

Hybrid high-energy factorization at NLO

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in collaboration with

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and at larger scope with

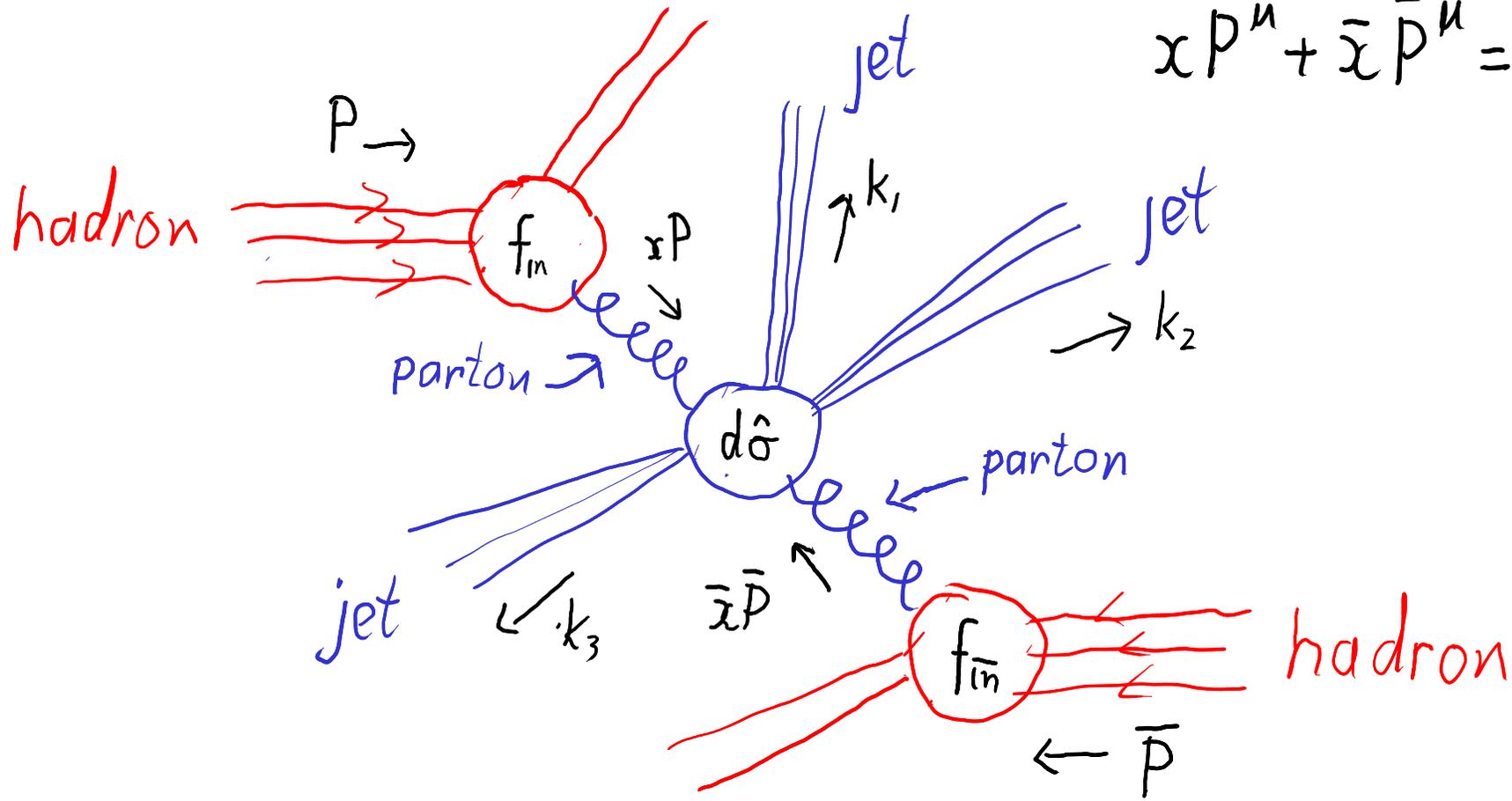
Etienne Blanco, Alessandro Giachino, Piotr Kotko

