Inclusive $J/\Psi$ and $\Upsilon$ emissions from single-parton fragmentation in hybrid high-energy/collinear factorization

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Outline

Introduction and motivations
  BFKL resummation
  Hybrid collinear/high-energy factorization

$J/\psi$ and $\Upsilon$ production
  $J/\psi$ plus jet production at low-$p_T$
  $J/\psi$ and $\Upsilon$ production from single parton fragmentation

Summary and outlook
Motivation

- Heavy flavor physics has long been considered as a perfect framework for testing perturbative QCD at colliders, due to the smallness of the running coupling
- However, at modern colliders, heavy-flavor production enters a two-scale regime, called semi-hard
- Semi-hard collision process, featuring the scale hierarchy

\[ s \gg Q^2 \gg \Lambda_{\text{QCD}}^2, \quad Q^2 \text{ a hard scale,} \]

\[ \alpha_s(Q^2) \ln \left( \frac{s}{Q^2} \right) \sim 1 \implies \text{all-order resummation needed} \]

- The Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach is the general framework for this resummation
  - Leading-logarithm-Approximation (LLA): \((\alpha_s \ln s)^n\)
  - Next-to-leading-logarithm-Approximation (NLA): \(\alpha_s(\alpha_s \ln s)^n\)
**BFKL resummation**

- Diffusion $A + B \rightarrow A' + B'$ in the Regge kinematical region
- Gluon Reggeization
- BFKL factorization for $\mathcal{A}_{AB}^{A'B'}$: convolution of a Green function (process independent) with the Impact factors of the colliding particles (process dependent)

$$\mathcal{A}_{AB}^{A'B'} = \frac{s}{(2\pi)^D} \int \frac{d^{D-2}q_1}{q_1^2 (q_1 - q)^2} \frac{d^{D-2}q_2}{q_2^2 (q_2 - q)^2}$$

$$\times \sum_{\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1, \vec{q}, s_0) \int \frac{d\omega}{2\pi i} \left[ \left( \frac{s}{s_0} \right)^{\omega} G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}) \right] \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2, \vec{q}, s_0)$$
Pomeron channel

- **BFKL equation**: $\vec{q}^2 = 0$ and singlet color state representation
  

  Redefinition:
  
  $G_\omega (\vec{q}_1, \vec{q}_2) \equiv \frac{G_\omega (0, \vec{q}_1^2, \vec{q}_2^2)}{\vec{q}_1^2 \vec{q}_2^2}$, $\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}(0, \vec{q}_1^2, \vec{q}_2^2)}{\vec{q}_1^2 \vec{q}_2^2}$

  $\omega G_\omega (\vec{q}_1, \vec{q}_2) = \delta^{(D-2)} (\vec{q}_1 - \vec{q}_2) + \int d^{D-2} q_r \mathcal{K}(\vec{q}_1, \vec{q}_r) G(\vec{q}_r, \vec{q}_2)$

- **Elastic amplitude factorization**:

  $\Im A_{AB}^{AB} = \frac{s}{(2\pi)^{D-2}} \int d^{D-2} q_1 d^{D-2} q_2$

  $\times \frac{\Phi^{(0)}_{AA}(\vec{q}_1, s_0)}{\vec{q}_1^2} \int \frac{d\omega}{2\pi i} \left[ \left( \frac{s}{s_0} \right)^\omega G_\omega (\vec{q}_1, \vec{q}_2) \right] \Phi^{(0)}_{BB}(-\vec{q}_2, s_0)$

- **Optical Theorem**:

  $\sigma_{AB} = \frac{\Im A_{AB}^{AB}}{s}$

- **Impact factor in the color singlet state**:

  $\Phi^{(0)}_{PP} = \langle cc' | \hat{P} | 0 \rangle \sum_{\{f\}} \int \frac{ds_{PR}}{2\pi} dp \Gamma_c^{\{f\} P} \Gamma_{\{f\} P}^{c'} *$
Hybrid collinear/high-energy factorization

- Straightforward adaptation to partially inclusive processes: just restrict the summation over intermediate states

