

Heavy quarkonium production via matching of High Energy Factorization and NLO of Collinear Factorization¹

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Outline

1. η_c inclusive hadroproduction:

$$z = \frac{M^2}{\hat{s}} \ll 1$$

, resummation of:

$$\alpha_s^n \ln^{n-1} \frac{1}{z}$$

2. J/ψ **inclusive** photoproduction:

$$\eta = \frac{\hat{s} - M^2}{M^2} \gg 1$$

resummation of $\alpha_s^n \ln^{n-1} (1 + \eta)$

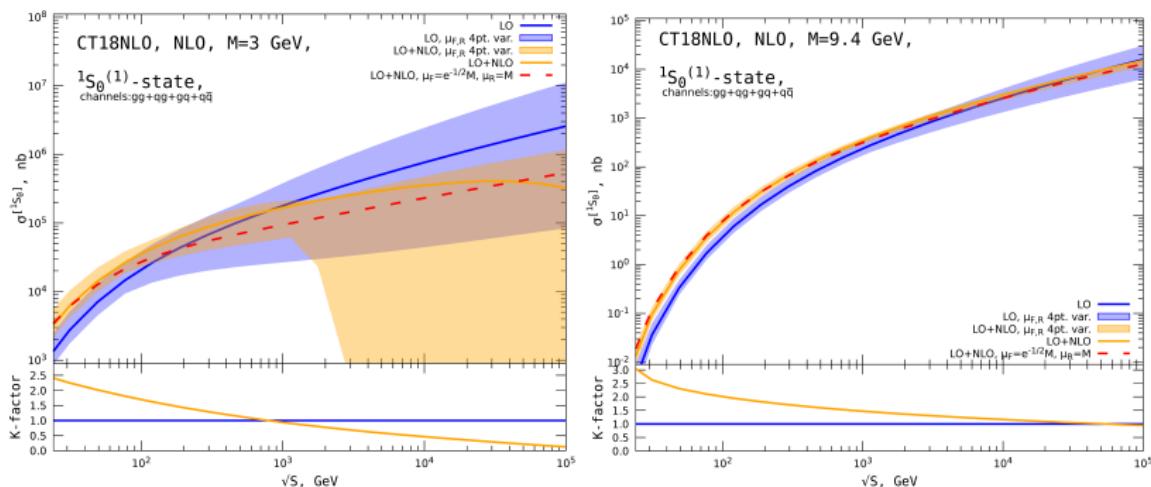
Part 1: η_c or η_b inclusive hadroproduction

Perturbative instability of the η_c total cross section

For the p_T -integrated cross section of η_Q hadroproduction, the LO partonic subprocess is simply:

$$g + g \rightarrow Q\bar{Q} \left[{}^1S_0^{(1)} \right].$$

The NLO correction can be computed in closed form [Kuhn, Mirkes, 93'; Petrelli *et.al.*, 98'], and:



Why?

Collinear factorization for total CS for the state $m =^{2S+1} L_J^{(0)}$:

$$\sigma^{[m]}(\sqrt{S}) = \int_{z_{\min}}^1 \frac{dz}{z} \mathcal{L}_{ij}(z, \mu_F) \hat{\sigma}_{ij}^{[m]}(z, \mu_F, \mu_R),$$

where $i, j = q, \bar{q}, g$, $z = M^2/\hat{s}$ and *partonic luminosity*:

$$\mathcal{L}_{ij}(z, \mu_F) = \int_{-y_{\max}}^{+y_{\max}} dy \tilde{f}_i\left(\frac{M}{\sqrt{Sz}} e^y, \mu_F\right) \tilde{f}_j\left(\frac{M}{\sqrt{Sz}} e^{-y}, \mu_F\right),$$

with $\tilde{f}_j(x, \mu_F)$ – momentum density PDFs.

NLO coefficient function [Kuhn, Mirkes, 93'; Petrelli *et.al.*, 98'] in the $z \rightarrow 0$ limit

$$\hat{\sigma}_{ij}^{[m]} = \sigma_{\text{LO}}^{[m]} \left[A_0^{[m]} \delta(1-z) + C_{ij} \frac{\alpha_s(\mu_R)}{\pi} \left(A_0^{[m]} \ln \frac{M^2}{\mu_F^2} + A_1^{[m]} \right) + O(z\alpha_s, \alpha_s^2) \right],$$

where $C_{gg} = 2C_A = 2N_c$, $C_{qg} = C_{gq} = C_F = (N_c^2 - 1)/(2N_c)$, $C_{q\bar{q}} = 0$ and $A_1^{[m]} < 0$.

