Unifying approaches: Balitsky hierarchy from the Lipatov effective action

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Based on:
S. Bondarenko, M. Zubkov,
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Lipatov's effective action setup

- Consider generating functional with Lipatov's effective action written in the form of interacting Wilson lines:

$$Z = \frac{1}{Z'} \int \mathcal{D}v \exp\left(\imath S^0[v] - \frac{\imath}{2\,C(R)} \int d^4x\, \mathcal{T}_+ \partial_\perp^2 \mathcal{T}_- \right)$$

here S^0 is the QCD gluon's action on which the averaging is performed, $C(R)$ is an eigenvalue of the Casimir operator for the corresponding gluons representation and in the simplest case (eikonal approximation)

$$\mathcal{T}_\pm(v_\pm) = \frac{1}{g}\partial_\pm O(v_\pm) = v_\pm O(v_\pm)\,,\; O(v_\pm) = P\,e^{g\int_{-\infty}^{x^\pm} dx^\pm\, v_\pm(x^\pm,\, x_\perp)}\,,\quad v_\pm = \imath T^a v_\pm^a$$

Introducing auxiliary currents and applying the light-cone gauge, we obtain:

$$Z[J] = \frac{1}{Z'} \int Dv \exp\left(\imath S^0[v] - \frac{\imath}{2\,C(R)} \int d^4x \left(\mathcal{T}_+ - J_+(x^+, x_\perp)\,\partial_\perp^{-2}\right) J_-(x^-, x_\perp)\right)$$

For the shock wave approximation we define also:

$$\int dx^\pm\, J_\pm(x^\pm, x_\perp) \rightarrow \int dx^\pm\, \delta(x^\pm) J_\pm\,(x_\perp)$$

Correlators of Wilson lines

- The construction above allows to define and calculate the correlators of Wilson lines, taking two derivatives of $\log Z[J]$ with respect to the currents we have:

$$-\,C(R)^2\,(2\,g)^2\left(\frac{\delta^2}{\delta J_{1\,-}^{a_1}\;\delta J_{2\,-}^{a_2}}\;\log Z[J]\right)_{J=0}=\int dx^-\,\delta(x^-)\int dy^-\,\delta(y^-)$$

$$<T^{a_1}\left(O_1(v_+)_{x^+=\infty}-O_1(v_+)_{x^+=-\infty}\right)\otimes T^{a_2}\left(O_2(v_+)_{x^+=\infty}-O_2(v_+)_{x^+=-\infty}\right)>=$$

$$=\int dx^-\,\delta(x^-)\int dy^-\,\delta(y^-)\;<\;(T^{a_1}\,V_1(v_+))\;\otimes\;(T^{a_2}\,V_2(v_+))\;>$$

The correlator is defined through the usual Wilson lines:

$$V(v_\pm)\;=\;P\,e^{g\,\int_{-\infty}^{\infty}\,dx^\pm\,v_\pm(x^\pm,\,x^\mp,\,x_\perp)}\;-\;1\,,\quad v_\pm\;=\;\imath\,T^a\,v_\pm^a$$

As usual, the gluon field is taken as background field (not Reggeon) plus the quantum fluctuation around:

$$v_+\;=\;\mathcal{B}_+(x^+,x_\perp)\;+\;\varepsilon_+$$

Hierarchy of correlators of Wilson lines

- An expansion of the Wilson lines around the background field:

$$V(v_+) = V(\mathcal{B}_+(x^+, x_\perp)) + g \int_{-\infty}^{\infty} dx^+ \left(O^T(\mathcal{B}_+(x^+, x_\perp))\, \varepsilon_+(x)\, O(\mathcal{B}_+(x^+, x_\perp)) \right) +$$

$$+\frac{g^2}{2} \int_{-\infty}^{\infty} dx^+\, O^T(\mathcal{B}_+(x^+, x_\perp))\, \varepsilon_+(x) \int d^4p\, G^+\,(x,p)\, \varepsilon_+(p)\, O(\mathcal{B}_+(p^+, p_\perp)) +$$

$$+\frac{g^2}{2} \int_{-\infty}^{\infty} dx^+ \int d^4p\, O^T\,(\mathcal{B}_+(p^+, p_\perp))\, \varepsilon_+(p)\, G^+\,(p,x)\, \varepsilon_+(x)\, O(\mathcal{B}_+(x^+, x_\perp)) + \cdots$$

