Two-photon transitions of charmonia on the light front

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Outline

- **Introduction**: two-photon transition of charmonium
- **Method**: light-front Hamiltonian formalism
- **Results**: transition width and transition form factors
- **Summary

Introduction: two-photon transition

- Charmonium provides an ideal testing ground for various investigations to understand QCD
  - Challenging: relativistic, non-perturbative effects
- The two-photon transition, $H_{c\bar{c}} \rightarrow \gamma^* + \gamma$, provides a clean and important probe to hadron states
- Experimental measurements
  - Diphoton width $\Gamma_{H\rightarrow\gamma\gamma}$: extensive measurements for $\eta_c, \eta_c', \chi_{c0}, \chi_{c2}$
  - Transition form factors $F_{H\gamma}(Q^2)$: $F_{\eta_c\gamma}(Q^2)$ by BABAR 2010; $F_{\chi_{cJ}\gamma}(Q^2)$ by Belle 2017
**Method: light-front Hamiltonian formalism**

Light-front Hamiltonian formalism is a natural framework for addressing relativistic bound-state and scattering problems in QCD.

**Light-front quantization**

- The quantum field is quantized on the equal light-front time $x^+ = 0$

<table>
<thead>
<tr>
<th>instant form</th>
<th>front form</th>
<th>point form</th>
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<tbody>
<tr>
<td>time variable</td>
<td>$t = x^0$</td>
<td>$x^+ \triangleq x^0 + x^3$</td>
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<tr>
<td>quantization surface</td>
<td></td>
<td>$\tau = 0$</td>
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### Hamiltonian

- kinematical: $P^0, \vec{P}, \vec{J}$
- dynamical: $\vec{K}, P^0$

### Dispersion relation

$$p^0 = \sqrt{\not{p}^2 + m^2}$$

### Invariant variables

- $p^\pm \triangleq P^0 - P^3$
- $\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J^-$
- $\vec{F}^\perp, P^-$
- $p^- = (\not{p}^\perp + m^2)/p^+$

### Momentum

$$p^\mu = mv^\mu \ (v^2 = 1)$$
Method: light-front Hamiltonian formalism

- Hamiltonian formalism
  - the invariant masses and the boost invariant wavefunctions can be obtained directly by solving the eigenvalue equation
    \[ (P^+ \hat{P}^- - \hat{P}^2) \psi_h(P, j, m_j) = M_h^2 \psi_h(P, j, m_j) \]
  - the light-front wavefunction encodes the information of the system, and provides direct access to observables

- Basis representation
  - basis can encode an analytical approximation to the solution
  - optimal basis is the key to numerical efficiency

→ Basis Light-Front Quantization (BLFQ)
The charmonium light-front wavefunction by BLFQ

The charmonium light-front wavefunction is solved using the BLFQ approach in the $|q\bar{q}\rangle$ sector\(^1\),

\[
H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \frac{\partial}{\partial x} \left( x(1-x) \frac{\partial}{\partial x} \right) + \frac{V_g}{\text{one-gluon exchange}}
\]

- **Confinement**
  - Transverse (QCD holography)\(^2\)
  - Longitudinal (completes the transverse confinement, and produces desirable distribution amplitudes)

- **One-gluon exchange**

\[
V_g = -\frac{4}{3} \frac{4\pi\alpha_s(Q^2)}{Q^2} \bar{u}_{\sigma'}\gamma\mu u_\sigma \bar{v}_s \gamma_{\mu'} v_{s'}
\]

- **Basis representation:** basis functions are eigenfunctions of $H_0$

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The charmonium light-front wavefunction by BLFQ

- The charmonium light-front wavefunction is solved using the BLFQ approach in the $|q\bar{q}\rangle$ sector$^1$,
  
  - Light-front wavefunctions:
    
    e.g., $\eta_c(1S)$

  - Mass spectra:

- Access to a variety of observables:
  
  Form factors [Li, PRD '18; Mondal, PRD '20],
  PDFs/GPDs [Lan, PRL '19, PRD '20; Adhikari, PRC '18, '21],
  radiative transitions [M. Li, PRD '18 & '19],
  diffractive production [Chen, PLB '17 & PRC '18]

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Two-photon transitions of charmonia on the light front
The two-photon transition

- The amplitude of the hadron-to-two-photon transition, \( H_{cc}(j, p, \lambda) \rightarrow \gamma^*(q_1, \lambda_1) + \gamma(q_2, \lambda_2) \), is related to the hadron matrix element,

\[
\epsilon^*_\mu(q_1, \lambda_1)\epsilon^*_\nu(q_2, \lambda_2)e_\alpha(p, \lambda)M^{\mu\nu\alpha} = \]

\[
H_{\lambda_1, \lambda_2, \lambda}(q_1, q_2) = \epsilon^*_\nu(q_2, \lambda_2) \left( \gamma^*(q_1, \lambda_1) | J^\gamma(0) | H(p, \lambda) \right)
\]

- The decay width, can be measured from experiments

\[
\Gamma_{H\rightarrow\gamma\gamma} = \frac{1}{2} \frac{1}{16\pi} \frac{1}{m_H} \frac{1}{2j + 1} \sum_{\lambda=-j}^{j} \sum_{\lambda_1, \lambda_2 = \pm 1} |H_{\lambda_1, \lambda_2, \lambda}|^2
\]
Results: two-photon decay widths

Without any parameter tuning, our results have reasonable agreement with experimental data\(^1\).

