

# Probing Bose Correlation in Deep Inelastic Scattering: Trijet Production

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*Alex Kovner, Ming Li and Vladimir Skokov, arXiv: 2105.14971; Accepted to PRL.*

# Outline

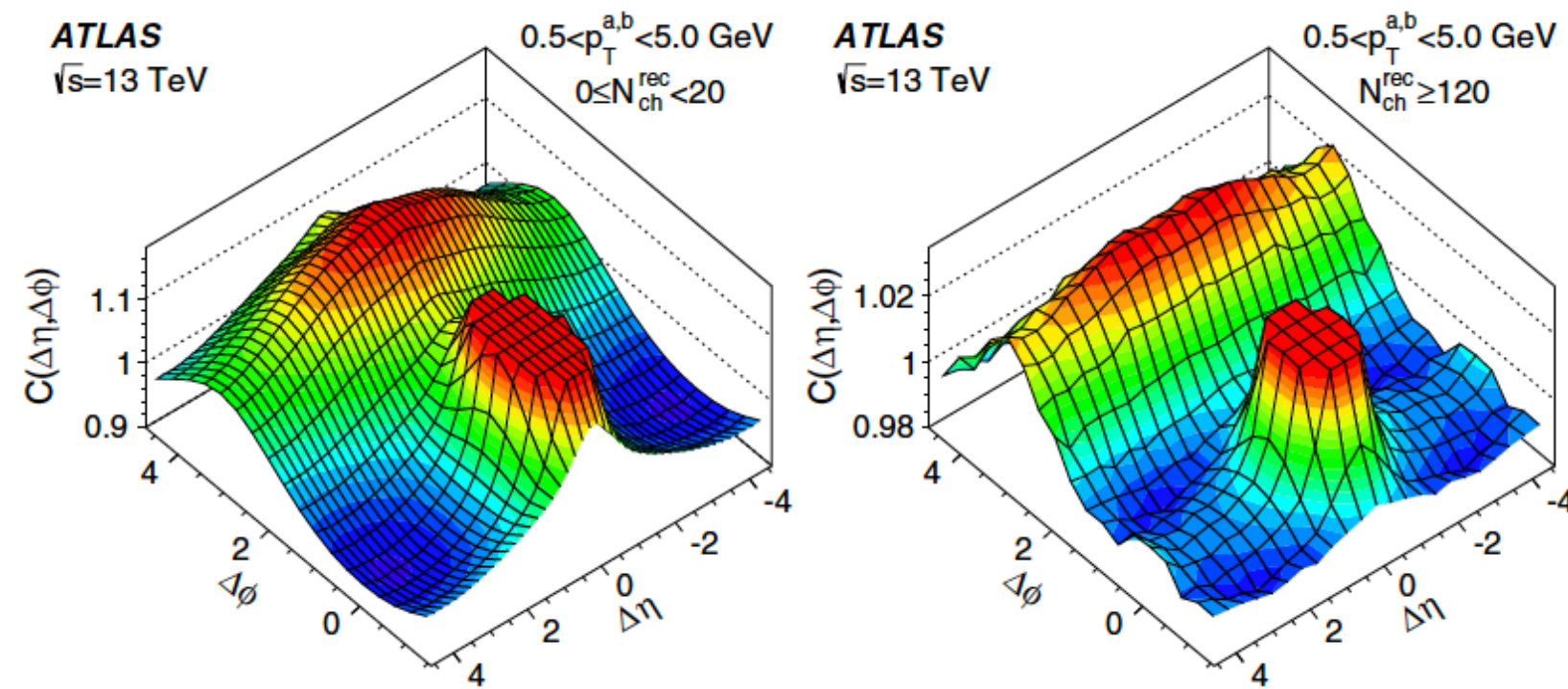
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- **Introduction and Motivation.** (*Ridge structure, Color Glass Condensate and multiparton correlations in nuclear wavefunction*)
- **Trijet Production in DIS.** (*Diffraction dijet + gluon jet, ensemble averaging, beyond correlation limit*)
- **Numerical Results and Discussions.**
- **Summary and Outlook.**



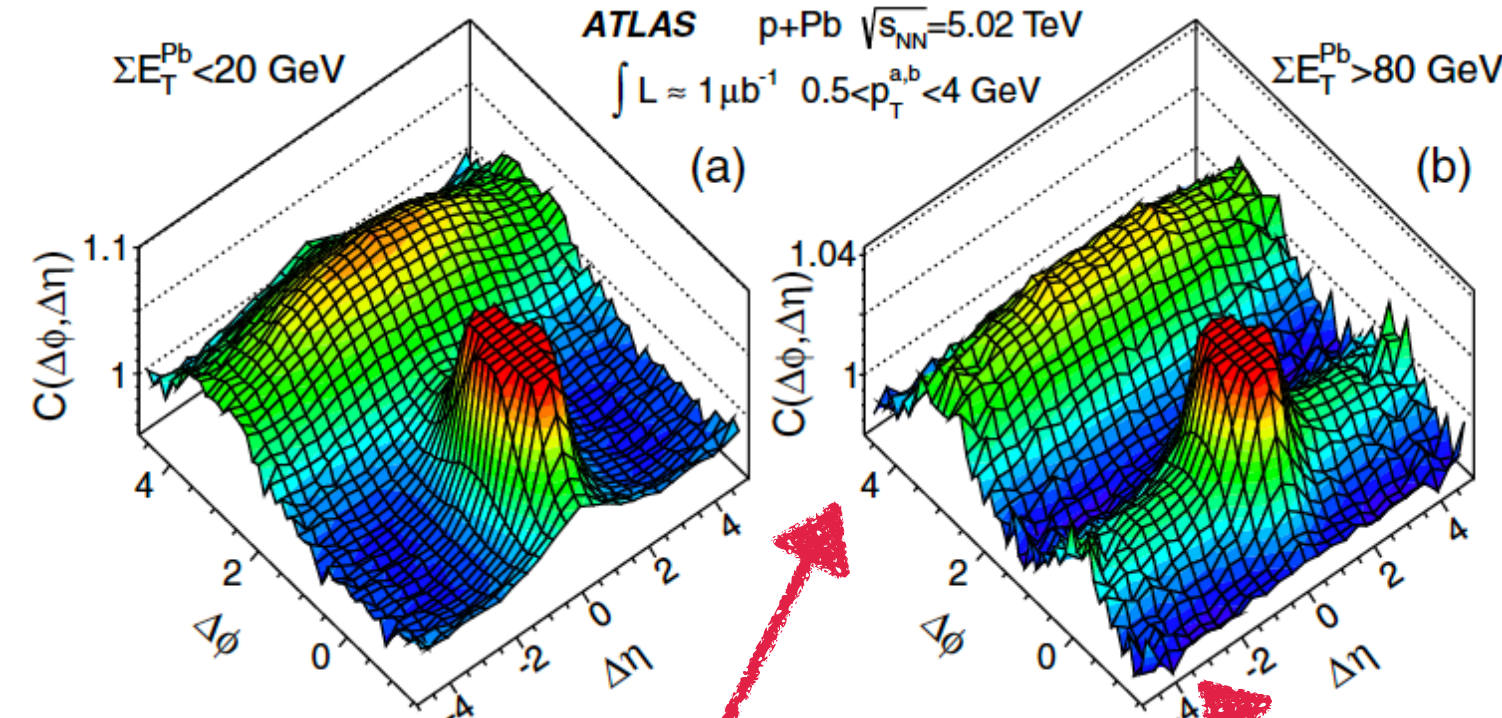
# Two-particle long range rapidity correlations (ridge structure, high multiplicity events)

## p-p collision



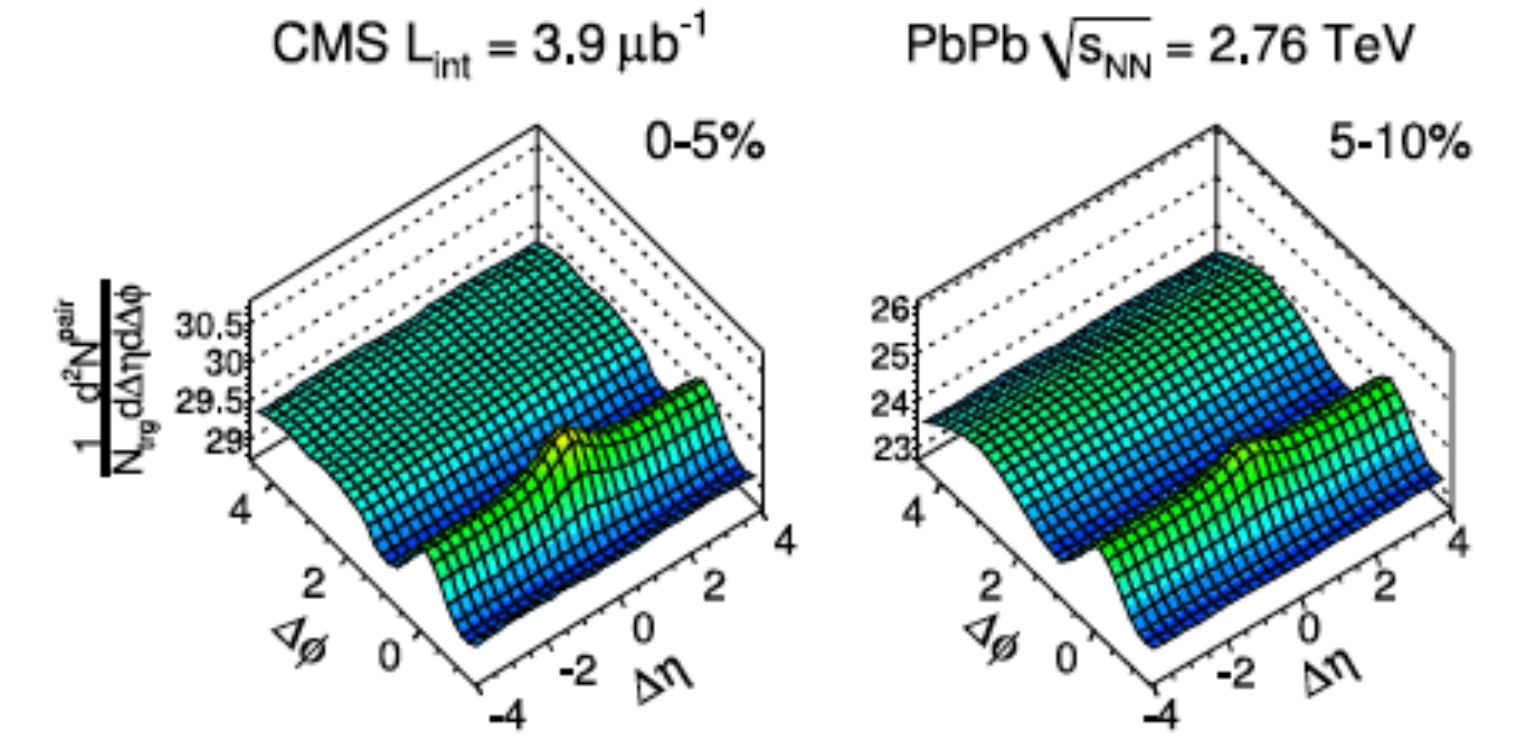
ATLAS Collaboration, PRL (2016), CMS Collaboration, JHEP (2010).

## p-Pb collision



ATLAS Collaboration, PRL (2013), CMS Collaboration, PLB(2013), ALICE Collaboration, PLB(2013).

## Pb-Pb collision



CMS Collaboration, EPJC (2012).

Away Side Ridge:  $\Delta\phi = \pi$ .

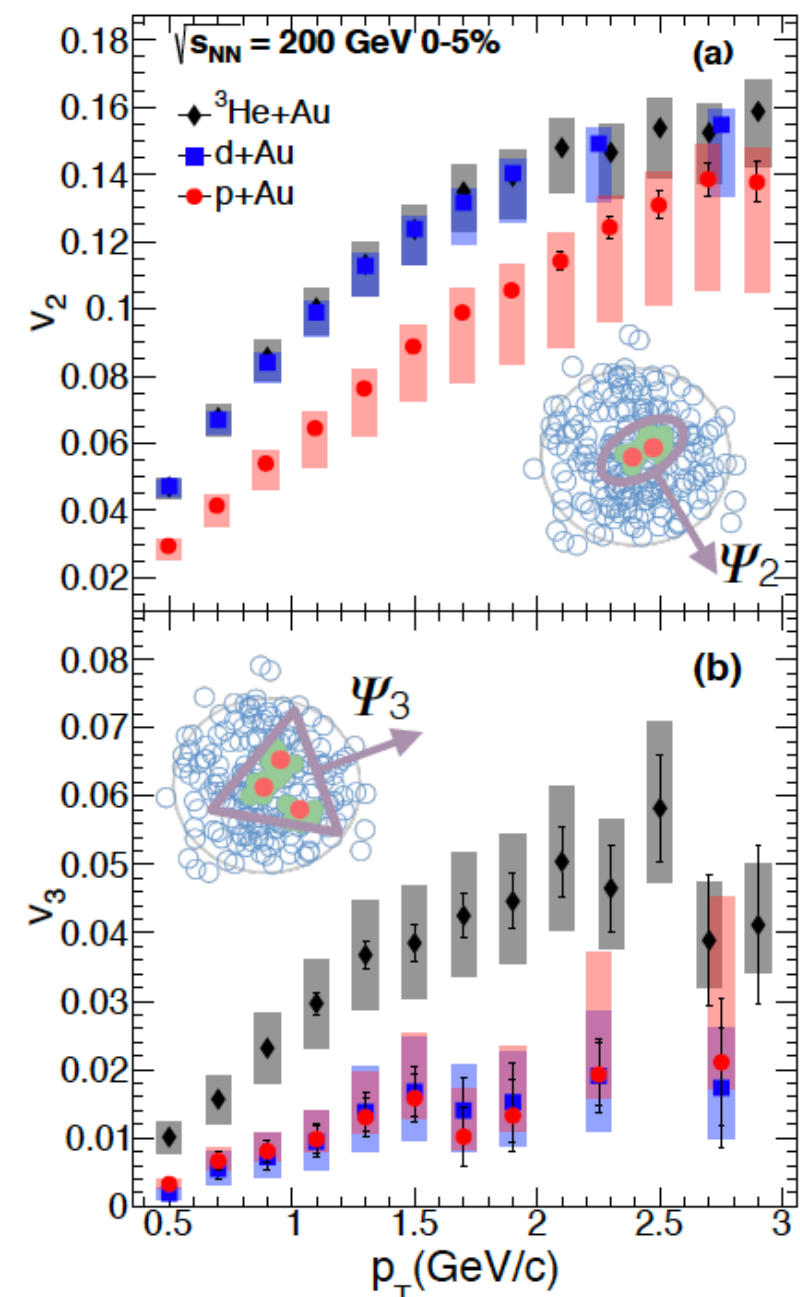
Near Side Ridge:  $\Delta\phi = 0$ ;

$$C(\Delta\phi, \Delta\eta) \equiv \frac{1}{N_{\text{trk}}} \frac{dN^{\text{pair}}}{d\Delta\phi d\Delta\eta}$$

Two particles are more likely to be emitted with same azimuthal angles even when they have large difference in rapidities.

