

# Report on the progress of TMD PDF calculations of the proton and heavy baryons within the BLFQ framework

HU Zhi

[huzhi@impcas.ac.cn](mailto:huzhi@impcas.ac.cn)

[tianluoqi42@gmail.com](mailto:tianluoqi42@gmail.com)

Institute of Modern Physics, Chinese Academy of Science

School of Nuclear Physics, University of Chinese Academy of Sciences

DIS2022 2022/5/4

Based on

ZH, S. Xu, C. Mondal, X. Zhao, J. Vary, Transverse momentum structure of proton in basis light-front quantization, *will be submitted to arXiv in a few days*

Z. Zhu, ZH, S. Xu, C. Mondal, X. Zhao, J. Vary, Transverse structure of heavy baryons in the momentum space: a light-front Hamiltonian approach, *in preparation*

# Outline

- 1 BLFQ framework
- 2 TMDs in this study
- 3 Proton TMDs
- 4 Comparisons between TMDs of Proton,  $\Lambda$  and  $\Lambda_c$ —mass effect analysis
- 5 Conclusions
- 6 Appendix



# Outline

- 1 BLFQ framework
- 2 TMDs in this study
- 3 Proton TMDs
- 4 Comparisons between TMDs of Proton,  $\Lambda$  and  $\Lambda_c$ —mass effect analysis
- 5 Conclusions
- 6 Appendix



# Basis Light-front Quantization is ...

a light-front framework starting to simultaneously get the mass spectrum and the corresponding internal information of many quantum bound states in a numerical manner and within a feasible computation time.



# Basis Light-front Quantization is ...

a light-front framework starting to simultaneously get the mass spectrum and the corresponding internal information of many quantum bound states in a numerical manner and within a feasible computation time.

$\mathcal{L} \rightarrow H \rightarrow H |P, \Lambda\rangle = M^2 |P, \Lambda\rangle \rightarrow$  light-front wavefunction



# Basis Light-front Quantization is ...

a light-front framework starting to simultaneously get the mass spectrum and the corresponding internal information of many quantum bound states in a numerical manner and within a feasible computation time.

$\mathcal{L} \rightarrow H \rightarrow H |P, \Lambda\rangle = M^2 |P, \Lambda\rangle \rightarrow$  light-front wavefunction

{ Hamiltonian light-front formalism  
From LFWF to internal structure  
GPCFs, GPDs, TMDs, PDFs, FFs



# Basis Light-front Quantization is ...

a light-front framework starting to simultaneously get the mass spectrum and the corresponding internal information of many quantum bound states in a numerical manner and within a feasible computation time.

$\mathcal{L} \rightarrow H \rightarrow H|P, \Lambda\rangle = M^2|P, \Lambda\rangle \rightarrow$  light-front wavefunction

{ Hamiltonian light-front formalism  
From LFWF to internal structure  
GPCFs, GPDs, TMDs, PDFs, FFs

Basis and truncations, Supercomputer, Quantum computing



# Basis Light-front Quantization is ...

a light-front framework starting to simultaneously get the mass spectrum and the corresponding internal information of many quantum bound states in a numerical manner and within a feasible computation time.

$\mathcal{L} \rightarrow H \rightarrow H|P, \Lambda\rangle = M^2|P, \Lambda\rangle \rightarrow$  light-front wavefunction

{ Hamiltonian light-front formalism  
From LFWF to internal structure  
GPCFs, GPDs, TMDs, PDFs, FFs

Basis and truncations, Supercomputer, Quantum computing

Electron, positronium, proton, light meson, heavy meson ...



# In this study ...

We study the proton,  $\Lambda$  and  $\Lambda_c$  system;



## In this study ...

$$H_{\text{eff.}} = \sum_{i=1}^3 \frac{m_i^2 + (p_i^\perp)^2}{x_i} + \frac{1}{2} \sum_{i,j=1}^3 V_{i,j}^{\text{conf.}} + \frac{1}{2} \sum_{i,j=1}^3 V_{i,j}^{\text{OGE}};$$

We study the proton,  $\Lambda$  and  $\Lambda_c$  system;



## In this study ...

$$H_{\text{eff.}} = \sum_{i=1}^3 \frac{m_i^2 + (p_i^\perp)^2}{x_i} + \frac{1}{2} \sum_{i,j=1}^3 V_{i,j}^{\text{conf.}} + \frac{1}{2} \sum_{i,j=1}^3 V_{i,j}^{\text{OGE}};$$

Currently we work within leading three-quark Fock sector;

We study the proton,  $\Lambda$  and  $\Lambda_c$  system;



## In this study ...

$$H_{\text{eff.}} = \sum_{i=1}^3 \frac{m_i^2 + (p_i^\perp)^2}{x_i} + \frac{1}{2} \sum_{i,j=1}^3 V_{i,j}^{\text{conf.}} + \frac{1}{2} \sum_{i,j=1}^3 V_{i,j}^{\text{OGE}};$$

We focus on the TMD PDF calculations;

Currently we work within leading three-quark Fock sector;

We study the proton,  $\Lambda$  and  $\Lambda_c$  system;



# Outline

- 1 BLFQ framework
- 2 TMDs in this study
- 3 Proton TMDs
- 4 Comparisons between TMDs of Proton,  $\Lambda$  and  $\Lambda_c$ —mass effect analysis
- 5 Conclusions
- 6 Appendix



# TMDs in this study

- Leading twist  $\mathcal{W} \approx \mathbb{1}$  + only valence quark  $\Rightarrow$  6 T-even quark TMDs