Resummed coefficient function

Small parameter:
$$z = \frac{M^2}{\hat{s}}.$$

LLA ($\alpha_s^n \ln^{n-1} \frac{1}{z}$) in High-Energy Factorization [Collins, Ellis, 91'; Catani, Ciafaloni, Hautmann, 91', 94']:

$$\begin{aligned} \hat{\sigma}_{ij}^{[m], \text{ HEF}}(z, \mu_F, \mu_R) &= \int_{-\infty}^{\infty} d\eta \int_0^{\infty} d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2 \mathcal{C}_{gi} \left(\frac{M_T}{M} \sqrt{z} e^{\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R \right) \\ &\times \mathcal{C}_{gj} \left(\frac{M_T}{M} \sqrt{z} e^{-\eta}, \mathbf{q}_{T2}^2, \mu_F, \mu_R \right) \int_0^{2\pi} \frac{d\phi}{2} \frac{H^{[m]}(\mathbf{q}_{T1}^2, \mathbf{q}_{T2}^2, \phi)}{M_T^4} + O(z) + \text{NLL}, \end{aligned}$$

The coefficient functions $H^{[m]}$ are known at LO in α_s [Hagler *et.al*, 2000; Kniehl, Vasin, Saleev 2006] for $m = {}^1S_0^{(1,8)}, {}^3P_J^{(1,8)}, {}^3S_1^{(8)}$.

The $H^{[m]}$ is a tree-level “squared matrix element” of the $2 \rightarrow 1$ -type process:

$$R_+(\mathbf{q}_{T1}, q_1^+) + R_-(\mathbf{q}_{T2}, q_2^-) \rightarrow c\bar{c}[m].$$

LLA evolution w.r.t. $\ln 1/z$

In the LL($\ln 1/z$)-approximation, the $Y = \ln 1/z$ -evolution equation for *collinearly un-subtracted* $\tilde{\mathcal{C}}$ -factor has the form:

$$\tilde{\mathcal{C}}(x, \mathbf{q}_T) = \delta(1-z)\delta(\mathbf{q}_T^2) + \hat{\alpha}_s \int_x^1 \frac{dz}{z} \int d^{2-2\epsilon} \mathbf{k}_T K(\mathbf{k}_T^2, \mathbf{q}_T^2) \tilde{\mathcal{C}}\left(\frac{x}{z}, \mathbf{q}_T - \mathbf{k}_T\right)$$

with $\hat{\alpha}_s = \alpha_s C_A / \pi$ and

$$K(\mathbf{k}_T^2, \mathbf{p}_T^2) = \delta^{(2-2\epsilon)}(\mathbf{k}_T) \frac{(\mathbf{p}_T^2)^{-\epsilon}}{\epsilon} \frac{(4\pi)^\epsilon \Gamma(1+\epsilon) \Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} + \frac{1}{\pi (2\pi)^{-2\epsilon} \mathbf{k}_T^2}.$$

It is convenient to go from (z, \mathbf{q}_T) -space to (N, \mathbf{x}_T) -space:

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = \int d^{2-2\epsilon} \mathbf{q}_T e^{i\mathbf{x}_T \cdot \mathbf{q}_T} \int_0^1 dx x^{N-1} \tilde{\mathcal{C}}(x, \mathbf{q}_T),$$

because:

- ▶ Mellin convolutions over z turn into products: $\int \frac{dz}{z} \rightarrow \frac{1}{N}$
- ▶ Large logs map to poles at $N = 0$: $\boxed{\alpha_s^{k+1} \ln^{\textcolor{red}{k}} \frac{1}{z} \rightarrow \frac{\alpha_s^{k+1}}{N^{\textcolor{red}{k+1}}}}$
- ▶ All *collinear divergences* are contained inside \mathcal{C} in \mathbf{x}_T -space.

Exact LL solution

In (N, \mathbf{q}_T) -space, subtracted \mathcal{C} , which resums all terms $\propto (\hat{\alpha}_s/N)^n$ has the form:

$$\mathcal{C}(N, \mathbf{q}_T, \mu_F) = R(\gamma_{gg}(N, \alpha_s)) \frac{\gamma_{gg}(N, \alpha_s)}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_{gg}(N, \alpha_s)},$$

where $\gamma_{gg}(N, \alpha_s)$ is the solution of [Jaroszewicz, 82]:

$$\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}(N, \alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma),$$

where $\psi(\gamma) = d \ln \Gamma(\gamma) / d\gamma$ – Euler's ψ -function. The first few terms:

$$\gamma_{gg}(N, \alpha_s) = \underbrace{\frac{\hat{\alpha}_s}{N}}_{\text{DLA}} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \dots$$

$\overbrace{\hspace{10em}}$
LLA

The function $R(\gamma)$ is

$$R(\gamma_{gg}(N, \alpha_s)) = 1 + O(\alpha_s^3).$$

Does this work?

