

# *Inclusive $J/\Psi$ and $\Upsilon$ emissions from single-parton fragmentation in hybrid high-energy/collinear factorization*

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in collaboration with  
F.G. Celiberto  
based on:  
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DIS2022, Santiago De Compostela, 5 May 2022



# *Outline*

## *Introduction and motivations*

BFKL resummation

Hybrid collinear/high-energy factorization

## *J/ $\psi$ and $\Upsilon$ production*

J/ $\psi$  plus jet production at low- $p_T$

J/ $\psi$  and  $\Upsilon$  production from single parton fragmentation

## *Summary and outlook*

# Motivation

- Heavy flavor physics has long been considered as a perfect framework for testing perturbative QCD at colliders, due to the smallness of the running coupling
- However, at modern colliders, heavy-flavor production enters a two-scale regime, called semi-hard
- Semi-hard collision process, featuring the scale hierarchy

$$s \gg Q^2 \gg \Lambda_{\text{QCD}}^2, \quad Q^2 \text{ a hard scale,}$$

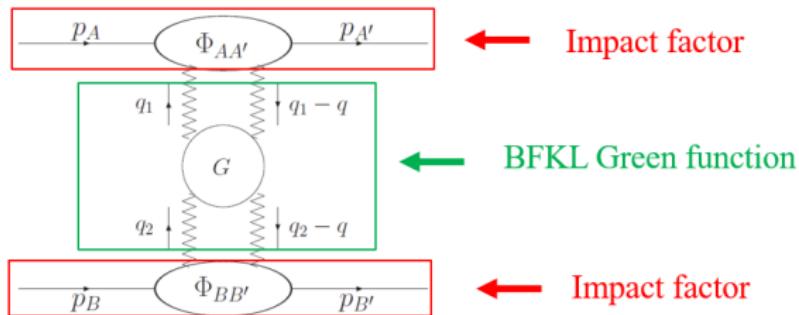
↗  
Regge kinematical region

$$\alpha_s(Q^2) \ln \left( \frac{s}{Q^2} \right) \sim 1 \implies \text{all-order resummation needed}$$

- The **Balitsky-Fadin-Kuraev-Lipatov (BFKL)** approach is the general framework for this resummation
  - Leading-logarithm-Approximation (LLA):  $(\alpha_s \ln s)^n$
  - Next-to-leading-logarithm-Approximation (NLA):  $\alpha_s (\alpha_s \ln s)^n$

BFKL resummation

- Diffusion  $A + B \rightarrow A' + B'$  in the **Regge kinematical region**
  - Gluon Reggeization
  - BFKL factorization for  $\Im \mathcal{A}_{AB}^{A'B'}$ : convolution of a **Green function** (process independent) with the **Impact factors** of the colliding particles (process dependent)



$$\Im \mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^2(\vec{q}_1 - \vec{q})^2} \frac{d^{D-2}q_2}{\vec{q}_2^2(\vec{q}_2 - \vec{q})^2} \\ \times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[ \left( \frac{s}{s_0} \right)^\omega G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}) \right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2, \vec{q}, s_0)$$

# Pomeron channel

- **BFKL equation:**  $\vec{q}^2 = 0$  and singlet color state representation  
[Ya. Ya. Balitsky, V. S. Fadin, E.A Kuraev, L.N Lipatov (1975)]

Redefinition :  $G_\omega(\vec{q}_1, \vec{q}_2) \equiv \frac{G_\omega^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^2 \vec{q}_2^2}, \quad \mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^2 \vec{q}_2^2}$

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2} q_r \mathcal{K}(\vec{q}_1, \vec{q}_r) G(\vec{q}_r, \vec{q}_2)$$

- **Elastic amplitude** factorization:

$$\begin{aligned} \Im \mathcal{A}_{AB}^{AB} &= \frac{s}{(2\pi)^{D-2}} \int d^{D-2} q_1 d^{D-2} q_2 \\ &\times \frac{\Phi_{AA}^{(0)}(\vec{q}_1, s_0)}{\vec{q}_1^2} \int \frac{d\omega}{2\pi i} \left[ \left( \frac{s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2) \right] \frac{\Phi_{BB}^{(0)}(-\vec{q}_2, s_0)}{\vec{q}_2^2} \end{aligned}$$

- **Optical Theorem:**

$$\sigma_{AB} = \frac{\Im \mathcal{A}_{AB}^{AB}}{s}$$

- Impact factor in the color singlet state:

