

# Non-linear evolution in the NLO vs experimental HERA data

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# Outline

- Current problems in theoretical description.
- Leading twist nonlinear evolution in NLO.
- Confronting exp. data: phenomenological input.
- Confronting exp. data: DIS structure functions.
- Conclusions.

# Current problems in theoretical description.

- LO is an apparent contradiction with the experimental data.
- NLO is needed to describe: BFKL Pomeron intercept and energy behavior of saturation scale.
- NLO of leading evolution = previous attempts [Albacete et all 2011, Contreras et all 2016, Ducloe et all 2017,2019, Beuf et all 2020].
- Differences in the nonlinear NLO corrections.

# Balitsky-Kovchegov (BK) Non-linear equation

$$\begin{aligned}\frac{\partial}{\partial Y} N(x_{10}, \mathbf{b}, Y; R) = \\ \bar{\alpha}_S \int \frac{d^2 x_2}{2\pi} K(x_{02}, x_{12}; x_{10}) \left( N\left(x_{12}, \mathbf{b} - \frac{1}{2}x_{20}, Y; R\right) + N\left(x_{20}, \mathbf{b} - \frac{1}{2}x_{12}, Y; R\right) - N(x_{10}, \mathbf{b}, Y; R) \right. \\ \left. - N\left(x_{12}, \mathbf{b} - \frac{1}{2}x_{20}, Y; R\right) N\left(x_{20}, \mathbf{b} - \frac{1}{2}x_{12}, Y; R\right) \right)\end{aligned}$$

$$K^{\text{LO}}(x_{02}, x_{12}; x_{10}) = \frac{x_{10}^2}{x_{02}^2 x_{12}^2}$$

For the kernel of the LO BFKL equation the eigenvalues take the form:

Lipatov et all 1975,  
1977, 1978

$$\omega_{\text{LO}}(\bar{\alpha}_S, \gamma) = \bar{\alpha}_S \chi^{LO}(\gamma) = \bar{\alpha}_S (2\psi(1) - \psi(\gamma) - \psi(1-\gamma))$$

# Resummed anomalous dimensions in NLO.

## General solution to linear equation

$$N(r, b, Y; R) = \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{d\gamma}{2\pi i} e^{\omega(\bar{\alpha}_S, \gamma)Y} \phi_\gamma(r, R, b) \phi_{\text{in}}(\gamma, R)$$

Eigenvalues in the NLO  
Fadin-Lipatov (1998)  
Ciafaloni-Camici (1998)

$$\omega_{\text{NLO}}(\bar{\alpha}_S, \gamma) = \bar{\alpha}_S \chi^{LO}(\gamma) + \bar{\alpha}_S^2 \chi^{NLO}(\gamma)$$



Turns out to be singular at  $\gamma \rightarrow 1$

Re-sum has been done: Ref. [Salam and collaborators arXiv: 9806482, 9905566, 0307188]

# Resulting spectrum of BFKL eq in NLO

**Khoze, Martin, Ryskin and Stirling suggested an economic form to**  $\chi_1(\omega, \gamma)$  **arXiv:04061305**

$$\omega^{\text{KMRS}} = \bar{\alpha}_S (1 - \omega^{\text{KMRS}}) \left( \frac{1}{\gamma} + \frac{1}{1 - \gamma + \omega^{\text{KMRS}}} + \underbrace{(2\psi(1) - \psi(2 - \gamma) - \psi(1 + \gamma))}_{\text{high twist contributions}} \right)$$

