Hybrid high-energy factorization at NLO

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Abstract

We promote to next-to-leading order (NLO) the hybrid high-energy factorization formula, in which one initial-state parton momentum is space-like and carries non-vanishing transverse components while the other is on-shell. We identify all non-cancelling soft and collinear divergencies in the real and virtual contribution for the partonic cross section, and observe that they force to change the interpretation of the factorization formula. Coincidentally, we recover expressions for inclusive NLO quark-and gluon impact factor corrections known in literature.

1 Hybrid k_T-factorization and the auxiliary parton method

The hybrid high-energy, or k_T-, factorization formula for cross sections in hadron scattering looks, at lowest order, schematically like

$$d\sigma^{(0)} = \int dx_{\mathrm{in}}\, d^2k_\perp\, d\bar{x}_{\overline{\mathrm{in}}}\, F(x_{\mathrm{in}}, |k_\perp|)\, f(\bar{x}_{\overline{\mathrm{in}}})\, dB^\star(x_{\mathrm{in}}, k_\perp, \bar{x}_{\overline{\mathrm{in}}})\ . \tag{1}$$

The functions $F(x_{\mathrm{in}}, |k_\perp|)$ and $f(\bar{x}_{\overline{\mathrm{in}}})$ are parton density functions (PDFs). They both depend on a longitudinal momentum fraction, but only one of them depends on transverse momentum. The Born-level differential partonic cross section $dB^\star$ is special compared to collinear factorization, as highlighted by the '$\star$', in the sense that one of its initial-state partons is space-like, and has momentum

$$k_{\mathrm{in}}^\mu = x_{\mathrm{in}} P_A^\mu + k_\perp^\mu\ , \tag{2}$$

where P_A is the light-like hadron momentum and $k_\perp$ is transverse, *i.e.* $k_\perp \cdot P_A = 0$. We consider the general case, for which the scattering process may involve a number of final-state jets, and/or massive quarks, and/or a Higgs boson etc. $dB^\star$ is implied to depend on the momenta of those, and to include the differential phase space.

It is straightforward to define and calculate the tree-level amplitudes necessary to construct $dB^\star$ with the auxiliary parton method [1]. We need a Sudakov decomposition of momenta as

$$K^\mu = \xi_K P^\mu + \bar{x}_K \bar{P}^\mu + K_\perp^\mu \tag{3}$$

where the momenta P and $\bar{P}$ have positive energy and satisfy

$$P^2 = \bar{P}^2 = 0 \quad , \quad 2P\cdot\bar{P} = \nu^2 > 0 \quad , \quad P\cdot K_\perp = \bar{P}\cdot K_\perp = 0 \,. \tag{4}$$

We introduce the symbols ξ and P to have the possibility of defining

$$x = \lambda^{-1}\xi \quad \text{and} \quad P_A = \lambda P \tag{5}$$

via a scaling yet to be determined. Now let the desired parton-level process be

$$g^\star(k_{\mathrm{in}})\ \omega_{\overline{\mathrm{in}}}(k_{\overline{\mathrm{in}}}) \ \to\ \omega_1(p_1)\ \omega_2(p_2)\ \cdots\ \omega_n(p_n) \tag{6}$$

where $g^\star$ represents the space-like gluon, and the ω_i represent the other partons or particles involved in the process. In the auxiliary parton method, this process is obtained from the quark scattering process

$$q\big(k_1(\Lambda)\big)\ \omega_{\overline{\mathrm{in}}}(k_{\overline{\mathrm{in}}}) \ \to\ q\big(k_2(\Lambda)\big)\ \omega_1(p_1)\ \omega_2(p_2)\ \cdots\ \omega_n(p_n) \tag{7}$$

where

$$k_1^\mu = \Lambda P^\mu \quad , \quad k_2^\mu = p_\Lambda^\mu = (\Lambda - \xi_{\mathrm{in}})P^\mu - k_\perp^\mu + \frac{|k_\perp|^2}{(\Lambda-\xi_{\mathrm{in}})\nu^2}\,\bar{P}^\mu \,, \tag{8}$$

so $k_1^2 = k_2^2 = 0$ and $k_1 - k_2 = k_{\mathrm{in}} + \mathcal{O}\big(\Lambda^{-1}\big)$ with $k_{\mathrm{in}} = \xi_{\mathrm{in}}P + k_\perp$. The process of Eq. (7) with the auxiliary quarks is on-shell, and its squared matrix element is well-defined and it is known how to calculate it. The squared matrix element of the desired process with the space-like gluon is obtained by taking

$$\Lambda \to \infty \,. \tag{9}$$

In [2] and [3] it was noted that instead of an auxiliary scattering quark, also an auxiliary scattering gluon can be used. At the level of squared matrix elements summed over color, one just needs to include a different overall factor to correct for the difference in color representation:

$$\frac{1}{g_s^2 C_{\mathrm{aux}}}\,\frac{\xi_{\mathrm{in}}^2|k_\perp|^2}{\Lambda^2}\,\big|\overline{\mathcal{M}}^{\mathrm{aux}}\big|^2\big(\Lambda P, k_{\overline{\mathrm{in}}}; p_\Lambda, \{p_i\}_{i=1}^n\big) \ \stackrel{\Lambda\to\infty}{\longrightarrow}\ \big|\overline{\mathcal{M}}^\star\big|^2\big(k_{\mathrm{in}}, k_{\overline{\mathrm{in}}}; \{p_i\}_{i=1}^n\big) \tag{10}$$

with

$$C_{\text{aux-q}} = \frac{N_c^2-1}{N_c} \quad , \quad C_{\text{aux-g}} = 2N_c \,. \tag{11}$$

The other factors on the left-hand side of Eq. (10) assure the correct power of the coupling constant, and the correct on-shell limit. In particular, because of the factor ξ_{in}^2, the right-hand side does not depend on the Sudakov representation of the momenta, and is invariant under the scaling of Eq. (5).

2 Promotion to NLO

The NLO contribution to the cross section is expected to schematically look like

$$d\sigma^{(1)} = \int dx_{\text{in}}\, d^2k_\perp\, d\bar{x}_{\overline{\text{in}}}\Big\{F(x_{\text{in}},|k_\perp|)\, f(\bar{x}_{\overline{\text{in}}})\Big[dV^\star(x_{\text{in}},k_\perp,\bar{x}_{\overline{\text{in}}}) + dR^\star(x_{\text{in}},k_\perp,\bar{x}_{\overline{\text{in}}})\Big] + \Big[F^{(1)}(x_{\text{in}},|k_\perp|)\, f(\bar{x}_{\overline{\text{in}}}) + F(x_{\text{in}},|k_\perp|)\, f^{(1)}(\bar{x}_{\overline{\text{in}}})\Big] dB^\star(x_{\text{in}},k_\perp,\bar{x}_{\overline{\text{in}}})\Big\}\,, \quad (12)$$

where, $dV^\star$ represents the virtual contribution involving one-loop amplitudes, and $dR^\star$ the real contribution involving an extra final-state parton. Both $dV^\star$ and $dR^\star$ are suppressed by an extra strong coupling constant compared to $dB^\star$. The functions $f^{(1)}$, $F^{(1)}$ in the second line of Eq. (12) are higher-order corrections to the PDFs, and carry an extra power of the coupling constant compared to the leading-order ones.

The loop integrals in $dV^\star$ and the phase space integrals in $dR^\star$ cause soft and collinear divergences. Not all of these cancel, a well-known phenomenon in collinear factorization, for which the remnant divergences are absorbed by what is denoted $f^{(1)}$ in Eq. (12). In the auxiliary parton approach, extra divergences appear caused by the limit of $\Lambda \to \infty$. Firstly, Λ effectively acts as a regulator of divergences associated with linear denominators in the loop integrals. Secondly, one cannot just take tree-level space-like gluon amplitudes with a radiative gluon to get the whole real contribution. Also the situation for which the radiative gluon shares the large Λ component with the auxiliary final-state parton must be included.

This was recently worked out in [4], where it was found that the difference between using auxiliary quarks or gluons goes beyond a simple color factor. Furthermore, the scaling invariance of Eq. (5) is broken. It turns out that in order to obtain a consistent result, one must take $\lambda = \Lambda$ and assume that $x_{\text{in}} = \mathcal{O}\big(\Lambda^{-1}\big)$. While such a criterion in inherently absent at LO, it does appear at NLO that x_{in} must be small, and that considering leading powers of Λ is equivalent to considering lowest powers in x_{in}. Under this restriction, the non-cancelling divergences can be organized within the NLO cross section as follows:

$$d\sigma^{(1)} = \int dx_{\text{in}}\, d^2k_\perp\, d\bar{x}_{\overline{\text{in}}}\Big\{F(x_{\text{in}},|k_\perp|)\, f(\bar{x}_{\overline{\text{in}}})\Big[dV^\star(x_{\text{in}},k_\perp,\bar{x}_{\overline{\text{in}}}) + dR^\star(x_{\text{in}},k_\perp,\bar{x}_{\overline{\text{in}}})\Big]_{\text{cancelling}} + \Big[F^{(1)}(x_{\text{in}},|k_\perp|) + \Delta^\star_{\text{coll}} + F(x_{\text{in}},|k_\perp|)\Delta_{\text{unf}}\Big] f(\bar{x}_{\overline{\text{in}}})\, dB^\star(x_{\text{in}},k_\perp,\bar{x}_{\overline{\text{in}}}) + F(x_{\text{in}},|k_\perp|)\Big[f^{(1)}(\bar{x}_{\overline{\text{in}}}) + \Delta_{\overline{\text{coll}}}\Big] dB^\star(x_{\text{in}},k_\perp,\bar{x}_{\overline{\text{in}}})\Big\}\,. \quad (13)$$