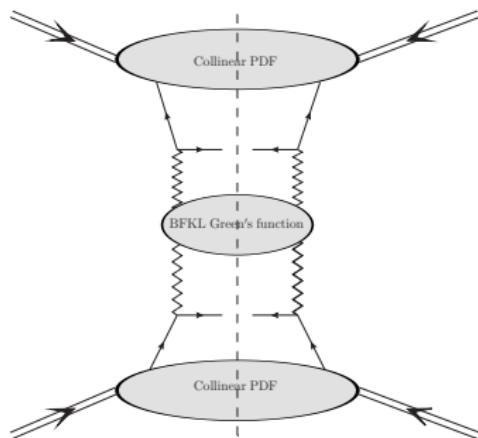
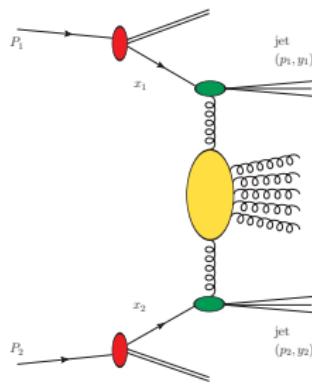
$$\Phi_{PP}^{(0)} = \langle cc' | \hat{\mathcal{P}} | 0 \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} d\rho_f \Gamma_{\{f\}P}^c (\Gamma_{\{f\}P}^{c'})^*$$

# Hybrid collinear/high-energy factorization

- Straightforward adaptation to partially inclusive processes: just restrict the summation over intermediate states

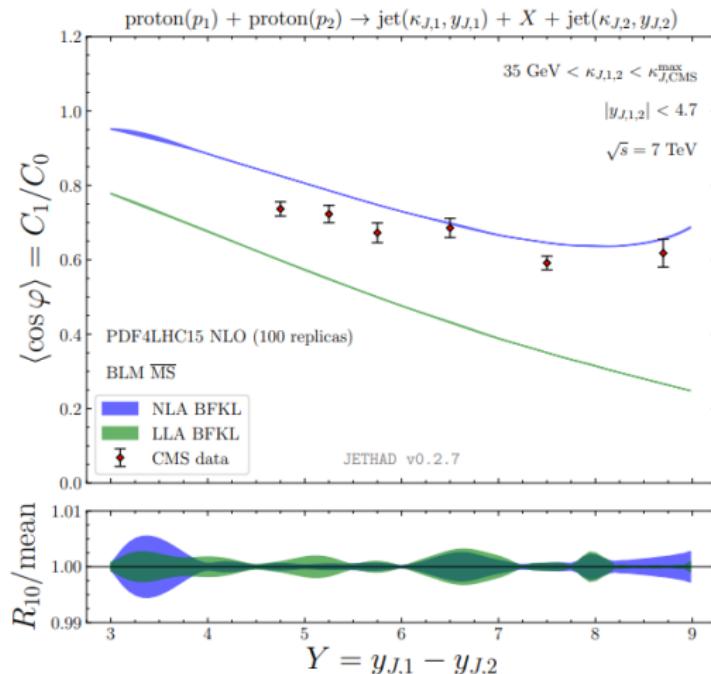
## Mueller-Navelet jets

- Inclusive production of two rapidity-separated jets in proton-proton collision
- Large energy logarithms  $\rightarrow$  BFKL resummed partonic cross section
- Moderate values of parton  $x \rightarrow$  collinear PDFs



- Hybrid formalism: can be extended to several type of semi-hard reactions

# Muller-Navelet: Theory vs Experiment



[B. Ducloué, L. Szymanowski, S. Wallon (2013)]

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

In this slide: [F.G. Celiberto (2021)]

# Mueller-Navelet: Theory vs Experiment

- CMS @7Tev with symmetric  $p_T$ -ranges, only!  
[CMS collaboration (2016)]
- LHC kinematic **domain** in between the sectors described by BFKL and DGLAP approaches
- Clearer manifestation of high-energy signatures expected at increasing energies (higher hadronic center-of-mass energy or higher rapidity difference between tagged jets)
- Need for more exclusive final states as well as more sensitive observables

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- Need for more exclusive final states as well as more sensitive observables
- Strong manifestation of higher-order **instabilities** via scale variation

NLA BFKL corrections to cross section with opposite sign with respect to the leading order (LO) result and large in absolute value...