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Collinear Factorization



$$xP^\mu + \bar{x}\bar{P}^\mu = k_1^\mu + k_2^\mu + k_3^\mu$$

factorization scale

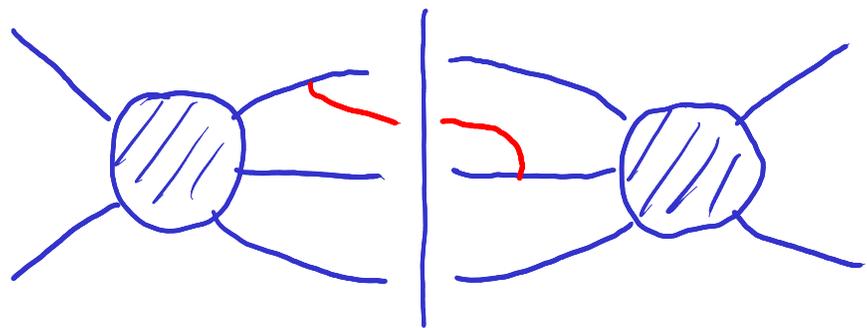
$$d\sigma = \int_0^1 dx f_{in}(x, \mu) \int_0^1 d\bar{x} f_{in}(\bar{x}, \mu) d\hat{\sigma}(x, \bar{x}, \{k_i\}_{i=1}^n, \mu)$$

universal PDFs, fit to data

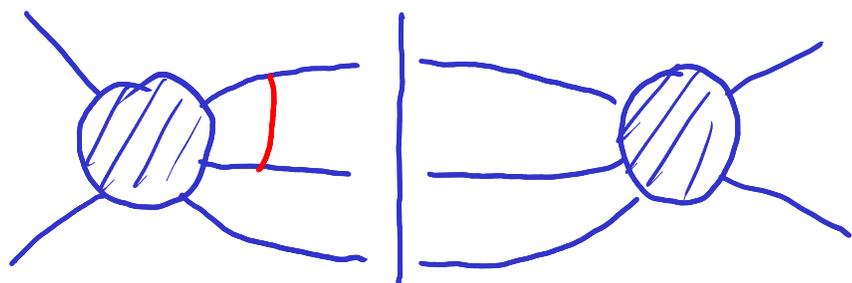
parton-level cross section
calculable perturbatively

Next-to-leading order

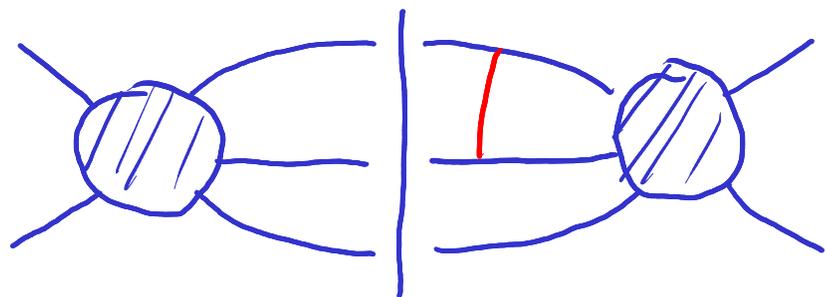
IR and UV divergencies appear



real contribution:
extra external parton can become soft, or collinear to another one



virtual contribution:
loop integral leads to soft, collinear, and UV divergencies



UV \rightarrow renormalization
IR \rightarrow cancel for a large part, but not completely

General structure

4

phase space
↓

Born (LO) $dB(k_{in}, k_{in}; \{k_i\}_{i=1}^n) = d\phi_n(\{k_i\}_{i=1}^n) \frac{M(k_{in}, k_{in}; \{k_i\}_{i=1}^n)}{4x\bar{x}P\cdot\bar{P}} J_B(\{k_i\}_{i=1}^n)$

$k_{in} = xP \quad k_{in} = \bar{x}\bar{P}$

squared and summed over color and spin
↓

jet definition
↓

same momenta

virtual $dV(k_{in}, k_{in}; \{k_i\}_{i=1}^n) = d\phi_n(\{k_i\}_{i=1}^n) \frac{2\text{Re} A_{\text{loop}} A_{\text{tree}}^\dagger}{4x\bar{x}P\cdot\bar{P}} J_B(\{k_i\}_{i=1}^n)$

real $dR(k_{in}, k_{in}; \{k_i\}_{i=1}^{n+1}) = d\phi_{n+1}(\{k_i\}_{i=1}^{n+1}) \frac{M(k_{in}, k_{in}, \{k_i\}_{i=1}^{n+1})}{4x\bar{x}P\cdot\bar{P}} J_R(\{k_i\}_{i=1}^{n+1})$