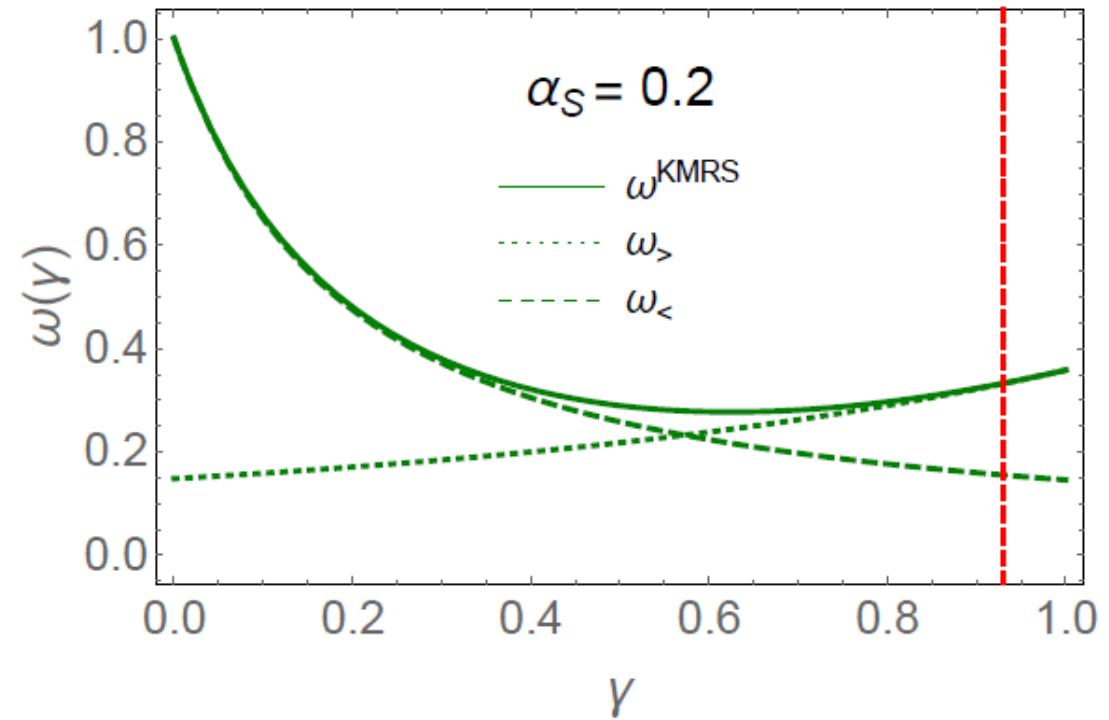
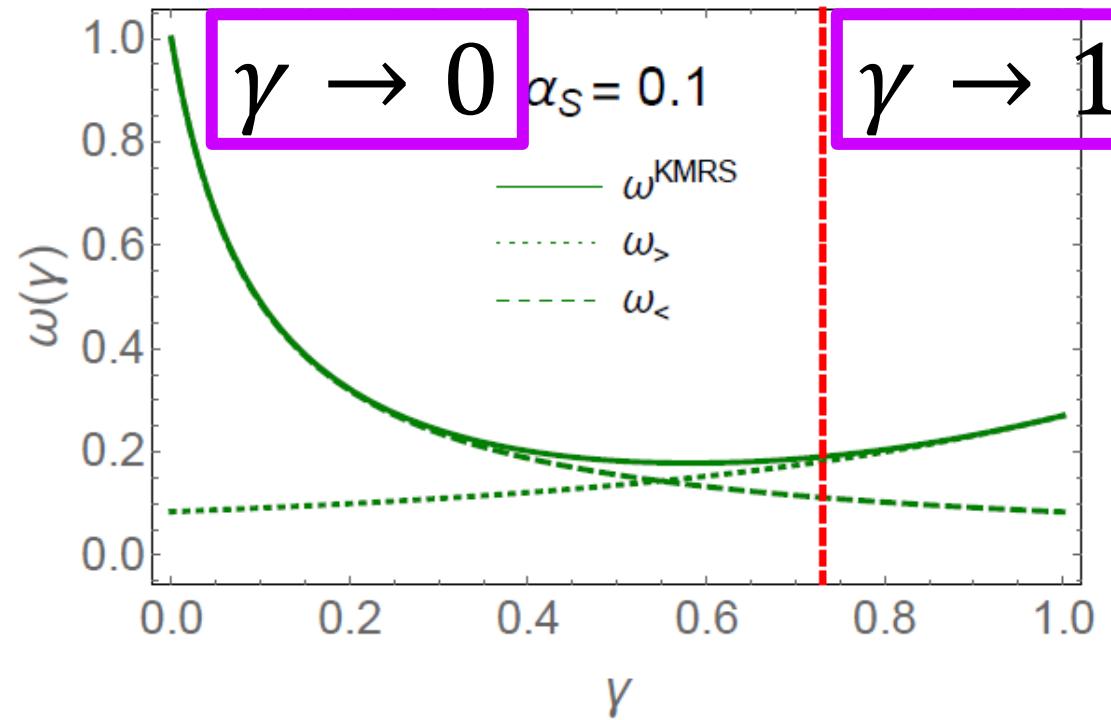
$$\omega \equiv \omega_> = \frac{1}{2} \left( -(1 - \gamma + \bar{\alpha}_S) + \sqrt{4\bar{\alpha}_S + (1 - \gamma + \bar{\alpha}_S)^2} \right)$$

