Two-photon transitions of charmonia on the light front

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IGFAE Instituto Galego de Física de Altas Enerxías

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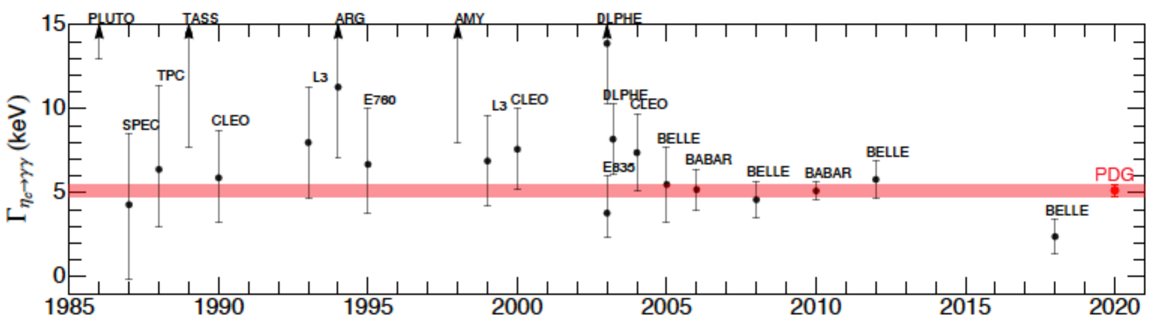
Outline

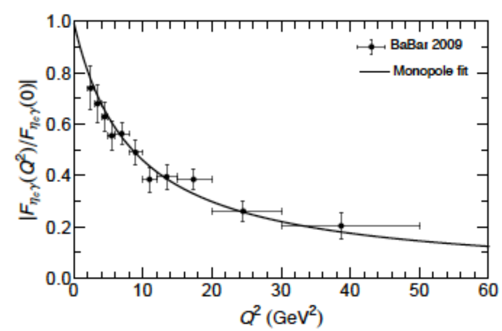
- **Introduction**: two-photon transition of charmonium
- **Method**: light-front Hamiltonian formalism
- **Results**: transition width and transition form factors
- **Summary**

Based on: Y. Li, M. Li and J.P. Vary, Phys. Rev. D 105 (2022) 7, L071901; arXiv:2111.14178 [hep-ph]

Introduction: two-photon transition

- Charmonium provides an ideal testing ground for various investigations to understand QCD
  - Challenging: relativistic, non-perturbative effects
- The two-photon transition, $H_{c\bar{c}} \to \gamma^* + \gamma$, provides a clean and important probe to hadron states
- Experimental measurements
  - Diphoton width $\Gamma_{H\to\gamma\gamma}$: extensive measurements for η_c, η_c', χ_{c0}, χ_{c2}
  - Transition form factors $F_{H\gamma}(Q^2)$: $F_{\eta_c\gamma}(Q^2)$ by BABAR 2010; $F_{\chi_{cj}\gamma}(Q^2)$ by Belle 2017

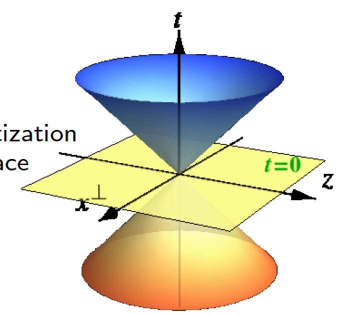
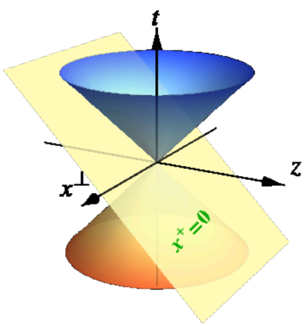
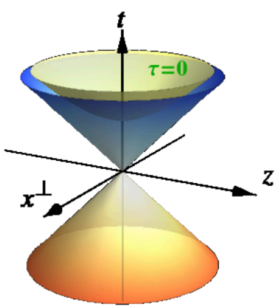




Method: light-front Hamiltonian formalism

Light-front Hamiltonian formalism is a natural framework for addressing relativistic bound-state and scattering problems in QCD