Non-cancelling collinear structure

virtual initial-state collinear divergence: $-\frac{1}{\epsilon} (\gamma_{in} + \gamma_{in}) dB$

real: $\frac{1}{\epsilon} \int_0^1 dz P_{in}(z) dB(zk_{in}, k_{in}, \{k_i\}_{i=1}^n) + \frac{1}{\epsilon} \int_0^1 dz P_{in}(z) dB(k_{in}, zk_{in}, \{k_i\}_{i=1}^n)$

← regular collinear splitting function →

Remember $k_{in} = xP$ and we integrate over x to obtain the cross section

→ $\int_0^1 dx f(x) dR_{\text{non-cancelling}}(xP, \dots) \rightarrow \int_0^1 dx \left[\frac{1}{\epsilon} \int_x^1 dz P_{in}(z) \frac{1}{z} f\left(\frac{x}{z}\right) \right] dB(xP, \dots)$

Factorization means renormalizing the PDF

$$d\sigma^{NLO+LO} = \alpha_s^{n+1} \int_0^1 dx \left\{ f(x) \left[dV_{cd}(x) + dR_{cd}(x) \right] + \left[\int_x^1 dz \frac{P(z)}{\epsilon} \frac{1}{z} f\left(\frac{x}{z}\right) \right] dB(x) \right\}$$

cancelling divergencies

$$+ \alpha_s^n \int_0^1 dx \left[f(x) + \alpha_s f^{(1)}(x) \right] dB(x)$$

NLO PDF correction ↗

⇒ claim $f^{(1)}(x)$ contains $\frac{1}{\epsilon} \int_x^1 dz P(z) \frac{1}{z} f\left(\frac{x}{z}\right)$

Works because non-cancelling div. $\propto dB$

Does a scheme like this,

to take care of all divergencies,
still work for initial-state partons with k_T ?

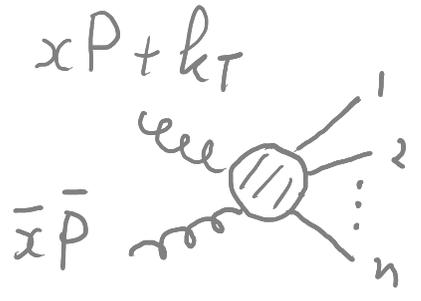
Hybrid k_T -factorization formula

$$P \cdot k_T = \bar{P} \cdot k_T = 0$$

$$d\sigma^{LO} = \int dx d^2k_T d\bar{x} F(x, |k_T|) f(\bar{x}) dB^*(x, k_T, \bar{x})$$

← k_T -dependent PDF

extra transverse initial-state d.o.f. only on 1 side



Necessary **space-like** amplitudes cannot be obtained by simply keeping one initial-state gluon off-shell (gauge invariance)

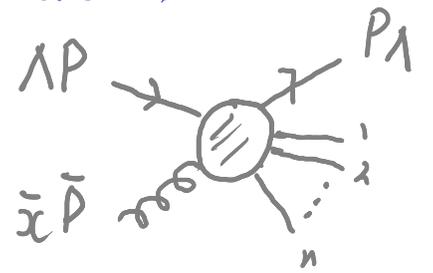
VAN HAMEREN, KUTAK, KOTKO 2013:

(anti) quark or gluon or other particle

want process $g^*(xP + k_T) \omega_{i\bar{n}}(k_{i\bar{n}}) \rightarrow \omega_1(k_1) \omega_2(k_2) \dots \omega_n(k_n)$

use process $q(\Lambda P) \omega_{i\bar{n}}(k_{i\bar{n}}) \rightarrow q(P_\Lambda) \omega_1(k_1) \omega_2(k_2) \dots \omega_n(k_n)$