- ◊ ...call for some optimization procedure...
- ◊ ...choose scales to mimic the most relevant subleading terms

- **BLM** [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]

- ✓ preserve the conformal invariance of an observable...
- ✓ ...by making vanish its  $\beta_0$ -dependent part

- "Exact" BLM:

suppress    NLO IFs    +    NLO Kernel     $\beta_0$ -dependent factors

# Partially inclusive processes in NLA

- Mueller-Navelet jet production

[J. Bartels, D. Colferai, G.P. Vacca (2003)]

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2011)]

[D.Yu. Ivanov, A. Papa (2012)]

[D. Colferai, A. Niccoli (2015)]

[B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

- Light hadron-light hadron production

[D.Yu. Ivanov, A. Papa (2012)]

[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]

- Light hadron-jet production

[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

- Heavy-light hadrons in VFNS

[F.G. Celiberto, M.F, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2021)]

[F.G. Celiberto, M.F, D.Yu. Ivanov, A. Papa (2021)]

# Other partially-inclusive reactions

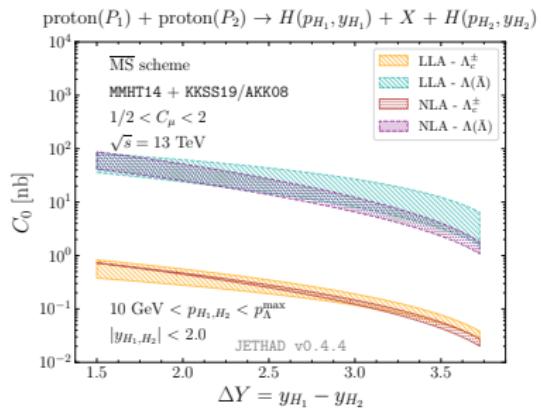
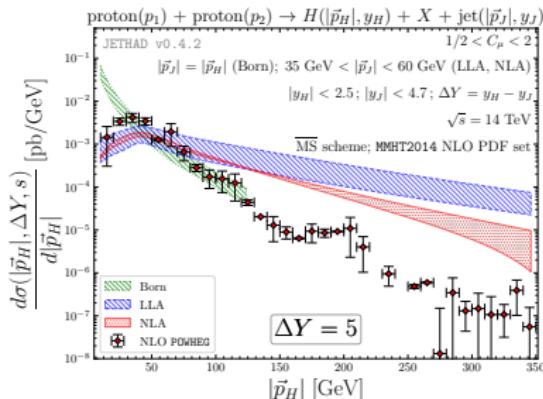
- Three / four jet production (partial NLA)
  - [F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2016)]
  - [F. Caporale, F.G. Celiberto, G. Chachamis, A. Sabio Vera (2016)]
  - [F. Caporale, F.G. Celiberto, G. Chachamis, D.G. Gomez, A. Sabio Vera (2016, 2017)]
- Drell-Yan pair - jet (partial NLA)
  - [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]
- Higgs - jet
  - Partial NLA
    - [F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2020)]
    - Full NLA,  $m_t \rightarrow \infty$  limit
      - [F.G. Celiberto, D.Yu. Ivanov, M.F., M.M.A. Mohammed, A. Papa (2022, in preparation)]
- Heavy-quark pair photo/hadro-production (partial NLA)
  - [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]
  - [A.D. Bolognino, F.G. Celiberto, M.F., D.Yu. Ivanov, A. Papa (2019)]
- $J/\Psi$  - jet production (partial NLA)
  - [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

# Stabilization effects

- Stabilization effects in Higgs and heavy flavor production

- $\Lambda$ -baryon FFs

- heavy species  $\longrightarrow \Lambda_c$   
KKSS19 [B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger (2020)]
- light species  $\longrightarrow \Lambda$   
AKK08 [S.Albino, B.A. Kniehl, and G. Kramer (2008)]

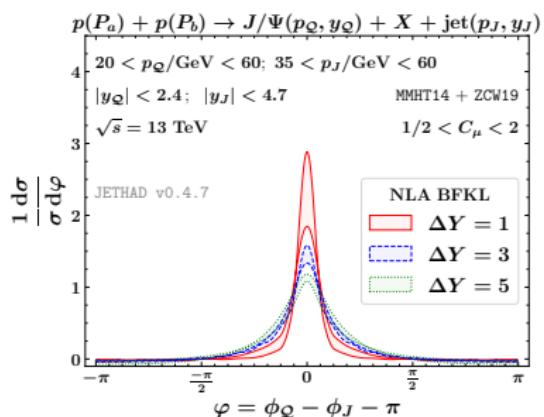
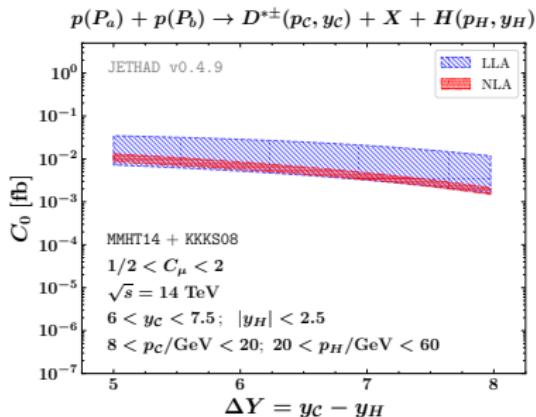