- Light-front quantization
  - The quantum field is quantized on the equal light-front time $x^+ = 0$

	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J^-$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^\perp, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = m v^\mu \ (v^2 = 1)$

Method: light-front Hamiltonian formalism

- Hamiltonian formalism
  - the invariant masses and the boost invariant wavefunctions can be obtained directly by solving the eigenvalue equation

$$\left(P^+\hat{P}^- - \vec{P}_\perp^2\right)|\psi_h(P,j,m_j)\rangle = M_h^2|\psi_h(P,j,m_j)\rangle$$

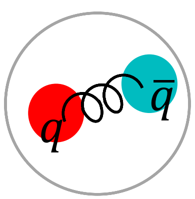
  - the light-front wavefunction encodes the information of the system, and provides direct access to observables
- Basis representation
  - basis can encode an analytical approximation to the solution
  - optimal basis is the key to numerical efficiency

→ Basis Light-Front Quantization (BLFQ)

The charmonium light-front wavefunction by BLFQ

- The charmonium light-front wavefunction is solved using the BLFQ approach in the $|q\bar{q}\rangle$ sector[1],

$$H_{\text{eff}} = \overbrace{\underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q+m_{\bar{q}})^2}\frac{\partial}{\partial x}\left(x(1-x)\frac{\partial}{\partial x}\right)}_{\text{confinement}}}^{H_0} + \underbrace{V_g}_{\text{one-gluon exchange}}$$



$x = p_q^+/P^+$

$\vec{k}_\perp = \vec{p}_{q\perp} - x\,\vec{P}_\perp$

- Confinement
  Transverse (QCD holography)[2]
  Longitudinal (completes the transverse confinement, and produces desirable distribution amplitudes)
- One-gluon exchange

$$V_g = -\frac{4}{3}\frac{4\pi\alpha_s(Q^2)}{Q^2}\bar{u}_{\sigma'}\gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$$

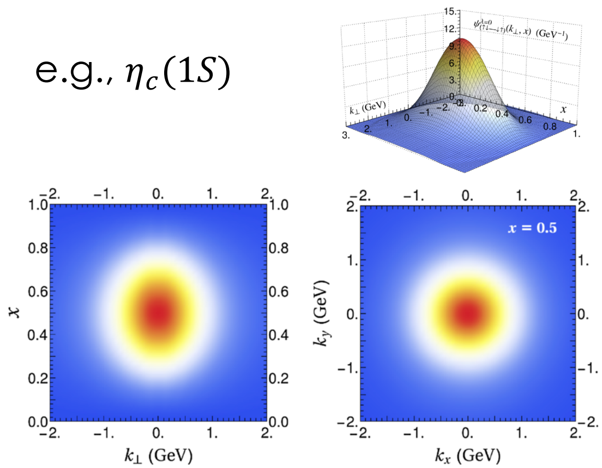
- Basis representation: basis functions are eigenfunctions of H_0

[1] *Y. Li, P. Maris, and J. P. Vary, Phys. Rev. D96, 016022 (2017).*
[2] *S. J. Brodsky, G. F. de Teramond, H. G. Dosch, and J. Erlich, Phys. Rept. 584, 1 (2015)*