**Mueller-Navelet jets**
- Inclusive production of two rapidity-separated jets in proton-proton collision
- Large energy logarithms $\rightarrow$ BFKL resummed partonic cross section
- Moderate values of parton $x \rightarrow$ collinear PDFs

- Hybrid formalism: can be extended to several type of semi-hard reactions
Muller-Navelet: Theory vs Experiment

\[
proton(p_1) + \text{proton}(p_2) \rightarrow \text{jet}(\kappa_{J,1}, y_{J,1}) + X + \text{jet}(\kappa_{J,2}, y_{J,2})
\]

\[35 \text{ GeV} < \kappa_{J,1,2} < \kappa_{J,\text{CMS}}^{\text{max}}\]
\[|y_{J,1,2}| < 4.7\]
\[\sqrt{s} = 7 \text{ TeV}\]

In this slide: [F.G. Celiberto (2021)]

[B. Ducloué, L. Szymanowski, S. Wallon (2013)]

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]
Mueller-Navelet: Theory vs Experiment

- CMS @7Tev with symmetric $p_T$-ranges, only!  
  [CMS collaboration (2016)]
- LHC kinematic **domain** in between the sectors described by BFKL and DGLAP approaches
- Clearer manifestation of high-energy signatures expected at increasing energies (higher hadronic center-of-mass energy or higher rapidity difference between tagged jets)
- Need for more exclusive final states as well as more sensitive observables
Mueller-Navelet: Theory vs Experiment

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- Need for more exclusive final states as well as more sensitive observables
- Strong manifestation of higher-order instabilities via scale variation

NLA BFKL corrections to cross section with opposite sign with respect to the leading order (LO) result and large in absolute value...
- ...call for some optimization procedure...
- ...choose scales to mimic the most relevant subleading terms

BLM [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
- preserve the conformal invariance of an observable...
- ...by making vanish its $\beta_0$-dependent part

* “Exact” BLM:
  suppress NLO IFs + NLO Kernel $\beta_0$-dependent factors
Partially inclusive processes in NLA

• Mueller-Navelet jet production

  [J. Bartels, D. Colferai, G.P. Vacca (2003)]
  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2011)]
  [D.Yu. Ivanov, A. Papa (2012)]
  [D. Colferai, A. Niccoli (2015)]
  [B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]
  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

• Light hadron-light hadron production

  [D.Yu. Ivanov, A. Papa (2012)]

• Light hadron-jet production


• Heavy-light hadrons in VFNS

Other partially-inclusive reactions

- Three / four jet production (partial NLA)
  [F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2016)]
  [F. Caporale, F.G. Celiberto, G. Chachamis, A. Sabio Vera (2016)]

- Drell-Yan pair - jet (partial NLA)
  [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]

- Higgs - jet
  - Partial NLA
  - Full NLA, $m_t \rightarrow \infty$ limit

- Heavy-quark pair photo/hadro-production (partial NLA)
  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

- $J/\Psi$ - jet production (partial NLA)
  [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
Stabilization effects

- Stabilization effects in Higgs and heavy flavor production
- Λ-baryon FFs
  - heavy species → Λ_c
    KKSS19 [B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger (2020)]
  - light species → Λ
    AKK08 [S. Albino, B.A. Kniehl, and G. Kramer (2008)]

\[ \int \frac{d\sigma}{dp_T^H} (|p_T^H|, \Delta Y, s) \]

\[ C_0 \text{ [nb]} \]

[MS scheme, MMHT2014 NLO PDF set]

\[ \sqrt{s} = 13 \text{ TeV} \]

\[ 10 \text{ GeV} < p_{h_1,h_2} < p_{h_1,h_2}^{\text{max}} \]

\[ |y_{h_1,h_2}| < 2.0 \]

\[ \Delta Y = y_{h_1} - y_{h_2} \]


[F.G. Celiberto, D.Yu. Ivanov, M. F., A. Papa (2021)]
D* plus Higgs production at the FPF

- ATLAS + FPF ultra-forward regime: Strong high-energy enhancement
- Final state with two stabilizers: Heavy-flavor and Higgs
- FPF studies on high-energy QCD: see yesterday’s WG6 talk by J. Rojo

\[ p(P_a) + p(P_b) \rightarrow D^\pm (p_C, y_C) + X + H(p_H, y_H) \]

\[ p(P_a) + p(P_b) \rightarrow J/\Psi(p_Q, y_Q) + X + \text{jet}(p_J, y_J) \]

\[ \Delta Y = y_C - y_H \]

\[ \varphi = \phi_Q - \phi_J - \pi \]