Is Quark-Gluon Plasma formed in high multiplicity p-Pb collisions and p-p collisions?

Final State Interaction: Initial State Interaction Geometry and Collective motion of particles in the fluid velocity field of QGP.



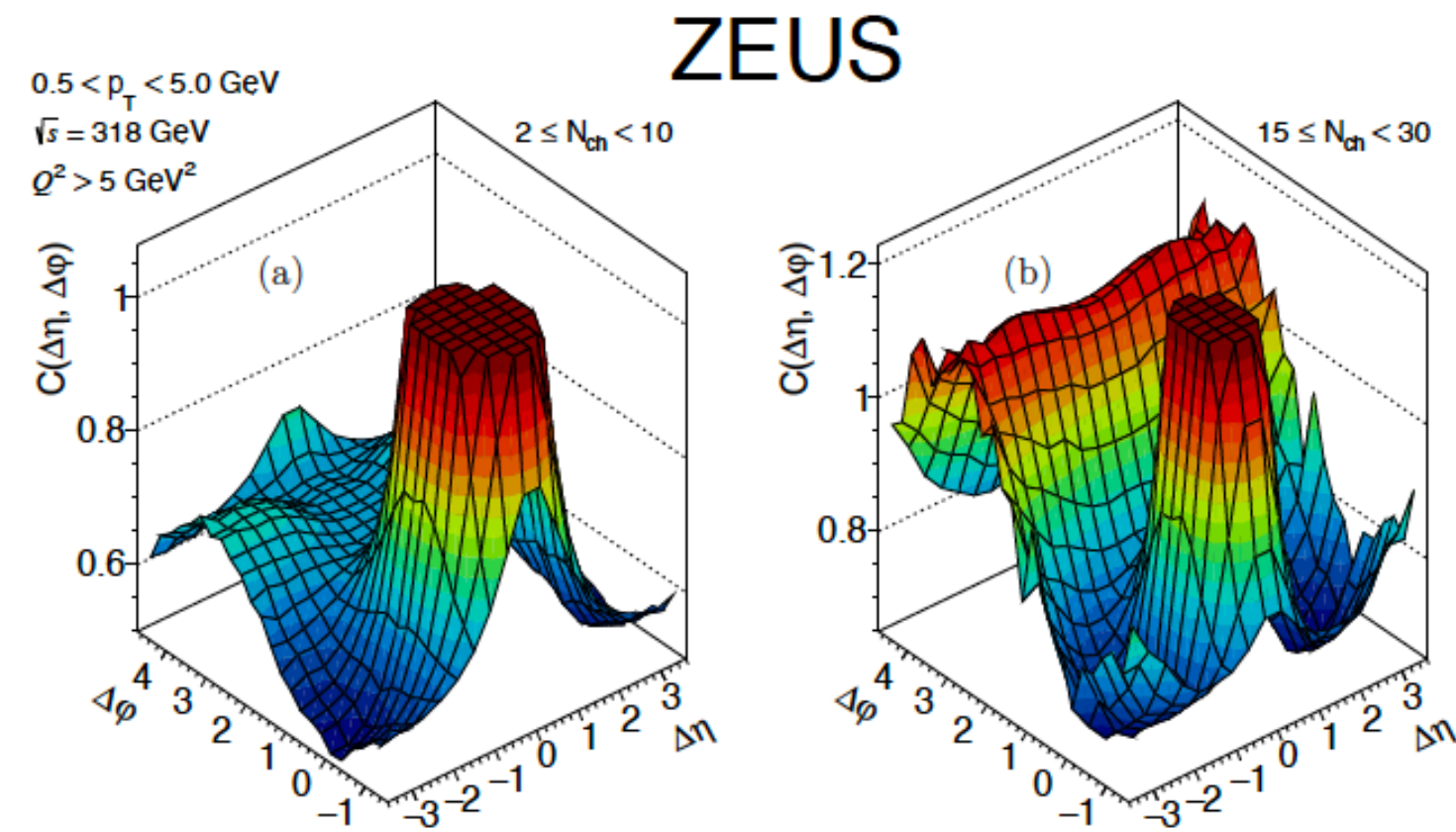
PHENIX Collaboration, Nature Physics (2018).



# Ridge Structure in Even Smaller Collision Systems?

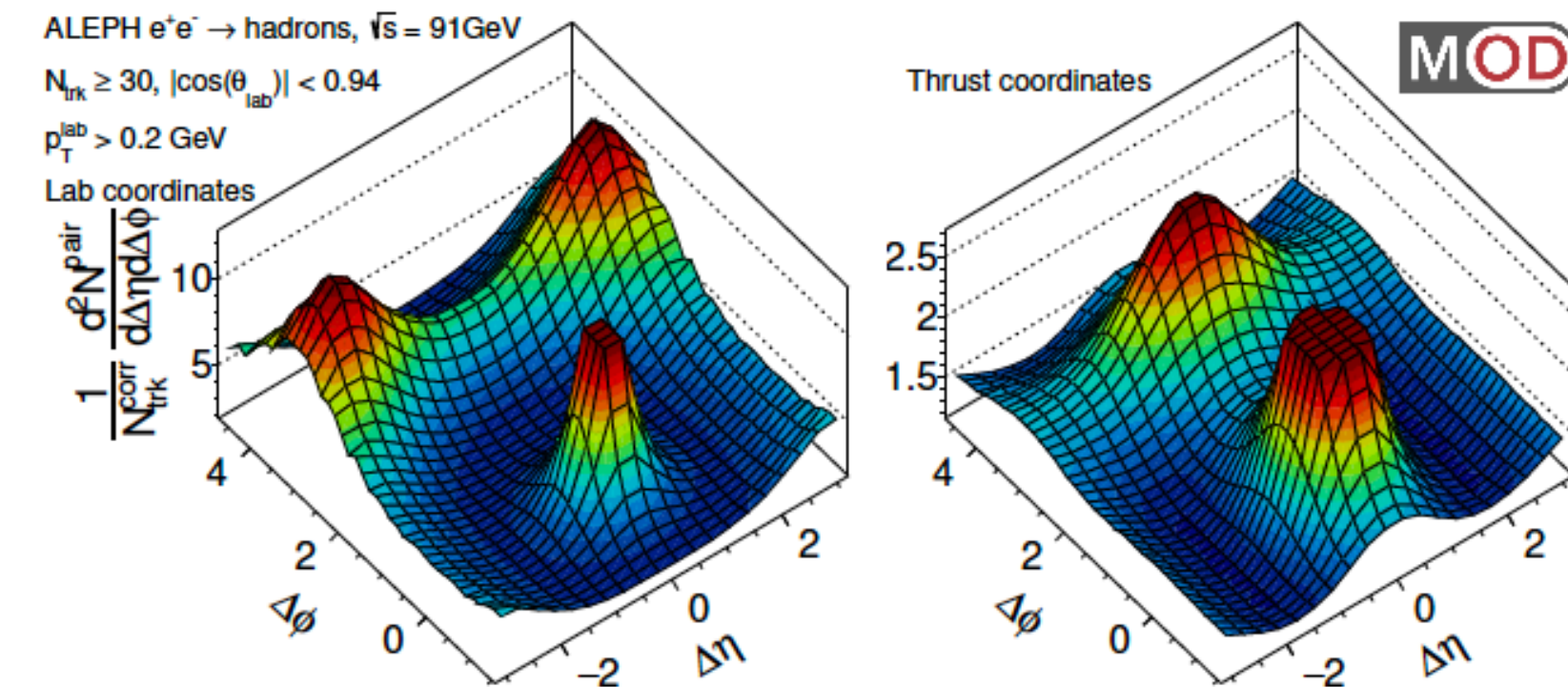
e-p collision

*The ZEUS Collaboration, JHEP (2020)*

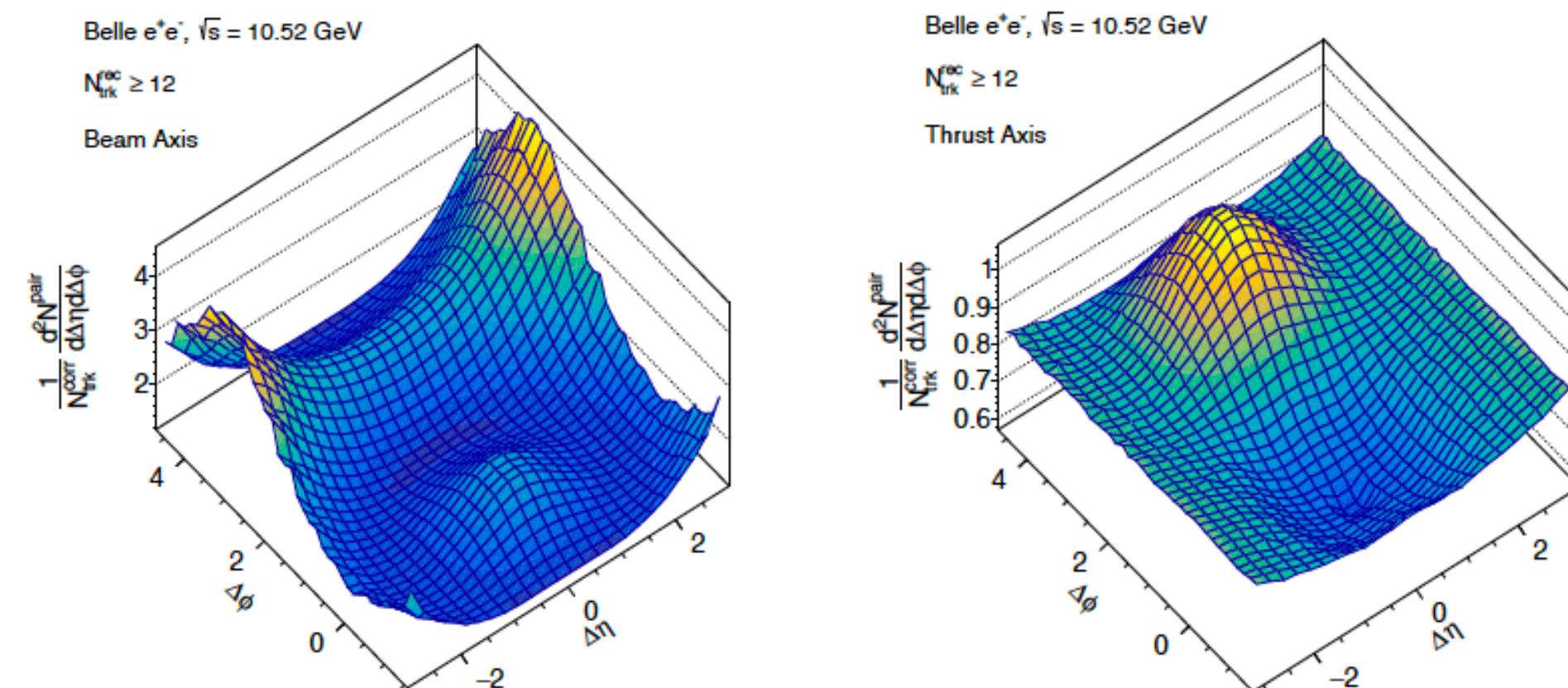


e-e collision

*Badea, et al. PRL (2019)*



*The Belle Collaboration, PRL (2022)*



Not Observed!