$$\gamma = \frac{\bar{\alpha}_S}{1 + \bar{\alpha}_S} \frac{1}{\omega}$$

$$\gamma \rightarrow 1$$

$$\gamma \rightarrow 0$$

$$\gamma > 1 - \gamma_{cr} = \bar{\gamma}$$



- Is needed for the description of the HERA data

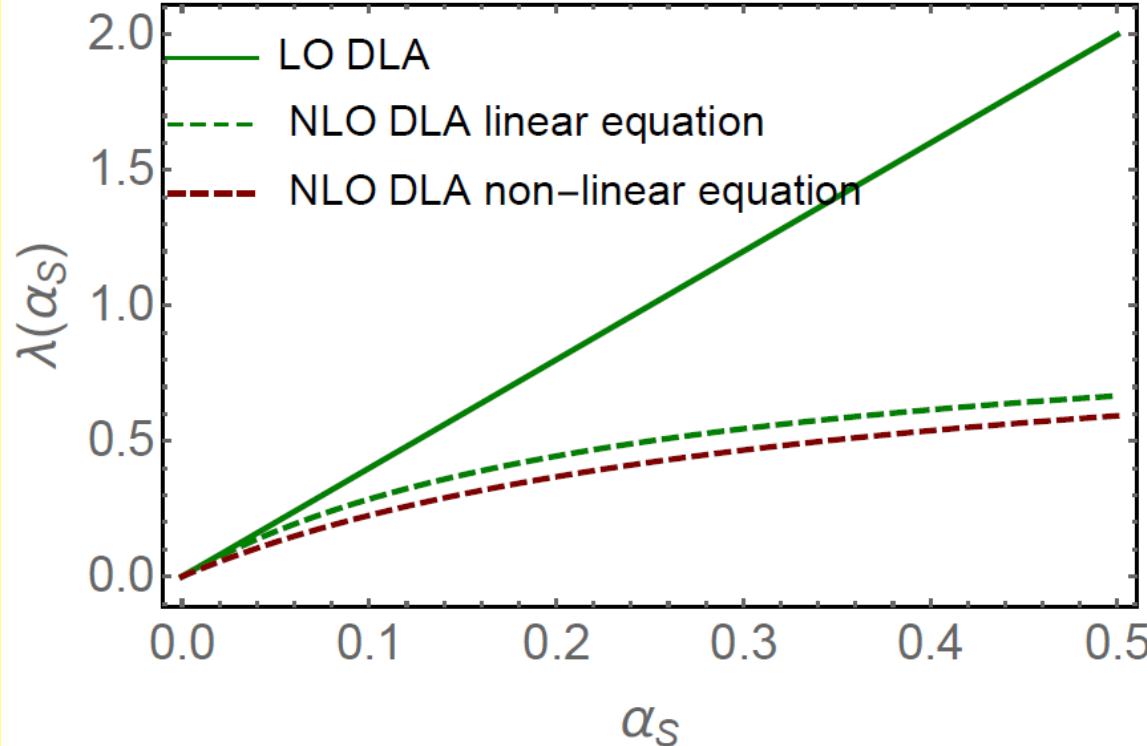


Fig. 5-a

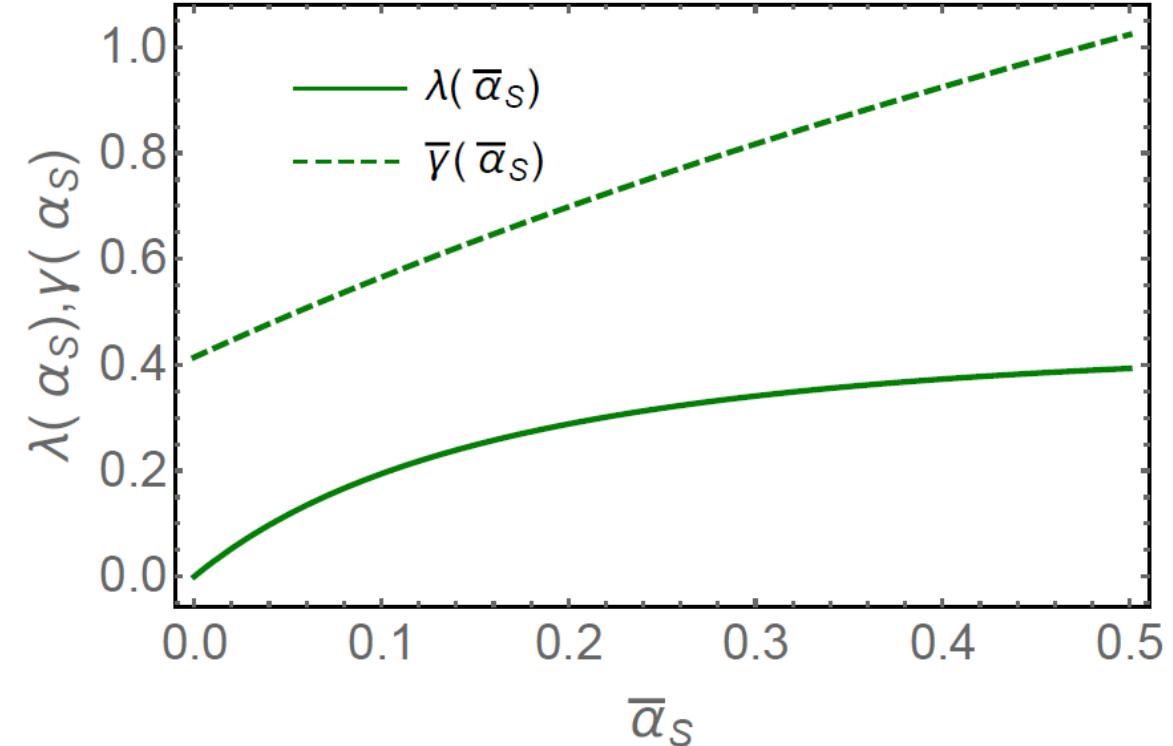


Fig. 5-b

$$\lambda = \frac{1}{2} \bar{\alpha}_S \frac{1 + \sqrt{2}}{\sqrt{2} - 1 + \frac{1}{2} \bar{\alpha}_S (1 + \sqrt{2})}$$

$$\bar{\gamma} = \sqrt{2} - 1 + \frac{1}{2} \bar{\alpha}_S (1 + \sqrt{2})$$

# Non linear evolution that we used

$$\frac{\partial^2 \Omega(\xi_s; \xi)}{\partial \xi_s \partial \xi} = \frac{\bar{\alpha}_S}{\lambda(\bar{\alpha}_S) (1 + \bar{\alpha}_S)} \left( 1 - \exp(-\Omega(\xi_s; \xi)) \right) \equiv \sigma \left( 1 - \exp(-\Omega(\xi_s; \xi)) \right)$$

$$N(Y, \xi) = 1 - \exp(-\Omega(Y, \xi))$$

**Levin-Tuchin 2000**

$$\xi_s = \ln(Q_s^2(Y) / Q_s^2(Y=0; \mathbf{b}, \mathbf{R})) = \lambda Y \quad \xi = \ln(r^2 Q_0^2)$$

# We found numerical solution:

$$N(z) = a \left( 1 - \exp(-\Omega(z)) \right) + (1 - a) \frac{\Omega(z)}{1 + \Omega(z)}$$

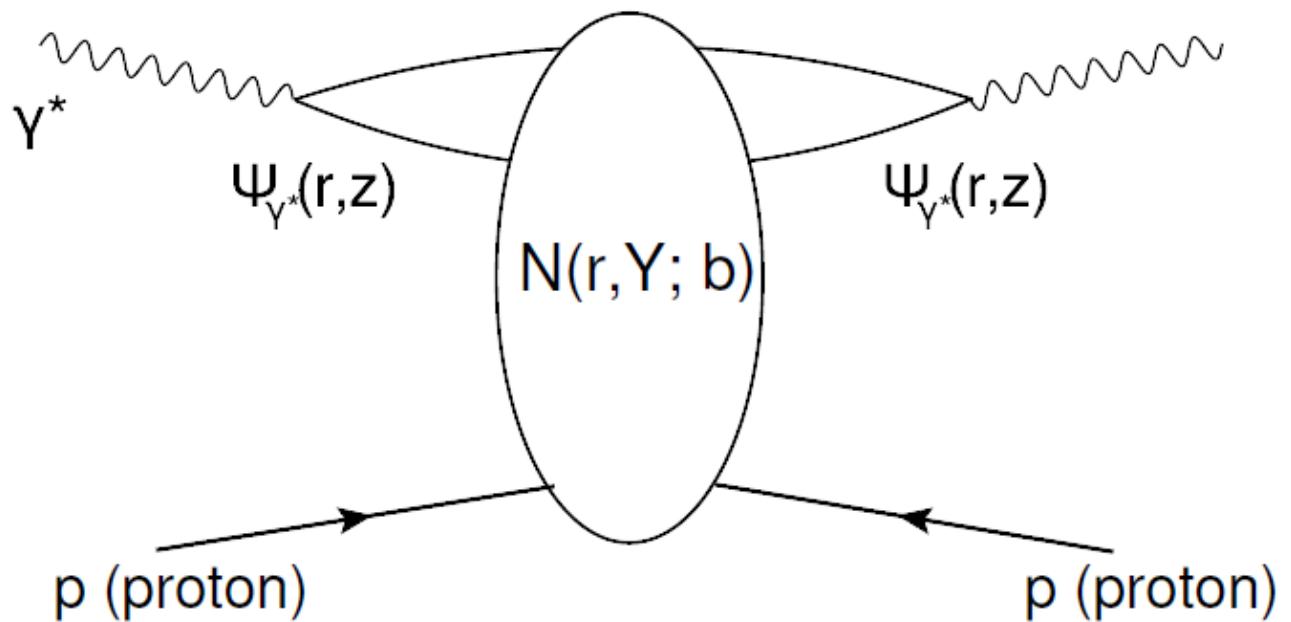
$$\Omega = \Omega_0 \left\{ \cosh(\sqrt{\sigma}(\xi_s + \xi)) + \frac{\bar{\gamma}}{\sqrt{\sigma}} \sinh(\sqrt{\sigma}(\xi_s + \xi)) \right\}$$