[FPF Collaboration (2022)]

[F. G. Celiberto, M.F., M. M. A. Mohammed (in preparation)]
**J/ψ plus jet production at low-\( p_T \)**

- Process: proton\( (p_1) \) + proton\( (p_2) \) → \( J/\psi + X + \) jet

- **hybrid collinear/BFKL approach**
  - high-energy hadroproduction of a \( J/\Psi \) meson and a jet, with a remnant \( X \)
  - both the \( J/\Psi \) and the jet emitted with large transverse momenta and well separated in rapidity

- NLA BFKL + NLO jet + LO \( J/\Psi \)
  - LO \( J/\Psi \) IF calculated in **NRQCD** (Color-singlet and Color-octect)
  - LO \( J/\Psi \) IF calculated in **color evaporation model (CEM)**

- Realistic CMS and CASTOR rapidity ranges, fixed \( p_T \) final states

[R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
$J/\psi$ plus jet production at low-$p_T$

- Realistic CMS and CASTOR rapidity ranges, fixed $p_T$ final states

\[
\frac{d\sigma}{d|k_{J/\psi}|d|k_{\text{jet}}|dY} [\text{nb.GeV}^{-2}]
\]

\[
\frac{d\sigma}{d|k_{J/\psi}|d|k_{\text{jet}}|dY} [\text{nb.GeV}^{-2}]
\]

$|k_{J/\psi}| = |k_{\text{jet}}| = 10 \text{ GeV}$

$|k_{J/\psi}| = |k_{\text{jet}}| = 20 \text{ GeV}$

[R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
$J/\psi$ and $\Upsilon$ production from single parton fragmentation

- **Process**
  \[ p(P_a) + p(P_b) \rightarrow Q(p_Q, y_Q) + X + \text{jet}(p_J, y_J) \]

- **Hybrid cross section**
  \[
  \frac{d\sigma}{dy_Q dy_J d^2 \vec{p}_Q d^2 \vec{p}_J} = \frac{1}{(2\pi)^2}
  \times \int \frac{d^2 \vec{q}_1}{\vec{q}_1^2} V_Q(\vec{q}_1, x_g, \vec{p}_Q) \int \frac{d^2 \vec{q}_2}{\vec{q}_2^2} V_J(\vec{q}_2, x_J, \vec{p}_J)
  \times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left( \frac{x_g x_J s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2)
  \]

- **Impact factors**
  \[
  V_Q(\vec{q}_1, x_Q, \vec{p}_J) = f_{q/g} \otimes H \otimes D^{Q}_{q/g}
  \]
  \[
  V_J(\vec{q}_2, x_J, \vec{p}_J) = f_{q/g} \otimes H \otimes J_{q/g}
  \]
Differential cross section in terms of azimuthal-angle coefficients

\[
\frac{d\sigma}{dy_Q dy_J d|p_Q| d|p_J| d\phi_Q d\phi_J} = \frac{1}{(2\pi)^2} \left[ C_0 + 2 \sum_{n=1}^{\infty} \cos(n\varphi) C_n \right]
\]

Unintegrated azimuthal-angle coefficients at NLO

\[
C_n = \int_0^{2\pi} d\phi_Q \int_0^{2\pi} d\phi_J \cos(n\varphi) \frac{d\sigma_{\text{NLA}}}{dy_Q dy_J d|p_Q| d|p_J| d\phi_Q d\phi_J}
\]

Azimuthal-angle coefficients at NLO
Single parton fragmentation function for $J/\psi$ and $\Upsilon$ at $\mu_0 = 3m_Q$

$$D_Q^Q(z, \mu_F \equiv \mu_0) = D_Q^{Q, \text{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3} |\mathcal{R}_Q(0)|^2 \Gamma_Q^{\text{NLO}}(z)$$