The resummation has to reproduce the $A_1^{[m]}$ NLO coefficient when expanded up to NLO in α_s . And it does. We have performed expansion up to NNLO:

State	$A_0^{[m]}$	$A_1^{[m]}$	$A_2^{[m]}$	$B_2^{[m]}$
1S_0	1	$\textcolor{red}{-1}$	$\frac{\pi^2}{6}$	$\frac{\pi^2}{6}$
3S_1	0	1	0	$\frac{\pi^2}{6}$
3P_0	1	$\textcolor{red}{-\frac{43}{27}}$	$\frac{\pi^2}{6} + \frac{2}{3}$	$\frac{\pi^2}{6} + \frac{40}{27}$
3P_1	0	$\frac{5}{54}$	$-\frac{1}{9}$	$-\frac{2}{9}$
3P_2	1	$\textcolor{red}{-\frac{53}{36}}$	$\frac{\pi^2}{6} + \frac{1}{2}$	$\frac{\pi^2}{6} + \frac{11}{9}$

for e.g.

$$\hat{\sigma}_{gg}^{[m], \text{HEF}}(z \rightarrow 0) = \sigma_{\text{LO}}^{[m]} \left\{ A_0^{[m]} \delta(1-z) + \frac{\alpha_s}{\pi} 2C_A \left[A_1^{[m]} + A_0^{[m]} \ln \frac{M^2}{\mu_F^2} \right] + \left(\frac{\alpha_s}{\pi} \right)^2 \ln \frac{1}{z} \cdot C_A^2 \left[2A_2^{[m]} + B_2^{[m]} + 4A_1^{[m]} \ln \frac{M^2}{\mu_F^2} + 2A_0^{[m]} \ln^2 \frac{M^2}{\mu_F^2} \right] + O(\alpha_s^3) \right\},$$

Matching with NLO of CF

The HEF works only at $z \ll 1$, misses power corrections $O(z)$, while NLO CF is exact in z , but only NLO in α_s . **We need to match them.**

- ▶ Simplest prescription: just subtract the overlap at $z \ll 1$:

$$\begin{aligned}\sigma_{\text{NLO+HEF}}^{[m]} = & \sigma_{\text{LO CF}}^{[m]} + \int_{z_{\min}}^1 \frac{dz}{z} \left[\check{\sigma}_{\text{HEF}}^{[m],ij}(z) \right. \\ & \left. + \hat{\sigma}_{\text{NLO CF}}^{[m],ij}(z) - \hat{\sigma}_{\text{NLO CF}}^{[m],ij}(0) \right] \mathcal{L}_{ij}(z),\end{aligned}$$

- ▶ Or introduce **smooth weights**:

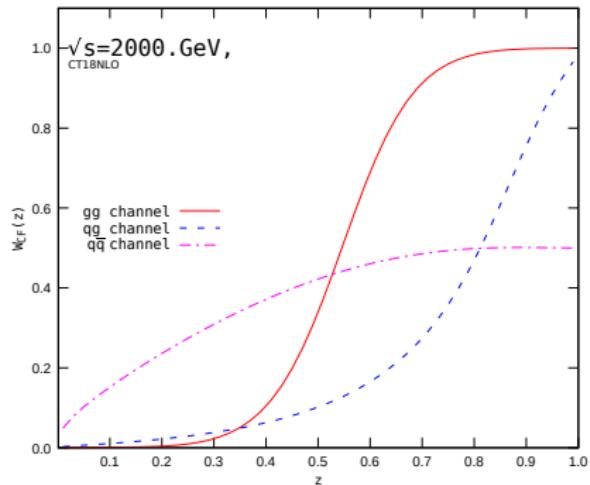
$$\begin{aligned}\sigma_{\text{NLO+HEF}}^{[m]} = & \sigma_{\text{LO CF}}^{[m]} + \int_{z_{\min}}^1 dz \left\{ \left[\check{\sigma}_{\text{HEF}}^{[m],ij}(z) \frac{\mathcal{L}_{ij}(z)}{z} \right] w_{\text{HEF}}^{ij}(z) \right. \\ & \left. + \left[\hat{\sigma}_{\text{NLO CF}}^{[m],ij}(z) \frac{\mathcal{L}_{ij}(z)}{z} \right] (1 - w_{\text{HEF}}^{ij}(z)) \right\},\end{aligned}$$

Inverse error weighting method

In the InEW method [Echevarria, et.al., 2018] the weights are calculated from **estimates of the error** of each contribution:

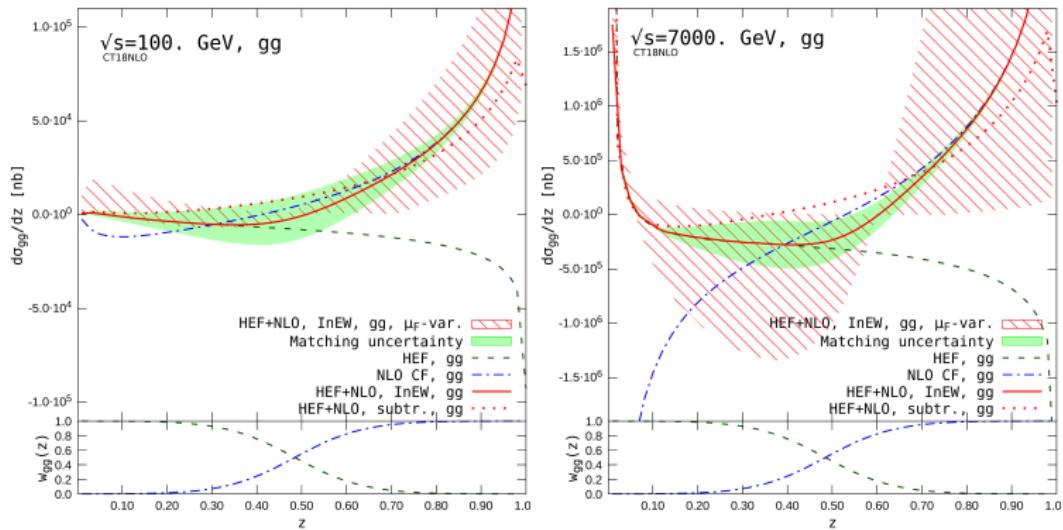
$$w_{\text{HEF}}^{ij}(z) = \frac{[\Delta\sigma_{\text{HEF}}^{ij}(z)]^{-2}}{[\Delta\sigma_{\text{HEF}}^{ij}(z)]^{-2} + [\Delta\sigma_{\text{CF}}^{ij}(z)]^{-2}},$$

- ▶ For $\Delta\sigma_{\text{CF}}$ we take the NNLO $\alpha_s^2 \ln \frac{1}{z}$ term of $\hat{\sigma}(z)$ predicted by HEF,
- ▶ For $\Delta\sigma_{\text{HEF}}$ we take the $\alpha_s O(z)$ part of the NLO CF result for $\hat{\sigma}(z)$.
- ▶ In both cases, stability against $O(\alpha_s^2)$ (constant in z , unknown) corrections is checked