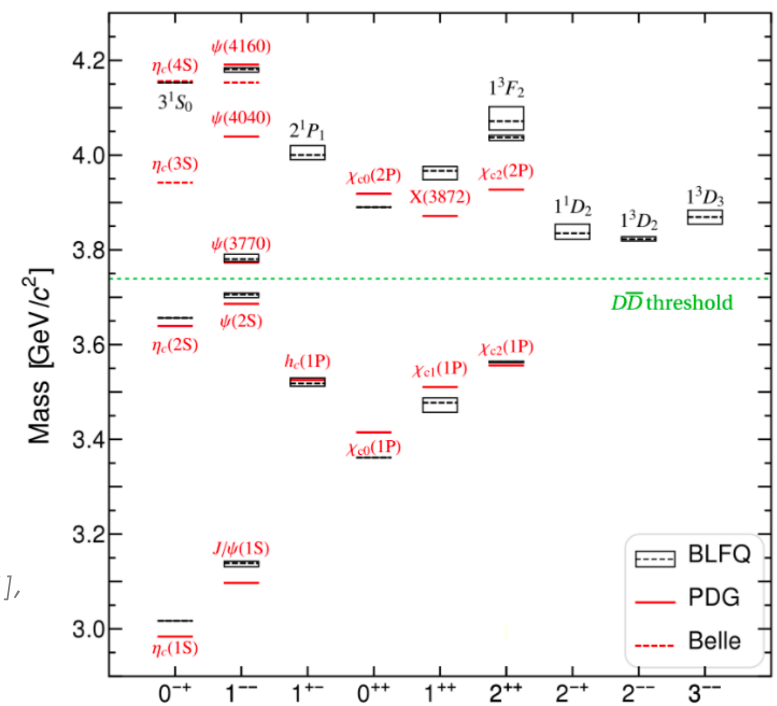
The charmonium light-front wavefunction by BLFQ

- The charmonium light-front wavefunction is solved using the BLFQ approach in the $|q\bar{q}\rangle$ sector[1],
  - Light-front wavefunctions:

e.g., $\eta_c(1S)$



  - Mass spectra:



✓ Access to a variety of observables:

Form factors *[Li, PRD '18; Mondal, PRD '20]*,
PDFs/GPDs *[Lan, PRL '19, PRD '20; Adhikari, PRC '18, '21]*,
radiative transitions *[M. Li, PRD '18 & '19]*,
diffractive production *[Chen, PLB '17 & PRC '18]*

[1] Y. Li, P. Maris, and J. P. Vary, Phys. Rev. D96, 016022 (2017).

The two-photon transition

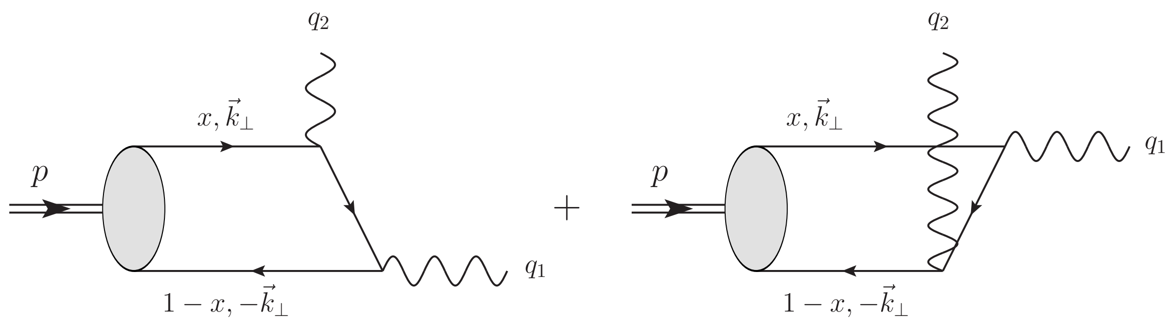
- The amplitude of the hadron-to-two-photon transition, $H_{c\bar{c}}(j,p,\lambda) \to \gamma^*(q_1,\lambda_1) + \gamma(q_2,\lambda_2)$, is related to the hadron matrix element,

The EM current

$$\epsilon^*_\mu(q_1,\lambda_1)\epsilon^*_\nu(q_2,\lambda_2)e_\alpha(p,\lambda)\mathcal{M}^{\mu\nu\alpha} =$$
$$H_{\lambda_1,\lambda_2;\lambda}(q_1,q_2) = \epsilon^*_\nu(q_2,\lambda_2)\langle\gamma^*(q_1,\lambda_1)|\, J^\nu(0)\,|H(p,\lambda)\rangle$$

The photon light-front wavefunction (by LO perturbation)

The hadron light-front wavefunction (by BLFQ)