[F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2020)]

[F.G. Celiberto, D.Yu. Ivanov, M. F., A. Papa (2021)]

# $D^*$ plus Higgs production at the FPF

- ATLAS + FPF ultra-forward regime: Strong high-energy enhancement
- Final state with two stabilizers: Heavy-flavor and Higgs
- FPF studies on high-energy QCD: see yesterday's WG6 talk by **J. Rojo**



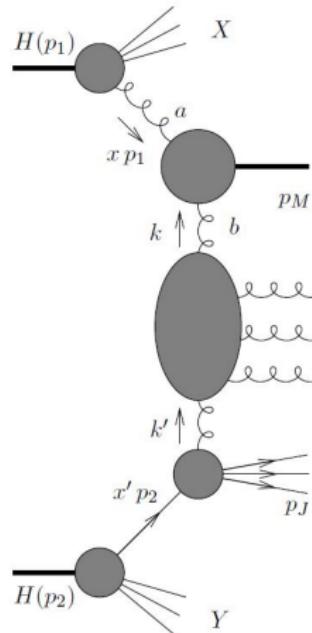
[FPF Collaboration (2022)]

[F. G. Celiberto, M.F., M. M. A. Mohammed (in preparation)]

# $J/\psi$ plus jet production at low- $p_T$

- Process: proton( $p_1$ ) + proton( $p_2$ )  $\rightarrow J/\psi + X + \text{jet}$

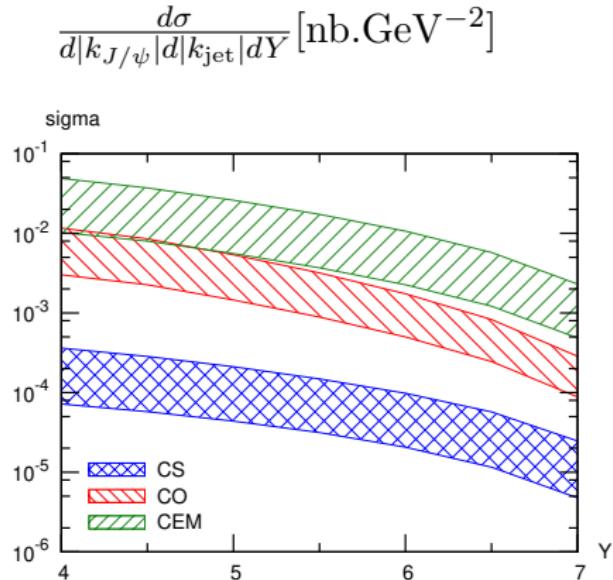
- **hybrid collinear/BFKL approach**
- high-energy hadroproduction of a  $J/\Psi$  meson and a jet, with a remnant  $X$
- both the  $J/\Psi$  and the jet emitted with large transverse momenta and well separated in rapidity
- NLA BFKL + NLO jet + LO  $J/\Psi$ 
  - LO  $J/\Psi$  IF calculated in **NRQCD** (Color-singlet and Color-octet)
  - LO  $J/\Psi$  IF calculated in **color evaporation model (CEM)**
- Realistic CMS and CASTOR rapidity ranges, fixed  $p_T$  final states



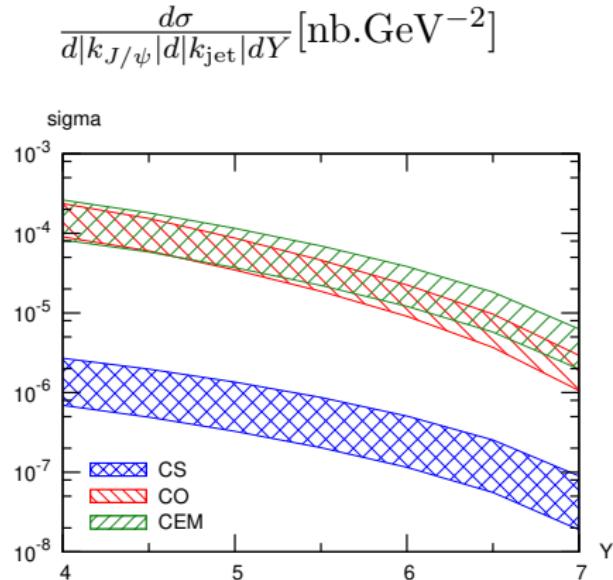
[R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

# $J/\psi$ plus jet production at low- $p_T$

- Realistic CMS and CASTOR rapidity ranges, fixed  $p_T$  final states



$$|k_{J/\psi}| = |k_{\text{jet}}| = 10 \text{ GeV}$$



$$|k_{J/\psi}| = |k_{\text{jet}}| = 20 \text{ GeV}$$

[R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]