# TMDs in this study

- Leading twist  $\mathcal{W} \approx \mathbb{1}$   $\rightarrow$  only valence quark  $\Rightarrow$  6 T-even quark TMDs
- In the current study, we find all TMDs expressed by the following helicity distributions

$$\Phi_{\lambda'_1, \lambda_1}^{\Lambda', \Lambda}(\{x, p^\perp\}) = N_{\text{all}} \int \sum_{\lambda_2, \lambda_3} \psi_{\lambda'_1, \lambda_2, \lambda_3}^{\Lambda', *}(\{x, p^\perp\}) \psi_{\lambda_1, \lambda_2, \lambda_3}^{\Lambda}(\{x, p^\perp\}) \quad (1)$$



# TMDs in this study

- Leading twist  $\mathcal{W} \approx \mathbb{1}$   $\rightarrow$  only valence quark  $\Rightarrow$  6 T-even quark TMDs
- In the current study, we find all TMDs expressed by the following helicity distributions

$$\Phi_{\lambda'_1, \lambda_1}^{\Lambda', \Lambda}(\{x, p^\perp\}) = N_{\text{all}} \int \sum_{\lambda_2, \lambda_3} \psi_{\lambda'_1, \lambda_2, \lambda_3}^{\Lambda', *}(\{x, p^\perp\}) \psi_{\lambda_1, \lambda_2, \lambda_3}^{\Lambda}(\{x, p^\perp\}) \quad (1)$$

**Q** In total 16 independent helicity distributions?



## TMDs in this study

- Leading twist  $+\mathcal{W} \approx \mathbb{1} +$  only valence quark  $\Rightarrow$  6 T-even quark TMDs
- In the current study, we find all TMDs expressed by the following helicity distributions

$$\Phi_{\lambda'_1, \lambda_1}^{\Lambda', \Lambda}(\{x, p^\perp\}) = N_{\text{all}} \int \sum_{\lambda_2, \lambda_3} \psi_{\lambda'_1, \lambda_2, \lambda_3}^{\Lambda', *}(\{x, p^\perp\}) \psi_{\lambda_1, \lambda_2, \lambda_3}^{\Lambda}(\{x, p^\perp\}) \quad (1)$$

**Q In total 16 independent helicity distributions?**

**! Actually only 6.**



# TMDs in this study

- Leading twist  $\mathcal{W} \approx \mathbb{1}$  + only valence quark  $\Rightarrow$  6 T-even quark TMDs
- In the current study, we find all TMDs expressed by the following helicity distributions

$$\Phi_{\lambda'_1, \lambda_1}^{\Lambda', \Lambda}(\{x, p^\perp\}) = N_{\text{all}} \int \sum_{\lambda_2, \lambda_3} \psi_{\lambda'_1, \lambda_2, \lambda_3}^{\Lambda', *}(\{x, p^\perp\}) \psi_{\lambda_1, \lambda_2, \lambda_3}^{\Lambda}(\{x, p^\perp\}) \quad (1)$$

## ! In total 6 independent helicity distributions

- Our calculations of the leading twist T-even TMDs don't support the previously found model-dependent relations. [[10.1103/PhysRevD.81.074035](#); [10.1103/PhysRevD.84.034039](#)]



# TMDs in this study

- Leading twist  $\mathcal{W} \approx \mathbb{1}$  + only valence quark  $\Rightarrow$  6 T-even quark TMDs
- In the current study, we find all TMDs expressed by the following helicity distributions

$$\Phi_{\lambda'_1, \lambda_1}^{\Lambda', \Lambda}(\{x, p^\perp\}) = N_{\text{all}} \int \sum_{\lambda_2, \lambda_3} \psi_{\lambda'_1, \lambda_2, \lambda_3}^{\Lambda', *}(\{x, p^\perp\}) \psi_{\lambda_1, \lambda_2, \lambda_3}^{\Lambda}(\{x, p^\perp\}) \quad (1)$$

## ! In total 6 independent helicity distributions

- Our calculations of the leading twist T-even TMDs don't support the previously found model-dependent relations. [[10.1103/PhysRevD.81.074035](#); [10.1103/PhysRevD.84.034039](#)]
- Still, some model-dependent relations
- Soffer-type bounds [[10.1103/PhysRevLett.85.712](#)] are satisfied.

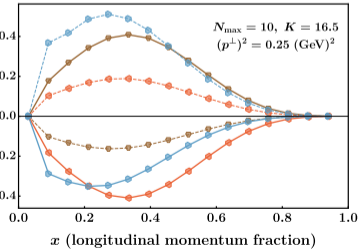
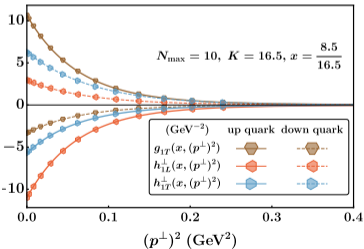
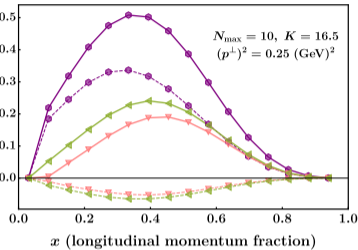
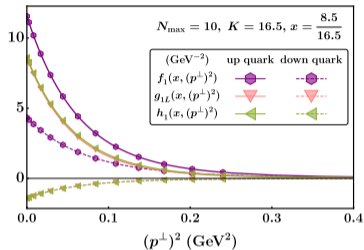


# Outline

- 1 BLFQ framework
- 2 TMDs in this study
- 3 Proton TMDs
- 4 Comparisons between TMDs of Proton,  $\Lambda$  and  $\Lambda_c$ —mass effect analysis
- 5 Conclusions
- 6 Appendix