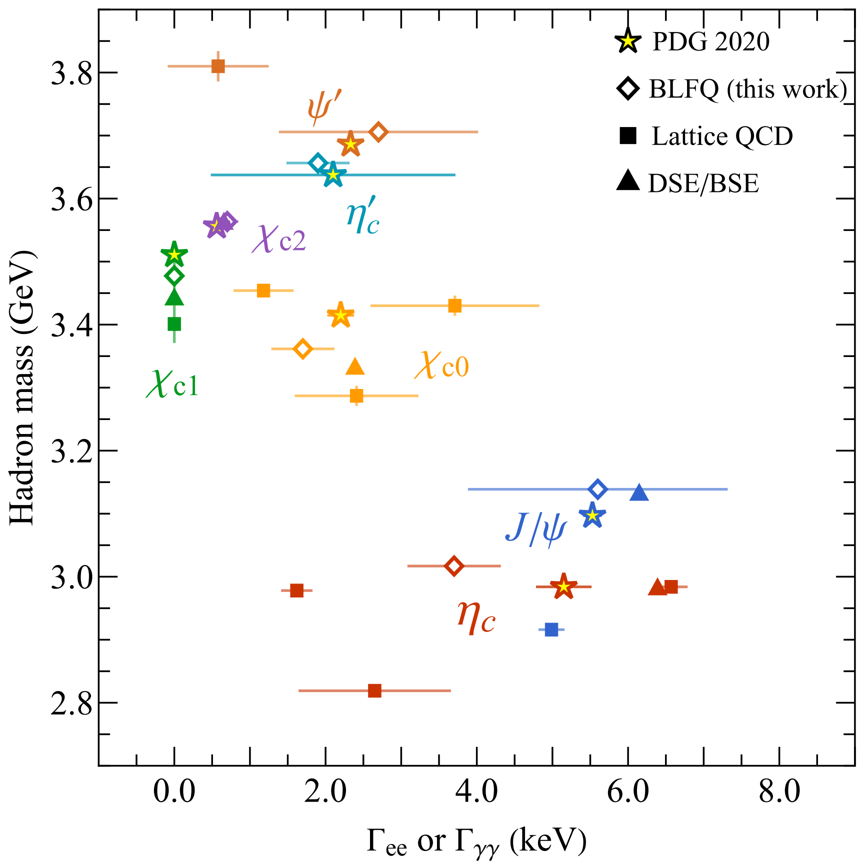
Light-cone dominance is manifest in the frame $q_2^+ = q_1^- = 0$

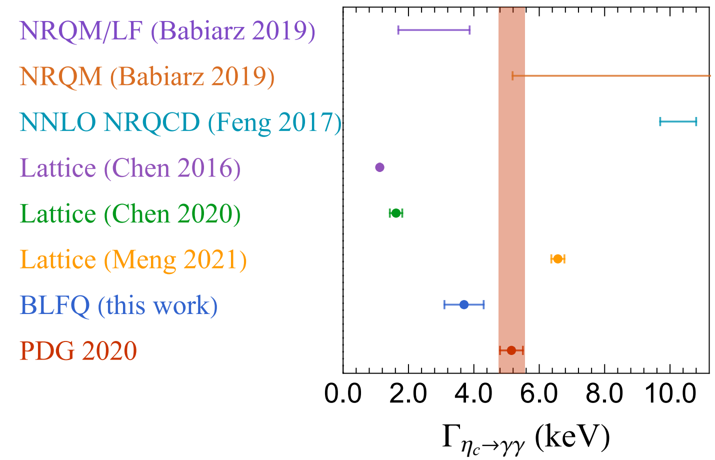
- The decay width, can be measured from experiments