# $J/\psi$ and $\Upsilon$ production from single parton fragmentation

- Process

$$p(P_a) + p(P_b) \rightarrow \mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}}) + X + \text{jet}(p_J, y_J)$$

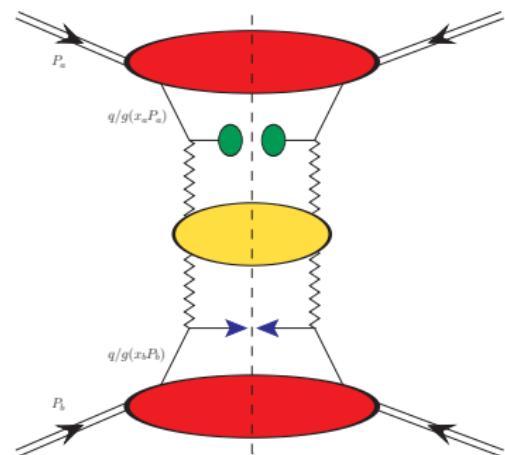
- Hybrid cross section

$$\begin{aligned} \frac{d\sigma}{dy_{\mathcal{Q}} dy_J d^2 \vec{p}_{\mathcal{Q}} d^2 \vec{p}_J} &= \frac{1}{(2\pi)^2} \\ &\times \int \frac{d^2 \vec{q}_1}{\vec{q}_1^2} V_{\mathcal{Q}}(\vec{q}_1, x_g, \vec{p}_{\mathcal{Q}}) \int \frac{d^2 \vec{q}_2}{\vec{q}_2^2} V_J(\vec{q}_2, x_J, \vec{p}_J) \\ &\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left( \frac{x_g x_J s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2) \end{aligned}$$

- Impact factors

$$V_{\mathcal{Q}}(\vec{q}_1, x_{\mathcal{Q}}, \vec{p}_J) = f_{q/g} \otimes H \otimes D_{q/g}^{\mathcal{Q}}$$

$$V_J(\vec{q}_2, x_J, \vec{p}_J) = f_{q/g} \otimes H \otimes \mathcal{J}_{q/g}$$



# $J/\psi$ and $\Upsilon$ production from single parton fragmentation

- Differential cross section in terms of azimuthal-angle coefficients

$$\frac{d\sigma}{dy_Q dy_J d|p_Q| d|p_J| d\phi_Q d\phi_J} = \frac{1}{(2\pi)^2} \left[ C_0 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) C_n \right]$$

- Unintegrated azimuthal-angle coefficients at NLO

$$\begin{aligned} C_n &= \int_0^{2\pi} d\phi_Q \int_0^{2\pi} d\phi_J \cos(n\varphi) \frac{d\sigma_{\text{NLA}}}{dy_Q dy_J d|\vec{p}_Q| d|\vec{p}_J| d\phi_Q d\phi_J} \\ &= \frac{e^{\Delta Y}}{s} \int_{-\infty}^{+\infty} d\nu e^{\Delta Y \bar{\alpha}_s(\mu_R) \left\{ \chi(n, \nu) + \bar{\alpha}_s(\mu_R) \left[ \bar{\chi}(n, \nu) + \frac{\beta_0}{8N_c} \chi(n, \nu) \left[ -\chi(n, \nu) + \frac{10}{3} + 4 \ln \left( \frac{\mu_R}{\sqrt{|\vec{p}_Q| |\vec{p}_J|}} \right) \right] \right] \right\}} \\ &\quad \times \alpha_s^2(\mu_R) \left[ c_Q^{\text{NLO}}(n, \nu, |\vec{p}_Q|, x_1) [c_J^{\text{NLO}}(n, \nu, |\vec{p}_J|, x_2)]^* + \bar{\alpha}_s^2(\mu_R) \Delta Y \frac{\beta_0}{4N_c} \chi(n, \nu) f(\nu) \right] \end{aligned}$$

- Azimuthal-angle coefficients at NLO

$$C_n = \int_{y_Q^{\min}}^{y_Q^{\max}} dy_Q \int_{y_J^{\min}}^{y_J^{\max}} dy_J \int_{p_Q^{\min}}^{p_Q^{\max}} d|\vec{p}_Q| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \delta(\Delta Y - y_Q - y_J) C_n(|\vec{p}_Q|, |\vec{p}_J|, y_Q, y_J)$$