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Results: the transition form factor (1) pseudoscalar $0^+-$

- The pseudoscalar transition amplitude is parameterized as
  \[ \mathcal{M}^{\mu\nu} = 4\pi\alpha_{em} e^{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma F_{\gamma\gamma}(q_1^2, q_2^2) \]
  - Single-tagged transition form factor: \( F_{\gamma\gamma}(Q^2 = -q^2) = F_{\gamma\gamma}(q^2, 0) = F_{\gamma\gamma}(0, q^2) \)

- The transition form factor in the light-front wavefunction representation reads
  \[ F_{\gamma\gamma}(Q^2) = 2Q^2 \sqrt{2N_c} \int \frac{d^2 k_\perp}{(2\pi)^3} \int_0^1 \frac{dx}{2\sqrt{x(1-x)}} \frac{\psi_{\uparrow\downarrow\downarrow\uparrow/p}(x, \vec{k}_\perp)}{k_\perp^2 + m_f^2 + x(1-x)Q^2} \]
  - At large \( Q \), it reduces to the partonic interpretation in terms of light-cone distribution amplitude (LCDA)
  \[ F_{\gamma\gamma}(Q^2) = \frac{e_f^2 f_p}{Q^2} \int_0^1 dx \frac{\phi_p(x, Q)}{x(1-x)} \]
Results: the transition form factor

(1) pseudoscalar $0^{+-}$

- $\eta_c$

![Graph 1](image1)

- $\eta_c'$

![Graph 2](image2)

The calculated transition form factors are in a reasonable agreement with experimental data

- BLFQ (this work), uncertainty is calculated from basis sensitivity
- BLFQ/DA, using the LCDA obtained from the BLFQ wavefunction
- Monopole fit, vector meson dominance model
Results: the transition form factor

The transition amplitude is parameterized as

\[ M^{\mu\nu}(q_1, q_2) = 4\pi\alpha_{em} \left\{ [(q_1 \cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu]F_{S_{\gamma\gamma,1}}(q_1^2, q_2^2) \right. \]
\[ 
+ \frac{1}{m_s^2} \left[ q_1^2 q_2^2 g^{\mu\nu} + (q_1 \cdot q_2)q_1^\mu q_2^\nu - q_1^2 q_2^2 q_1^\nu - q_2^2 q_1^2 q_1^\nu \right] F_{S_{\gamma\gamma,2}}(q_1^2, q_2^2) \}
\]

- Single-tagged transition form factor: \( F_{S_r}(Q^2 = -q^2) = F_{S_{\gamma\gamma,1}}(q^2, 0) = F_{S_{\gamma\gamma,1}}(0, q^2) \)

The transition form factor in the light-front wavefunction representation reads

\[ F_{S_r}(Q^2) = e^2 f^2 2\sqrt{2N_C} \int_0^1 \frac{dx}{2\sqrt{x(1-x)}} \int \frac{d^2k_\perp}{(2\pi)^3} \]
\[ \times \left\{ \psi_{\uparrow\downarrow\downarrow\uparrow}/S(x, \vec{k}_\perp) \frac{(1 - 2x)[x(1 - x)Q^2 + m_f^2]}{[k_\perp^2 + x(1-x)Q^2 + m_f^2]^2} \right. \]
\[ \left. + \psi_{\uparrow\downarrow\uparrow\downarrow}/S(x, \vec{k}_\perp) \frac{\sqrt{2m_f(k_x + ik_y)}}{[k_\perp^2 + x(1-x)Q^2 + m_f^2]^2} \right\} \]

- At large \( Q \), in terms of distribution amplitude (LCDA)

\[ F_{S_r}(Q^2) = e^2 f^2 \int_0^1 dx \frac{(1 - 2x)\phi_S(x, \mu)}{x(1-x)Q^2 + m_f^2} \]
Results: the transition form factor (2) scalar $0^{++}$ and tensor $2^{++}$

- $\chi_{c0} (0^{++})$

- $\chi_{c2} (2^{++})$

The calculated transition form factors are in a reasonable agreement with experimental data

- BLFQ (this work), uncertainty is calculated from basis sensitivity
- BLFQ/DA, using the LCDA obtained from the BLFQ wavefunction
Summary and outlook

- We investigated the **two-photon transitions** of charmonia, \( \eta_c, \eta_c', \chi_{c0}, \) and \( \chi_{c2} \), in the light-front Hamiltonian approach
  - We derived the **formulas** of transition form factors in the light-front wavefunction representation
    - Universal for other hadron light-front wavefunctions
  - We computed the decay widths, and the transition form factors, both in **good agreements** with experimental measurements
    - Reveal relativistic nature of charmonia

**Based on:** Y. Li, M. Li and J.P. Vary, Phys. Rev. D 105 (2022) 7, L071901; arXiv:2111.14178 [hep-ph]. LFWFs available on Mendeley Data

- Ongoing and future works on radiative transitions
  - A comprehensive study on different leptonic and radiative transitions
  - Extension to bottomonia, heavy-light mesons, and light mesons

Thank you!