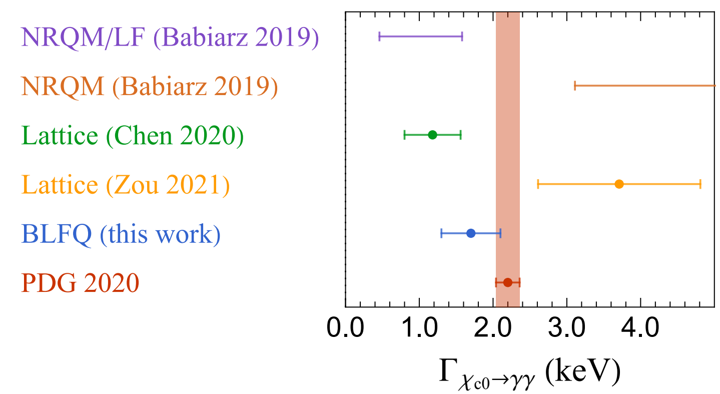
$$\Gamma_{H\to\gamma\gamma} = \frac{1}{2}\frac{1}{16\pi}\frac{1}{m_H}\frac{1}{2j+1}\sum_{\lambda=-j}^{j}\sum_{\lambda_1,\lambda_2=\pm1}|H_{\lambda_1,\lambda_2;\lambda}|^2$$

Results: two-photon decay widths

- Without any parameter tuning, our results have reasonable agreement with experimental data[1]







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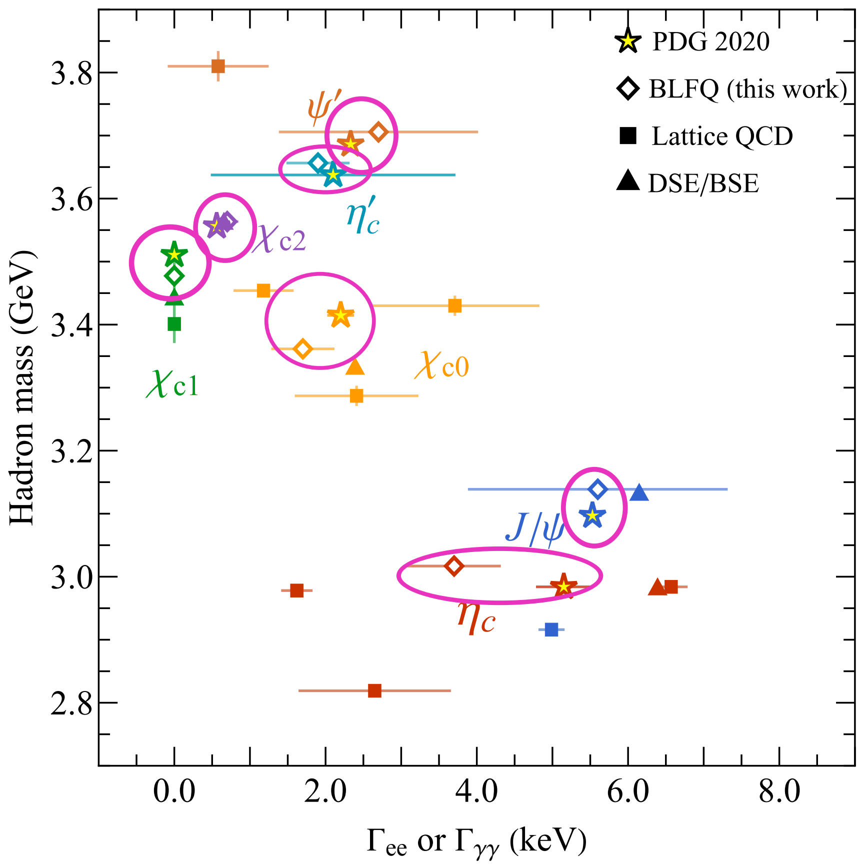
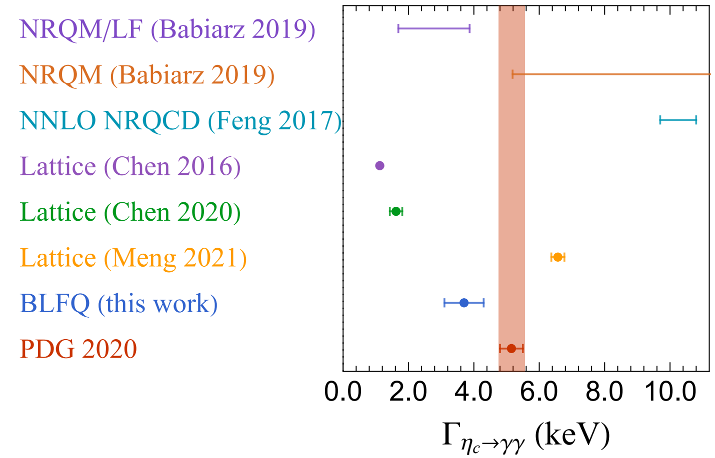
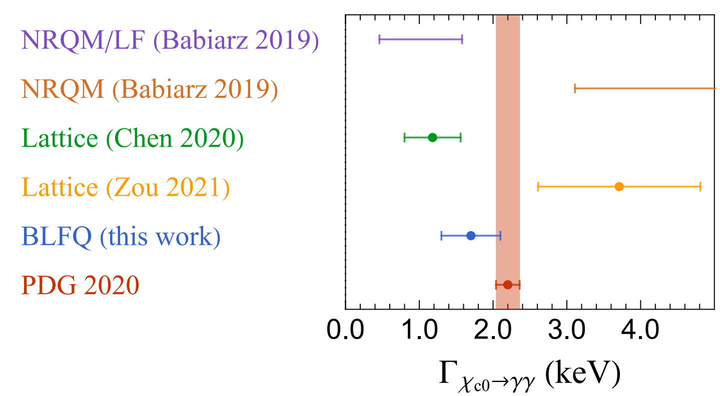