LO fragmentation function

$$D_Q^{Q, \text{LO}}(z) = \frac{\alpha_s^2(\mu_R)}{m_Q^3} \frac{8z(1-z)^2}{27\pi(2-z)^6} |\mathcal{R}_Q(0)|^2 (5z^4 - 32z^3 + 72z^2 - 32z + 16)$$

The NLO correction is given by a polynomial function, e.g.

$$\Gamma^{J/\Psi, \text{NLO}}(z) = -9.01726z^{10} + 18.22777z^9 + 16.11858z^8 - 82.54936z^7 + 106.57565z^6 - 72.30107z^5 + 28.85798z^4 - 6.70607z^3 + 0.84950z^2 - 0.05376z - 0.00205$$


FFs are evolved from the initial scale through the DGLAP evolution equations
\( \Delta Y \)-behaviour of the cross section

\[
C_0 = \int_{y_Q^{\text{min}}}^{y_Q^{\text{max}}} dy_Q \int_{y_J^{\text{min}}}^{y_J^{\text{max}}} dy_J \int_{p_Q^{\text{min}}}^{p_Q^{\text{max}}} d|p_Q| \int_{p_J^{\text{min}}}^{p_J^{\text{max}}} d|p_J| \delta(\Delta Y - y_Q - y_J) C_0 \left(|p_Q|, |p_J|, y_Q, y_J\right)
\]

\[ p(P_a) + p(P_b) \rightarrow J/\Psi(p_Q, y_Q) + X + \text{jet}(p_J, y_J) \]

**J/\psi and \Upsilon production from single parton fragmentation**

- MS scheme
  - MMHT14 + ZCW19
  - \( 1/2 < C_\mu < 2 \)
  - \( \sqrt{s} = 13 \text{ TeV} \)
  - \( |y_Q| < 2.4; \ |y_J| < 4.7 \)
  - \( 20 < p_Q/\text{GeV} < 60; \ 35 < p_J/\text{GeV} < 60 \)

\( \Delta Y = y_Q - y_J \)

- LLA
- NLA
Azimuthal distribution

\[
\frac{1}{\sigma} \frac{d\sigma}{d\varphi} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi)\langle\cos(n\varphi)\rangle \right\} = \frac{1}{2\pi} \left\{ 1 + 2 \sum_{n=1}^{\infty} \cos(n\varphi)R_n \right\}.
\]
\( J/\psi \) and \( \Upsilon \) production from single parton fragmentation

- \( \Delta Y \)-behaviour of the cross section under scale variation

\[
C_\mu = \mu_{R,F}/\mu_N
\]

\[
p(P_a) + p(P_b) \rightarrow J/\Psi(p_Q, y_Q) + X + \text{jet}(p_J, y_J)
\]

\[
\Delta Y = y_Q - y_J
\]

\[\begin{array}{c|c|c|c}
\hline
C_\mu & 1 & 2 & 30 \\
\hline
C_\mu = 1 & \text{red} & \text{red} & \text{red} \\
C_\mu = 2 & \text{green} & \text{green} & \text{green} \\
C_\mu = 4 & \text{blue} & \text{blue} & \text{blue} \\
C_\mu = 10 & \text{black} & \text{black} & \text{black} \\
C_\mu = 20 & \text{orange} & \text{orange} & \text{orange} \\
C_\mu = 30 & \text{purple} & \text{purple} & \text{purple} \\
\hline
\end{array}\]

\[\sqrt{s} = 13 \text{ TeV}\]

\[20 < p_Q/\text{GeV} < 60; \; 35 < p_J/\text{GeV} < 60\]

\[|y_Q| < 2.4; \; |y_J| < 4.7\]

[F. G. Celiberto, M.F. (2022)]
Conclusions and outlook

Conclusions

• Heavy flavored emissions represent a promising channel to investigate the semi-hard regime of QCD, providing with a fair stability of the BFKL series

• Some stability effects also occur for $J/\psi$ FF calculated in the NRQCD framework

Outlook and related topics

• Inclusion of subleading corrections from the heavy-quark pair impact factors, needed to produce full-NLA predictions