# BLFQ results of six T-even leading twist TMDs when $\mathcal{W} = 1$

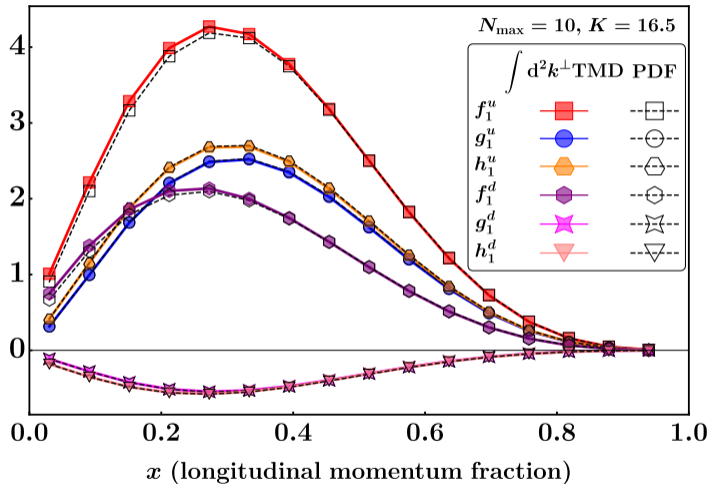


qualitative agreement with other theoretical calculations

[[10.1103/PhysRevD.81.074035](https://arxiv.org/abs/10.1103/PhysRevD.81.074035);  
[10.1103/PhysRevD.80.014021](https://arxiv.org/abs/10.1103/PhysRevD.80.014021);  
[10.1103/PhysRevD.103.014024](https://arxiv.org/abs/10.1103/PhysRevD.103.014024);  
[10.1103/PhysRevD.78.074010](https://arxiv.org/abs/10.1103/PhysRevD.78.074010);  
[10.1103/PhysRevD.95.074009](https://arxiv.org/abs/10.1103/PhysRevD.95.074009);  
[10.1103/PhysRevD.78.034025](https://arxiv.org/abs/10.1103/PhysRevD.78.034025);  
[10.1103/PhysRevD.83.094507](https://arxiv.org/abs/10.1103/PhysRevD.83.094507);  
[10.1103/PhysRevD.85.094510](https://arxiv.org/abs/10.1103/PhysRevD.85.094510);  
[10.1103/PhysRevD.96.094508](https://arxiv.org/abs/10.1103/PhysRevD.96.094508)]



# Comparisons with PDFs obtained within the BLFQ framework



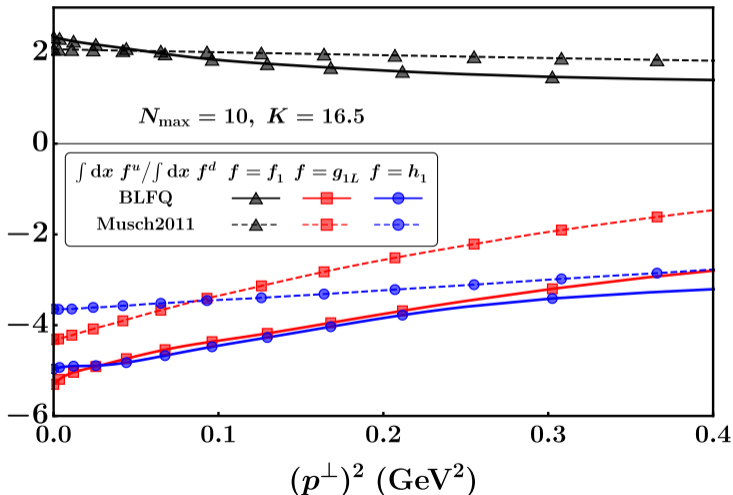
Integrating TMDs  $\rightarrow$  PDFs [[10.1103/PhysRevD.104.094036](https://arxiv.org/abs/10.1103/PhysRevD.104.094036)]



# Flavor-ratios compared with the lattice QCD calculations

- comparison with lattice results Musch2011

[[10.1103/PhysRevD.83.094507](https://arxiv.org/abs/10.1103/PhysRevD.83.094507)] via favor ratio



$$\frac{\int dx f^u}{\int dx f^d}$$

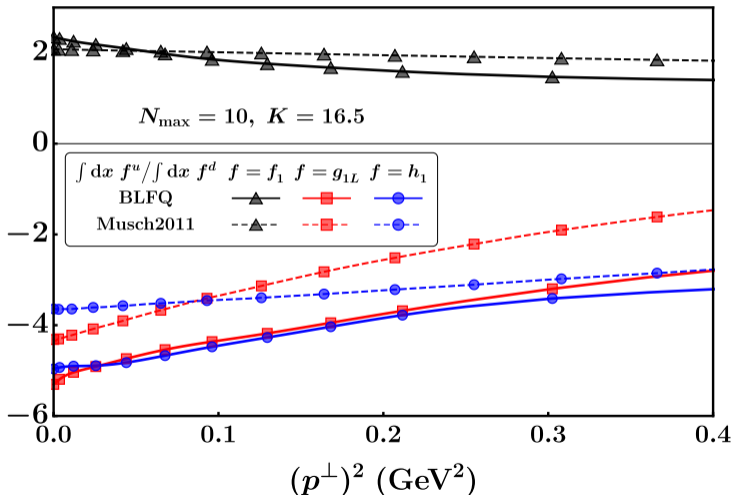


# Flavor-ratios compared with the lattice QCD calculations

- comparison with lattice results Musch2011

[[10.1103/PhysRevD.83.094507](https://arxiv.org/abs/10.1103/PhysRevD.83.094507)] via favor ratio