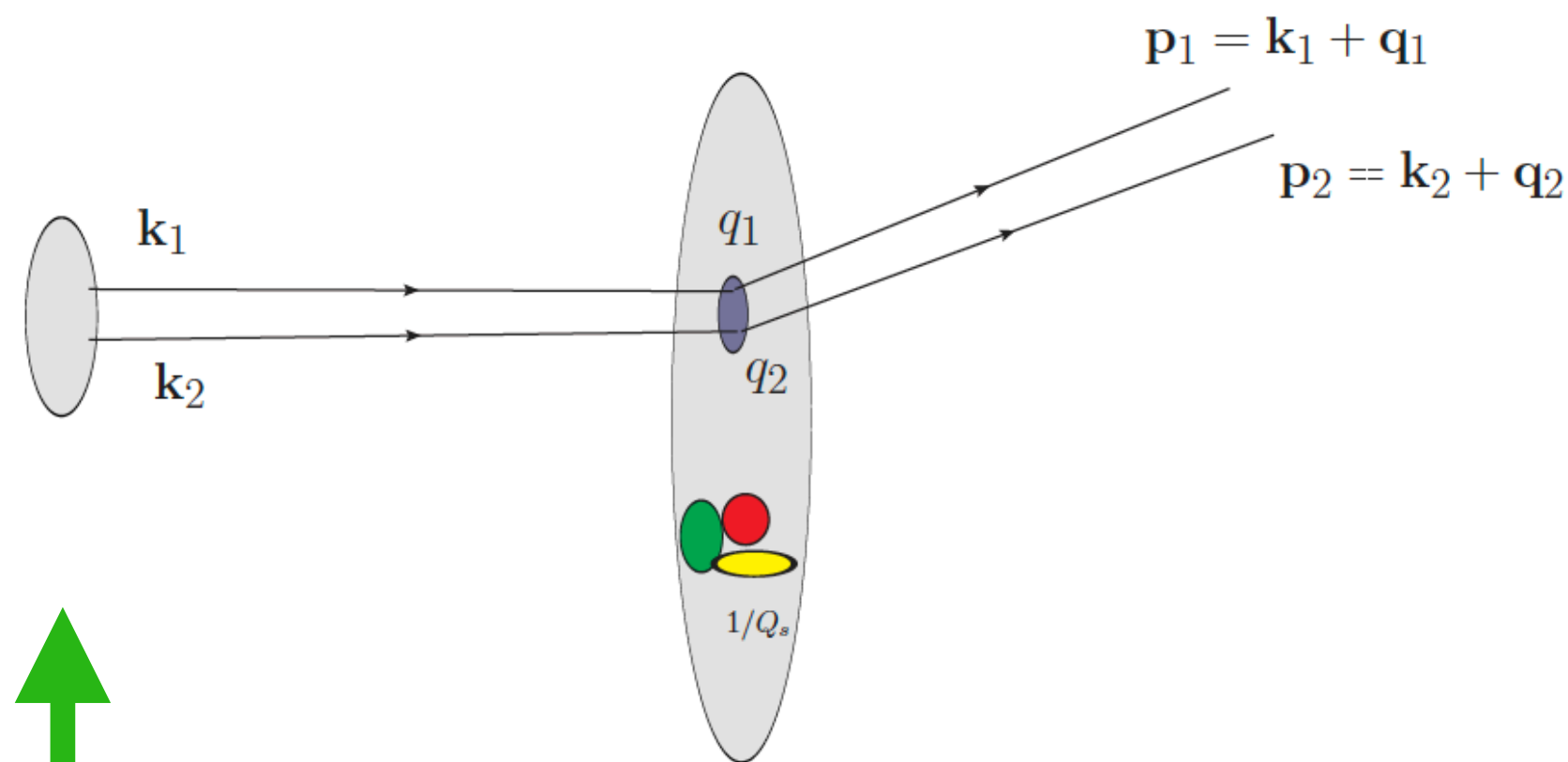
# Ridge Structure from Color Glass Condensate Approach

1. Long Range Correlation in Rapidity: Universal Property of High Energy Collisions, Boost Invariance.

2. Near Side and Away Side Azimuthal Correlation.

Two partons scatter off the same color domain of the nucleus receive similar transverse momentum kicks  
 $|\mathbf{q}_1| \sim |\mathbf{q}_2| \gg |\mathbf{k}_1|, |\mathbf{k}_2|$

A simplified fixed-configuration picture



Nucleus with Gluon Saturation.  
 Typical color domain size  $1/Q_s$

In **dilute target limit** (AKA two-gluon exchange approximation)

$$\frac{dN}{d^2\mathbf{p}_1 d^2\mathbf{p}_2} = \int_{\mathbf{q}_1, \mathbf{q}_2} \tilde{F}(\mathbf{p}_1, \mathbf{q}_1) \tilde{F}(\mathbf{p}_2, \mathbf{q}_2) \left( 1 + \frac{(2\pi)^2}{(N_c^2 - 1)S_\perp} [\delta(\mathbf{q}_1 + \mathbf{q}_2) + \delta(\mathbf{q}_1 - \mathbf{q}_2)] \right)$$

Bose correlation  $\longrightarrow$  near-side ridge

There are also quantum correlations from projectile wave function;

Assuming the two partons from the projectile are uncorrelated in this talk.

We are interested in Bose Correlation in the Target!

**Not HBT!**

$$\frac{dN}{d^2\mathbf{p}_1 d^2\mathbf{p}_2} \sim \delta(\mathbf{p}_1 + \mathbf{p}_2) + \delta(\mathbf{p}_1 - \mathbf{p}_2)$$

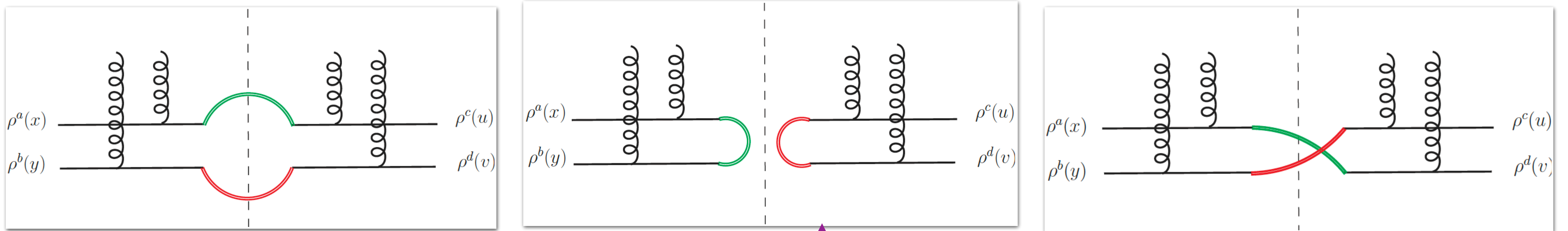


# Ridge Structure from Color Glass Condensate Approach

1. In Color Glass Condensate framework, the fundamental degrees of freedom are the color charge densities  $\rho^a(x^-, \mathbf{x}_\perp)$ .
2. In McLerran-Venugopalan model, the color charge densities are assumed to follow Gaussian distribution.

$$\langle \rho^a(x^-, \mathbf{x}_\perp) \rho^b(y^-, \mathbf{y}_\perp) \rangle = \delta^{ab} \delta(x^- - y^-) \delta^{(2)}(\mathbf{x} - \mathbf{y}) g^2 \mu^2(x^-, \mathbf{x}_\perp)$$

$$\langle \rho^a(x) \rho^b(y) \rho^c(u) \rho^d(v) \rangle = \langle \rho^a(x) \rho^c(u) \rangle \langle \rho^b(y) \rho^d(v) \rangle + \langle \rho^a(x) \rho^b(y) \rangle \langle \rho^c(u) \rho^d(v) \rangle + \langle \rho^a(x) \rho^d(v) \rangle \langle \rho^b(y) \rho^c(u) \rangle$$



$$\frac{dN}{d^2\mathbf{p}_1 d^2\mathbf{p}_2} = \int_{\mathbf{q}_1, \mathbf{q}_2} \tilde{F}(\mathbf{p}_1, \mathbf{q}_1) \tilde{F}(\mathbf{p}_2, \mathbf{q}_2) \left[ 1 + \frac{(2\pi)^2}{(N_c^2 - 1) S_\perp} [\delta(\mathbf{q}_1 + \mathbf{q}_2) + \delta(\mathbf{q}_1 - \mathbf{q}_2)] \right]$$

# Multi-Parton Correlations in Nuclear Wavefunction

Proton/Nucleus is a many-parton system, how to probe multi-parton correlations in the nuclear wavefunction?

Multi-Dimensional Imaging of Single Parton Distribution in the Nucleon

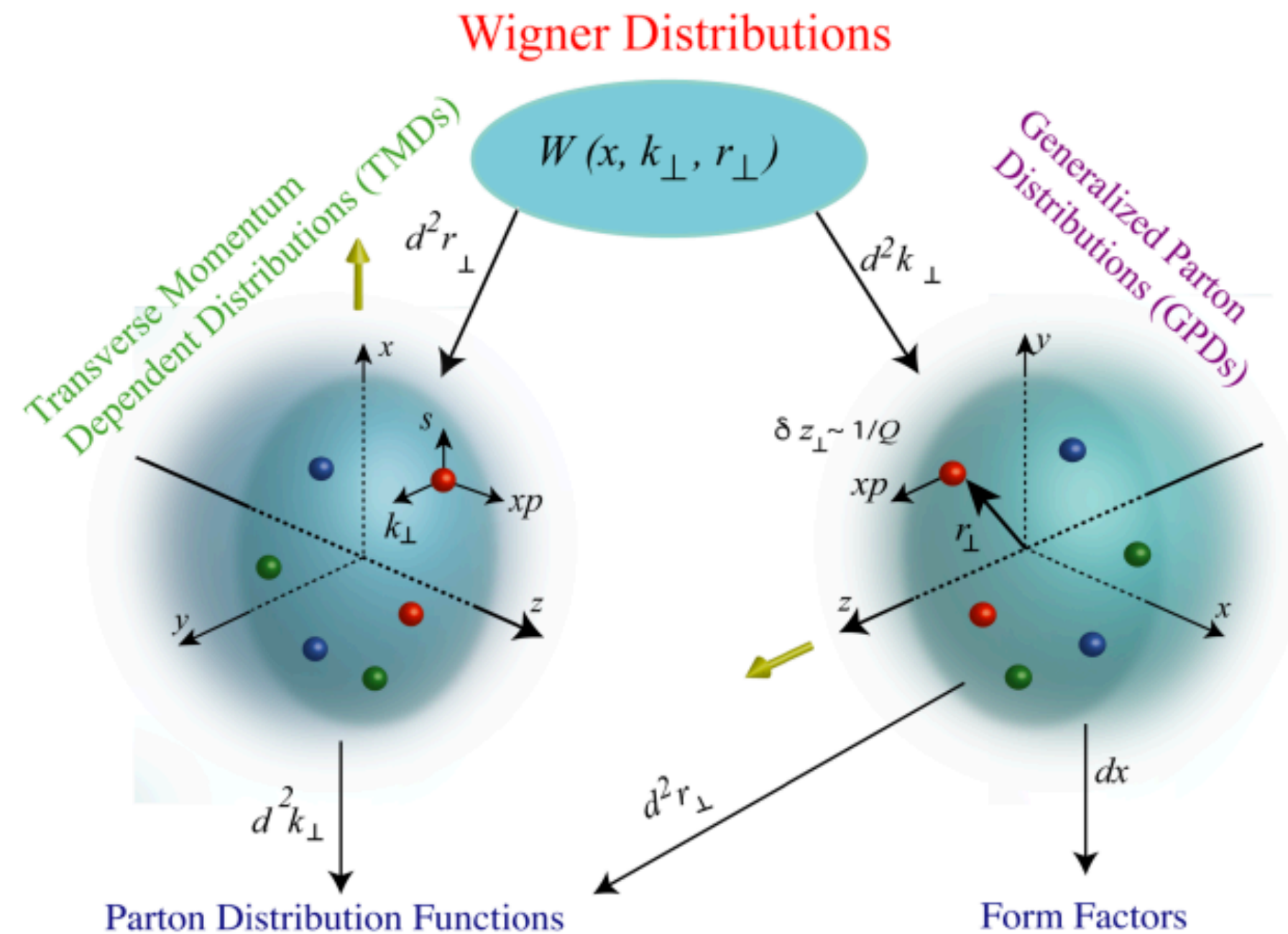


Figure from INT Program INT-17-3

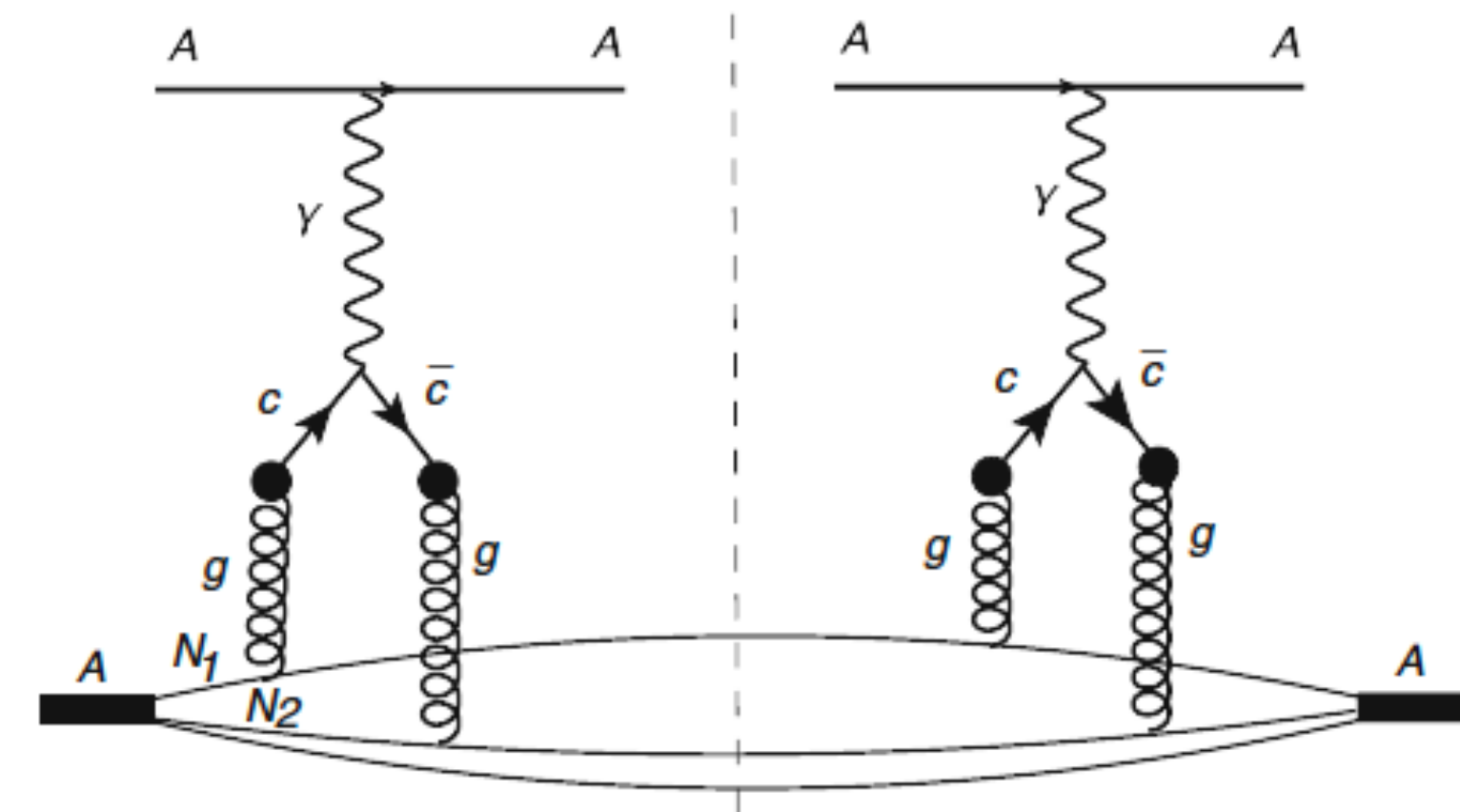
B. Blok and M. Strikman, EPJC (2014)