Here we defined:

$$G^+_{xy} = G^{+\,0}_{xy} + g\, G^{+\,0}_{xz}\, v_{+z}\, G^+_{zy}\,, \quad D_{+\,xy}\, G^+_{yz} = \delta^4_{xz}$$

$$G^{+\,0}_{xy} = \theta(x^+ - y^+)\, \delta^3_{xy}\,, \quad \partial_{+\,x}\, G^{+\,0}_{xy} = \delta^4_{xy}$$

$$O^T(v_\pm) = P\, e^{g \int_{x^\pm}^{\infty} dx^\pm\, v_\pm(x^\pm,\, x_\perp)}$$

with D as a covariant derivative operator.

Hierarchy of correlators of Wilson lines

- Finally we can define the correlator of interest:

$$< V(x) \otimes V(y) >= V_{ik}(x_\perp) V_{lj}(y_\perp) + g V_{ik}(x_\perp) \int_{-\infty}^{\infty} dy^+ \left(O_y^T (\imath T^c) O_y \right)_{lj} < \varepsilon_+^c(y) > +$$

$$+g \int_{-\infty}^{\infty} dx^+ \left(O_x^T (\imath T^c) O_x \right)_{ik} < \varepsilon_+^c(x) > V_{lj}(y_\perp) +$$

$$+\frac{g^2}{2} \int_{-\infty}^{\infty} dx^+ \int d^4p \left(O_x^T (\imath T^c) G^+(x,p) (\imath T^d) O_p \right)_{ik} < \varepsilon_+^c(x) \varepsilon_+^d(p) > V_{lj}(y_\perp) +$$

$$+\frac{g^2}{2} \int_{-\infty}^{\infty} dx^+ \int d^4p \left(O_p^T (\imath T^c) G^+(p,x) (\imath T^d) O_x \right)_{ik} < \varepsilon_+^c(p) \varepsilon_+^d(x) > V_{lj}(y_\perp) +$$

$$+\frac{g^2}{2} V_{ik}(x_\perp) \int_{-\infty}^{\infty} dy^+ \int d^4p \left(O_p^T (\imath T^c) G^+(p,y) (\imath T^d) O_y \right)_{lj} < \varepsilon_+^c(p) \varepsilon_+^d(y) > +$$

$$+\frac{g^2}{2} V_{ik}(x_\perp) \int_{-\infty}^{\infty} dy^+ \int d^4p \left(O_y^T (\imath T^c) G^+(y,p) (\imath T^d) O_p \right)_{lj} < \varepsilon_+^c(y) \varepsilon_+^d(p) > +$$

$$+g^2 \int_{-\infty}^{\infty} dx^+ \left(O_x^T (\imath T^c) O_x \right)_{ik} \int_{-\infty}^{\infty} dy^+ \left(O_y^T (\imath T^d) O_y \right)_{lj} < \varepsilon_+^c(x) \varepsilon_+^d(y) > + \cdots .$$

it is similar (almost) to the Balitsky hierarchy of the correlators.

Correlator (propagator) of gluon´s fluctuations

- An expansion of the gluon field around classical solution read as:

$$
\begin{aligned}
v_{+}^{a} &= v_{+}^{a\,cl} + \varepsilon_{+}^{a} = \mathcal{B}_{+}^{a}(x^{+}, x_{\perp}) + \varepsilon_{+}^{a} \\
v_{i}^{a} &= v_{i}^{a\,cl} + \varepsilon_{i}^{a} = tr\,[f^{\,a}\, O(x^{+}) f^{\,b}\, O^{T}(x^{+})]\, \rho_{\,bi}\left(x^{-}, x_{\perp}\right) + \varepsilon_{i}^{a}\,, \quad \rho_{i}^{b} = \left(\partial_{-}^{-1}\left(\partial_{i}\, \mathcal{B}_{-}^{\,b}\right)\right)
\end{aligned}
$$

