Next to Leading Order Corrections to dihadron production in DIS at small x *

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Abstract

We calculate the one-loop corrections to dihadron production in DIS at small x using the Color Glass Condensate formalism. Spinor helicity methods are used to evaluate the Dirac structure of the production amplitudes. Using power counting it is shown that all UV and soft divergences cancel while rapidity divergences are absorbed into rapidity evolution of the dipoles and quadrupoles. The remaining collinear divergences are absorbed into parton-hadron fragmentation functions which renders the final result finite.

1 Introduction

The Color Glass Condensate formalism (see [1] for reviews) is an effective theory of QCD at high energy (equivalently small x) which encodes the dynamics of gluon saturation, expected to occur in the wave function of a high energy hadron probed at small x. Since the experimental observation of the fast rise of parton distribution functions at small x there has been an intense interest in gluon saturation and its experimental signatures. It is expected that les inclusive observables provide a more sensitive signature. Dihadron production and angular correlation at small x is such a process which has been measured and studied in proton-nucleus collisions at RHIC [2]. While there is tangential evidence for gluon saturation at RHIC and the LHC there is no clear and convincing experimental evidence for it. To make the predictions of the Color Glass Condensate more robust it is therefore essential to improve the theoretical accuracy of the calculations. The first step in this direction is to calculate the one-loop corrections

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to the tree level results. Here we do the calculations for the case of a longitudinal photon splitting into a quark anti-quark pair which then multiply scatters on a proton or nucleus target. The Leading Order (LO) cross section for production of a quark and anti-quark with transverse momenta $\mathbf{p}$ and $\mathbf{q}$ and rapidities y_1, y_2 is given by

$$\begin{aligned}\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, dy_2} \;&=\; \frac{e^2 Q^2 (z_1 z_2)^2 N_c}{(2\pi)^7}\delta(1-z_1-z_2)\int d^8\boldsymbol{x}\, [S_{122'1'} - S_{12} - S_{1'2'} + 1] \\ & e^{i\mathbf{p}\cdot(\mathbf{x}'_1-\mathbf{x}_1)} e^{i\mathbf{q}\cdot(\mathbf{x}'_2-\mathbf{x}_2)} \Bigg[4 z_1 z_2 K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1) + \\ & (z_1^2+z_2^2)\,\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2'}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2'}|}\, K_1(|\mathbf{x}_{12}|Q_1) K_1(|\mathbf{x}_{1'2'}|Q_1)\Bigg] \end{aligned} \tag{1}$$

where the first term is the contribution of longitudinal polarization of the virtual photon while the second term is the contribution of transverse polarizations. Also, z_1, z_2 are the longitudinal momentum fractions of the photon carried by quark and anti-quark and $Q_1^2 \equiv z_1\,(1-z_1)\,Q^2$ with photon virtuality Q^2. The quark and anti-quark pass through the target and multiply scatter from it at transverse coordinates $\mathbf{x}_1$ and $\mathbf{x}_2$ in the amplitude (primed in the complex conjugate amplitude). All the QCD dynamics of the target is encoded in the dipoles and quadrupole, normalized trace of two- and four-point functions of Wilson lines which efficiently resum multiple scattering of a quark or anti-quark from the target.

One loop corrections to the above Leading Order cross section involves radiation of a gluon by the quark or anti-quark in the amplitude which is then absorbed either in the complex conjugate amplitude (real corrections) or in the amplitude (virtual corrections), and similarly if the radiation is done by a quark or anti-quark in the complex conjugate amplitude. This has been done in [3] using spinor helicity formalism. The resulting expressions are long, therefore here we show a small sample,

$$\begin{aligned}\frac{d\sigma^L_{1\times 1}}{d^2\boldsymbol{p}\, d^2\boldsymbol{q}\, dy_1\, dy_2} =& \frac{2e^2 g^2 Q^2 N_c^2 z_2^3 (1-z_2)^2 (z_1^2 + (1-z_2)^2)}{(2\pi)^{10} z_1}\int \frac{dz}{z}\int d^{10}x\, K_0(|\boldsymbol{x}_{12}|Q_2) \\ & K_0(|\boldsymbol{x}_{1'2'}|Q_2)\Delta^{(3)}_{11'}\, [S_{122'1'} - S_{12} - S_{1'2'} + 1]\, e^{i\boldsymbol{p}\cdot(\boldsymbol{x}'_1-\boldsymbol{x}_1)} e^{i\boldsymbol{q}\cdot(\boldsymbol{x}'_2-\boldsymbol{x}_2)} e^{i\frac{z}{z_1}\boldsymbol{p}\cdot(\boldsymbol{x}'_1-\boldsymbol{x}_1)}\end{aligned} \tag{2}$$

where

$$\Delta^{(3)}_{ij} = \frac{\boldsymbol{x}_{3i}\cdot\boldsymbol{x}_{3j}}{\boldsymbol{x}_{3i}^2\boldsymbol{x}_{3j}^2} \tag{3}$$

is the gluon radiation kernel. This is the contribution of the diagram where the gluon is radiated by the quark in the amplitude and absorbed by the quark in the complex conjugate amplitude. We also show the result for one of the virtual terms,