B. Blok, Yu. Dokshitzer, L. Frankfurt and M. Strikman, EPJC (2012)

Four jets production through two hard processes

to probe two-parton GPD:  ${}_2D(x_1, x_2, p_{1t}^2, p_{2t}^2)$

$$\gamma + p/A \longrightarrow c + \bar{c} + g_1 + g_2 + X$$



Event rate: Four-Jet/ Dijet  $\lesssim 0.05\%$  with jet  $p_t > 5 \text{ GeV}$

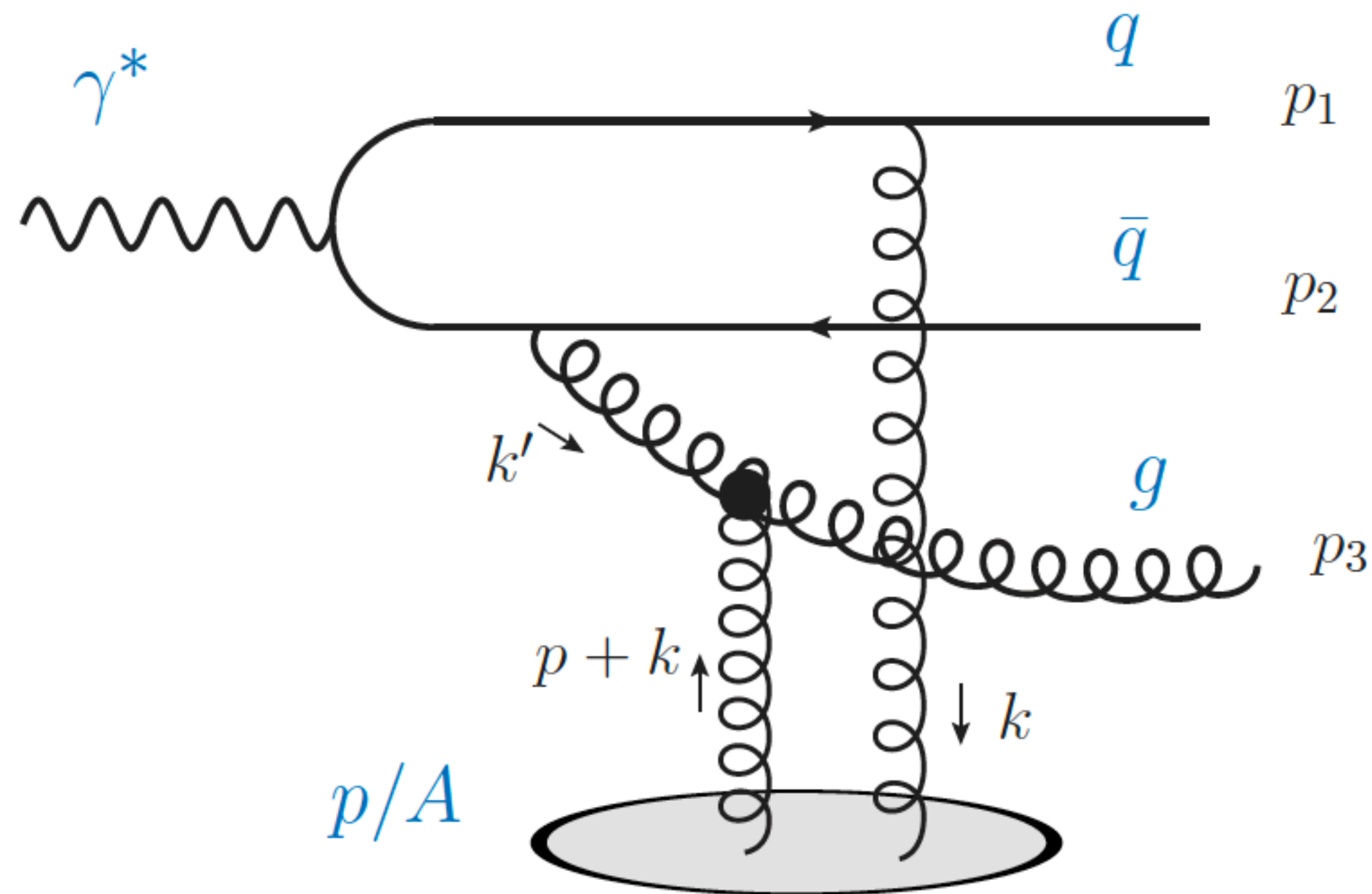
# Trijet Production in Deep Inelastic Scatterirng

Goal: find some observable that is

1. Sensitive to two particle correlation in the nuclear wave function
2. Manifests itself in near-side ridge correlation.

$$\gamma^* + p/A \longrightarrow q + \bar{q} + g + X$$

- (a) Quark and antiquark are in color singlet state and locate in the forward rapidity region;
- (b) The gluon jet locates in the central rapidity region.



Trijet production under two-gluon exchange

Gluon momentum

Momentum imbalance of Dijet

**Measure the correlation  $C(\mathbf{p}_3, \Delta)$**

Transverse Momentum Conservation

$$\mathbf{k}' + \mathbf{k} + \mathbf{p}_1 + \mathbf{p}_2 = 0$$

Notation:  $\mathbf{p} \equiv \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$ ,  $\Delta \equiv \mathbf{p}_1 + \mathbf{p}_2$ .

For phase space region  $|\mathbf{k}'| \ll |\mathbf{p}_3|$

$$\mathbf{p} + \mathbf{k} \sim \mathbf{p}_3, \quad -\mathbf{k} \sim \Delta$$



# Diffractive Quark Antiquark Dijet + Gluon Jet in DIS

## Trijet Production

$$\frac{d^3N}{d^3p_1 d^3p_2 d^3p_3} = \sum_{\alpha_1, \alpha_2, \alpha_3} \langle \psi_F | \hat{d}_{\alpha_1}^\dagger(p_1) \hat{d}_{\alpha_1}(p_1) \hat{b}_{\alpha_2}^\dagger(p_2) \hat{b}_{\alpha_2}(p_2) \hat{a}_{\alpha_3}^\dagger(p_3) \hat{a}_{\alpha_3}(p_3) | \psi_F \rangle$$

**Final state wavefunction**  $|\psi_F\rangle = \hat{C}^\dagger \hat{S} \hat{C} |\gamma^*\rangle \otimes |N\rangle.$

## Coherent operator (Soft gluon radiation operator)

$$\hat{C} = \exp \left\{ i \int d^2\mathbf{x} b_i^a(\mathbf{x}) \int_{\Lambda^- e^{\Delta y}}^{\Lambda^-} \frac{dk^-}{\sqrt{2\pi} |k^-|} \left( \hat{a}_i^{a\dagger}(k^-, \mathbf{x}) + \hat{a}_i^a(k^-, \mathbf{x}) \right) \right\}$$

## S-Matrix operator

$$\hat{S} = \exp \left\{ i \int d^2\mathbf{x} \hat{j}(\mathbf{x}) \alpha_T(\mathbf{x}) \right\}.$$

## Virtual Photon Fock Space Approximation

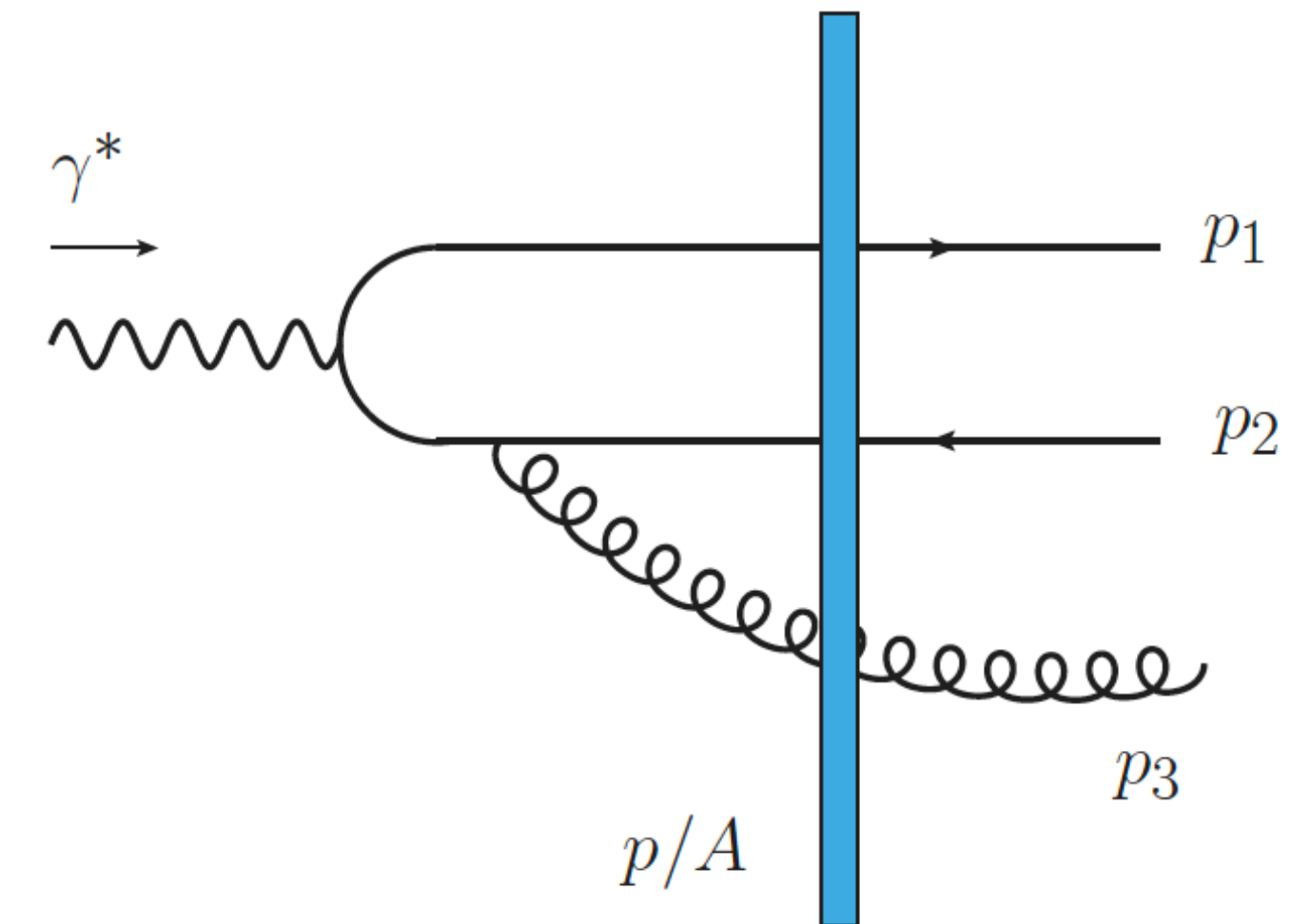
$$|\gamma^*\rangle \simeq \sum_{q\bar{q}} \Psi_{q\bar{q}} |q\bar{q}\rangle$$

## Test NLO QCD and measure $\alpha_s$ , single particle PDF

Z. Nagy and Z. Trocsanyi, PRL(2001)  
ZEUS Collaboration, PLB (1998), (2001)  
H1 Collaboration, PLB (2001)  
.....

## Trijet and small-x in the literature:

A. Kovner and U. A. Wiedemann, PRD (2001)  
A. Ayala, M. Hentschinski, J. Jalilian-Marian and M.E.Tejada-Yeomans, PLB (2016)  
R. Boussarie, A.V. Grabovsky, L. Szymanowski and S. Wallon, JHEP (2016)  
T. Altinoluk, R. Boussarie, C. Marquet and P. Taels, JHEP(2020)  
E. Iancu, A. H. Mueller and D.N. Triantafyllopoulos, arXiv: 2112.06353

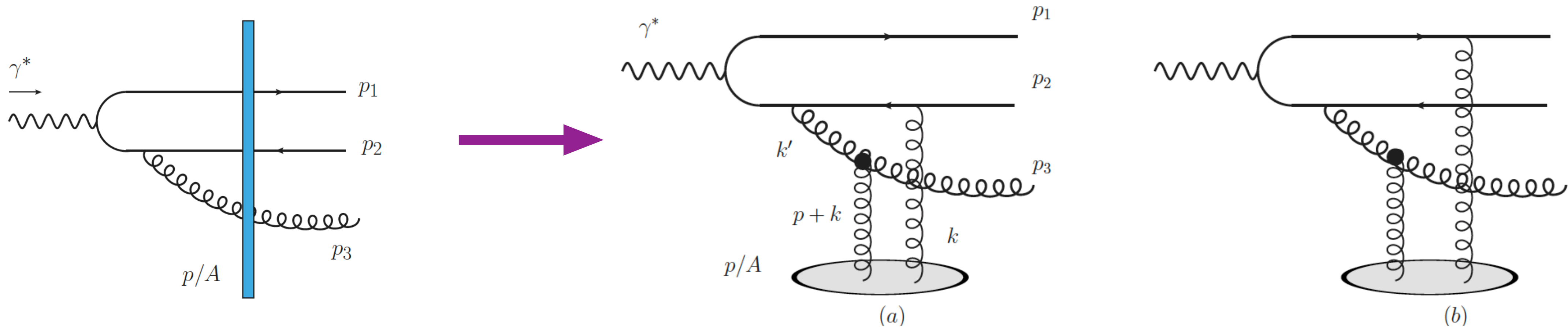


# The Two-Gluon Exchange Approximation (Dilute Limit)

**Trijet Production**

$$\frac{d^3N}{d^3p_1 d^3p_2 d^3p_3} = \left| M_{\text{diff}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \right|^2$$