$$\frac{\int dx f^u}{\int dx f^d}$$



- ratio cancel possible overall factors and effects from scale evolution

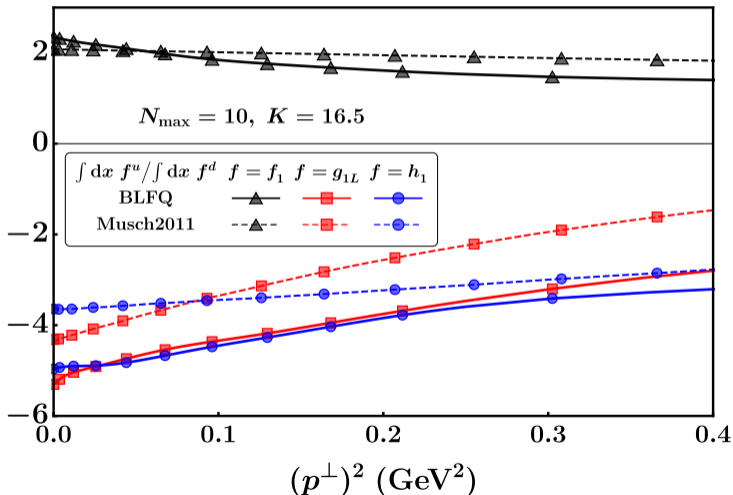


# Flavor-ratios compared with the lattice QCD calculations

- comparison with lattice results Musch2011

[[10.1103/PhysRevD.83.094507](https://arxiv.org/abs/10.1103/PhysRevD.83.094507)] via  
favor ratio

$$\frac{\int dx f^u}{\int dx f^d}$$



- ratio cancel possible overall factors and effects from scale evolution
- our  $d$  quark distributions extend to higher  $(p^\perp)^2 \rightarrow$  our flavor ratios decrease faster

# $\langle (p^\perp)^2 \rangle$ and Gaussian ansatz

## Overview

- Some popular simplifications

- $f^q(x, (p^\perp)^2) = f^q(x) \frac{e^{-\frac{(p^\perp)^2}{\langle (p^\perp)^2 \rangle_f}}}{\pi \langle (p^\perp)^2 \rangle_f}$

- no  $x$  and flavour dependence of  $\langle (p^\perp)^2 \rangle_f$

[[10.1007/JHEP04\(2014\)005](#); [2110.10253](#); [10.1103/PhysRevD.92.114023](#); [10.1016/j.physletb.2020.135347](#); [10.1103/PhysRevD.102.054002](#); [10.1103/PhysRevD.91.034010](#)]



# $\langle (p^\perp)^2 \rangle$ and Gaussian ansatz

## Overview

- Some popular simplifications

- $f^q(x, (p^\perp)^2) = f^q(x) \frac{e^{-\frac{(p^\perp)^2}{\langle (p^\perp)^2 \rangle_f}}}{\pi \langle (p^\perp)^2 \rangle_f}$

- no  $x$  and flavour dependence of  $\langle (p^\perp)^2 \rangle_f$

[[10.1007/JHEP04\(2014\)005](#); [2110.10253](#); [10.1103/PhysRevD.92.114023](#); [10.1016/j.physletb.2020.135347](#); [10.1103/PhysRevD.102.054002](#); [10.1103/PhysRevD.91.034010](#)]

- More sophisticated researches don't adopt those simplifications

[[10.1007/JHEP06\(2017\)081](#); [10.1007/JHEP06\(2019\)028](#); [10.1007/JHEP11\(2013\)194](#); [10.1007/JHEP06\(2020\)137](#); [10.1103/PhysRevD.93.014009](#)]



# $\langle (p^\perp)^2 \rangle$ and Gaussian ansatz

## Overview

- Some popular simplifications

- $f^q(x, (p^\perp)^2) = f^q(x) \frac{e^{-\frac{(p^\perp)^2}{\langle (p^\perp)^2 \rangle_f}}}{\pi \langle (p^\perp)^2 \rangle_f}$

- no  $x$  and flavour dependence of  $\langle (p^\perp)^2 \rangle_f$

[[10.1007/JHEP04\(2014\)005](#); [2110.10253](#); [10.1103/PhysRevD.92.114023](#); [10.1016/j.physletb.2020.135347](#); [10.1103/PhysRevD.102.054002](#); [10.1103/PhysRevD.91.034010](#)]

- More sophisticated researches don't adopt those simplifications

- **[?] What about in BLFQ?**

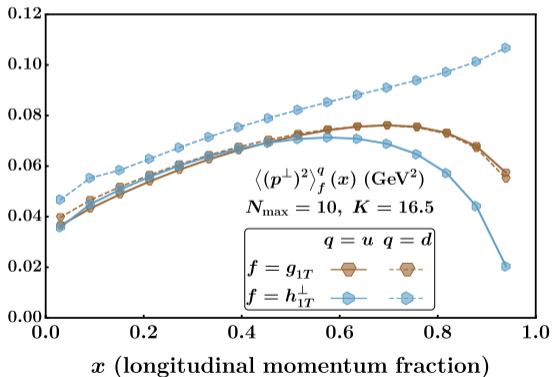
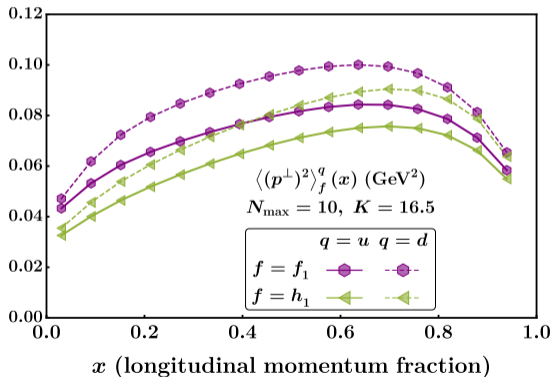
[[10.1007/JHEP06\(2017\)081](#); [10.1007/JHEP06\(2019\)028](#); [10.1007/JHEP11\(2013\)194](#); [10.1007/JHEP06\(2020\)137](#); [10.1103/PhysRevD.93.014009](#)]