$$\Omega_0 = 0.1$$

$$a = 0.65$$

# Observables in DIS

$$N(Q, Y; b) = \int \frac{d^2 r}{4\pi} \int_0^1 dz |\Psi_{\gamma^*}(Q, r, z)|^2 N(r, Y; b)$$



$$\sigma_{T,L} = 2 \int d^2 b N_{T,L}(Q, Y, ; b)$$

$$F_2(Q, Y) = \frac{Q^2}{4\pi^2 \alpha_{\text{e.m.}}} \{\sigma_T + \sigma_L\}$$

$$F_2^{cc}(Q, Y) = \frac{Q^2}{4\pi^2 \alpha_{\text{e.m.}}} \{\sigma_T^{cc} + \sigma_L^{cc}\}$$

$$F_L(Q, Y) = \frac{Q^2}{4\pi^2 \alpha_{\text{e.m.}}} \sigma_L$$

# Scattering amplitude in three different kinematic regions

- $Q_s^2 r^2 < 1$  (Perturbative region)

$$\frac{\partial^2}{\partial Y \partial \xi'} N(\xi', Y) + \frac{\partial}{\partial Y} N(\xi', Y) = \frac{1}{2} \bar{\alpha}_S N(\xi', Y) - \frac{1}{2} \bar{\alpha}_S \frac{\partial}{\partial \xi'} N(\xi', Y)$$

- $Q_s^2 r^2 \sim 1$  (Vicinity of saturation scale)

$$N(r, \eta; b) = N_0 (Q_s^2(Y, b) r^2)^{\bar{\gamma}}$$

- $Q_s^2 r^2 \gg 1$  (Saturation region)

$$N(z) = a \left( 1 - \exp(-\Omega(z)) \right) + (1 - a) \frac{\Omega(z)}{1 + \Omega(z)}$$

# Two approaches: saturation scale

- We used phenomenological input: nonperturbative behavior at large  $b$  only in  $Q_s$ . (Kovner and Wiedemann, 2002)
- We introduced exponential suppression which follows from Froissart theorem.

$$1. \quad Q_s^{(1)2} (Y, b) = Q_s^{(1)2} (Y = 0, b = 0) e^{-m b} e^{\lambda Y} = Q_0^2 e^{-m b} e^{\lambda Y}$$

- Different approach summing pion loops in  $Q_s$ . (Gotsman-Levin 2020)

$$2. \quad Q_s^{(2)2} (Y, b) = Q_s^{(2)2} (Y = 0, b = 0) e^{-\frac{3}{4} \mathcal{Z}} e^{\lambda Y} = Q_0^2 e^{\lambda Y} e^{-\frac{3}{4} \mathcal{Z}}$$