At least two gluons have to be exchanged for the quark + antiquark to be in a color singlet state.



$$M_{\text{diff}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = g^3 \sqrt{2\pi} T_{ab}^{c_3} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \alpha_T^a(\mathbf{p} + \mathbf{k}) \alpha_T^b(-\mathbf{k}) L_j(-(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k}), \mathbf{p}_3) \psi_{\sigma_1 \sigma_2}^D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k})$$

$$\psi_{\sigma_1 \sigma_2}^D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) = \left[ \Psi_{\sigma_1 \sigma_2}(p_1^+, \mathbf{p}_1; p_2^+, -\mathbf{p}_1) - \Psi_{\sigma_1 \sigma_2}(p_1^+, \mathbf{p}_1 + \mathbf{k}; p_2^+, -\mathbf{p}_1 - \mathbf{k}) - \Psi_{\sigma_1 \sigma_2}(p_1^+, -\mathbf{p}_2 - \mathbf{k}; p_2^+, \mathbf{p}_2 + \mathbf{k}) + \Psi_{\sigma_1 \sigma_2}(p_1^+, -\mathbf{p}_2; p_2^+, \mathbf{p}_2) \right]$$

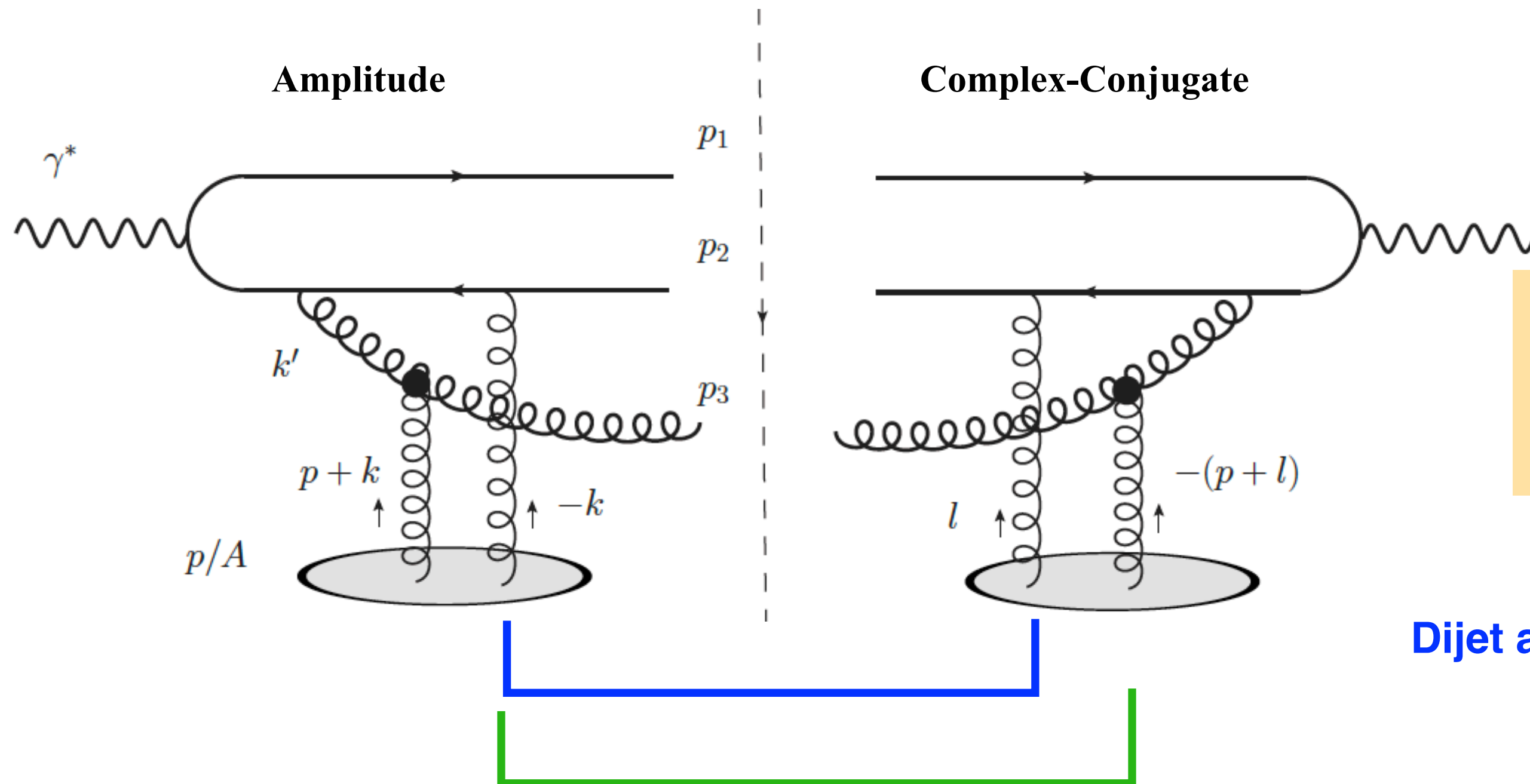
Four different ways of attaching two gluon lines to quark and antiquark



# The Two-Gluon Exchange Approximation (Dilute Limit)

**Trijet Production**

$$\frac{d^3N}{d^3p_1 d^3p_2 d^3p_3} = -(2\pi)g^6 \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d^2\mathbf{l}}{(2\pi)^2} L_j(-(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k}), \mathbf{p}_3) L_j(-(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{l}), \mathbf{p}_3) \left[ \psi_{\sigma_1\sigma_2}^D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) \psi_{\sigma_1\sigma_2}^{D*}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{l}) \right] \\ \times T_{ab}^{c_3} T_{cd}^{c_3} \left\langle \alpha_T^a(\mathbf{p} + \mathbf{k}) \alpha_T^b(-\mathbf{k}) \alpha_T^c(-\mathbf{p} - \mathbf{l}) \alpha_T^d(\mathbf{l}) \right\rangle$$



**Only two non vanishing contractions of color charge densities due to the color singlet state requirement for the quark and the antiquark.**

$$T_{ab}^{c_3} T_{cd}^{c_3} \left\langle \rho_T^a(\mathbf{p} + \mathbf{k}) \rho_T^b(-\mathbf{k}) \rho_T^c(-\mathbf{p} - \mathbf{l}) \rho_T^d(\mathbf{l}) \right\rangle \\ = -N_c(N_c^2 - 1)g^4\mu_T^4 S_\perp \left[ (2\pi)^2\delta^{(2)}(\mathbf{k} - \mathbf{l}) - (2\pi)^2\delta^{(2)}(\mathbf{p} + \mathbf{k} + \mathbf{l}) \right]$$

**Dijet and Gluon jet uncorrelated production**

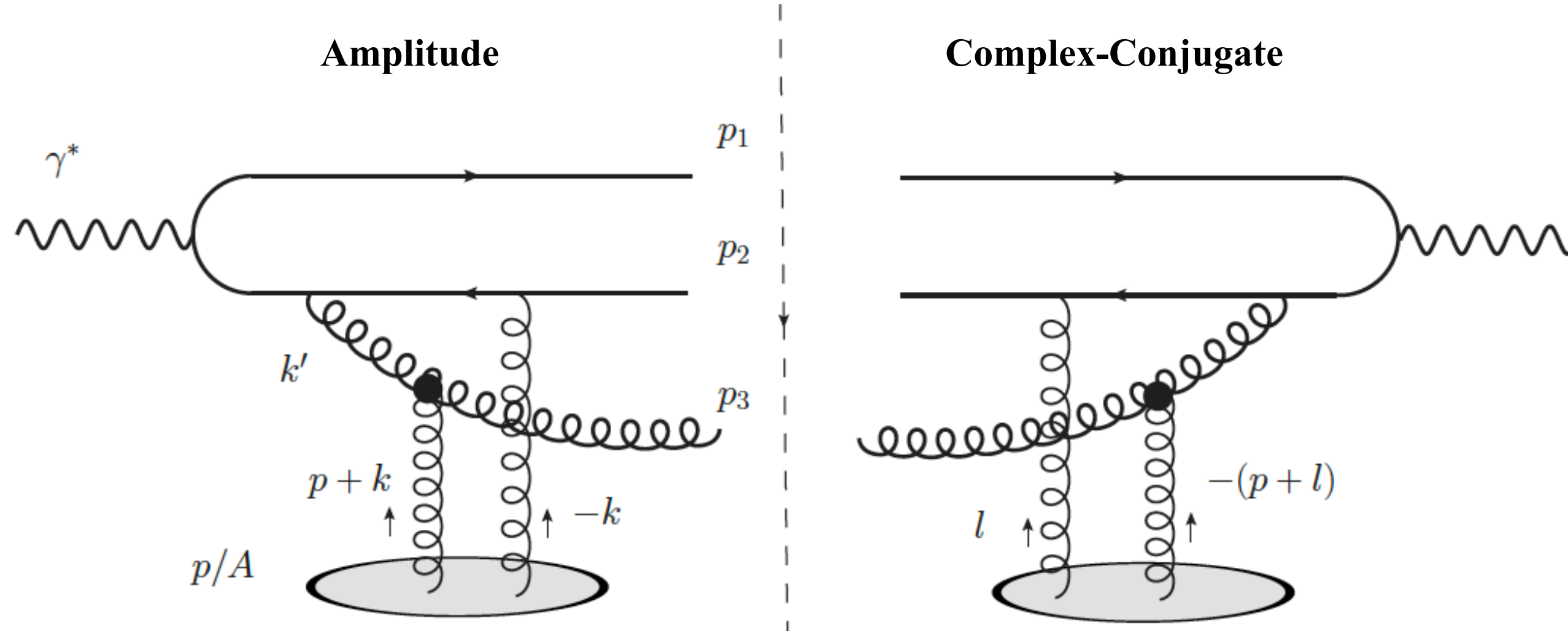
**Dijet and Gluon jet near-side ridge correlation**

**Opposite sign and no color or area suppressions.**

# Beyond the Correlation Limit: Charge Neutrality of Dipole Wavefunction

**Trijet Production**

$$\frac{d^3N}{d^3p_1 d^3p_2 d^3p_3} = -(2\pi)g^6 \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d^2\mathbf{l}}{(2\pi)^2} L_j(-(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k}), \mathbf{p}_3) L_j(-(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{l}), \mathbf{p}_3) \left[ \psi_{\sigma_1\sigma_2}^D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) \psi_{\sigma_1\sigma_2}^{D*}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{l}) \right] \\ \times T_{ab}^{c_3} T_{cd}^{c_3} \left\langle \alpha_T^a(\mathbf{p} + \mathbf{k}) \alpha_T^b(-\mathbf{k}) \alpha_T^c(-\mathbf{p} - \mathbf{l}) \alpha_T^d(\mathbf{l}) \right\rangle$$



$$\psi_{\sigma_1\sigma_2}^D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) = \left[ \Psi_{\sigma_1\sigma_2}(p_1^+, \mathbf{p}_1; p_2^+, -\mathbf{p}_1) - \Psi_{\sigma_1\sigma_2}(p_1^+, \mathbf{p}_1 + \mathbf{k}; p_2^+, -\mathbf{p}_1 - \mathbf{k}) \right. \\ \left. - \Psi_{\sigma_1\sigma_2}(p_1^+, -\mathbf{p}_2 - \mathbf{k}; p_2^+, \mathbf{p}_2 + \mathbf{k}) + \Psi_{\sigma_1\sigma_2}(p_1^+, -\mathbf{p}_2; p_2^+, \mathbf{p}_2) \right]$$

The part containing dipole wave function vanishes when  $\mathbf{k} = 0$  or  $-\mathbf{k}' = \mathbf{k} + \mathbf{p}_1 + \mathbf{p}_2 = 0$ .

Or physically speaking  $|\mathbf{k}'|, |\mathbf{k}| \ll |\mathbf{p}_1| \sim |\mathbf{p}_2| \sim Q$   
The **CORRELATION LIMIT**

The exchanged gluon has much smaller momentum,  
Thus cannot resolve the finer structure of the dipole.