- QCD Lagrangian with respect to the fluctuations, we obtain the following expression:

$$
\begin{aligned}
S_{\varepsilon^2} &= -\frac{1}{2}\int d^4x\, \Big(\varepsilon_{i}^{a}\,(\delta_{ac}\,(\delta_{ij}\,\Box + \partial_i\,\partial_j) - \\
&- \quad 2g f_{abc}\left(\delta_{ij}\, v_{k}^{b\,cl}\partial_k - 2\, v_{j}^{b\,cl}\partial_i + v_{i}^{b\,cl}\partial_j - \delta_{ij}\, v_{+}^{b\,cl}\partial_{-}\right) - \\
&- \quad g^2\, f_{abc_1}\, f_{c_1 b_1 c}\left(\delta_{ij}\, v_{k}^{b\,cl}\, v_{k}^{b_1\,cl} - v_{i}^{b_1\,cl}\, v_{j}^{b\,cl}\right)\Big)\, \varepsilon_{j}^{c} + \\
&+ \quad \varepsilon_{+}^{a}\left(-2\,\delta^{ac}\,\partial_{-}\partial_i - 2g f_{abc}\left(v_{i}^{b\,cl}\partial_{-} - \left(\partial_{-} v_{i}^{b\,cl}\right)\right)\right)\varepsilon_{i}^{c} + \varepsilon_{+}^{a}\,\delta_{ac}\,\partial_{-}^{2}\,\varepsilon_{+}^{c}\,\Big) = \\
&= \quad -\frac{1}{2}\,\varepsilon_{\mu}^{a}\left((M_0)_{\mu\,\nu}^{ac} + (M_1)_{\mu\,\nu}^{ac} + (M_2)_{\mu\,\nu}^{ac}\right)\varepsilon_{\nu}^{c}
\end{aligned}
$$

Correlator (propagator) of gluon´s fluctuations

- The full gluon propagator:

$$G^{ac}_{\mu\nu} = \left[(M_0)^{ac}_{\mu\nu} + (M_1)^{ac}_{\mu\nu} + (M_2)^{ac}_{\mu\nu} \right]^{-1}$$

or

$$G^{ac}_{\mu\nu}(x,y) = G^{ac}_{0\,\mu\nu}(x,y) - \int d^4z\, G^{ab}_{0\,\mu\rho}(x,z) \left((M_1(z))^{bd}_{\rho\gamma} + (M_2(z))^{bd}_{\rho\gamma} \right) G^{dc}_{\gamma\mu}(z,y)$$

For our purposes we need:

$$\begin{aligned} G^{ac}_{++}(x,y) &= G^{ac}_{0\,++}(x,y) - \int d^4z\, G^{ab}_{0\,+i}(x,z)\ (M_1(z))^{bd}_{ij}\ G^{dc}_{j+}(z,y) \\ G^{ac}_{j+}(x,y) &= G^{ac}_{0\,j+}(x,y) - \int d^4z\, G^{ab}_{0\,ji}(x,z)\ (M_1(z))^{bd}_{ik}\ G^{dc}_{k+}(z,y) \end{aligned}$$

Correlator (propagator) of gluon´s fluctuations

- We reproduce the Balitsky-Belitsky result (I. I. Balitsky and A. V. Belitsky, Nucl. Phys. B **629** (2002)) performing the full re-summation over transverse vertex $M_{1\,ij}$:

$$G^{ac}_{++}(x,y) = G^{ac}_{0\,++}(x,y) - 4\pi\,\imath \int \frac{d^4p}{(2\pi)^4}\, e^{-\,\imath\, p\, x} \int \frac{d^4k}{(2\pi)^4} e^{\,\imath\, k\, y}\, \frac{1}{p^2\, k^2}\, \frac{p_i k_i}{p_-}$$
$$\delta(p_- - k_-) \int d^2 z_\perp\, e^{\imath\, z^i\,(p_i - k_i)} \Big(\theta(p_-)\, V^{ac}(\mathsf{v}^{cl}_+) - \theta(-p_-)\, V^{ac}(\mathsf{v}^{cl}_+) \Big)$$

with

$$V^{ab}_{\pm}(\mathsf{v}_\pm) = \Big(P\, e^{g\, \int_{-\infty}^{\infty}\, dx^{\pm}\, \mathsf{v}_{\pm}(x^{\pm},\, x^{\mp},\, x_\perp)} - 1 \Big)^{ab}$$

for the case of adjoint representation of the gluon field. The second part of the propagator is called "shock wave" propagator, usually the Wilson line correlaros are accounted in respect to the interactions provided by this part of the full G_{++}.