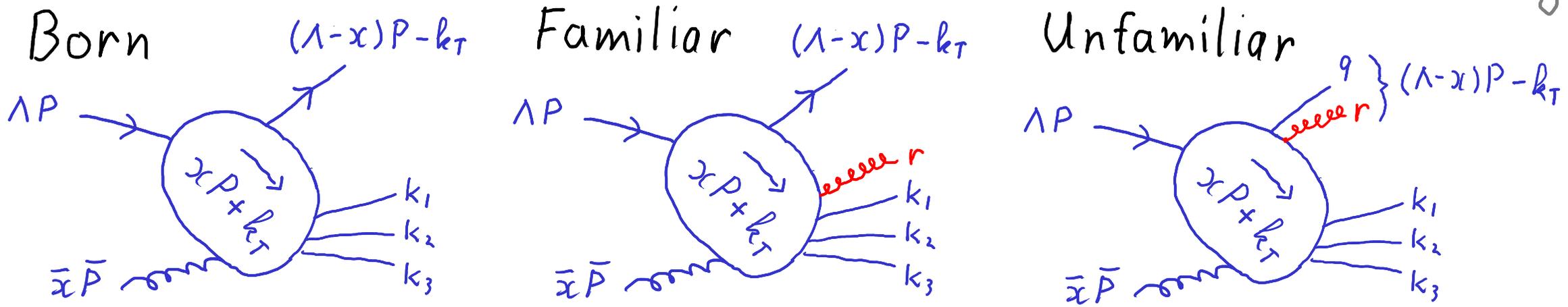
$$P_\Lambda = (\Lambda - x)P - k_T + \frac{|k_T|^2}{(\Lambda - x)v^2} \bar{P} \quad (v^2 = 2P \cdot \bar{P})$$



$$\frac{1}{g_s^2 C_{aux-g}} \cdot \frac{x^2 |k_T|^2}{\Lambda^2} M_{tree}^{aux-g}(\Lambda P, k_{i\bar{n}}; P_\Lambda, \{k_i\}_{i=1}^n) \xrightarrow{\Lambda \rightarrow \infty} M_{tree}^*(xP + k_T, k_{i\bar{n}}; \{k_i\}_{i=1}^n)$$

correcting color factor only dependence on type of auxiliary partons

in agreement with Lipatov



The unfamiliar real contribution is proportional to the Born contribution, and involves the triple- Λ limit:

$$\frac{1}{C_{aux}} M_{tree}^{aux}(\Lambda, \bar{x}; x_r \Lambda P + r_T, x_q \Lambda P + q_T; \{k_i\}_{i=1}^n) \quad x_q + x_r = 1$$

$$\xrightarrow{\Lambda \rightarrow \infty} Q_{aux}(x_q, q_T, x_r, r_T) \frac{\Lambda^2 M_{tree}^*(x, -q_T - r_T, \bar{x}; \{k_i\}_{i=1}^n)}{x |q_T + r_T|^2}$$

Q_{aux} depends on auxiliary/radiative parton type \rightarrow ~~universal~~
 contains collinear splitting function as factor

As defined, unfamiliar-real includes small- ξ_r region reserved for familiar \rightarrow must be subtracted.

Corresponds to $1/(1-z)$ in splitting function.

Unfamiliar real contribution

~~smooth~~ large- Λ limit auxiliary parton ~~universality~~

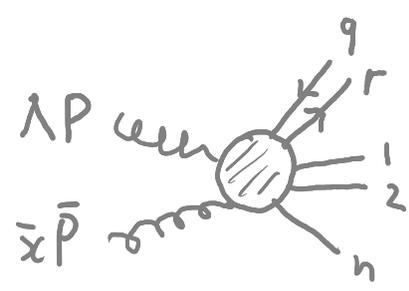
$$dR^{unf} = \frac{\alpha_s}{2\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} N_c \left(\frac{\mu^2}{|k_T|^2} \right)^\epsilon \left[\frac{4}{\epsilon} \ln \frac{1-x}{\Lambda} + R_{aux} \right] dB^*$$

\swarrow ~~smooth~~ on-shell limit \swarrow

$$R_{aux-g} = \frac{3}{\epsilon} - \frac{2\pi^2}{3} + \frac{7}{2} - 2Li_2 \left(\frac{|k_T|^2}{v^2 \bar{x}(1-x)} \right) - \frac{1}{N_c^2} \left[\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} + 4 \right]$$

$2P \cdot \bar{P} \nearrow$

$$R_{aux-g} = \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \frac{11}{2} - \frac{2\pi^2}{3} + \frac{67}{9} - 2Li_2 \left(\frac{|k_T|^2}{v^2 \bar{x}(1-x)} \right)$$



number of quark types \rightarrow

$$= \frac{n_f}{N_c} \left[\frac{2}{3\epsilon} + \frac{10}{9} - \frac{1}{N_c^2} \left(\frac{1}{3\epsilon} - \frac{1}{6} \right) \right]$$

Unfamiliar virtual contribution

If the $1/\epsilon$ poles in dR^{unf} are supposed to cancel against virtual ones, then those should also come with $(\mu^2/|k_T|^2)^\epsilon$.