The first line of Eq. (13) is free of any divergences, and also independent of the type of auxiliary partons used. The third line is also independent, but contains the well-known divergence also appearing in collinear factorization, in the $\overline{\text{MS}}$ scheme given by

$$\Delta_{\overline{\text{coll}}} = -\frac{a_\epsilon}{\epsilon}\int_{\bar{x}_{\overline{\text{in}}}}^{1} dz\,\Big[\mathcal{P}^{\text{reg}}_{\overline{\text{in}}}(z) + \gamma_{\overline{\text{in}}}\delta(1-z)\Big]\frac{1}{z} f\!\left(\frac{\bar{x}_{\overline{\text{in}}}}{z}\right) \quad (14)$$

where, restricting ourselves to a radiative gluon,

$$\mathcal{P}_q^{\mathrm{reg}}(z) = \frac{N_c^2-1}{2N_c}\left(\frac{2}{[1-z]_+} - 1 - z - \epsilon(1-z)\right) \quad , \quad \gamma_q = \frac{3}{2}\frac{N_c^2-1}{2N_c} \quad , \tag{15}$$

$$\mathcal{P}_g^{\mathrm{reg}}(z) = N_c\left(\frac{2}{[1-z]_+} + \frac{2}{z} + 2z(1-z) - 4\right) \quad , \quad \gamma_g = \frac{11N_c - 2n_f}{6} \quad , \tag{16}$$

depend on the type of on-shell initial-state parton, and

$$a_\epsilon = \frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \quad , \quad \epsilon = \frac{4-\dim}{2} \,. \tag{17}$$

The symbol n_f represents the number of light quark families included, and $N_c = 3$. Within collinear factorization, $\Delta_{\overline{\mathrm{coll}}}$ is imagined to be absorbed into $f^{(1)}$, or formulated differently, $f^{(1)}$ is imagined to contain a divergent contribution that cancels against $\Delta_{\overline{\mathrm{coll}}}$.

The second line of Eq. (13) contains divergences that are different than the ones appearing in collinear factorization. Still similar is

$$\Delta_{\mathrm{coll}}^\star = -\frac{a_\epsilon}{\epsilon}\int_{x_{\mathrm{in}}}^1 \frac{dz}{z}\left(\frac{2N_c}{[1-z]_+} + \gamma_g\delta(1-z)\right)F\left(\frac{x_{\mathrm{in}}}{z}, |k_\perp|\right) \tag{18}$$

where the plus-distribution now only acts on $F\big(x_{\mathrm{in}}/z, |k_\perp|\big)$ and not on the $1/z$ in front of it. Also appearing, however, is

$$\Delta_{\mathrm{unf}} = \frac{a_\epsilon N_c}{\epsilon}\left(\frac{\mu^2}{|k_\perp|^2}\right)^\epsilon \left[\mathcal{J}_{\mathrm{univ}} + \mathcal{J}_{\mathrm{univ}} + \mathcal{J}_{\mathrm{aux}}\right] , \tag{19}$$

where

$$\mathcal{J}_{\mathrm{univ}} = \frac{11}{6} - \frac{n_f}{3N_c} - \frac{\mathcal{K}}{N_c}(-\epsilon) \quad \text{with} \quad \mathcal{K} = N_c\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5n_f}{9} \,, \tag{20}$$

and

$$\mathcal{J}_{\text{aux-q}} = \frac{3}{2} - \frac{1}{2}(-\epsilon) \quad , \quad \mathcal{J}_{\text{aux-g}} = \frac{11}{6} + \frac{n_f}{3N_c^3} + \frac{n_f}{6N_c^3}(-\epsilon) \,. \tag{21}$$

As indicated by the label of $\mathcal{J}_{\mathrm{aux}}$, it *does* depend on the type of auxiliary partons used. It turns out, however, that $\mathcal{J}_{\mathrm{univ}} + \mathcal{J}_{\text{aux-q}}$ and $\mathcal{J}_{\mathrm{univ}} + \mathcal{J}_{\text{aux-g}}$ are identical to the expressions in equation (4.9) and equation (5.11) of [5] for NLO corrections to quark and gluon impact factors. We introduced the symbol $\mathcal{K}$ for the same quantity defined in equation (4.10) of [5]. The remaining $\mathcal{J}_{\mathrm{univ}}$ in Eq. (19), which is independent of the type of auxiliary partons used, is a correction related to the proper UV renormalization regarding the space-like gluon.

3 Conclusion

In conclusion, while $\Delta_{\mathrm{coll}}^\star$ could still be interpreted like in the collinear case and to be absorbed by the PDF correction $F^{(1)}$, the appearance of Δ_{unf} goes beyond such an interpretation. The factorized form can still be maintained at NLO, but not anymore purely into PDFs and partonic cross section. The auxiliary parton dependent color factor at LO must be interpreted as related to the target impact factor, and at NLO its non-trivial corrections appear.

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