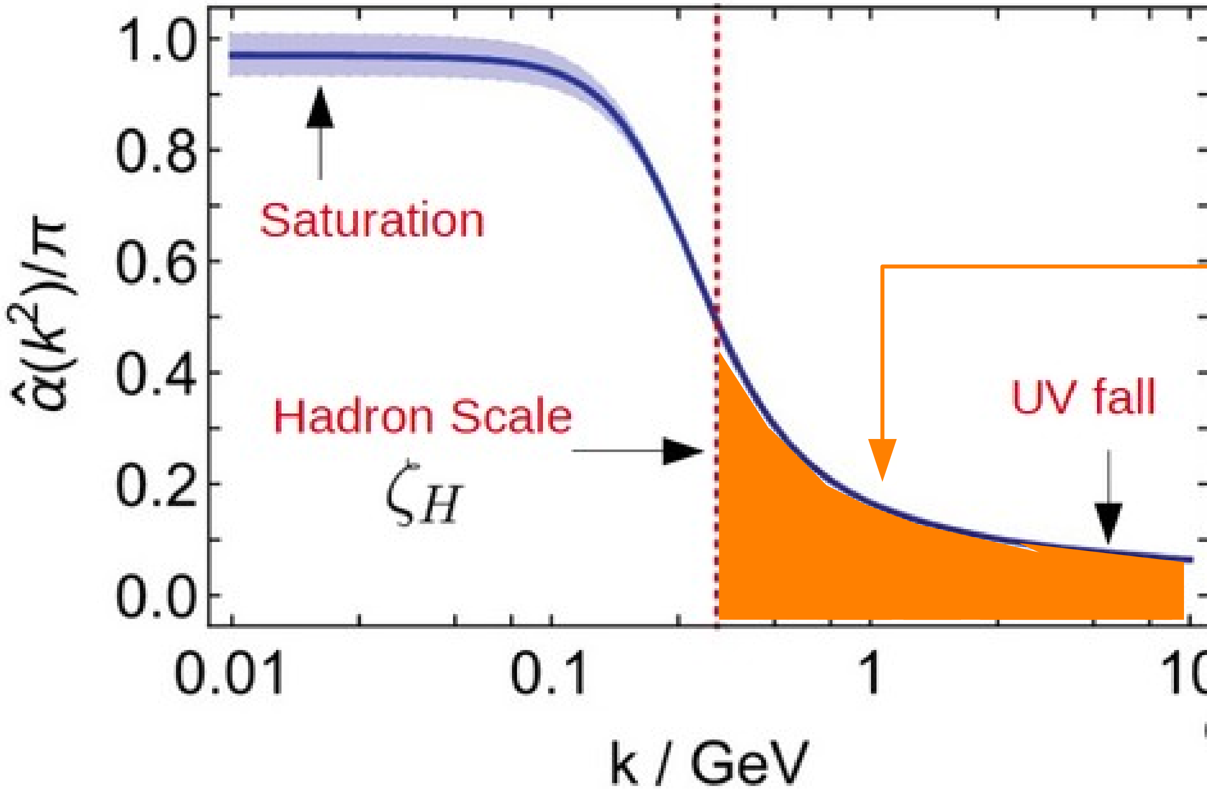


The strength of the charge defines de input for the evolution

$$S(\zeta_H, \zeta_f) = \int_{2\ln(\zeta_H/\Lambda_{QCD})}^{2\ln(\zeta_f/\Lambda_{QCD})} dt\, \widehat{\alpha}(t)$$

$$\langle x(\zeta_5)\rangle_q^\pi = \frac{1}{2}\exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_5)\right) = 0.20(2)$$

QCD effective charge



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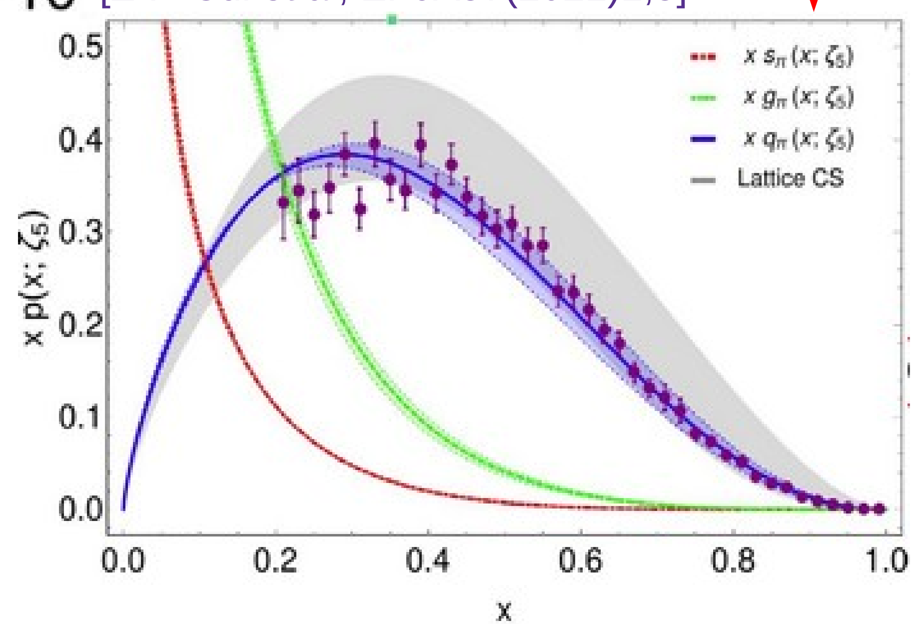
$$\langle x(\zeta_5)\rangle_q^\pi = \frac{1}{2}\exp\left(-\frac{8}{9\pi}S(\zeta_H,\zeta_5)\right) = \boxed{0.20(2)}$$

[Z-F. Cui et al, EPJC80(2020)11,1064]
[Z-F. Cui et al, EPJA57(2021)1,5]



Then, the glue, valence- and sea-quark DFs can be predicted, with no tuned parameter, on the ground of the effective charge definition, from the LFWF (or, equivalentely, from a symmetry-preserving DSE/BSE computation of the valence-quarks Mellin moments

[M. Ding et al, CPC44(2020)3,031002]