Size of gluon probe  $\gg$  Size of quark-antiquark dipole  $\sim 1/Q$

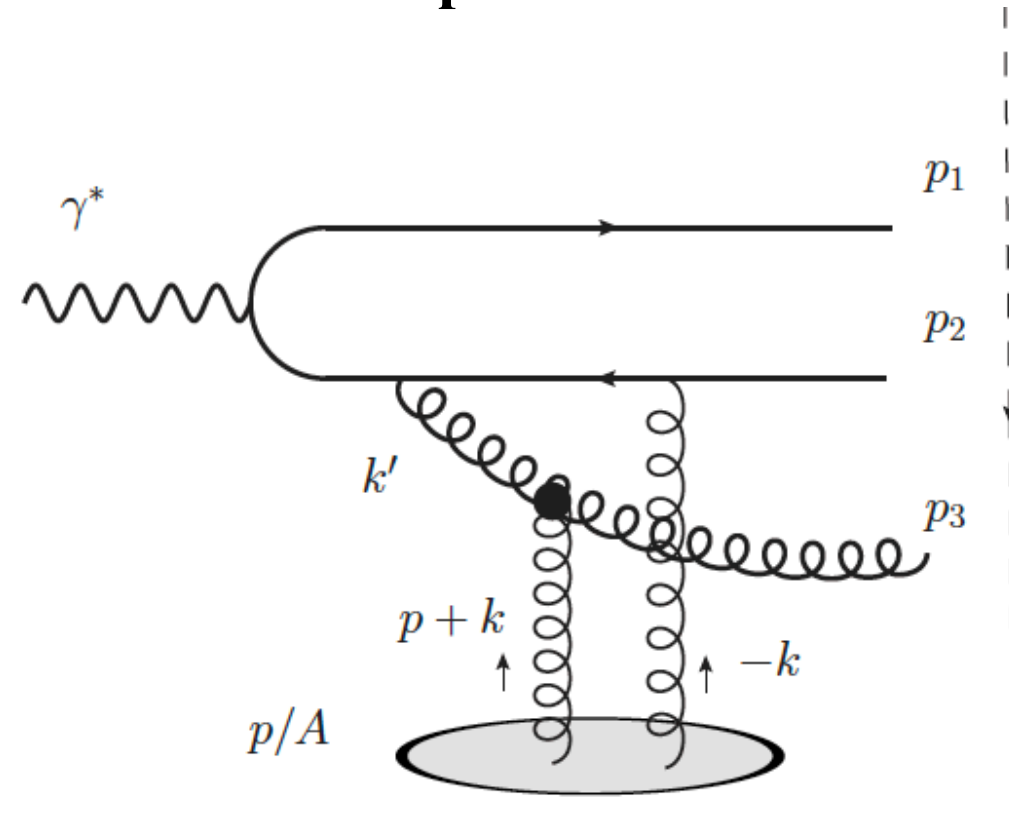


# Event Selection

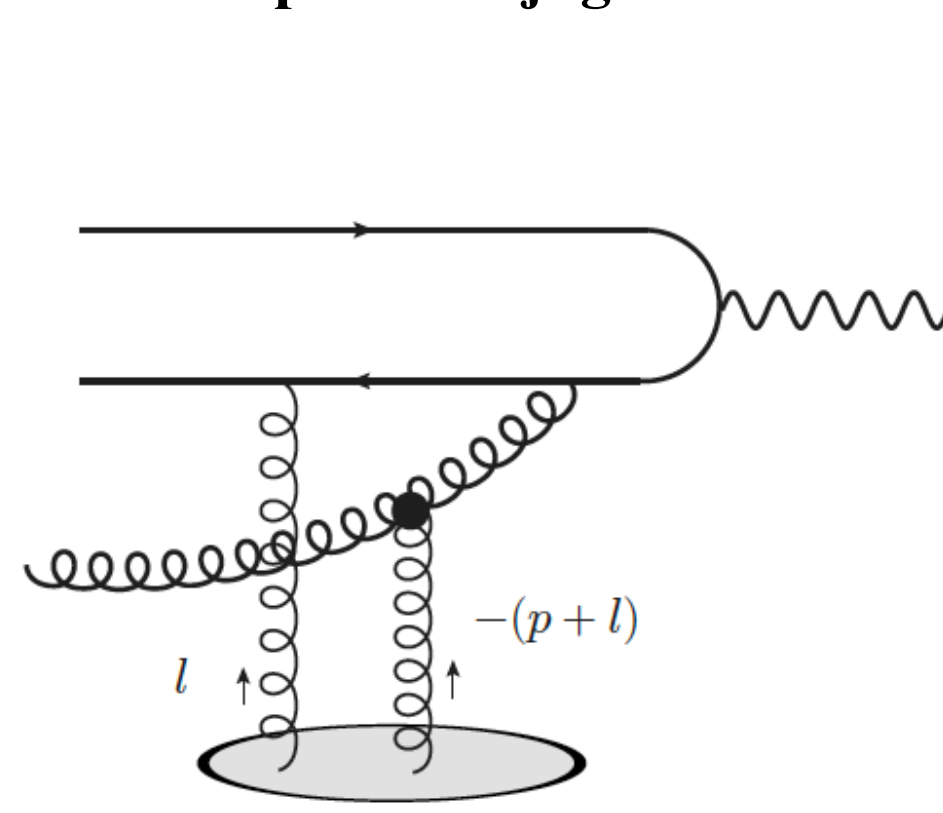
## Trijet Production

$$\frac{d^3N}{d^3p_1 d^3p_2 d^3p_3} = -(2\pi)g^6 \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{d^2\mathbf{l}}{(2\pi)^2} L_j(-(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{k}), \mathbf{p}_3) L_j(-(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{l}), \mathbf{p}_3) \left[ \psi_{\sigma_1\sigma_2}^D(\mathbf{p}_1, \mathbf{p}_2, \mathbf{k}) \psi_{\sigma_1\sigma_2}^{D*}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{l}) \right] \\ \times T_{ab}^{c_3} T_{cd}^{c_3} \left\langle \alpha_T^a(\mathbf{p} + \mathbf{k}) \alpha_T^b(-\mathbf{k}) \alpha_T^c(-\mathbf{p} - \mathbf{l}) \alpha_T^d(\mathbf{l}) \right\rangle$$

Amplitude



Complex-Conjugate



The trijet production depends on three external momenta  $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ . These six variables characterize all the possible trijet events.

We explore events  $\mathbf{p}_3 \approx \Delta = \mathbf{p}_1 + \mathbf{p}_2$  to see if there are enhancement.

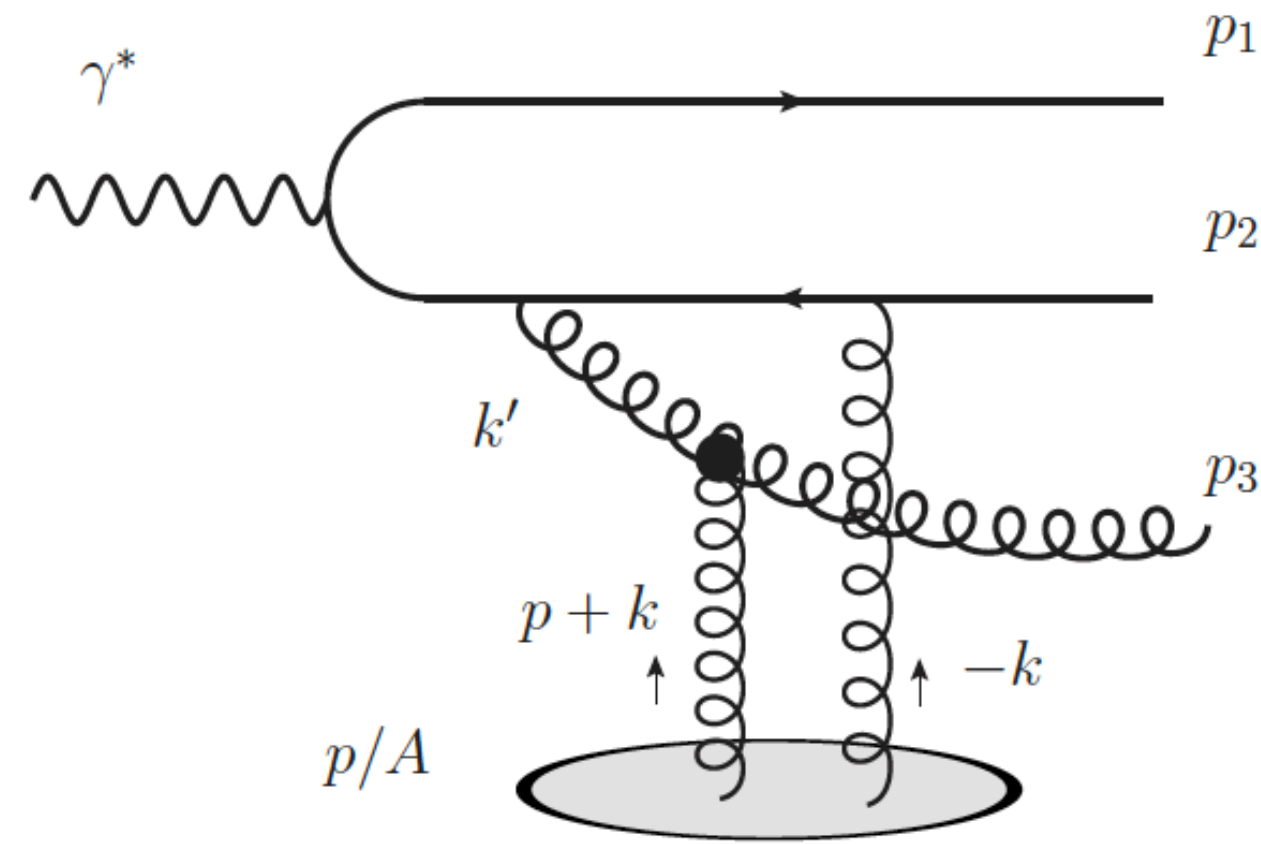
The correlation limit:  $|P_\perp| \gg |\Delta|$  with  $P_\perp = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$

To go beyond the correlation limit, simply specify  $|\mathbf{p}_1| \gg |\mathbf{p}_2|$ . Do not integrate over  $|\mathbf{p}_1|, |\mathbf{p}_2|$ .

$$C(|\mathbf{p}_3|, |\Delta|, \theta; |\mathbf{p}_1|, |\mathbf{p}_2|) \equiv \frac{1}{\mathcal{N}} \int d\beta_3 \frac{d^3N}{d^3p_1 d^3p_2 d^3p_3},$$

$$\cos \theta = \frac{\mathbf{p}_3 \cdot \Delta}{|\mathbf{p}_3| |\Delta|} \quad \beta_3 : \text{azimuthal angle of } \mathbf{p}_3.$$

# Numerical Results: Zero-Angle Peak



**Input parameters:**

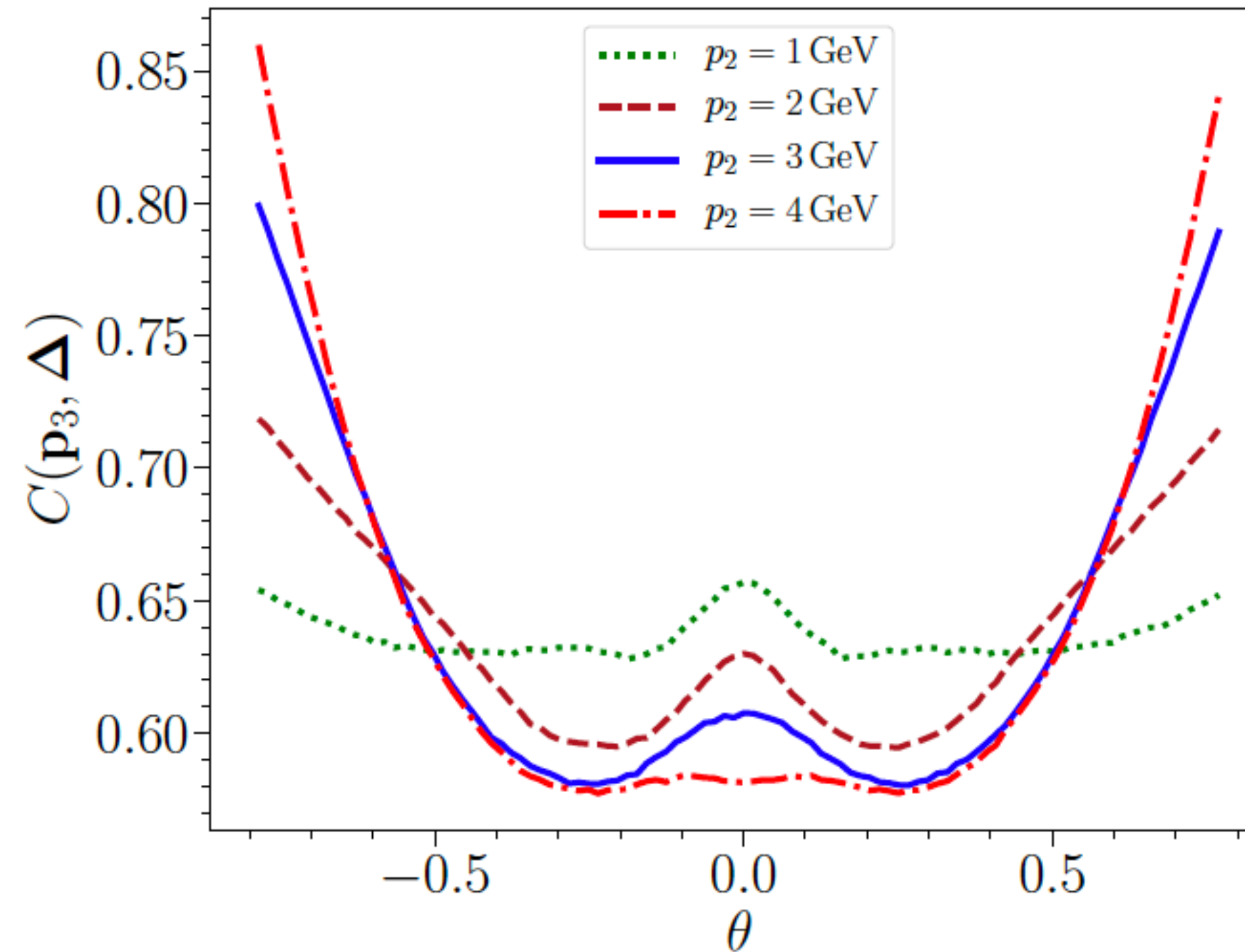
$|\mathbf{p}_3| = |\Delta| = 10 \text{ GeV}; |\mathbf{p}_1| = 10 \text{ GeV};$