[1] *Y. Li, M. Li and J.P. Vary, Phys. Rev. D 105 (2022) 7, L071901; arXiv:2111.14178 [hep-ph]*

Results: the transition form factor (1) pseudoscalar 0^{+-}

- The pseudoscalar transition amplitude is parameterized as

$$\mathcal{M}^{\mu\nu} = 4\pi\alpha_{\rm em}\varepsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}F_{P\gamma\gamma}(q_1^2, q_2^2)$$

  - Single-tagged transition form factor: $F_{\mathcal{P}\gamma}(Q^2 = -q^2) = F_{\mathcal{P}\gamma\gamma}(q^2, 0) = F_{\mathcal{P}\gamma\gamma}(0, q^2)$

- The transition form factor in the light-front wavefunction representation reads

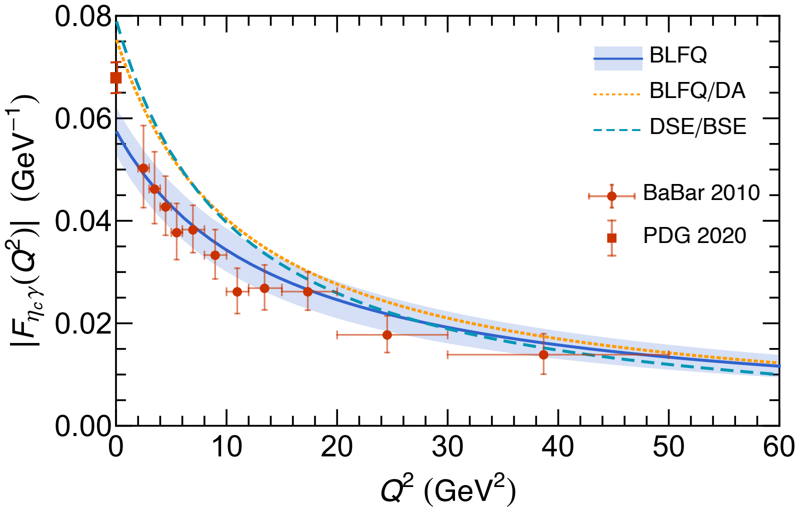
$$F_{\mathcal{P}\gamma}(Q^2) = 2Q_f^2\sqrt{2N_c}\int\frac{d^2k_\perp}{(2\pi)^3}\int_0^1\frac{dx}{2\sqrt{x(1-x)}}\frac{\psi_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2 + m_f^2 + x(1-x)Q^2}$$

  - At large Q, it reduces to the partonic interpretation in terms of light-cone distribution amplitude (LCDA)