Balitsky hierarchy (BK equation)

- In order to obtain now the familiar form of Balitsky hierarchy expression we apply the shock wave approximation and using

$$< \varepsilon_{\mu}(x)\, \varepsilon_{\nu}(y) >= -\imath\, G_{\mu\,\nu}(x,y)$$

we obtain after some calculations

$$\begin{aligned}
&< V(x) \otimes V(y) >= V_{ik}(x_\perp)\, V_{lj}(y_\perp) \; - \\
- \quad & \frac{\alpha_s}{\pi^2} \left(((\imath T^c) U(x))_{ik} \left(U(y)(\imath T^d) \right)_{lj} + (U(x)(\imath T^c))_{ik} \left((\imath T^d) U(y) \right)_{lj} \right) \int \frac{dp_-}{p_-} \\
& \int d^2 z_\perp \, \frac{(x_i - z_i)\,(y_i - z_i)}{(z_i - x_i)^2\,(y_i - z_i)^2}\, U^{cd}(z) \; + \\
+ \quad & \frac{\alpha_s}{\pi^2} \left((\imath\, T^c) U(\mathcal{B}_+)(\imath\, T^d) \right)_{ik} V_{lj}(y_\perp) \int \frac{dp_-}{p_-} \int \frac{d^2 z_\perp}{(z_i - x_i)^2}\, U^{cd}(B_+) + \\
+ \quad & \frac{\alpha_s}{\pi^2} V_{ik}(x_\perp) \left((\imath\, T^d) U(\mathcal{B}_+)(\imath\, T^c) \right)_{lj} \int \frac{dp_-}{p_-} \int \frac{d^2 z_\perp}{(z_i - y_i)^2}\, U^{cd}(\mathcal{B}_+)
\end{aligned}$$

the expression is LO Balitsky's hierarchy for the correlators of Wilson lines in the background field with

$$U \; = \; P\, e^{g \int_{-\infty}^{\infty} dx^+ \, \mathrm{v}_+(x^+,\, x_\perp)}$$

Where the formalism can be used

- In an exploration of the BK equation structure, including any kind of corrections to LO BK and in an clarification (calculation) of the NNLO (three loops) BFKL kernel. The Balitsky hierarchy reproduces the BFKL equation:
  1. Tree re-summated propagator $\rightarrow$ LO (one loop) BFKL kernel);
  2. One loop propagator $\rightarrow$ NLO (two loops) BFKL kernel (mostly important contrinutions);
  3. Two loops propagator $\rightarrow$ NNLO (three loops) BFKL kernel (mostly important contrinutions)?;
- Ways of calculations of the two loops propagator:
  1. The first (hard one): in the framework of QFT calculate the propagator as is it.
  2. The second (can be much easier): the propagator of gluon's fluctuations is related to the propagator of the reggeized gluons. If it can be clarified how, the answer for the two loops G_{++} can be extracted from the NLO propagator of the reggeized gluons. In some extend, Balitsky hierarchy generates the next order kernel on the base of the lower order propagator, which in turn, can be related to the lower order kernel of the color propagator.

What do we calculate or about non-eikonal corrections

- In the action the Wilson line appears:

$$V(v_{\pm}) \,=\, P\, e^{g\, \int_{-\infty}^{\infty} dx^{\pm}\, v_{\pm}(x^{\pm},\, x^{\mp},\, x_{\perp})} \,-\, 1$$

Where it comes from? Consider the following S-matrix element for the general case of quark's propagation in an external field:

$$S_{fi} \,=\, -\int d^4x_i\, d^4x_f\, J_f(x_f)\, S(x_f,\, x_i)\, J_i(x_i)\,,$$

with the processes of creation and absorption of the quark in y and x correspondingly and $S(x,y)$ as quark's propagator in the external field. For example, for the quark asymptotically free at $x, y \rightarrow \pm\infty$ we have:

$$J_f(x_f) \,=\, \overline{u}(p)\, e^{\imath\, p\, x_f} \left(\imath\hat{\partial} \,-\, m\right)_{x_f}\,,\quad J_i(x_i) \,=\, \left(\imath\, \overleftarrow{\hat{\partial}} \,+\, m\right)_{x_i} e^{-\imath\, q\, x_i}\, u(q)\,.$$