# $\langle (p^\perp)^2 \rangle$ and Gaussian ansatz

$x$  and flavour dependence of  $\langle (p^\perp)^2 \rangle$  within the BLFQ framework

$$\langle (p^\perp)^2 \rangle_f^q(x) = \frac{\int d^2 p^\perp (p^\perp)^2 f_{\text{BLFQ}}^q(x, (p^\perp)^2)}{\int d^2 p^\perp f_{\text{BLFQ}}^q(x, (p^\perp)^2)}$$

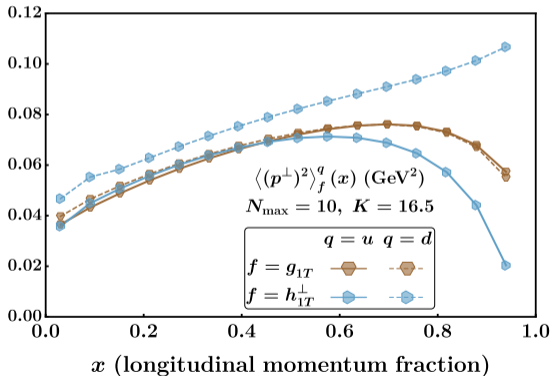
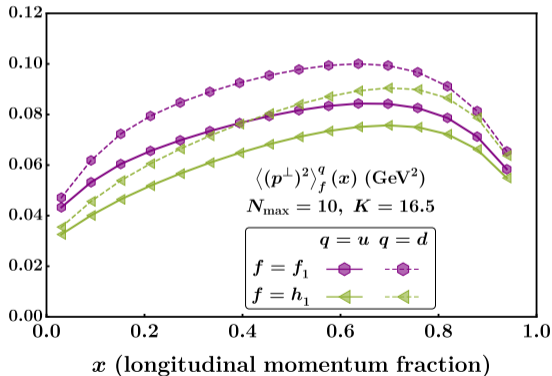


# $\langle(p^\perp)^2\rangle$ and Gaussian ansatz

$x$  and flavour dependence of  $\langle(p^\perp)^2\rangle$  within the BLFQ framework

- strong  $x$  and flavor dependence of  $\langle(p^\perp)^2\rangle$
- don't support  $x - p^\perp$  factorization
- peak structure

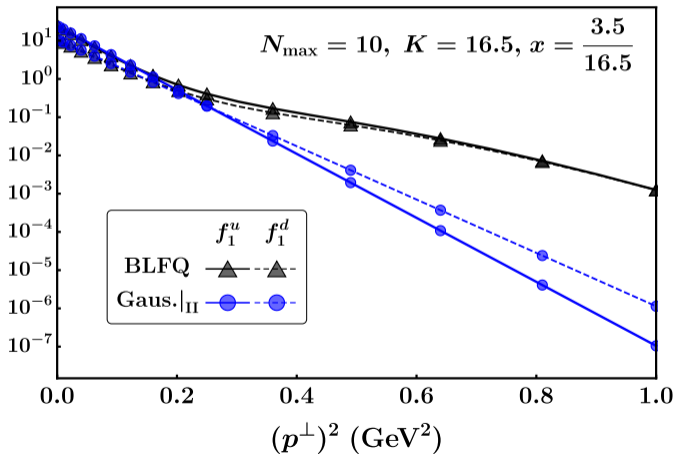
$$\langle(p^\perp)^2\rangle_f^q(x) = \frac{\int d^2p^\perp (p^\perp)^2 f_{\text{BLFQ}}^q(x, (p^\perp)^2)}{\int d^2p^\perp f_{\text{BLFQ}}^q(x, (p^\perp)^2)}$$



# $\langle (p^\perp)^2 \rangle$ and Gaussian ansatz

Compatibility between BLFQ calculations and Gaussian ansatz

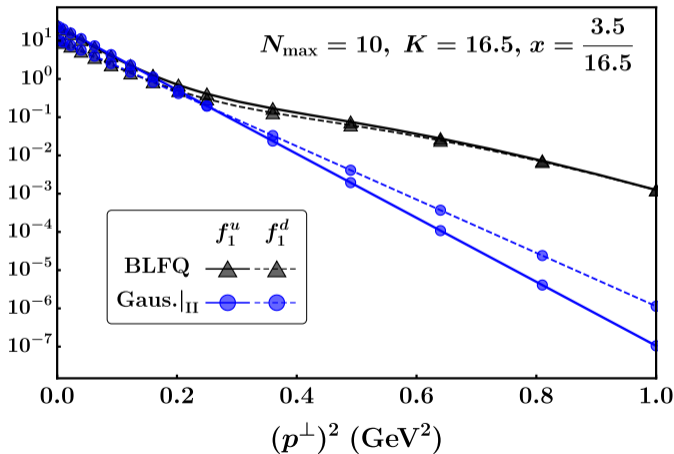
$$f_{\text{Gaus.}}^q(x, (p^\perp)^2) = f_{\text{BLFQ}}^q(x) \frac{\exp\left(-\frac{(p^\perp)^2}{\langle (p^\perp)^2 \rangle_f^q(x)}\right)}{\pi \langle (p^\perp)^2 \rangle_f^q(x)}$$



# $\langle (p^\perp)^2 \rangle$ and Gaussian ansatz

Compatibility between BLFQ calculations and Gaussian ansatz

$$f_{\text{Gaus.}}^q(x, (p^\perp)^2) = f_{\text{BLFQ}}^q(x) \frac{\exp\left(-\frac{(p^\perp)^2}{\langle (p^\perp)^2 \rangle_f^q(x)}\right)}{\pi \langle (p^\perp)^2 \rangle_f^q(x)}$$