\Rightarrow consider on-shell limit $|k_T| \rightarrow 0$

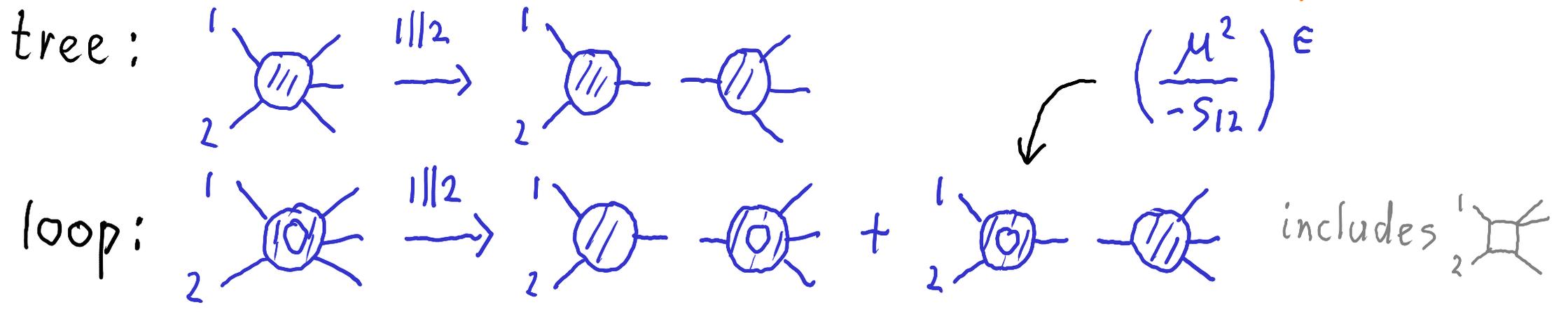
\Rightarrow collinear limit of aux. partons does it commute with $\Lambda \rightarrow \infty$?

At tree-level, $|k_T| \rightarrow 0$ commutes with $\Lambda \rightarrow \infty$:

$$A_{tree}^{aux}([\Lambda P]^\pm, [P \Lambda]^\mp, \dots) \xrightarrow{|k_T| \rightarrow 0} \sum_{h=\pm} \text{Split}^{\pm \mp h} \left(\frac{\Lambda}{x} \right) A_{tree}([x P]^h, \dots)$$

\uparrow color-ordered helicity amplitude $\quad \hookrightarrow (\Lambda-x)P + k_T + \mathcal{O}(\Lambda^{-1}) \quad \hookrightarrow \frac{\Lambda}{xK}, \frac{\Lambda}{xK^*}$

One-loop splitting formula Bern et al. 1995, 1999



Unfamiliar virtual contribution

- 
 ← contains $z = \frac{\Lambda}{x} \rightarrow \ln \Lambda$
 - 
 ← simply replace on-shell by space-like amplitude (can prove (numerically))
- nowhere else $\ln \Lambda$ (conjecture)
 nowhere else $\frac{1}{\epsilon} \ln \Lambda$ (can prove)

$$dV^{unf} = a_\epsilon N_c \left(\frac{\mu^2}{|k_T|^2} \right)^\epsilon \left[\frac{2}{\epsilon} \left(\ln \frac{\Lambda}{x} - i\pi \right) + V_{aux} \right] dB^*$$

$$V_{aux-g} = \frac{1}{\epsilon} \frac{13}{6} + \frac{\pi^2}{3} + \frac{80}{18} + \frac{1}{N_c^2} \left[\frac{1}{\epsilon^2} + \frac{3}{2} \frac{1}{\epsilon} + 4 \right] - \frac{n_f}{N_c} \left[\frac{2}{3} \frac{1}{\epsilon} + \frac{10}{9} \right]$$

$$V_{aux-g} = -\frac{1}{\epsilon^2} + \frac{\pi^2}{3}$$

Blanco, Giachino, Kotko

Confirmed by explicit calculations $g^*g \rightarrow g, g^*g \rightarrow H, g^*g \rightarrow q\bar{q}$

$$A_{loop}^* = \left(\overset{(unf)}{\text{Singular}} + \overset{(fam)}{\text{Singular}} \right) A_{tree}^* + \text{Finite}$$

as above ↗

↖ familiar on-shell limit

Combined unfamiliar

$$dR^{\text{unf}} + dV^{\text{unf}} = \frac{a_e N_c}{\epsilon} \left(\frac{\mu^2}{|k_T|^2} \right)^\epsilon \left[2I_{\text{univ}} + I_{\text{aux}} + 2I_{\text{remn}} \right] dB^*$$