Integration by parts provides in turn:

$$\begin{aligned} S_{fi} &= \overline{u}(p)\, (\hat{p} \,-\, m\,) \int d^4x_f\, d^4x_i\, e^{\imath\, p\, x_f}\, S(x_f,\, x_i)\, e^{-\imath\, q\, x_i}\, (\hat{q} \,-\, m\,)\, u(q) \,= \\ &= \overline{u}(p)\, (\hat{p} \,-\, m\,)\, S(p,\, q)\, (\hat{q} \,-\, m\,)\, u(q) \end{aligned}$$

What do we calculate or about non-eikonal corrections

- Make the calculations easier-use the two component spinors:

$$u(q) = \frac{1}{m}(\hat{q} + m)\, u(q)_L = \sqrt{m} \begin{pmatrix} 1 \\ \frac{q^0 + \sigma^i q_i}{m} \end{pmatrix} \Psi_L(q)$$

$$u(q)_L = \frac{1}{2}(1 - \gamma^5)\, u(q) = \frac{\sqrt{m}}{2}(1 - \gamma^5) \begin{pmatrix} \Psi_L(q) \\ \Psi_R(q) \end{pmatrix} = \sqrt{m} \begin{pmatrix} \Psi_L(q) \\ 0 \end{pmatrix}$$

with the chiral basis for the gamma matrices used:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \sigma^\mu = (1, \sigma^i), \quad \bar{\sigma}^\mu = (1, -\sigma^i), \quad \gamma^5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}.$$

Similarly a right-handed spinor can be introduced:

$$u(q)_R = \frac{1}{2}(1 + \gamma^5)\, u(q) = \frac{\sqrt{m}}{2}(1 + \gamma^5) \begin{pmatrix} \Psi_L(q) \\ \Psi_R(q) \end{pmatrix} = \sqrt{m} \begin{pmatrix} 0 \\ \Psi_R(q) \end{pmatrix}$$

What do we calculate or about non-eikonal corrections

- Write the quarks propagator in the two component form as well:

$$S(p,\, q) \,=\, \left(\begin{array}{cc} G_L(p,q) & G_1(p,q) \\ G_2(p,q) & G_R(p,q) \end{array} \right) \,,$$

for the case of the propagation of a quark with chirality preservation the only G_L or G_R component will be requested, the change of chirality is provided by matrix element of the $\overline{\Psi}_{R,L}\,\cdots\,\Psi_{L,R}$ with more complicated propagator. The T-matrix element correspondingly:

$$T_{fi}\,=\,\frac{1}{m}\,\overline{\Psi}_R(p)\,(p^2\,-\,m^2)\,\left(G_R(p,q)\,-\,G_{R\,0}(p,q)\,\right)\,(q^2\,-m^2)\,\Psi_R(q)\,.$$

or when the quark is created at some y and asymptotically free at x:

$$T_{fi}\,=\,-\,\frac{1}{m}\,\overline{\Psi}_R(p)\,(p^2\,-\,m^2)\,\int\,d^4q\,\left(G_R(p,q)\,-\,G_{R\,0}(p,q)\,\right)\,\tilde{J}_i(q)$$

What do we calculate or about non-eikonal corrections

- The expression for $G_R(x, y)$ component of $S(x,\, y)$ can be found from

$$\left(\imath\, \hat{D} \,+\, \hat{\mathsf{v}} \,-\, m\right) \begin{pmatrix} G_L & G_1 \\ G_2 & G_R \end{pmatrix} \,=\, I_4\, \delta^4_{xy}\,,$$

with v as an external gluon field. Resolving the equation we obtain:

$$\left((\partial_\mu \,-\, \mathsf{v}_\mu)^2 \,+\, \frac{1}{2}\,\sigma^{\mu\nu}\, F_{\mu\nu} \,+\, m^2\right) G_R \,=\, -\,\delta^2$$

with

$$\sigma^{\mu\nu} \,=\, \frac{1}{2}\, \left(\sigma^\mu\, \bar{\sigma}^\nu \,-\, \sigma^\nu\, \bar{\sigma}^\mu\right)$$

and with final standard expression

$$G_R(x_f, x_i) \,=$$

$$=\; -\,\frac{\imath}{2} \int_{T_0}^{\infty} dT \int_{x_i = z(T_0)}^{x_f = z(T)} \mathcal{D}x\; PExp\left[\imath \int_{T_0}^{T} dt\, \left(\frac{\dot{x}^2}{2} \,-\, \frac{m^2}{2} \,+\, \dot{x}_\mu\, \mathsf{v}^\mu \,+\, \frac{1}{4}\,\sigma^{\mu\nu}\, F_{\mu\nu}\right)\right]$$