$$\begin{aligned}\frac{d\sigma^L_5}{d^2\boldsymbol{p}\, d^2\boldsymbol{q}\, dy_1\, dy_2} =& \frac{2e^2 g^2 Q^2 N_c^2 z_2^3 z_1}{(2\pi)^{10}}\int_0^{z_1}\frac{dz}{z} d^{10}\boldsymbol{x}\, [S_{322'1'}S_{13} - S_{13}S_{23} - S_{1'2'} + 1](z_1^2 + (z_1-z)^2) \\ & \frac{K_0(QX_5)K_0(|\boldsymbol{x}_{1'2'}|Q_1)}{\boldsymbol{x}_{31}^2} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}'_1-\boldsymbol{x}_1)} e^{i\boldsymbol{q}\cdot(\boldsymbol{x}'_2-\boldsymbol{x}_2)} e^{-i\frac{z}{z_1}\boldsymbol{p}\cdot(\boldsymbol{x}_3-\boldsymbol{x}_1)}\end{aligned} \tag{4}$$

$$\tag{5}$$

which corresponds to contribution of the virtual diagram where the quark radiates and then absorbs a gluon where both quark and gluon are scattering from the target.

The one-loop corrections to the LO terms contain divergences; it is shown in [3] that all real corrections are UV finite while the UV divergences in the virtual corrections cancel each other as follows,

$$\begin{aligned}
&[d\sigma_5 + d\sigma_{11}]_{\rm UV} = 0,\\
&[d\sigma_6 + d\sigma_{12}]_{\rm UV} = 0,\\
&\left[d\sigma_9 + d\sigma_{10} + d\sigma_{14(1)} + d\sigma_{14(2)}\right]_{\rm UV} = 0.
\end{aligned} \tag{6}$$

The soft divergences on the other hand cancel between the real and virtual corrections, for instance,

$$\begin{aligned}
&[d\sigma_{1\times 1} + 2\, d\sigma_9]_{\rm soft} = 0,\\
&[d\sigma_{2\times 2} + 2\, d\sigma_{10}]_{\rm soft} = 0,\\
&\left[d\sigma_{1\times 2} + d\sigma_{13(1)} + d\sigma_{13(2)}\right]_{\rm soft} = 0,\\
&[d\sigma_{3\times 3} + d\sigma_{4\times 4} + 2\, d\sigma_{3\times 4}]_{\rm soft} = 0,\\
&[d\sigma_{1\times 3} + d\sigma_{1\times 4}]_{\rm soft} = 0,\\
&[d\sigma_{2\times 3} + d\sigma_{2\times 4}]_{\rm soft} = 0,\\
&[d\sigma_5 + d\sigma_7]_{\rm soft} = 0,\\
&[d\sigma_6 + d\sigma_8]_{\rm soft} = 0,\\
&\left[d\sigma_{11} + d\sigma_{14(1)}\right]_{\rm soft} = 0,\\
&\left[d\sigma_{12} + d\sigma_{14(2)}\right]_{\rm soft} = 0.
\end{aligned} \tag{7}$$

The rapidity divergences appear as

$$\int \frac{dz}{z} \tag{8}$$

and lead to evolution (rapidity renormalization) of dipoles and quadrupoles as governed by the JIMWLK equation [5]. The remaining collinear divergences are then absorbed into DGLAP evolution [6] of quark-hadron (and anti quark-hadron) fragmentation function $D_{h_1/q}(z_{h_1}, \mu^2)$ defined as (in dimensional regularization)

$$D_{h_1/q}(z_{h_1}, \mu^2) = \int_{z_{h_1}}^1 \frac{d\xi}{\xi} D^0_{h_1/q}\left(\frac{z_{h_1}}{\xi}\right) \left[\delta(1-\xi) + \frac{\alpha_s}{\pi} P_{qq}(\xi) \left(\frac{1}{\epsilon} - \log\left(\pi e^{\gamma_E} \mu |\boldsymbol{x}_1' - \boldsymbol{x}_1|\right)\right)\right]. \tag{9}$$

Symbolically, the final result can be written as

$$d\sigma^{\gamma^* A \to h_1 h_2 X} = d\sigma_{LO} \otimes \text{JIMWLK} + d\sigma_{LO} \otimes D_{h_1/q}(z_{h_1}, \mu^2) D_{h_2/\bar{q}}(z_{h_2}, \mu^2) + d\sigma_{NLO}^{\rm finite} \tag{10}$$

where μ^2 is the renormalization scale.

The obtained results can be used to investigate dihadron production and angular correlations in DIS at small x as in [7]. In practice one expects Sudakov effects to make sizable contributions to this observable [8] in the strict back to back limit so that one will need to include those before a completely reliable quantitative study is undertake. Furthermore, as the kinematic phase space at the proposed Electron-Ion Collider is somewhat limited (for dihadron production) it will be important to go beyond the eikinal approximation used in CGC and include finite energy effects in the scatterings of the partons on the target [9] which may lead to a more general formalism encompassing CGC at small x and pQCD and collinear factorization at high p_t. Such a development would be of enormous significance and would allow one to investigate an array of diverse processes [10], such as ultra-high energy neutrino-nucleon cross section [11], using the same formalism.

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