# $J/\psi$ and $\Upsilon$ production from single parton fragmentation

- Single parton fragmentation function for  $J/\psi$  and  $\Upsilon$  at  $\mu_0 = 3m_Q$

$$D_Q^{\mathcal{Q}}(z, \mu_F \equiv \mu_0) = D_Q^{\mathcal{Q}, \text{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3} |\mathcal{R}_{\mathcal{Q}}(0)|^2 \Gamma^{\mathcal{Q}, \text{NLO}}(z)$$

- LO fragmentation function

$$D_Q^{\mathcal{Q}, \text{LO}}(z) = \frac{\alpha_s^2(\mu_R)}{m_Q^3} \frac{8z(1-z)^2}{27\pi(2-z)^6} |\mathcal{R}_{\mathcal{Q}}(0)|^2 (5z^4 - 32z^3 + 72z^2 - 32z + 16)$$

- The NLO correction is given by a polynomial function, e. g.

$$\begin{aligned} \Gamma^{J/\Psi, \text{NLO}}(z) &= -9.01726z^{10} + 18.22777z^9 + 16.11858z^8 - 82.54936z^7 \\ &+ 106.57565z^6 - 72.30107z^5 + 28.85798z^4 - 6.70607z^3 \\ &+ 0.84950z^2 - 0.05376z - 0.00205 \end{aligned}$$

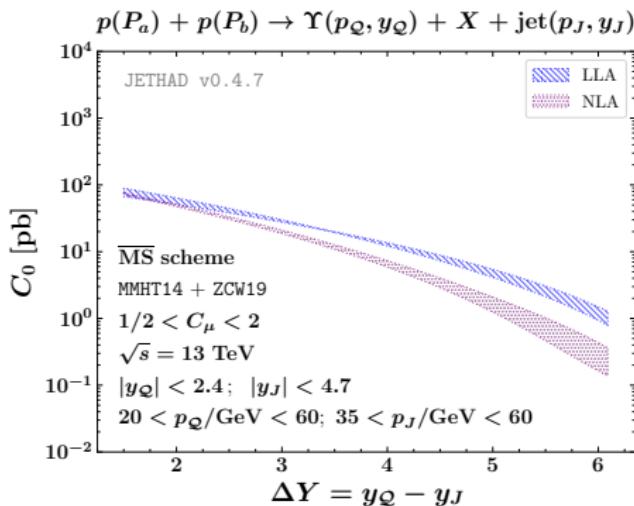
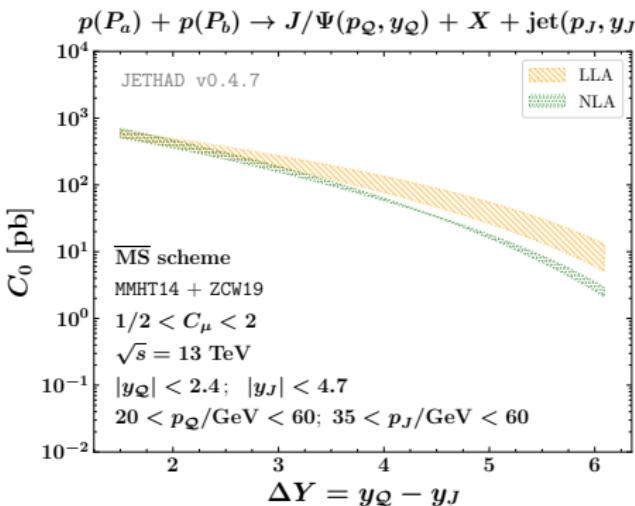
[X.-C. Zheng, C.-H. Chang, and X.-G. Wu (2019)]

- FFs are evolved from the initial scale through the DGLAP evolution equations

*J/ψ and Υ production from single parton fragmentation*

- $\Delta Y$ -behaviour of the cross section

$$C_0 = \int_{y_Q^{\min}}^{y_Q^{\max}} dy_Q \int_{y_J^{\min}}^{y_J^{\max}} dy_J \int_{p_Q^{\min}}^{p_Q^{\max}} d|\vec{p}_Q| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \delta(\Delta Y - y_Q - y_J) C_0(|\vec{p}_Q|, |\vec{p}_J|, y_Q, y_J)$$