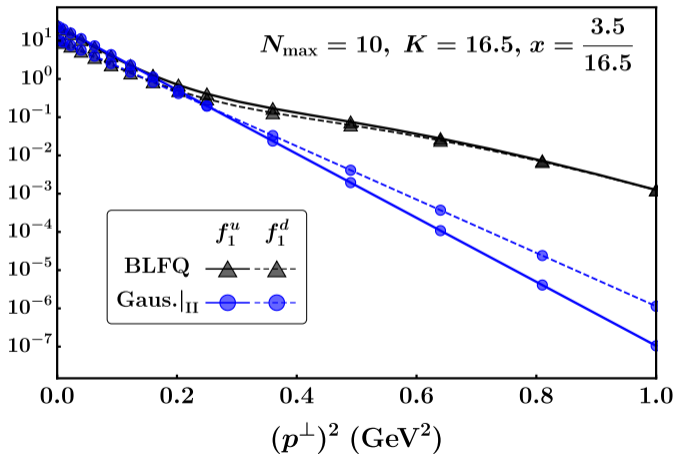
- **small  $(p^\perp)^2$  region:**  
Gaussian-type distributions with  $x$ -dependent Gaussian width



# $\langle (p^\perp)^2 \rangle$ and Gaussian ansatz

Compatibility between BLFQ calculations and Gaussian ansatz

$$f_{\text{Gaus.}}^q(x, (p^\perp)^2) = f_{\text{BLFQ}}^q(x) \frac{\exp\left(-\frac{(p^\perp)^2}{\langle (p^\perp)^2 \rangle_f^q(x)}\right)}{\pi \langle (p^\perp)^2 \rangle_f^q(x)}$$



- **small  $(p^\perp)^2$  region:**  
Gaussian-type distributions with  $x$ -dependent Gaussian width
- **large  $(p^\perp)^2$  region:** BLFQ results decrease slower than Gaussian-type distributions

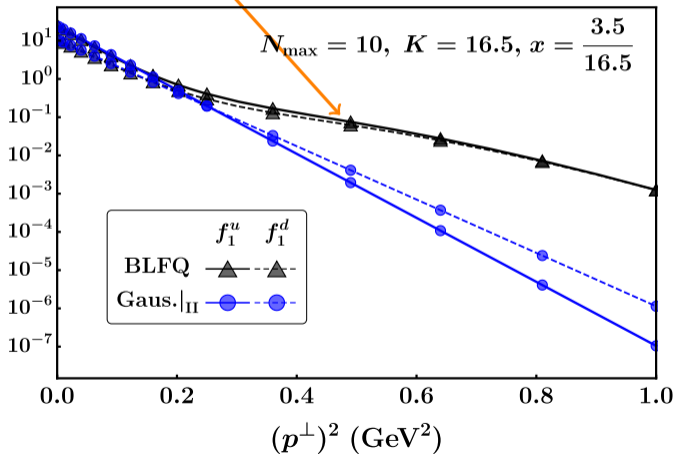


# $\langle (p^\perp)^2 \rangle$ and Gaussian ansatz

## Compatibility between BLFQ calculations and Gaussian ansatz

Our calculations

$$\sim 1/(p^\perp)^{3.5} \quad f_{\text{Gaus.}}^q(x, (p^\perp)^2) = f_{\text{BLFQ}}^q(x) \frac{\exp\left(-\frac{(p^\perp)^2}{\langle (p^\perp)^2 \rangle_f^q(x)}\right)}{\pi \langle (p^\perp)^2 \rangle_f^q(x)}$$



- **small  $(p^\perp)^2$  region:**  
Gaussian-type distributions with  $x$ -dependent Gaussian width
- **large  $(p^\perp)^2$  region:** BLFQ results decrease slower than Gaussian-type distributions
- **perturbative results:**  
 $f_1 \sim 1/(p^\perp)^2$  in the large  $(p^\perp)^2$  region

[[10.1088/1126-6708/2008/08/023](https://doi.org/10.1088/1126-6708/2008/08/023)]



# Outline

- 1 BLFQ framework
- 2 TMDs in this study
- 3 Proton TMDs
- 4 Comparisons between TMDs of Proton,  $\Lambda$  and  $\Lambda_c$ —mass effect analysis
- 5 Conclusions
- 6 Appendix

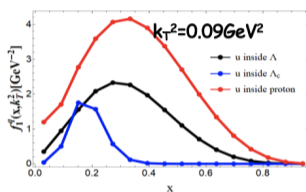
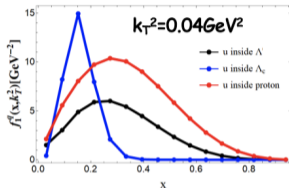
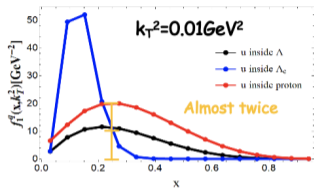


# Comparisons in the longitudinal direction

Electromagnetic structure of heavy baryons, 5 May 2022, 16:40

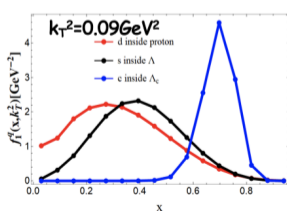
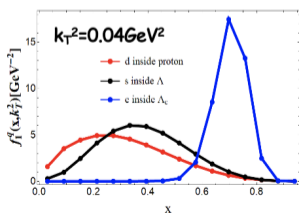
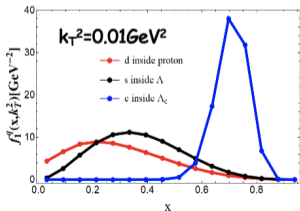
lighter quark

