Kinematical higher-twist corrections in
\[ \gamma^* + \gamma \rightarrow M_1 + M_2 \]

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In collaboration with Cédric Lorcé and Bernard Pire, arXiv:2205.xxxx
Outline

- Motivation and Introduction
- Kinematical higher-twist corrections in DVCS
- Kinematical higher-twist corrections in $\gamma^* + \gamma \rightarrow M_1 + M_2$
Generalized Parton distributions (GPDs)

GPDs can be measured in Deeply Virtual Compton Scattering (DVCS).

- Proton spin puzzle

GPDs

- Energy momentum tensor (EMT) form factors of hadrons
- mass radius, mass distribution, pressure distribution and shear force distribution

Pressure distribution of proton was extracted from JLAB measurements.

Cédric Lorcé, Hervé Moutarde and Arkadiusz Trawiński, EPJC 79 (2019) 1, 89.
EMT form factors and mass radius of pions?

The GPDs of pions can not be measured by DVCS, since there is no such a facility.

\[ \gamma^* + \pi \to \gamma + \pi \]

How to obtain EMT form factors of pions?

Option 1: Model calculations of EMT form factors.

- Hyeon-Dong Son and Hyun-Chul Kim, PRD 90 (2014), 111901(R).

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Option 2: EMT form factors can be obtained from generalized distribution amplitudes of pions

- M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.
EMT form factors extracted from GDAs of pions

- GDAs → Timelike matrix elements of EMT → Timelike EMT form factors
  - Dispersion relation
  - Spacelike matrix elements of EMT → Spacelike EMT form factors
    - Mass radius
    - Mass distribution
    - Pressure distribution
    - Shear force distribution
GDAs in $\gamma^* + \gamma \rightarrow M_1 + M_2$ at KEKB

Hard part: $\gamma^* + \gamma \rightarrow q + \bar{q}$
Soft part: $q + \bar{q} \rightarrow h + \bar{h}$, GDAs.

GDA of a scalar meson is defined as:

$$
\Phi(z, \cos \theta, s) = \int \frac{dx^-}{2\pi} e^{-ip^+ x^-} \langle h(p) \bar{h}(p') | \bar{q}(x^-) \gamma^- q(0) | 0 \rangle \quad \gamma^* + \gamma \rightarrow h + \bar{h}
$$

$$
z = \frac{k^+}{p^+}, \quad s = W^2 = (p + p')^2
$$


Belle measurements of $\gamma^* + \gamma \rightarrow \pi^0 + \pi^0$:

$8 \text{ GeV}^2 < Q^2 < 24 \text{ GeV}^2$, $0.2 \text{ GeV}^2 < s < 4 \text{ GeV}^2$

Given the kinematics of Belle measurements, kinematical higher-twist contributions of order $s/Q^2$ and $m^2/Q^2$ are important in the cross section.
Kinematical higher-twist contributions

DVCS: $\gamma^* + h \rightarrow \gamma + h$

Cross section = twist-2 contribution + higher-twist contributions

- **Twist-2 GPDs involved**
- **Kinematical higher-twist contributions**
- **Dynamical higher-twist contributions**

- Twist-2 GPDs involved, $t/Q^2$ and $m^2/Q^2$
- Genuine higher-twist GPDs involved

The theoretical cross section with kinematical higher-twist contributions can give a better description of experimental measurements *without introducing genuine higher-twist GPDs.*
Kinematical higher-twist contributions in DVCS

A separation of kinematical and dynamical contributions in the operator product of two electromagnetic currents was proven by Braun et al.

Kinematical contributions in two electromagnetic currents:

\[ T_{\mu\nu} = T\{j_\mu(z_1x)j_\nu(z_2x)\} \]

The kinematical corrections of order \( t/Q^2 \) and \( m^2/Q^2 \) were estimated for DVCS.

Scalar meson:

\( \gamma^* + \pi \rightarrow \gamma + \pi \)

Proton case:

\( \gamma^* + P \rightarrow \gamma + P \)
Kinematical higher-twist contributions in DIS

Kinematical higher-twist contributions in DVCS can be considered as a general case of the target mass corrections in DIS, \( \sim m^2/Q^2 \).

The total derivative of the leading twist operators contribute in DVCS.

\[
\left[ i P^\mu, [i P^\mu, \mathcal{O}^{t=2}] \right] \quad \text{DVCS} \quad \sim t/Q^2, \sim m^2/Q^2 \\
\left[ i P^\mu, \frac{\partial}{\partial x^\mu} \mathcal{O}^{t=2} \right] 
\]

Twist-2 operator:

\[
\mathcal{O}^{t=2}(z_1 x, z_2 x) = \frac{1}{2} [ O_V(z_1 x, z_2 x) - O_V(z_2 x, z_1 x) - O_A(z_1 x, z_2 x) - O_A(z_2 x, z_1 x) ]
\]

\[
O_V(z_1 x, z_2 x) = \bar{q}(z_1 x) \gamma^\alpha x_\alpha q(z_2 x)
\]

\[
O_A(z_1 x, z_2 x) = \bar{q}(z_1 x) \gamma^\alpha x_\alpha \gamma^5 q(z_2 x)
\]

In DIS, the matrix elements of total derivative operators vanish, only target mass corrections of \( m^2/Q^2 \) are available.
Kinematical contributions in $\gamma^* + \gamma \rightarrow M_1 + M_2$

We can also calculate the amplitudes of $\gamma^* + \gamma \rightarrow M_1 + M_2$ by using the operator results of the kinematical contributions in two electromagnetic currents.

$$T_{\mu \nu} = T\{j_\mu(z_1x)j_\nu(z_2x)\}$$

Helicity amplitudes of a scalar meson:

$$A_{\lambda_1 \lambda_2} = T_{\mu \nu} \epsilon^\mu(\lambda_1) \epsilon^\nu(\lambda_2)$$

There are three independent helicity amplitudes: $A_{++}$, $A_{0+}$ and $A_{+-}$.