**Photon virtuality:**  $Q = 1 \text{ GeV};$

**Gluon saturation scale:**  $Q_s = 2 \text{ GeV}$

**Azimuthal Angle Range:**  $-\pi/4 < \theta < \pi/4$

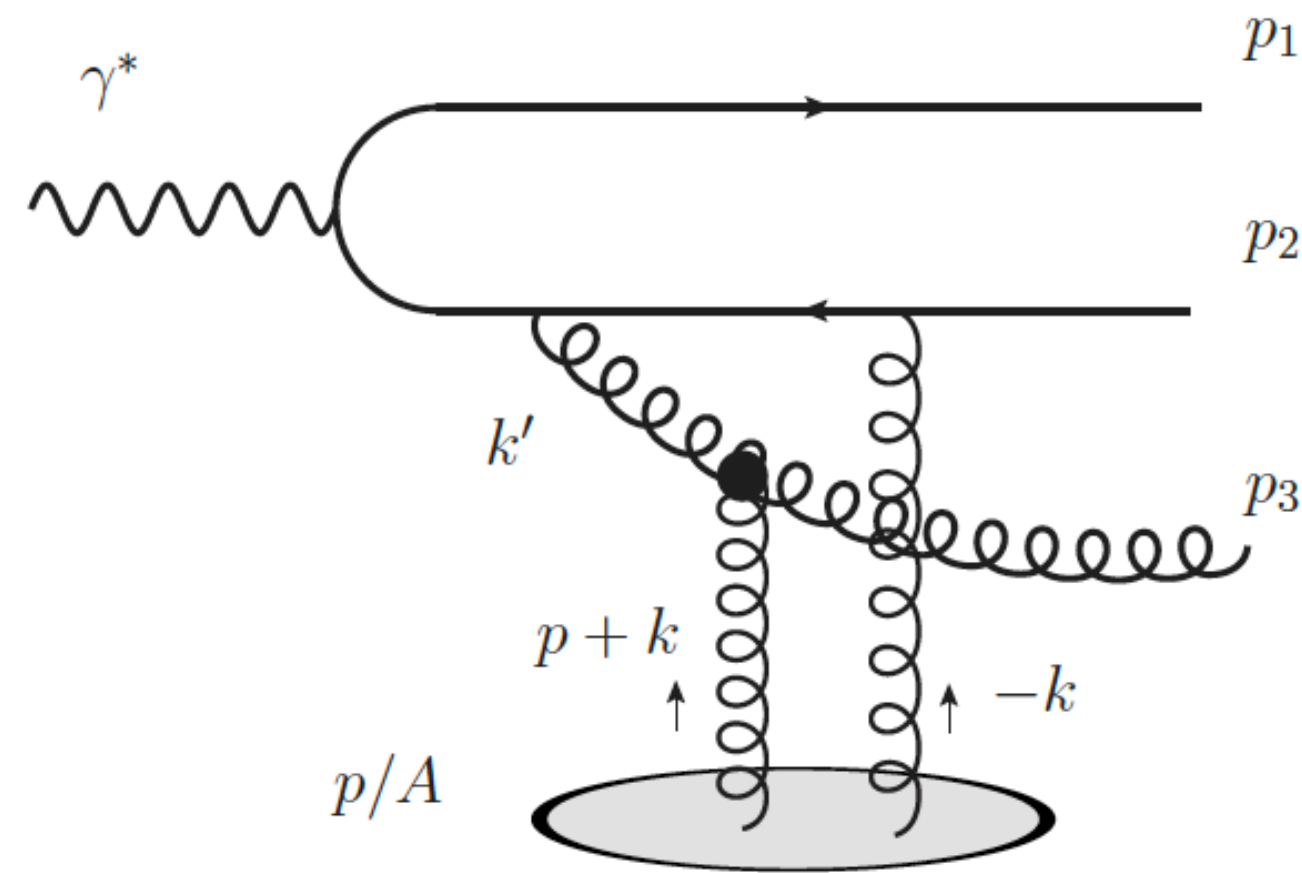
The zero-angle peak at  $\mathbf{p}_3 = \Delta$  is the salient feature of Bose correlation.



*One can vary values of  $|\mathbf{p}_3| = |\Delta|$  from 5 GeV – 15 GeV, as well of varying the corresponding values of  $|\mathbf{p}_1|, |\mathbf{p}_2|$  as long as  $|\mathbf{p}_1| \gg |\mathbf{p}_2|$ , the zero-angle peaks persist.*



# Zero-Angle Peak Comes from Gluon Bose Correlation



**Input parameters:**

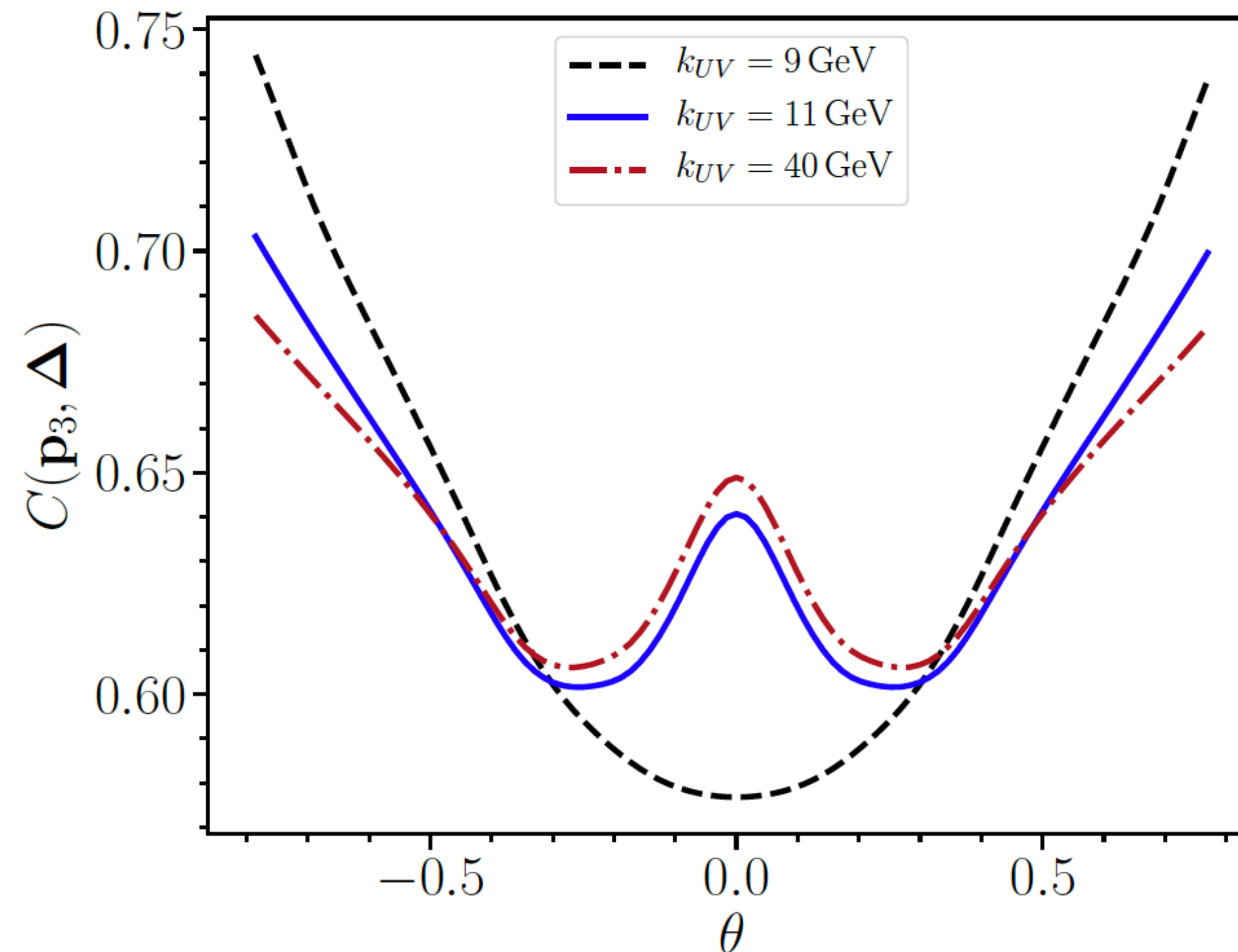
$|\mathbf{p}_3| = |\mathbf{\Delta}| = 10 \text{ GeV}; |\mathbf{p}_1| = 10 \text{ GeV};$

**Photon virtuality:**  $Q = 1 \text{ GeV};$

**Gluon saturation scale:**  $Q_s = 2 \text{ GeV}$

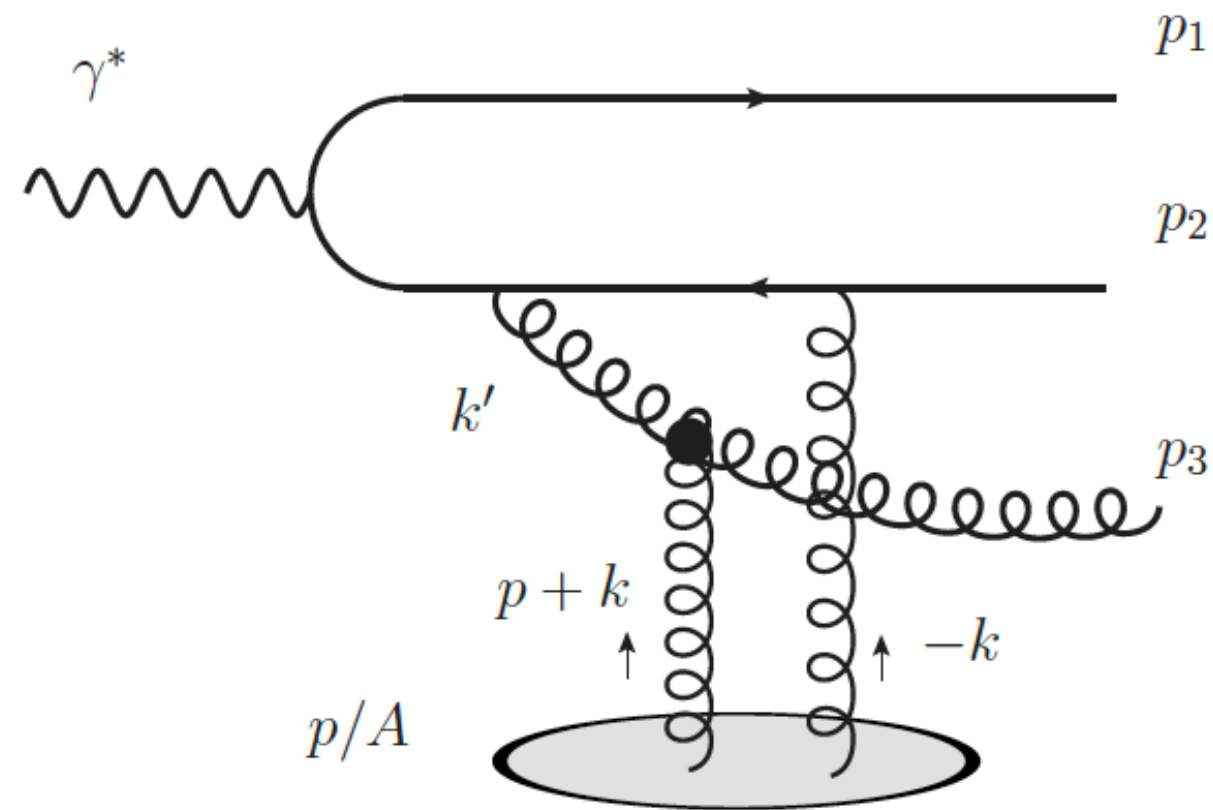
**Azimuthal Angle Range:**  $-\pi/4 < \theta < \pi/4$

The zero-angle peak mainly comes from the momentum region  $|\mathbf{k}| \sim |\mathbf{\Delta}|$ ,  
In this situation,  $-\mathbf{k} \approx \mathbf{\Delta}$ ,  $\mathbf{p} + \mathbf{k} \approx \mathbf{p}_3$ , corresponding to  
the Bose correlation of the two gluons from the target.



Note that the other possibility  $\mathbf{k} \approx \mathbf{\Delta}$  is extremely unlikely as then  $|\mathbf{k}'| \approx 2|\mathbf{\Delta}| \gg Q$ ,  
a quark (or antiquark) with momentum of magnitude  $Q$  can not radiate  
a gluon whose momentum is  $\mathbf{k}'$ .

# Numerical Results: Dependence on Gluon Saturation Scale



**Input parameters:**

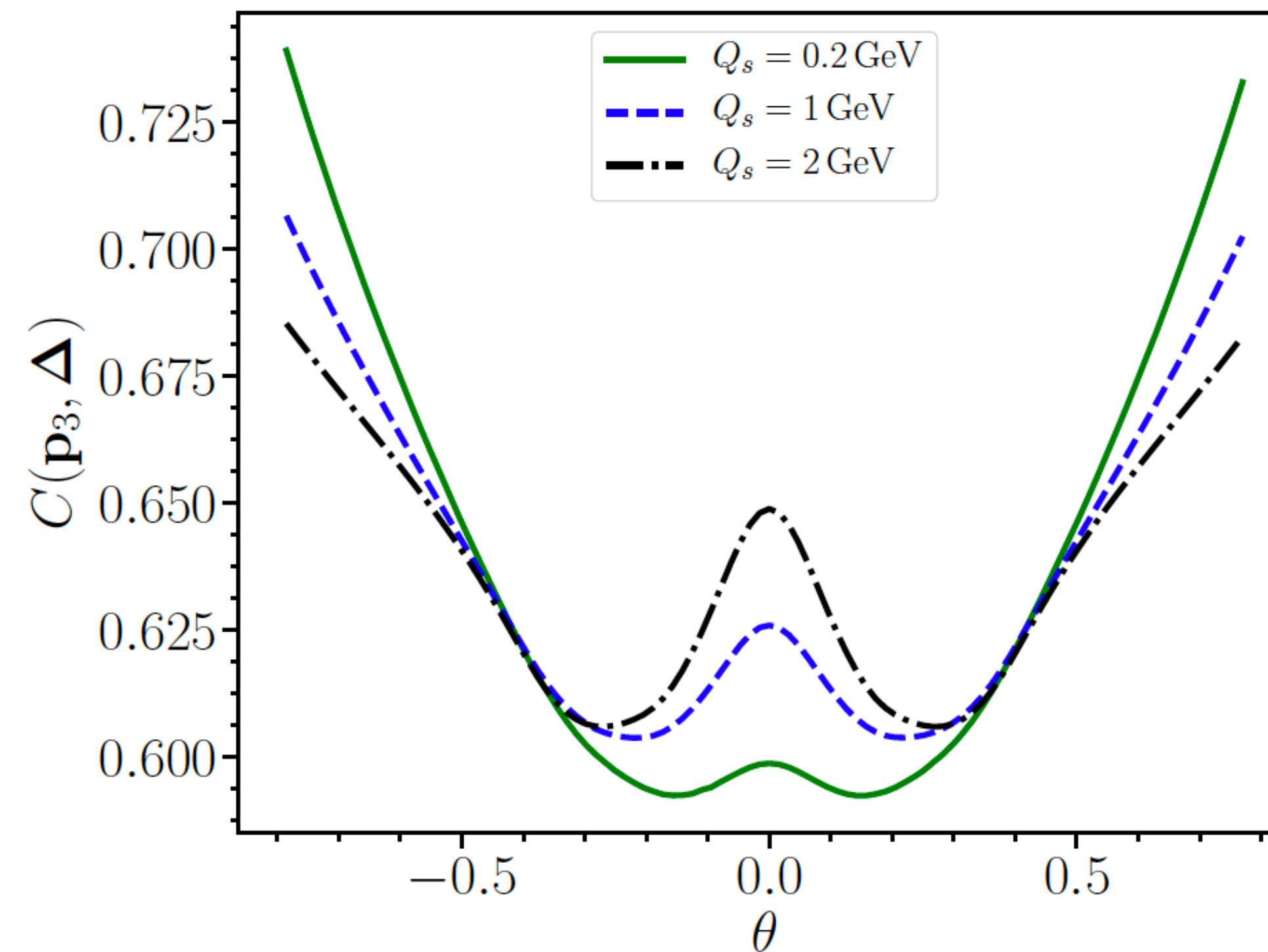
$|\mathbf{p}_3| = |\mathbf{\Delta}| = 10 \text{ GeV}; |\mathbf{p}_1| = 10 \text{ GeV};$