$$\mathcal{Z} = \left( \frac{b^4}{4\alpha'^2_{\text{eff}} Y} \right)^{1/3}$$

# Restriction of kinematics region and parameters of our fit.

$$0.85 \text{ GeV}^2 \leq Q^2 \leq 27 \text{ GeV}^2$$

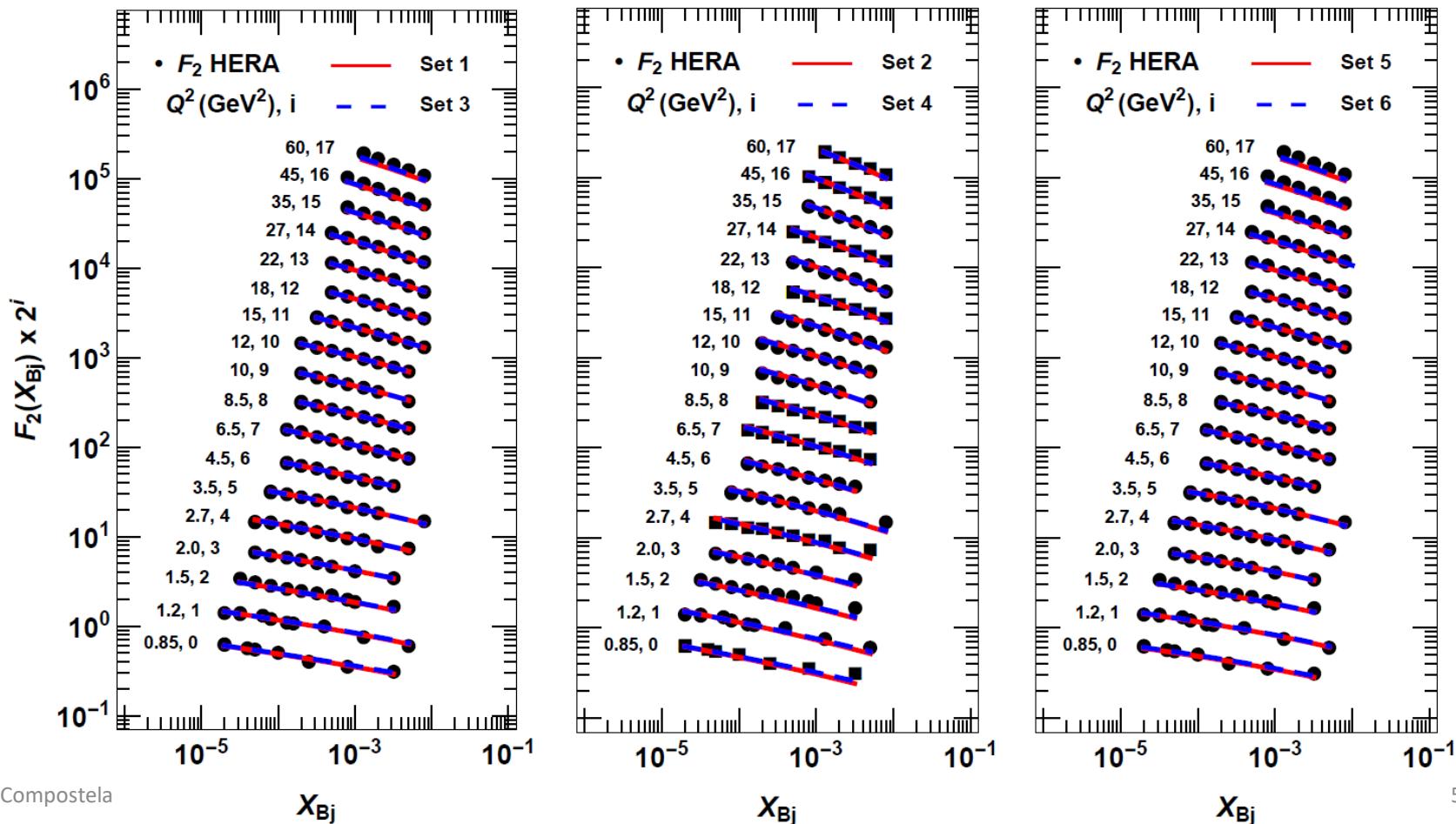
$$x \leq 0.01$$

	Dipole amplitude					Wave function				$\chi^2/d.o.f.$
Set	$\bar{\alpha}_S$	$N_0$	$Q_0^2(\text{GeV}^2)$	$m(\text{GeV})$	$\alpha_{eff}(\text{GeV}^{-2})$	$m_u(\text{MeV})$	$m_d(\text{MeV})$	$m_s(\text{MeV})$	$m_c(\text{GeV})$	$0.85 \leq Q^2 \leq 27 \text{ GeV}^2$
1	0.091	0.236	0.998	0.612	n/a	140	140	140	1.4	$124.9/133 = 0.93$
2	0.20	0.25	1.00	0.551	n/a	140	140	140	1.4	$61.99/66 = 0.93$
3	0.096	0.448	0.921	0.840	n/a	2.3	4.8	95	1.4	$117.2/133 = 0.88$
4	0.20	0.343	0.999	1.300	n/a	2.3	4.8	95	1.4	$91.74/66 = 1.39$
5	0.038	0.599	1.284	n/a	0.216	140	140	140	1.4	$175/133 = 1.31$
6	0.043	0.565	1.429	n/a	0.149	2.3	4.8	95	1.4	$143.7/133 = 1.08$