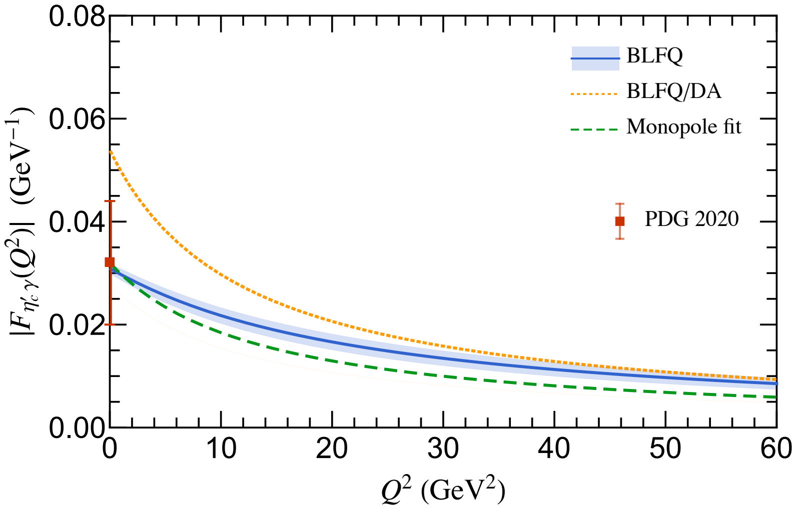
$$F_{P\gamma}(Q^2) = \frac{e_f^2 f_P}{Q^2}\int_0^1 dx\,\frac{\phi_P(x,Q)}{x(1-x)}$$

Results: the transition form factor (1) pseudoscalar 0^{+-}

- η_c



- η_c'



- The calculated transition form factors are in a reasonable agreement with experimental data
  - BLFQ (this work), uncertainty is calculated from basis sensitivity
  - BLFQ/DA, using the LCDA obtained from the BLFQ wavefunction
  - Monopole fit, vector meson dominance model

Results: the transition form factor (2) scalar 0^{++}

- The transition amplitude is parameterized as

$$\mathcal{M}^{\mu\nu}(q_1,q_2) = 4\pi\alpha_{em}\Big\{[(q_1\cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu]F_{S\gamma\gamma,1}(q_1^2,q_2^2) + \frac{1}{m_S^2}[q_1^2 q_2^2 g^{\mu\nu} + (q_1\cdot q_2)q_1^\mu q_2^\nu - q_1^2 q_2^\mu q_2^\nu - q_2^2 q_1^\mu q_1^\nu]F_{S\gamma\gamma,2}(q_1^2,q_2^2)\Big\}$$

  - Single-tagged transition form factor: $F_{S\gamma}(Q^2=-q^2) = F_{S\gamma\gamma,1}(q^2,0) = F_{S\gamma\gamma,1}(0,q^2)$

- The transition form factor in the light-front wavefunction representation reads

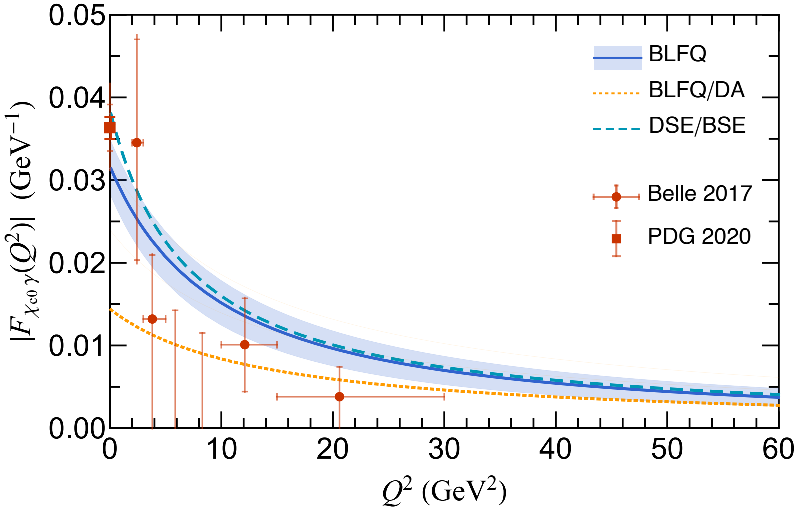
$$F_{S\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int_0^1 \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3} \times\Big\{\psi_{\uparrow\downarrow+\downarrow\uparrow/S}(x,\vec{k}_\perp)\frac{(1-2x)[x(1-x)Q^2+m_f^2]}{[k_\perp^2+x(1-x)Q^2+m_f^2]^2} + \psi_{\uparrow\uparrow/S}(x,\vec{k}_\perp)\frac{\sqrt{2}m_f(k_x+ik_y)}{[k_\perp^2+x(1-x)Q^2+m_f^2]^2}\Big\}$$