**Photon virtuality:**  $Q = 1 \text{ GeV};$

**Gluon saturation scale:**  $Q_s = 2 \text{ GeV}$

**Azimuthal Angle Range:**  $-\pi/4 < \theta < \pi/4$

Bose enhancement peaks become more prominent with increasing gluon saturation scale.



$$\langle \rho^a(\mathbf{k}) \rho^b(\mathbf{l}) \rangle = \delta^{ab} g^2 \mu^2 \frac{k^2}{k^2 + Q_s^2} \delta^{(2)}(\mathbf{k} + \mathbf{l})$$

E. Iancu, K. Itakura and L. McLerran, NPA (2003)

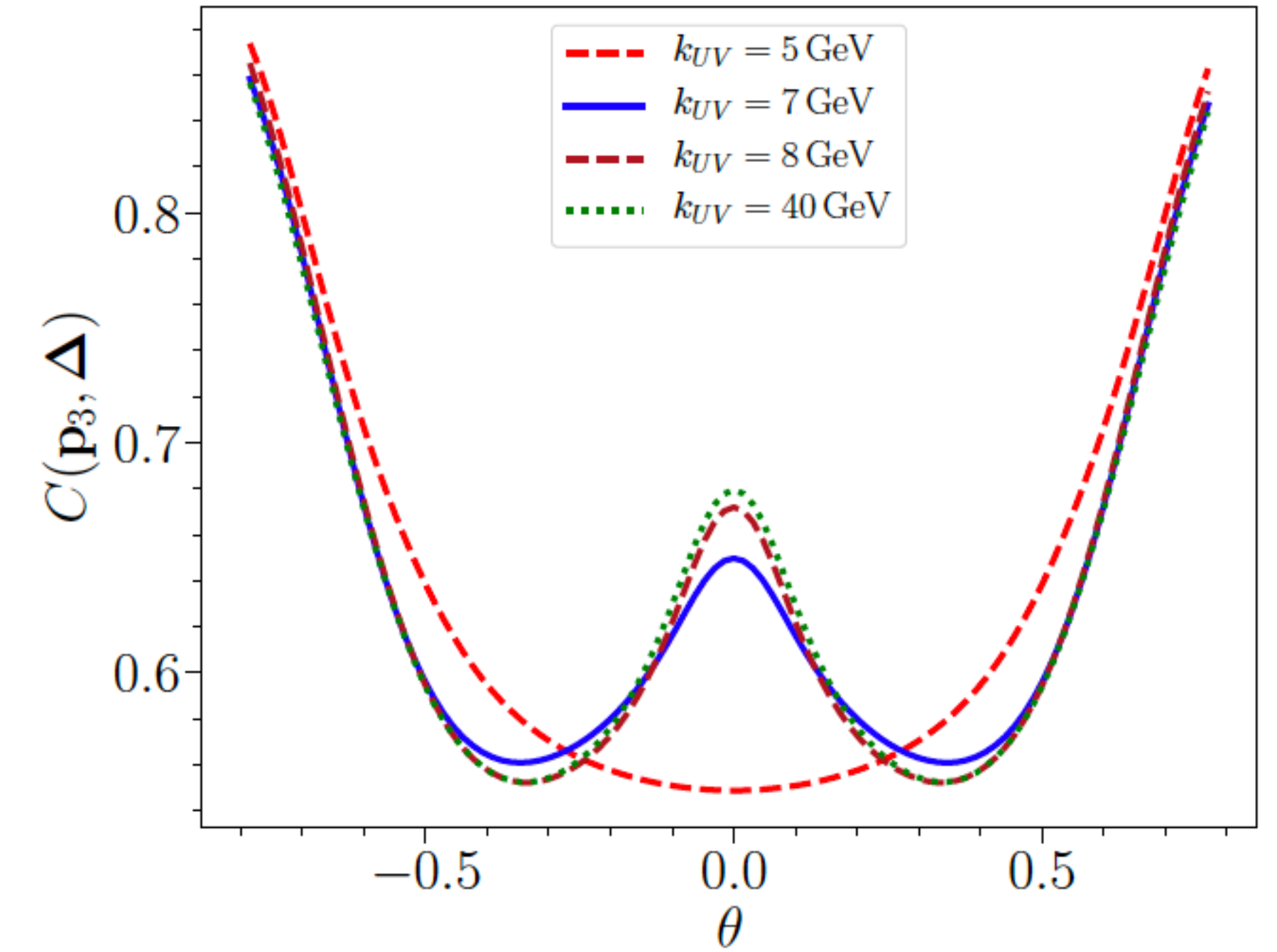
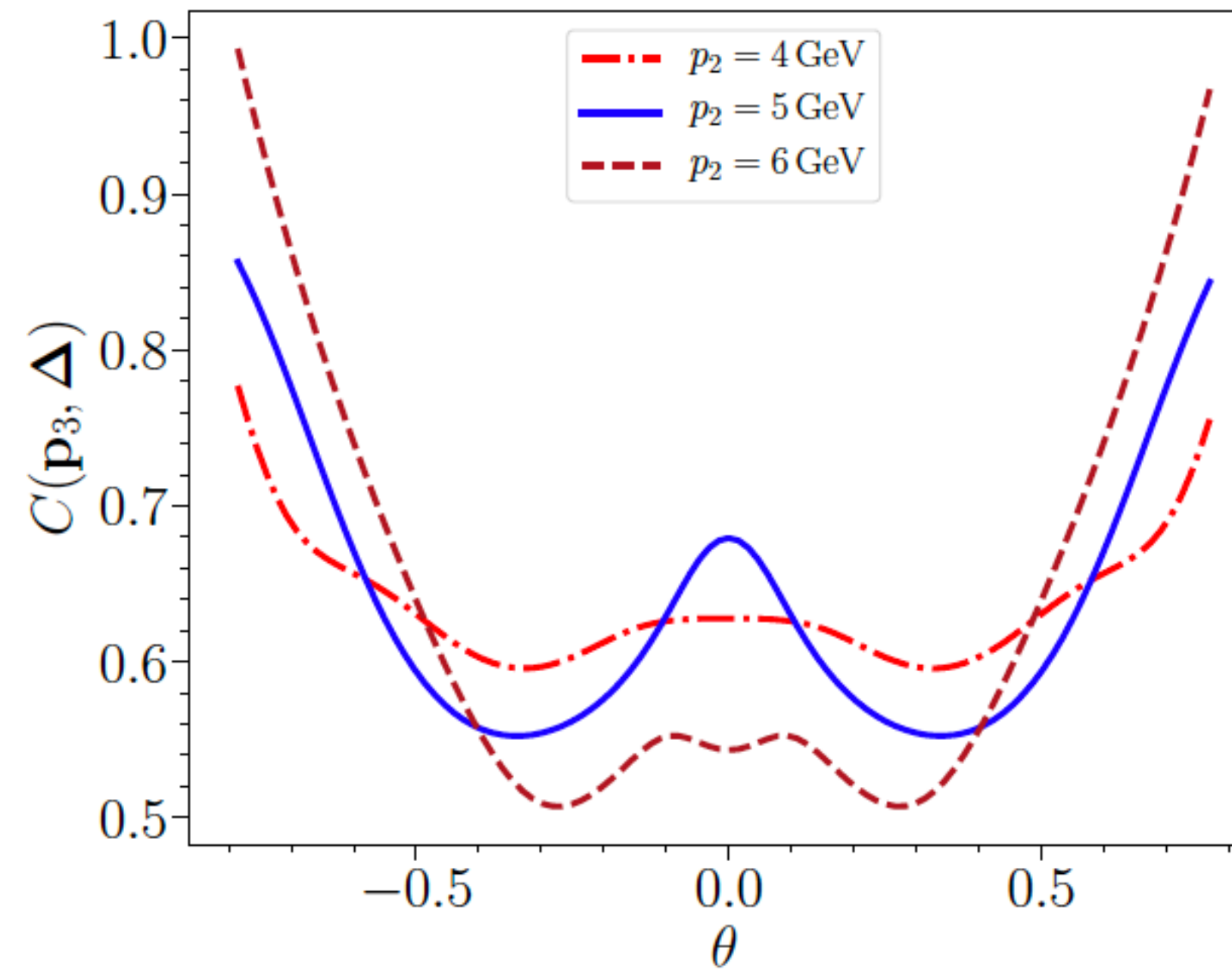
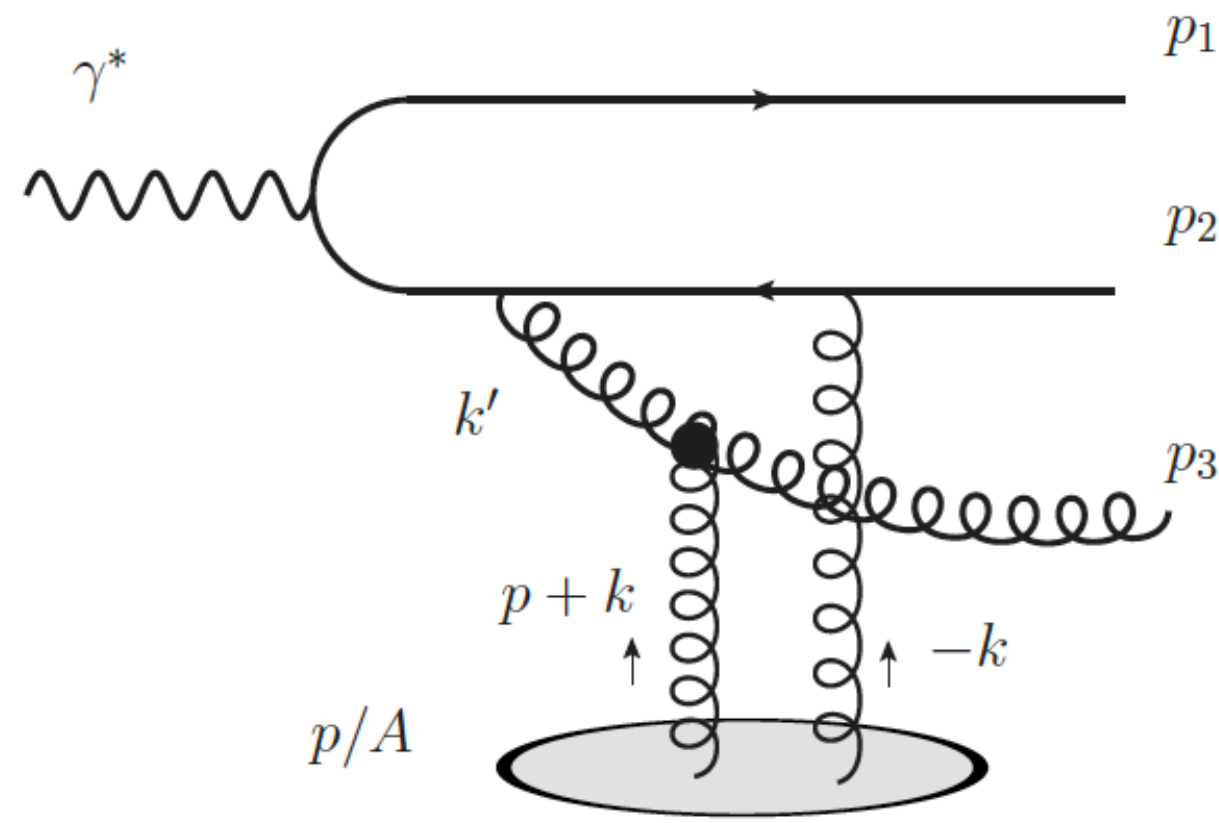


# Summary and Outlook

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- We proposed and demonstrated that diffractive quark antiquark dijet + gluon jet production in DIS has near-side ridge correlation which originates from the Bose correlations in the nuclear wave function.
- The correlation signals are prominent for events outside the conventional correlation limit.
- The correlation signals are more prominent for nuclear target that has larger saturation scale.
- The numerical computations are done in the two-gluon exchange approximation (dilute limit). It would be very interesting, although challenging, to analyze the general case of dense nuclear target.

# Backup



**Input parameters:**

$|\mathbf{p}_3| = |\Delta| = 6 \text{ GeV}; |\mathbf{p}_1| = 8 \text{ GeV};$

**Photon virtuality:**  $Q = 1 \text{ GeV};$

**Gluon saturation scale:**  $Q_s = 2 \text{ GeV}$

**Azimuthal Angle Range:**  $-\pi/4 < \theta < \pi/4$