$$I_{\text{univ}} = \frac{11}{6} - \frac{n_f}{3N_c} + \left[\frac{67}{18} - \frac{\pi^2}{6} - \frac{n_f}{N_c} \frac{5}{9} \right] \epsilon$$

$$I_{\text{aux-g}} = \frac{3}{2} + \frac{\epsilon}{2} \quad I_{\text{aux-g}} = \frac{11}{6} + \frac{n_f}{3N_c^3} - \frac{n_f}{6N_c^3} \epsilon$$

$$I_{\text{remn}} = 2 \ln(1-x) - \ln \Lambda - \ln x - \epsilon \text{Li}_2 \left(\frac{|k_T|^2}{v^2 \bar{x}(1-x)} \right)$$

$I_{\text{univ}} + I_{\text{aux}} \rightarrow$ correction to impact factor Ciafaloni Colferai 1999

$I_{\text{univ}} \rightarrow$ UV subtraction

I_{remn} must vanish $\rightarrow \Lambda = 1/x \rightarrow x \ll 1$

Power corrections in Λ^{-1} are power corrections in x .

Factorization at NLO

$$d\sigma^{NLO} = \int dx d^2k_T d\bar{x} \left\{ F(x, k_T) f(\bar{x}) [dR(x, k_T, \bar{x}) + dV(x, k_T, \bar{x})] \right. \\ \left. + \left[F(x, k_T) f^{NLO}(\bar{x}) + F^{NLO}(x, k_T) f(\bar{x}) \right] dB(x, k_T, \bar{x}) \right\}$$

depends on aux. partons, must cancel impact factor corrections
 → High-energy factorization emerges

What about collinear-type non-cancelling divergence in dR^{fam} ?

Collinear limit when $r = x_r P + \bar{x}_r \bar{P} + r_T \rightarrow x_r P$

$$M_{tree}^*(x, k_T, \bar{x}; r, \{k_i\}_{i=1}^n) \xrightarrow{r \parallel P} \frac{1}{P \cdot r} \cdot \frac{2N_c x^2}{x_r (x - x_r)^2} M_{tree}^*(x - x_r, k_T, \bar{x}; \{k_i\}_{i=1}^n)$$

usually $\frac{P(1 - x_r/x)}{x - x_r}$ →

$k_T - r_T$ ↗ ↖ $\bar{x} - \bar{x}_r$

“even more valid” with recoil in initial state like subtraction term

Collinear-type non-cancelling familiar real

$$\int dx d^2 k_T F(x, k_T) dR_{\text{coll}}^{\text{fam}}(x, k_T) = \int dx d^2 k_T \tilde{F}(x, k_T) dB^*(x, k_T)$$

$$\tilde{F}(x, k_T) = \frac{2a_\epsilon N_c}{\pi_\epsilon \mu^{-2\epsilon}} \int_x^1 \frac{dz}{z(1-z)} \int d^{D-2} r_T \frac{|k_T|^2}{|r_T|^2 |k_T+r_T|^2} \Theta_{|r_T|^2 < v^2 x \bar{x} \frac{1-z}{z}} F\left(\frac{x}{z}, k_T+r_T\right)$$

essentially identical to **Nefedov 2020**

Contains soft-collinear $1/\epsilon^2, 1/\epsilon$ terms \propto Born that do cancel.

$$\text{Non-cancelling term: } - \frac{2a_\epsilon N_c}{\epsilon} \int_x^1 \frac{dz}{[1-z]_+} \frac{1}{z} F\left(\frac{x}{z}, k_T\right)$$

Can be accounted for by collinear-type evolution via

$$F(x, k_T) = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) \phi(z, k_T) \quad \text{Nefedov 2020}$$

$$\rightarrow - \frac{2a_\epsilon N_c}{\epsilon} \int_x^1 \frac{dz}{z} \phi(z, k_T) \left[\int_{x/z}^1 \frac{dy}{[1-y]_+} \frac{1}{y} f\left(\frac{x}{zy}\right) + \ln \frac{1-x}{1-\frac{x}{z}} f\left(\frac{x}{z}\right) \right]$$

Conclusion

High-energy factorization emerges from "naïve" or "direct" approach to (hybrid) k_T -dependent factorization in the auxiliary parton method at NLO.

$$F \otimes M \otimes f \Leftrightarrow \phi_A \otimes G \otimes \phi_B$$

Next

Need to isolate $\ln x$