—●— u inside proton —●— u inside  $\Lambda$  —●— u inside  $\Lambda_c$



heavier quark

—●— d inside proton —●— s inside  $\Lambda$  —●— c inside  $\Lambda_c$

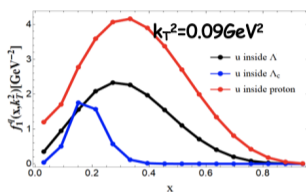
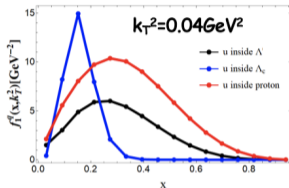
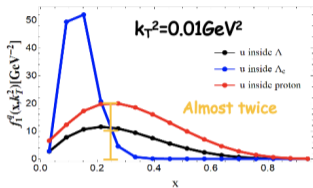


# Comparisons in the longitudinal direction

Electromagnetic structure of heavy baryons, 5 May 2022, 16:40

lighter quark

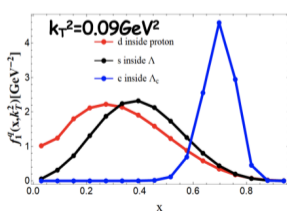
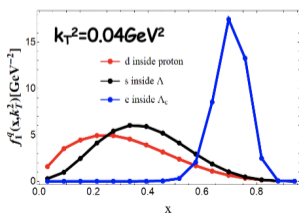
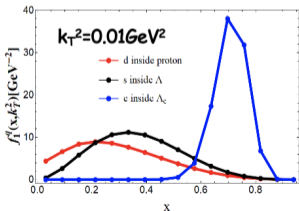
—●— u inside proton —●— u inside  $\Lambda$  —●— u inside  $\Lambda_c$



● Lighter quark vs heavier quark

heavier quark

—●— d inside proton —●— s inside  $\Lambda$  —●— c inside  $\Lambda_c$

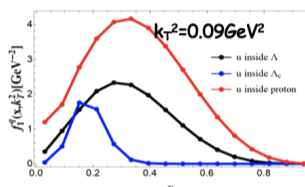
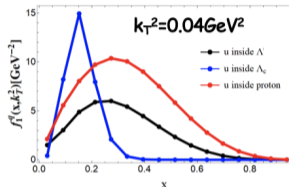
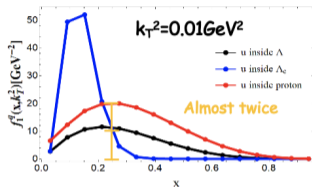


# Comparisons in the longitudinal direction

Electromagnetic structure of heavy baryons, 5 May 2022, 16:40

lighter quark

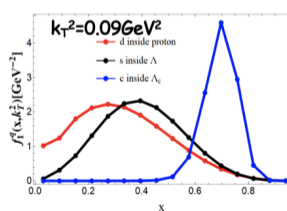
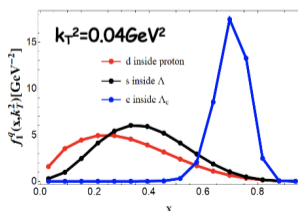
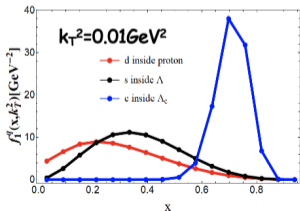
—●— u inside proton —●— u inside  $\Lambda$  —●— u inside  $\Lambda_c$



- Lighter quark vs heavier quark
- For proton, the difference of mass is small, only some MeV.

heavier quark

—●— d inside proton —●— s inside  $\Lambda$  —●— c inside  $\Lambda_c$

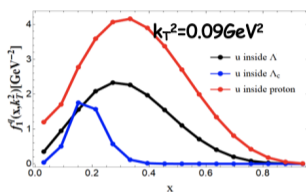
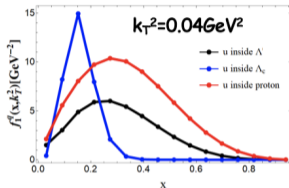
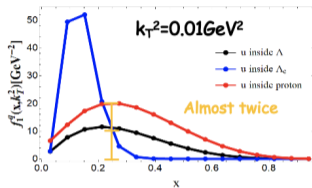


# Comparisons in the longitudinal direction

Electromagnetic structure of heavy baryons, 5 May 2022, 16:40

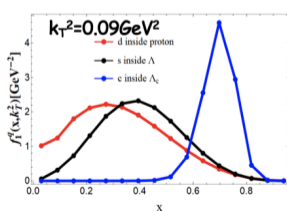
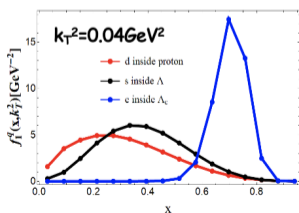
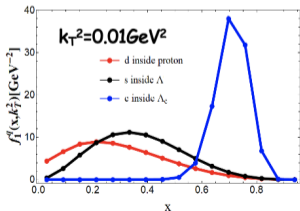
lighter quark

—●— u inside proton —●— u inside  $\Lambda$  —●— u inside  $\Lambda_c$



heavier quark

—●— d inside proton —●— s inside  $\Lambda$  —●— c inside  $\Lambda_c$



- Lighter quark vs heavier quark
- For proton, the difference of mass is small, only some MeV.
- Light partons in Proton and  $\Lambda$  are basically the same to within the factor of 2.