Leading twist amplitude: $A_{++}$
Higher twist amplitudes: $A_{0+}$ and $A_{+-}$.

Helicity amplitudes

\[ A^{(0)} = 2\chi \left\{ (1 - \frac{s}{2Q^2}) \int_0^1 dz \frac{\Phi(z, \cos \theta, s)}{1 - z} - \frac{s}{Q^2} \int_0^1 dz \frac{\Phi(z, \cos \theta, s)}{z} \ln (1 - z) \right. \]

\[ \left. - \left( \frac{2s}{Q^2} \cos + \frac{\Delta^2_T}{\beta_0^2 Q^2} \frac{\partial}{\partial \cos \theta} \right) \frac{\partial}{\partial \cos \theta} \int_0^1 dz \frac{\Phi(z, \cos \theta, s)}{z} \left[ \frac{\ln(1 - z)}{2} + Li_2(1 - z) - Li_2(1) \right] \right\}, \]

\[ A^{(1)} = \frac{4\chi}{\beta_0 Q} \frac{\partial}{\partial \cos \theta} \int_0^1 dz \Phi(z, \cos, s) \frac{\ln(1 - z)}{z}, \]

\[ A^{(2)} = \frac{4\chi}{\beta_0^2 Q^2} \frac{\partial}{\partial \cos \theta} \frac{\partial}{\partial \cos \theta} \int_0^1 dz \Phi(z, \cos, s) \frac{2z - 1}{z} \ln (1 - z). \]

A_{++} = A^{(0)}
A_{0+} = -A^{(1)} \Delta \cdot \epsilon^\mu (-) \quad \rightarrow \quad \propto \Delta_T \quad \Delta \text{ is the relative momentum of final meson pair.}
A_{-+} = -A^{(2)} [\Delta \cdot \epsilon^\mu (-)]^2 \quad \rightarrow \quad \propto (\Delta_T)^2

Asymptotic form of pion GDAs:

\[ \Phi(z, \cos \theta, s) = 18z(1 - z)(2z - 1)[\tilde{B}_{10}(s) + \tilde{B}_{12}(s) P_2(\cos \theta)] \]

The nonvanishing helicity-flip amplitudes \( A_{0+} \) and \( A_{+-} \) indicate the existence of the D-wave GDAs.
Numerical estimate of the cross section

\[
\frac{d\sigma(e + \gamma \to e + \pi + \pi)}{dQ^2 ds d\cos\theta} = \frac{\alpha^3 \beta_0}{8s_{e\gamma}^2 Q^2(1 - \varepsilon)} \left( |A_{++}|^2 + |A_{--}|^2 + 2\varepsilon|A_{0+}|^2 \right)
\]


The range of kinematics in the following plots are same with that of Belle measurements.

Solid lines: cross section with kinematical contributions, twist 2+twist 3+twist 4.
Dashed lines: twist-2 cross section
Cross section of $e + \gamma \to e + \pi + \pi$ is calculated at higher $Q^2$, the kinematical contributions become less important as $Q^2$ increases.

Numerical calculation of the cross section for KK and eta eta is in progress.
The ratio indicates the kinematical contributions are significant if $s > 1 \text{ GeV}^2$ where the cross section is necessary to study the EMT form factors.

$\Lambda \geq 3 \text{ GeV}^2$ is necessary for pion EMT form factor, PRD 97 (2018) 014020.
Experimental measurements of $\gamma^* + \gamma \rightarrow M_1 + M_2$

In 2016, the Belle Collaboration released the measurements of $\gamma^* \gamma \rightarrow \pi^0 \pi^0$.

Differential cross section of $\gamma^* \gamma \rightarrow \pi^0 \pi^0$

$8 \text{ GeV}^2 < Q^2 < 24 \text{ GeV}^2$

$0.2 \text{ GeV}^2 < s < 4 \text{ GeV}^2$

$s = W^2$

$\sim s/Q^2$, $\sim m^2/Q^2$

kinematical corrections

The errors are large, and statistical errors are dominant. This situation can be improved by the Belle II Collaboration.

Luminosity: $2 \times 10^{34}$ cm$^{-2}$ s$^{-1}$ → $8 \times 10^{35}$ cm$^{-2}$ s$^{-1}$

A precise description of the cross section requires the inclusion of kinematical higher-twist contributions!
Summary

- GDAs can be considered as an alternative way to investigate the EMT form factors of pions, since pion GPDs can be not measured by experiment.

- Kinematical higher-twist contributions are calculated for $\gamma^* + \gamma \rightarrow M_1 + M_2$, from which the GDAs can be extracted.

- The numerical calculation of kinematical contributions is also performed for $\gamma^* + \gamma \rightarrow \pi + \pi$, and the kinematical contributions are significant if $s > 1 \text{ GeV}^2$ where the cross section is necessary to study the EMT form factors.

- Belle II was upgraded with a higher luminosity in 2018, more precise measurements of GDAs can be expected, the accurate description of the amplitudes also requires the inclusion of kinematic contributions.

Thank you very much