  - At large Q, in terms of distribution amplitude (LCDA)

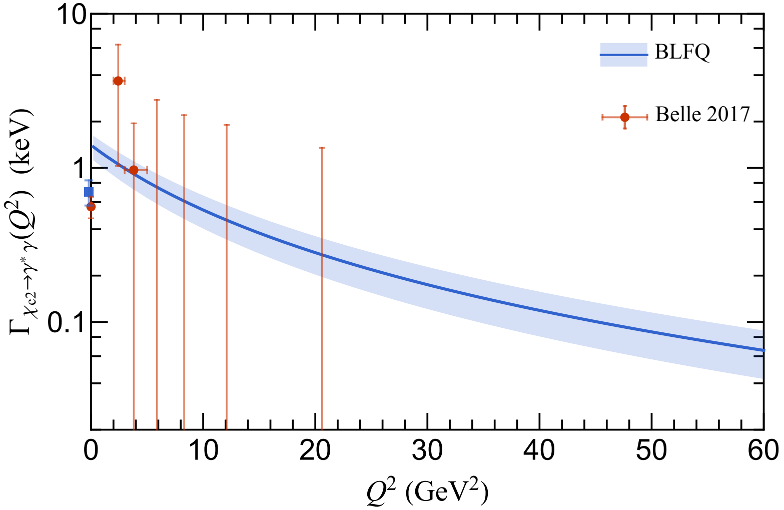
$$F_{S\gamma}(Q^2) = e_f^2 f_S\int_0^1 \mathrm{d}x\,\frac{(1-2x)\phi_S(x,\mu)}{x(1-x)Q^2+m_f^2}$$

Results: the transition form factor (2) scalar 0^{++} and tensor 2^{++}

- χ_{c0} (0^{++})



- χ_{c2} (2^{++})



➨ The calculated transition form factors are in a reasonable agreement with experimental data

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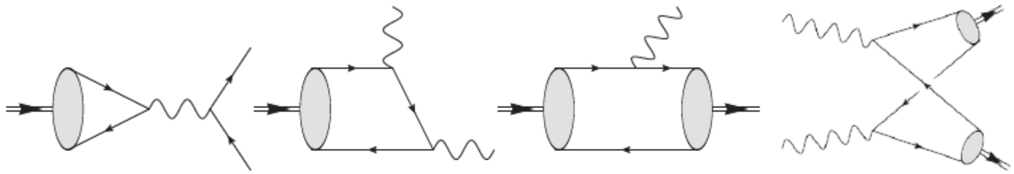
- BLFQ/DA, using the LCDA obtained from the BLFQ wavefunction

Summary and outlook

- We investigated the two-photon transitions of charmonia, η_c, $\eta_c{}'$, χ_{c0}, and χ_{c2}, in the light-front Hamiltonian approach
  - We derived the formulas of transition form factors in the light-front wavefunction representation
    - Universal for other hadron light-front wavefunctions
  - We computed the decay widths, and the transition form factors, **both** in good agreements with experimental measurements
    - Reveal relativistic nature of charmonia

Based on: Y. Li, M. Li and J.P. Vary, Phys. Rev. D 105 (2022) 7, L071901; arXiv:2111.14178 [hep-ph]. LFWFs available on Mendeley Data

- Ongoing and future works on radiative transitions
  - A comprehensive study on different leptonic and radiative transitions
  - Extension to bottomonia, heavy-light mesons, and light mesons



Thank you!