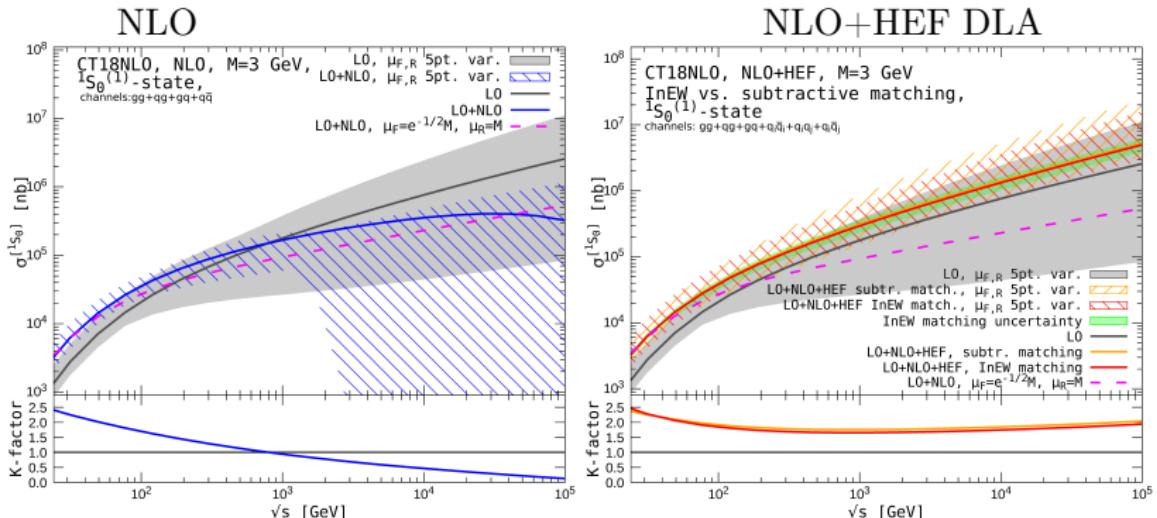


Matching plots

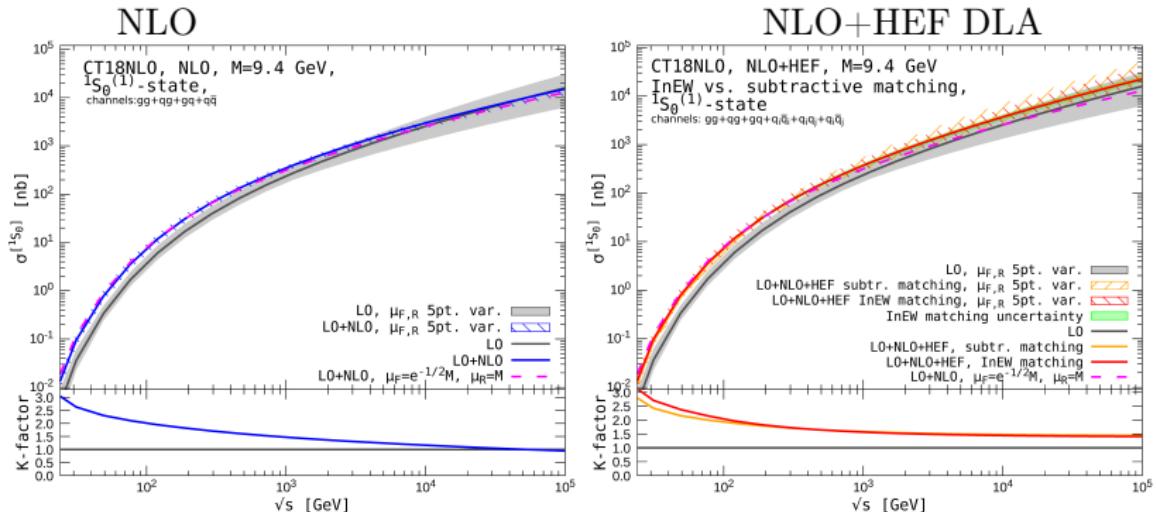
Plots of the integrand of the total cross section (gg channel) as function of $z = M^2/\hat{s}$:



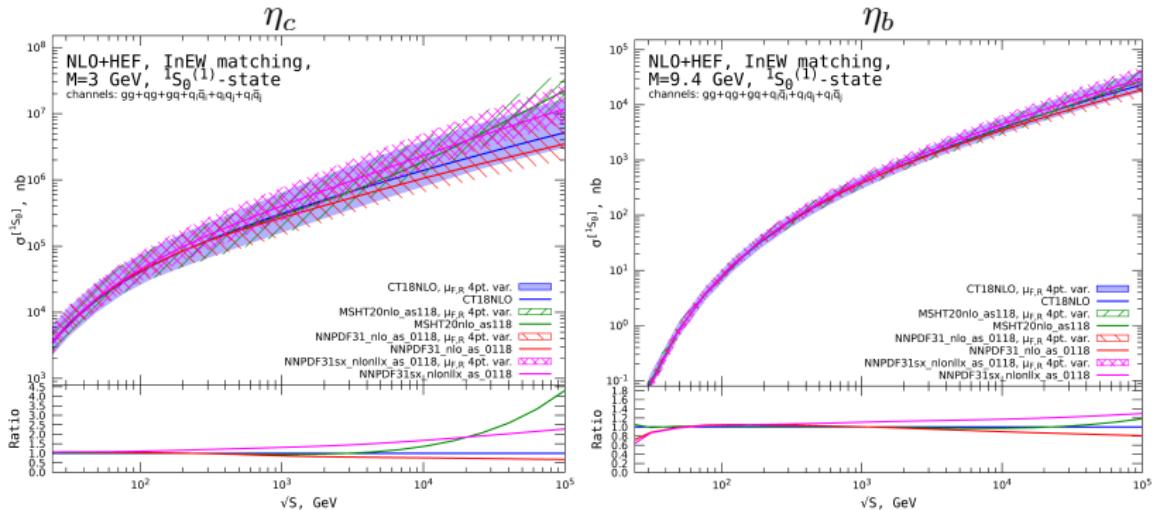
Matched results for η_c



Matched results for η_b



The PDF dependence



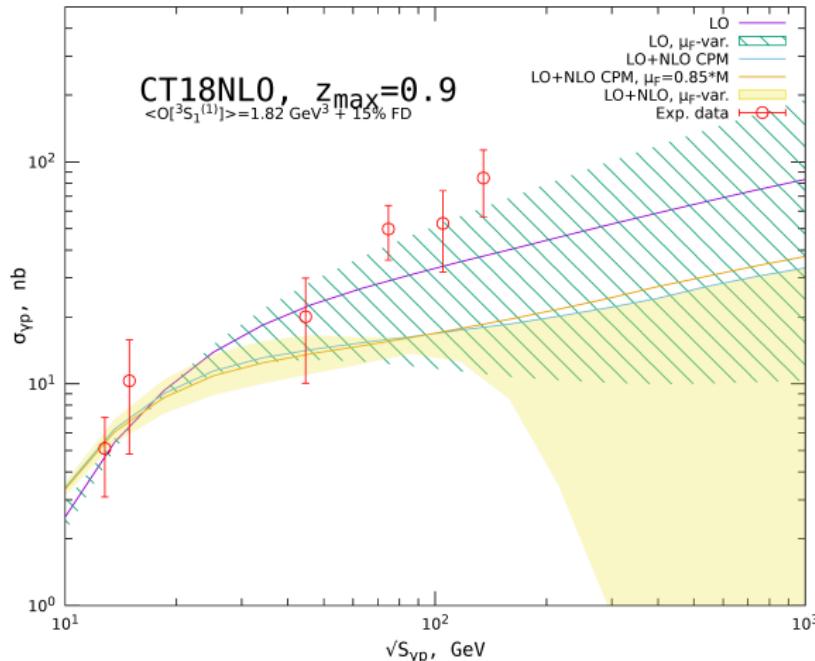
Part 2: J/ψ **inclusive** photoproduction