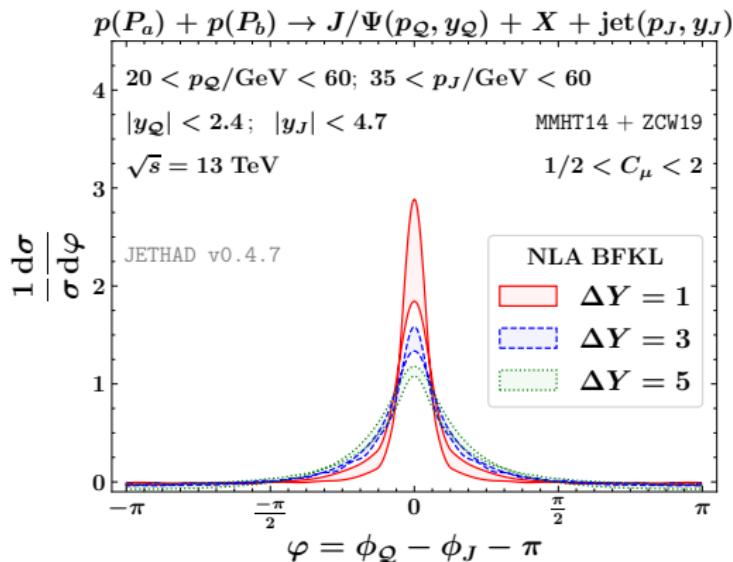


[F. G. Celiberto, M.F. (2022)]

# $J/\psi$ and $\Upsilon$ production from single parton fragmentation

- Azimuthal distribution

$$\frac{1}{\sigma} \frac{d\sigma}{d\varphi} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) \langle \cos(n\varphi) \rangle \right\} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) R_{n0} \right\} .$$

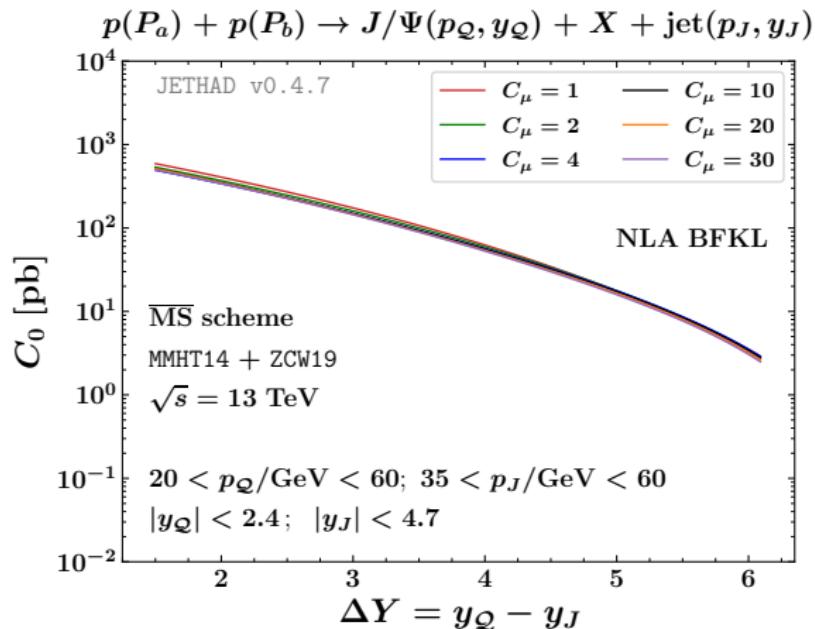


[F. G. Celiberto, M.F. (2022)]

# $J/\psi$ and $\Upsilon$ production from single parton fragmentation

- $\Delta Y$ -behaviour of the cross section under scale variation

$$C_\mu = \mu_{R,F}/\mu_N$$



[F. G. Celiberto, M.F. (2022)]

# *Conclusions and outlook*

## *Conclusions*

- Heavy flavored emissions represent a promising channel to investigate the semi-hard regime of QCD, providing with a **fair stability** of the BFKL series
- Some stability effects also occur for  $J/\psi$  FF calculated in the NRQCD framework

## *Outlook and related topics*

- Inclusion of **subleading corrections** from the heavy-quark pair impact factors, needed to produce full-NLA predictions
- Investigation of the single-forward  $J/\Psi$  photo/electro-production
  - [M. Hentschinski, E. P. Molina (2021)], **Martin's talk**
  - [H. Mäntysaari, K. Roy, F. Salazar, B. Schenke (2021)]
  - [H. Mäntysaari, J. Penttala (2022)], **Jani's talk**
- Matching of high-energy factorization and NLO of Collinear Factorization in quarkonium production

**Maxim's talk**

Thank you for the attention

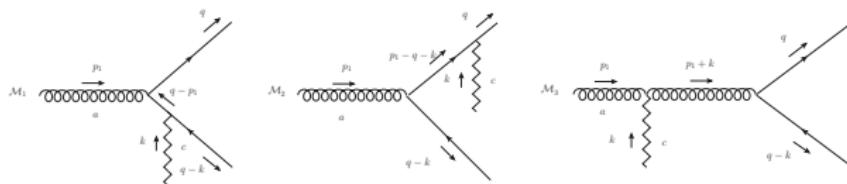
# Backup

# LO heavy-quark impact factors

- Gluon-initiated impact factor

[A.D. Bolognino, F.G. Celiberto, M. F., D.Yu. Ivanov, A. Papa (2019)]