• Investigation of the single-forward $J/\Psi$ photo/electro-production

  [M. Hentschinski, E. P. Molina (2021)], Martin’s talk
  [H. Mäntysaari, K. Roy, F. Salazar, B. Schenke (2021)]
  [H. Mäntysaari, J. Penttala (2022)], Jani’s talk

• Matching of high-energy factorization and NLO of Collinear Factorization in quarkonium production

  Maxim’s talk
Thank you for the attention
Backup
**LO heavy-quark impact factors**

- **Gluon-initiated impact factor**
  
  \[ \text{[A.D. Bolognino, F.G. Celiberto, M. F., D.Yu. Ivanov, A. Papa (2019)]} \]

- **Feynman diagrams**

- **Impact factor**

  \[
  d\Phi^{Q\bar{Q}}_{gg}(\vec{k}, \vec{q}, z) = \frac{\alpha_s^2 \sqrt{N_c^2 - 1}}{2\pi N_c} \left[ \left( \frac{m^2 (R + \bar{R})^2 + (z^2 + \bar{z}^2)(\vec{P} + \bar{\vec{P}})^2}{2m^2 R\bar{R} + (z^2 + \bar{z}^2)(\vec{P} \cdot \bar{\vec{P}})} \right) \right] d^2\vec{q} \, dz ,
  \]

- **Projection onto the LO BFKL eigenfunctions**

  \[
  \frac{d\Phi^{Q\bar{Q}}_{gg} (n, \nu, \vec{q}, z)}{d^2 \vec{q} \, dz} \equiv \int \frac{d^2\vec{k}}{\pi \sqrt{2}} (\vec{k} \cdot i\nu - \frac{3}{2} e^{i\theta} \frac{d\Phi^{Q\bar{Q}}_{gg} (\vec{k}, \vec{q}, z)}{d^2 \vec{q} \, dz}) \equiv \alpha_s^2 e^{i\varphi} c(n, \nu, \vec{q}, z)
  \]

- **Photon-initiated impact factor**

  \[ \text{[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]} \]
**Λ-baryon production**

- Process: proton($p_1$) + proton($p_2$) → $\Lambda + X + \Lambda$
  
  [F.G. Celiberto, M. F., Dmitry Yu. Ivanov, Alessandro Papa (2021)]

- Zero-mass variable flavor number scheme (ZM-VFNS)

- Light parton NLO impact factors → Heavy baryon NLO impact factor
  
  [M. Ciafaloni and G. Rodrigo (2000)]  [V.S. Fadin et al. (2000)]
  
  [D.Yu. Ivanov, A. Papa (2012)]

- Lambda FFs
  
  - heavy species → $\Lambda_c$
    
    KKSS19 [B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger (2020)]
  
  - light species → $\Lambda^0$
    
    AKK08 [S.Albino, B.A. Kniehl, and G. Kramer (2008)]

\[
\Delta Y = y_{H_1} - y_{H_2}
\]

```
proton($P_1$) + proton($P_2$) → $\Lambda^+_c(p_{H_1}, y_{H_1}) + X + \Lambda_c^-(p_{H_2}, y_{H_2})$
```

```
proton($P_1$) + proton($P_2$) → $\Lambda(\bar{\Lambda})(p_{H_1}, y_{H_1}) + X + \Lambda(\bar{\Lambda})(p_{H_2}, y_{H_2})$
```
The mass of light quarks \((q = u, d, s)\) is always set to zero. They are always present in the initial state.

The presence in the initial state and the way one must treat the mass of an heavy-quark \((Q = c, b, t)\) depends on kinematical conditions.

**Zero-mass variable flavor number scheme**
- \(m_Q = 0\)
- Heavy quark is present in the initial state above a fixed threshold.
- Powers of \(m_Q^2/p_{T,HQ}^2\) missed by the scheme
- It is appropriate in region of high \(p_{T,HQ}^2 \gg m_Q^2\)

**Fixed flavor number scheme**
- \(m_Q \neq 0\)
- Heavy quark is present only in the final state
- Logarithms of \(p_{T,HQ}^2/m_Q^2\) missed by the scheme
- It is appropriate in regions of moderate \(p_{T,HQ}^2\)

**General-mass variable flavor number schemes**
- It is a matching between the previous schemes
- There is some arbitrariness in the combination