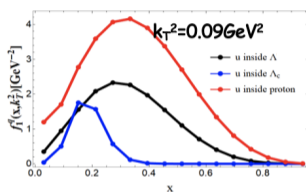
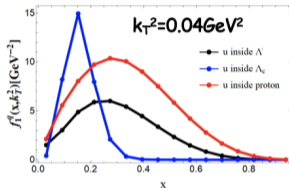
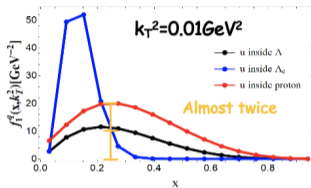


# Comparisons in the longitudinal direction

Electromagnetic structure of heavy baryons, 5 May 2022, 16:40

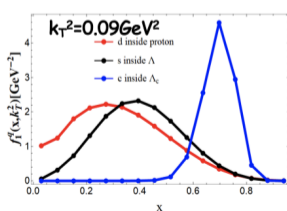
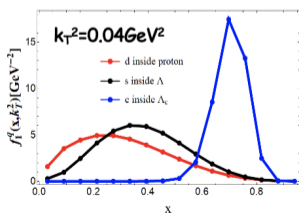
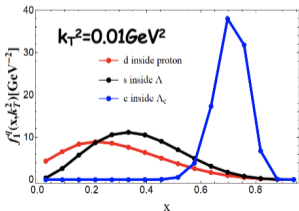
lighter quark

—●— u inside proton —●— u inside  $\Lambda$  —●— u inside  $\Lambda_c$



heavier quark

—●— d inside proton —●— s inside  $\Lambda$  —●— c inside  $\Lambda_c$



- Lighter quark vs heavier quark
- For proton, the difference of mass is small, only some MeV.
- Light partons in Proton and  $\Lambda$  are basically the same to within the factor of 2.
- For  $\Lambda$  and  $\Lambda_c$ , heavier parton have large  $x$ .



# Outline

- 1 BLFQ framework
- 2 TMDs in this study
- 3 Proton TMDs
- 4 Comparisons between TMDs of Proton,  $\Lambda$  and  $\Lambda_c$ —mass effect analysis
- 5 Conclusions
- 6 Appendix



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs
  - satisfy the universal Soffer-type bounds;



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs
  - satisfy the universal Soffer-type bounds;
  - don't support previously found model-dependent relations.



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs
  - satisfy the universal Soffer-type bounds;
  - don't support previously found model-dependent relations.
- After integrating over the transverse momentum we do get the correct PDFs from TMDs.



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs
  - satisfy the universal Soffer-type bounds;
  - don't support previously found model-dependent relations.
- After integrating over the transverse momentum we do get the correct PDFs from TMDs.
- Favour ratio comparison with lattice simulations are consistent.



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs
  - satisfy the universal Soffer-type bounds;
  - don't support previously found model-dependent relations.
- After integrating over the transverse momentum we do get the correct PDFs from TMDs.
- Favour ratio comparison with lattice simulations are consistent.
- Strong  $x$  and flavour dependence of  $\langle (p^\perp)^2 \rangle_f^q(x)$ .



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs
  - satisfy the universal Soffer-type bounds;
  - don't support previously found model-dependent relations.
- After integrating over the transverse momentum we do get the correct PDFs from TMDs.
- Favour ratio comparison with lattice simulations are consistent.
- Strong  $x$  and flavour dependence of  $\langle (p^\perp)^2 \rangle_f^q(x)$ .
- Our TMDs are



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs
  - satisfy the universal Soffer-type bounds;
  - don't support previously found model-dependent relations.
- After integrating over the transverse momentum we do get the correct PDFs from TMDs.
- Favour ratio comparison with lattice simulations are consistent.
- Strong  $x$  and flavour dependence of  $\langle (p^\perp)^2 \rangle_f^q(x)$ .
- Our TMDs are
  - consistent with Gaussian-type distributions in the small  $(p^\perp)^2$  region;



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs
  - satisfy the universal Soffer-type bounds;
  - don't support previously found model-dependent relations.
- After integrating over the transverse momentum we do get the correct PDFs from TMDs.
- Favour ratio comparison with lattice simulations are consistent.
- Strong  $x$  and flavour dependence of  $\langle (p^\perp)^2 \rangle_f^q(x)$ .
- Our TMDs are
  - consistent with Gaussian-type distributions in the small  $(p^\perp)^2$  region;
  - qualitatively consistent with the perturbative calculations in the large  $(p^\perp)^2$  region.



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs
  - satisfy the universal Soffer-type bounds;
  - don't support previously found model-dependent relations.
- After integrating over the transverse momentum we do get the correct PDFs from TMDs.
- Favour ratio comparison with lattice simulations are consistent.
- Strong  $x$  and flavour dependence of  $\langle (p^\perp)^2 \rangle_f^q(x)$ .
- Our TMDs are
  - consistent with Gaussian-type distributions in the small  $(p^\perp)^2$  region;
  - qualitatively consistent with the perturbative calculations in the large  $(p^\perp)^2$  region.
- For proton, the effect from mass of quark is small, and for heavy-light system, heavier quark possess larger  $x$ .



# Conclusions

- We start from an effective Hamiltonian to get the LFWF and thus the leading twist T-even TMDs of baryon.
- Our TMDs
  - satisfy the universal Soffer-type bounds;
  - don't support previously found model-dependent relations.
- After integrating over the transverse momentum we do get the correct PDFs from TMDs.
- Favour ratio comparison with lattice simulations are consistent.
- Strong  $x$  and flavour dependence of  $\langle (p^\perp)^2 \rangle_f^q(x)$ .
- Our TMDs are
  - consistent with Gaussian-type distributions in the small  $(p^\perp)^2$  region;
  - qualitatively consistent with the perturbative calculations in the large  $(p^\perp)^2$  region.
- For proton, the effect from mass of quark is small, and for heavy-light system, heavier quark possess larger  $x$ .
- Future research will focus on the inclusion of a non-trivial gauge link and also the calculations of cross section asymmetry.

# Thank You !