$$\gamma(q) + p(P) \rightarrow J/\psi(p) + X \text{ @ NLO}$$

The LO CS partonic process is

$$\gamma + g \rightarrow c\bar{c} \left[{}^3S_1^{(1)} \right] + g.$$

The CS contribution is > 50% of p_T -diff. cross section even at NLO.
For p_T -integrated CS one has:

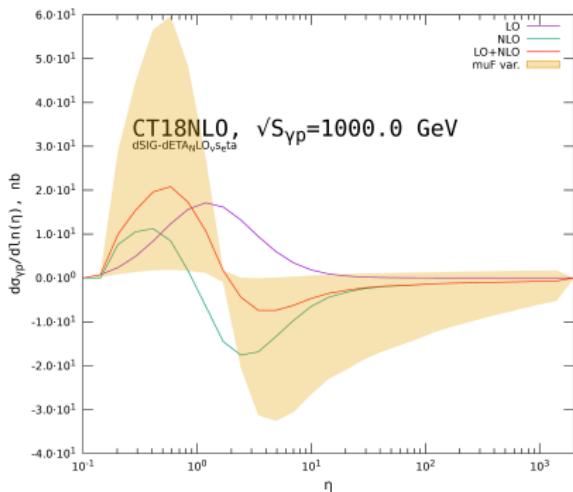
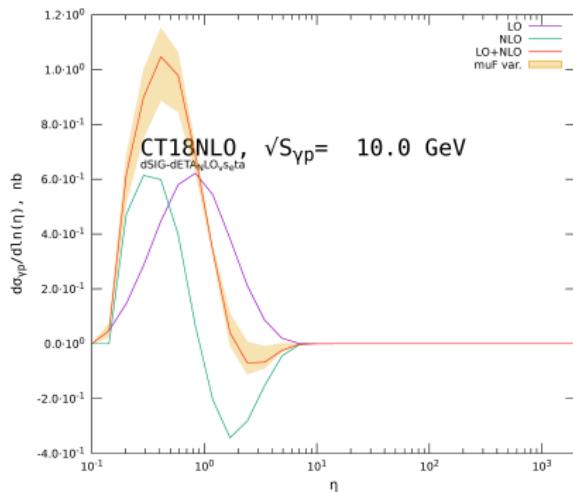


Why?

$$\frac{d\sigma_{\gamma p}}{dz} = \frac{M^2}{S_{\gamma p}} \int_0^{S_{\gamma p}/M^2 - 1} d\eta \ f_i \left(\frac{M^2}{S_{\gamma p}}(\eta + 1), \mu_F \right) \frac{d\hat{\sigma}_{i\gamma}(\eta, z)}{dz},$$

where $\boxed{z = \frac{Pp}{Pq} = \frac{p^-}{q^-}, \ \eta = \frac{\hat{s}}{M^2} - 1}$ with $\hat{s} = S_{\gamma p}x$.

Plots of the integrand:

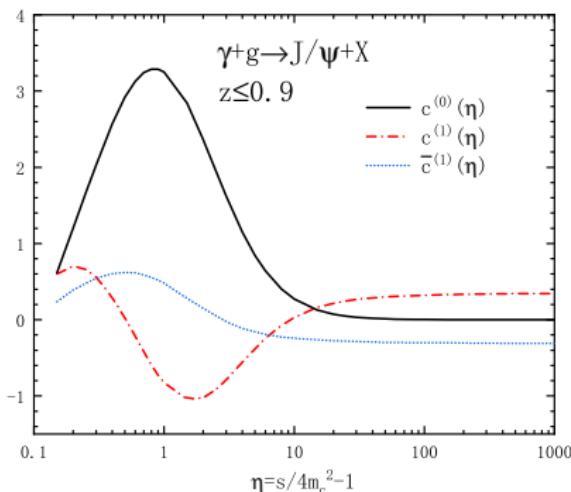


Asymptotics $\hat{\sigma}_{\text{NLO}}(\eta \rightarrow \infty)$

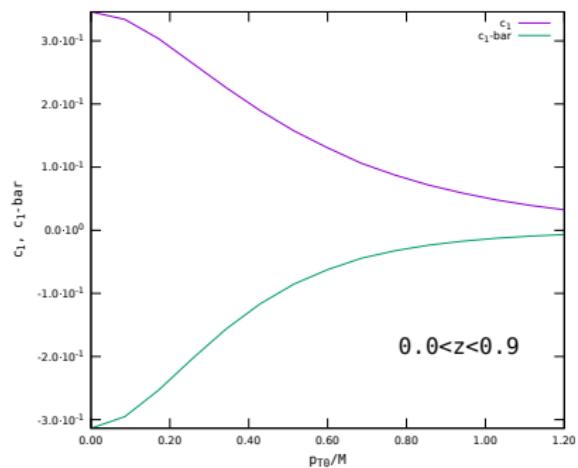
[Kraemer, 1995]:

$$\hat{\sigma}_{i\gamma} \propto c_{i\gamma}^{(0)}(\eta) + 4\pi\alpha_s \left[c_{i\gamma}^{(1)}(\eta) + \bar{c}_{i\gamma}^{(1)}(\eta) \ln \frac{\mu_F^2}{m_c^2} + \frac{\beta_0}{8\pi^2} c_{i\gamma}^{(0)}(\eta) \right]$$

Numerical NLO result
(FDC code, Yu Feng)



$c^{(1)}$ and $\bar{c}^{(1)}$ at $\eta \rightarrow \infty$ from HEF as function of $p_T^{J/\psi}$



$\gamma(q) + p(P) \rightarrow J/\psi(p) + X$ in HEF

HEF resummed partonic cross section:

$$\frac{d\hat{\sigma}_{i\gamma}^{\text{HEF}}(\eta, z)}{dz} = \frac{1}{2zM^2} \int_{1/z}^{1+\eta} \frac{dy}{y} \int_0^\infty d\mathbf{q}_T^2 \mathcal{C}_{gi} \left(\frac{y}{\eta+1}, \mathbf{q}_T^2, \mu_F, \mu_R \right) \mathcal{H}(y, \mathbf{q}_T^2, z),$$

actually resums $\ln \frac{1}{z_+} = \ln \frac{\eta+1}{y}$. Is resummation of only $\ln(1 + \eta)$ possible? Yes.

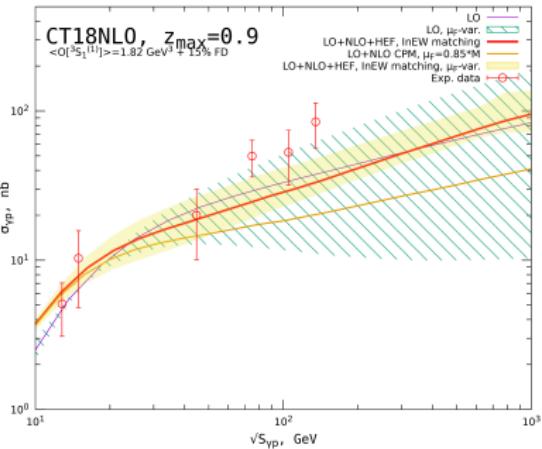
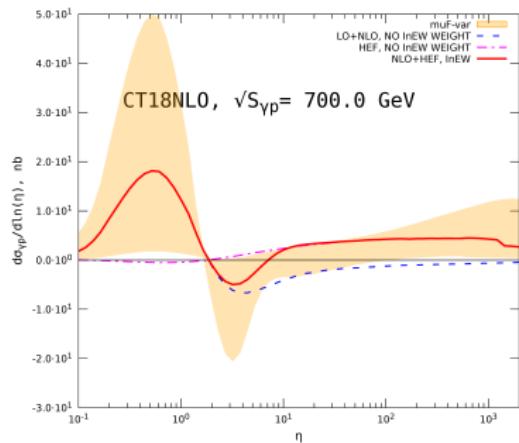
The \mathcal{H} is the integral of the HEF coefficient function (H):

$$R_+(\mathbf{q}_{T1}, q_1^+) + \gamma(q) \rightarrow c\bar{c} \left[{}^3S_1^{(1)} \right] (p) + g(k),$$

over the PS of the gluon ($y = q_1^+ q^- / M^2$):

$$\begin{aligned} \mathcal{H}(y, \mathbf{q}_{T1}^2, z) &= \int_0^\infty \frac{dk^+ dk^-}{2(2\pi)^2} \int d^2 \mathbf{p}_T H(\hat{s}, \hat{t}, \hat{u}, (\mathbf{q}_{T1} \cdot \mathbf{p}_T), \mathbf{q}_{T1}^2) \\ &\times \delta(q^-(1-z) - k^-) \delta(q_1^+ - \frac{M_T^2}{q_- z} - k^+) \delta(k^+ k^- - (\mathbf{q}_{T1} - \mathbf{p}_T^2)^2), \end{aligned}$$