What do we calculate or about non-eikonal corrections

- Now, assume straight line (high energy) of the trajectory

$$x \,=\, x_{straight} \,+\, \xi(t) \,=\, C \,+\, p\, t \,+\, \xi\,, \;\; \dot{C} \,=\, 0$$

and extracting the bare propagator from the expression we obtain:

$$\hat{G}_R(x_f, x_i) \,=\, -\,\frac{\imath}{2} \int_{T_0}^{\infty} dT\, e^{\frac{\imath}{2}(p^2 - m^2)\,(T - T_0)} \int \mathcal{D}\xi\, e^{\frac{\imath}{2}\,\int_{T_0}^{T}\, dt\, \dot{\xi}^2}$$

$$\left(PExp\left[\imath \int_{T_0}^{T} dt\, \left(\left(p^{\mu} + \dot{\xi}^{\mu} \right) \mathsf{v}_{\mu}(x) \,+\, \frac{1}{4}\, \sigma^{\mu\nu}\, F_{\mu\nu}(x) \right) \right] \,-\, 1 \right) \,=$$

$$= \quad -\,\frac{\imath}{2} \int_{T_0}^{\infty} dT\, e^{\frac{\imath}{2}(p^2 - m^2)\,(T - T_0)} \int \mathcal{D}\xi\, e^{\frac{\imath}{2}\,\int_{T_0}^{T}\, dt\, \dot{\xi}^2}$$

$$\left(PExp\left[\imath \int_{x(T_0) = x_i}^{x(T) = x_f} dx^{\mu}\, \mathsf{v}_{\mu}(x) \,+\, \imath \int_{T_0}^{T} dt\, \left(\dot{\xi}^{\mu}\, \mathsf{v}_{\mu}(x) \,+\, \frac{1}{4}\, \sigma^{\mu\nu}\, F_{\mu\nu}(x) \right) \right] \,-\, 1 \right)$$

What do we calculate or about non-eikonal corrections

- For the T matrix element of interest we need isolate one pole (cut one free propagator fro the general expression):

$$\hat{T}_{fi} = -\,(p^2 - m^2)\,(p^2 - m^2)^{-1} \int_{T_0}^{\infty} dT \left(\frac{d}{dT}\, e^{\frac{\imath}{2}(p^2-m^2)\,(T-T_0)} \right)$$

$$\int \mathcal{D}\xi\, e^{-\frac{1}{2}\,\varepsilon\,(T-T_0) + \frac{\imath}{2}\,\int_{T_0}^{T}\, dt\,\dot{\xi}^2} \left(PExp\left[\imath \int_{T_0}^{T} dt \left(\left(p^{\mu} + \dot{\xi}^{\mu}\right) \mathsf{v}_{\mu} + \frac{1}{4}\,\sigma^{\mu\nu}\, F_{\mu\nu}\right)\right] - 1\right) =$$

$$= -\int_{T_0}^{\infty} dT \left(\frac{d}{dT} e^{\frac{\imath}{2}(p^2-m^2)\,(T-T_0)} \right) e^{-\frac{1}{2}\,\varepsilon\,(T-T_0)}\, f(T_0,\,T) =$$

$$= -\left(- f(T_0,\,T_0) - \int_{T_0}^{\infty} dT\, e^{\frac{\imath}{2}(p^2-m^2)\,(T-T_0)}\, \frac{df(T,\,T_0)}{dT} \right) .$$

taking $p^2 \;\rightarrow\; m^2$ limit in the exponential in the integral obtain finally

$$\hat{T}_{fi} \;=\; f(\infty,\,T_0)$$

see details also in V. N. Pervushin, Teor. Mat. Fiz. **4** (1970), 22-32; B. M. Barbashov and V. V. Nesterenko, Teor. Mat. Fiz. **10** (1972), 196-203; B. M. Barbashov and V. V. Nesterenko, Teor. Mat. Fiz. **14** (1973), 27-35. E. Laenen, G. Stavenga and C. D. White, JHEP **03** (2009), 054.