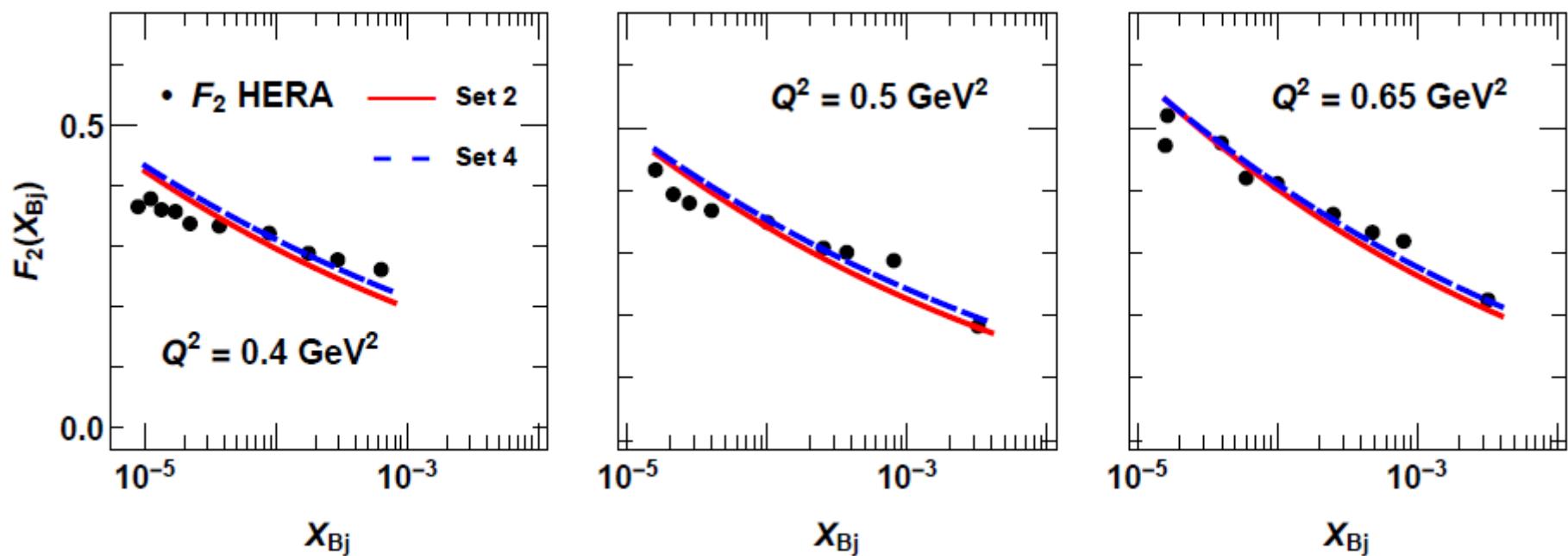
# Confronting the model with experimental data:

$F_2$  VS  $x_{Bj}$

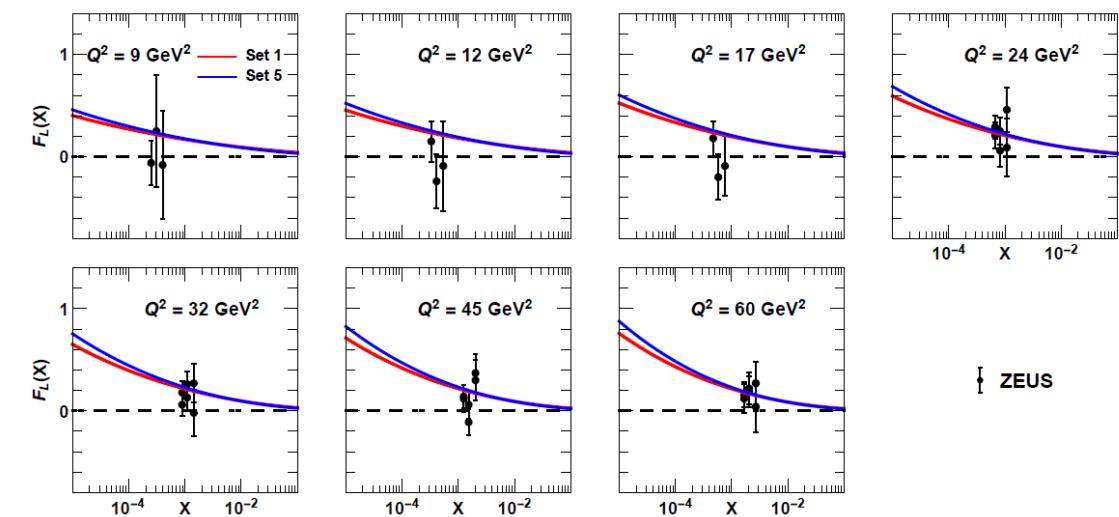
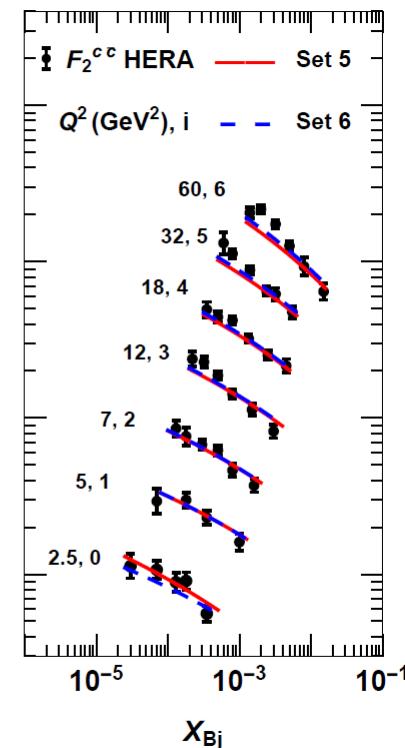
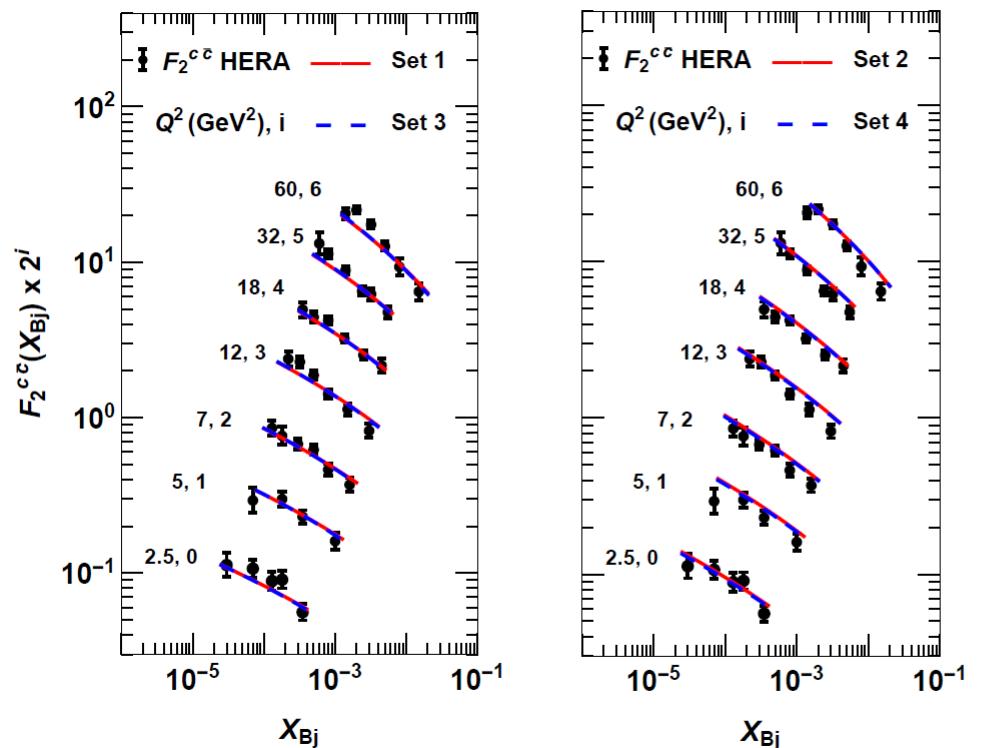
[arXiv:0911.0884 [hep-ex]]



# Confronting the model with experimental data: $F_2$ vs $x_{Bj}$ , (at low $Q^2$ )



# Confronting the model with experimental data: $F_L$ vs $x_{Bj}$ - $F_2^{c\bar{c}}$ vs $x_{Bj}$



[arXiv:0911.0884 [hep-ex]]

[arXiv:1211.1182 [hep-ex]]

# Conclusions

- We treated the non-linear evolution equation including NLO corrections.
- We have found numerical solution to BK equation.
- We used two approaches in Saturation momentum, in accordance with Froissart theorem.
- The results of our fits describe quite well the experimental data of DIS.