InEW matching results



Analytic results for $\eta \rightarrow \infty$ asymptotics of $d\hat{\sigma}/dp_T$

Can be derived via expansion of HEF formula up to NLO and applying **IBP-reduction** to \mathbf{q}_T -integrals.

$$\begin{aligned} & \frac{dc_1(z, \rho, \eta \rightarrow \infty)}{dz d\rho} = c_1^{(\text{R})}(z, \rho) \\ & + c_1^{(1)}(z, \rho) \ln \left[\frac{z^2(1-z)^2}{(\rho + (1-z)^2)^2} \right] + c_1^{(2)}(z, \rho) \ln \left[\frac{(\rho+1-z)^2}{(1-z)(\rho+2-z)} \right] \\ & + \frac{\tilde{c}_1^{(3)}(z, \rho)}{\sqrt{(1+\rho)((2-3z)^2 + (2-z)^2\rho)}} \\ & \times \ln \left[\frac{\rho(2-z) - (3-2z)z + 2 - \sqrt{(\rho+1)(\rho(z-2)^2 + (2-3z)^2)}}{\rho(2-z) - (3-2z)z + 2 + \sqrt{(\rho+1)(\rho(z-2)^2 + (2-3z)^2)}} \right], \end{aligned}$$

where $\rho = \mathbf{p}_T^2/M^2$ and

$$\begin{aligned} c_1^{(1)}(z, \rho) &= \frac{-z^3}{(\rho+1)^2 (\rho+(z-1)^2)^2 (\rho-2z+1)^4} \\ &\times \left\{ 5(\rho+1)^4 + 4(2\rho+1)z^6 - (\rho+1)(23\rho+31)z^5 + (\rho+1)(\rho(12\rho+77)+89)z^4 \right. \\ &- 2(\rho+1)(\rho+3)(\rho(\rho+18)+21)z^3 + 2(\rho+1)^2(\rho(3\rho+32)+47)z^2 \\ &\left. - (\rho+1)^3(11\rho+35)z \right\}, \end{aligned}$$

and so on...

Analytic results for $\eta \rightarrow \infty$, $\rho \gg 1$ asymptotics

was mentioned in my “pheno” talk on Tuesday:

$$\begin{aligned}\rho \rightarrow \infty \quad : \quad & \frac{dc_1(z, \rho, \eta \rightarrow \infty)}{dz d\rho} = -\frac{2(z-1)z}{\rho^3(z-2)^2} \\ & \times \left\{ z^4 - 2z^3 - z^2 \ln \left[\frac{(1-z)z^2}{\rho(2-z)^2} \right] - 4z + 4 \right\} + O(\rho^{-4}), \\ & \frac{d\bar{c}_1(z, \rho, \eta \rightarrow \infty)}{d\rho dz} = \frac{2z(1-z)}{\rho^4} (1 - z(1-z))^2 + O(\rho^{-5}),\end{aligned}$$

$$\hat{\sigma}_{i\gamma} \propto c_{i\gamma}^{(0)}(\eta) + 4\pi\alpha_s \left[c_{i\gamma}^{(1)}(\eta) + \bar{c}_{i\gamma}^{(1)}(\eta) \ln \frac{\mu_F^2}{m_c^2} + \frac{\beta_0}{8\pi^2} c_{i\gamma}^{(0)}(\eta) \right]$$

Conclusions and outlook

- ▶ **Message 1:** Quarkonium production at $p_T \sim M \ll \sqrt{S}$ is the unique part of collider phenomenology where BFKL-type resummation is not just desirable, but unavoidable.
- ▶ **Message 2:** NLO corrections with $\hat{s} \sim M^2$ are as numerically important as those with $\hat{s} \gg M^2$ at any \sqrt{S} . Matching between HEF and NLO CF is always required!
- ▶ The high-energy instability of the NLO cross section is related with lack of the $\alpha_s^n \ln^{n-1} \frac{\hat{s}}{M^2}$ corrections in $\hat{\sigma}$ at $\hat{s} \ll M^2$.
- ▶ The HEF at DLA is the formalism to solve this problem if the standard fixed-order PDFs are to be used.
- ▶ Matching between NLO CF and HEF has to be performed.
- ▶ NLO CF+NLL HEF calculation is in progress.
- ▶ Future plans:
 - ▶ χ_{cJ} production in DLA+NLO CF
 - ▶ rapidity distributions in DLA+NLO CF
 - ▶ p_T -distribution of J/ψ in DLA+NLO CF
 - ▶ next-to-DLA corrections...

Thank you for your attention!