What do we calculate or about non-eikonal corrections

- Here

$$f(\infty,T_0) = \int \mathcal{D}\xi e^{\frac{\imath}{2}\int_{T_0}^{\infty} dt\dot{\xi}^2}\left(PExp\left[\imath\int_{T_0}^{\infty} dt\left(\left(p^\mu+\dot{\xi}^\mu\right)\mathsf{v}_\mu+\frac{1}{4}\sigma^{\mu\nu}F_{\mu\nu}\right)\right]-1\right) =$$

$$= \int \mathcal{D}\xi e^{\frac{\imath}{2}\int_{T_0}^{\infty} dt\dot{\xi}^2}\left(PExp\left[\imath\int_{x_i}^{x_f} dx^\mu \mathsf{v}_\mu(x)+\imath\int_{T_0}^{\infty} dt\left(\dot{\xi}^\mu\mathsf{v}_\mu(x)+\frac{1}{4}\sigma^{\mu\nu}F_{\mu\nu}(x)\right)\right]-1\right)$$

is a "phase"operator which includes all kind of the non-eikonal corrections. Going beyond the eikonal approximation, we have instead usual Wilson line:

$$V(\mathsf{v}) = f(\infty,-\infty) = \mathcal{N}^{-1}$$

$$\int \mathcal{D}\xi e^{\frac{\imath}{2}\int_{-\infty}^{\infty} dt\dot{\xi}^2}\left(PExp\left[\imath\int_{x_i(-\infty)}^{x_f(\infty)} dx^\mu \mathsf{v}_\mu(x)+\imath\int_{-\infty}^{\infty} dt\left(\dot{\xi}^\mu\mathsf{v}_\mu(x)+\frac{1}{4}\sigma^{\mu\nu}F_{\mu\nu}(x)\right)\right]\right.$$

with

$$\mathcal{N}(\infty,-\infty) = \int \mathcal{D}\xi\, Exp\left[\frac{\imath}{2}\int_{-\infty}^{\infty} dt\,\dot{\xi}^2\right]$$

What do we calculate or about non-eikonal corrections

- In the case of Regge kinematics:

$$x \,=\, \sqrt{\frac{s}{2}}\, n_{LC}\, t \,+\, \xi(\lambda) \,=\, \frac{\lambda}{\sqrt{2}}\, n_{LC} \,+\, \xi(\lambda)\,,\, \xi(\infty) \,=\, \xi(-\infty) \,=\, 0\,,\;\; n^{\mu}_{LC} = (1,0,0_{\perp})$$

$$V(v) \,=\, P\left[\left(e^{g\,\int_{-\infty}^{\infty}\, dx^{\mu}\, v_{\mu} \,+\, \frac{g}{2\sqrt{s}}\,\int_{-\infty}^{\infty}\, d\lambda\, \sigma^{\mu\,\nu}\, F_{\mu\,\nu}}\right)_{\xi\,=\,0} S(\xi) \,-\, 1\right]\,,$$

The variable ξ in the expressions is a fluctuation of the trajectory around the straight line and

$$S(\xi) \,=\, \mathcal{N}^{-1} \int \mathcal{D}\xi\, Exp\left[\imath \int d\lambda \left(\frac{\sqrt{s}}{2}\, \dot{\xi}^{2} - \imath\, g\, \dot{\xi}^{\mu} \sum_{n=1}^{\infty} C(n)\, v_{\mu,\,\rho_1\cdots\rho_n}\, \xi^{\rho_1} \cdots \xi^{\rho_n} - \right.\right.$$

$$\left.\left. - \quad \frac{\imath\, g}{2\sqrt{s}}\, \sigma^{\mu\nu} \sum_{n=1}^{\infty} C(n) F_{\mu\nu,\,\rho_1\cdots\rho_n}\, \xi^{\rho_1} \cdots \xi^{\rho_n} \right)\right]$$

is a factor which accounts non-eikonal corrections related with deviation of the Wislon line from the straight one (segmentation).

Conclusion:

- Lipatov's effective action in combination with Balitsky approach to the calculation of correlators of Wilson lines can provide systematical calculations of different types of corrections to the scattering amplitudes at high energies and can be served as possible tool for the calculation and clarification of the NNLO BFKL kernel.
- The calculation of the NNLO BFKL kernel (mostly important part) requires a knowledge of only two-loops propagator of gluon field in an background shock wave instead direct three-loops calculation of the kernel in the BFKL approach.
- It will be interesting to understand whether there is a connection between the propagator that we need in the approach and the reggeized gluons propagator in the usual BFKL scheme, if there is, then the calculation of the kernel can be drastically simplified.