- Feynman diagrams



- Impact factor

$$d\Phi_{gg}^{\{Q\bar{Q}\}}(\vec{k}, \vec{q}, z) = \frac{\alpha_s^2 \sqrt{N_c^2 - 1}}{2\pi N_c} \left[ \left( m^2 (R + \bar{R})^2 + (z^2 + \bar{z}^2) (\vec{P} + \vec{\bar{P}})^2 \right) \right. \\ \left. - \frac{N_c^2}{N_c^2 - 1} \left( 2m^2 R \bar{R} + (z^2 + \bar{z}^2) 2\vec{P} \cdot \vec{\bar{P}} \right) \right] d^2 \vec{q} dz ,$$

- Projection onto the LO BFKL eigenfunctions

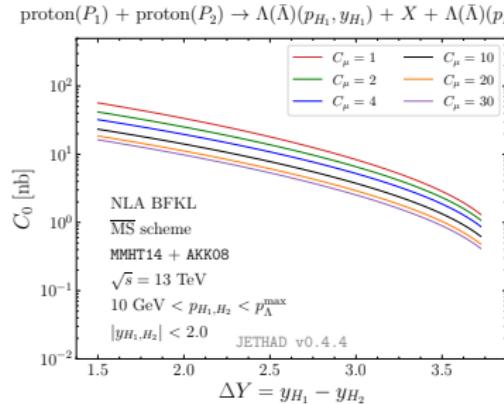
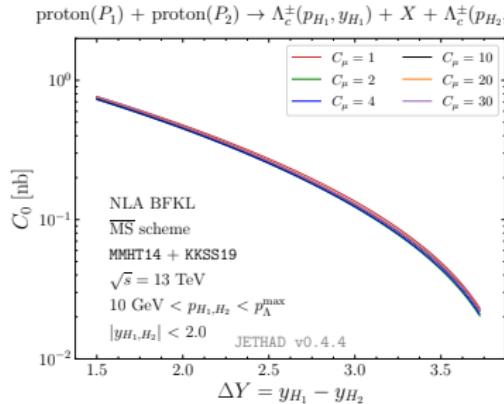
$$\frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(n, \nu, \vec{q}, z)}{d^2 \vec{q} dz} \equiv \int \frac{d^2 \vec{k}}{\pi \sqrt{2}} (\vec{k}^2)^{i\nu - \frac{3}{2}} e^{in\theta} \frac{d\Phi_{gg}^{\{Q\bar{Q}\}}(\vec{k}, \vec{q}, z)}{d^2 \vec{q} dz} \equiv \alpha_s^2 e^{in\varphi} c(n, \nu, \vec{q}, z)$$

- Photon-initiated impact factor

[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

*$\Lambda$ -baryon production*

- Process: proton( $p_1$ ) + proton( $p_2$ ) →  $\Lambda + X + \Lambda$   
[[F.G. Celiberto, M. F., Dmitry Yu. Ivanov, Alessandro Papa \(2021\)](#)]
  - Zero-mass variable flavor number scheme (ZM-VFNS)
  - Light parton NLO impact factors → Heavy baryon NLO impact factor  
[[M. Ciafaloni and G. Rodrigo \(2000\)](#)] [[V.S. Fadin et al. \(2000\)](#)]  
[[D.Yu. Ivanov, A. Papa \(2012\)](#)]
  - Lambda FFs
    - heavy species →  $\Lambda_c$   
KKSS19 [[B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger \(2020\)](#)]
    - light species →  $\Lambda^0$   
AKK08 [[S.Albino, B.A. Kniehl, and G. Kramer \(2008\)](#)]



# Towards bound states: Flavor number schemes

- The mass of light quarks ( $q = u, d, s$ ) is always set to zero. They are always present in the initial state
- The presence in the initial state and the way one must treat the mass of an heavy-quark ( $Q = c, b, t$ ) depends on kinematical conditions
- **Zero-mass variable flavor number scheme**
  - $m_Q = 0$
  - Heavy quark is present in the initial state above a fixed threshold.
  - Powers of  $m_Q^2/p_{T,HQ}^2$  missed by the scheme
  - It is appropriate in region of high  $p_{T,HQ}^2 \gg m_Q^2$
- **Fixed flavor number scheme**
  - $m_Q \neq 0$
  - Heavy quark is present only in the final state
  - Logarithms of  $p_{T,HQ}^2/m_Q^2$  missed by the scheme
  - It is appropriate in regions of moderate  $p_{T,HQ}^2$
- **General-mass variable flavor number schemes**
  - It is a matching between the previous schemes
  - There is some arbitrariness in the combination