Evidence for the maximally entangled low x proton in Deep Inelastic Scattering from H1 data

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Based on
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Motivation

Entropy and low x dynamics (and hadronic collisions) attracts considerable theoretical interest since it

- constraints the growth of PDFs with energy through quantum bounds
- links to other areas (thermodynamics, gravity, quantum information, string theory)
- links of entropy to saturation

K. Kutak ‘11, R. Peschanski ‘13, A. Kovner, M. Lublinsky ‘15, A. Kovner, M. Lublinsky, M. Serino ‘18, Hagiwara, Hatta, Xiao,Yuan’18, Z.
Armesto, Dominguez, Kovner, Lublinsky, Skokov’19 Tu, D. Kharzeev, T. Ulrich ’20, C. Akkaya, H. Duan , A. Kovner, V. Skokov ’20 K. Zhang, K. Hao, D. Kharzeev, V. Korepin’ 21, E. Levin, D. Kharzeev ’21,
D. Kharzeev ‘21, M. Hentschinski, K. Kutak ’21, Dvali, Venugopalan’ 21, Liu, Nowak, Zahed, 21,
H. Duan , A. Kovner, V. Skokov ’22

Boltzman and von Neuman entropy formulas – reminder

The entropy S of macrostate is given by the log of number W of distinct microstates that compose it

$$S = -\sum_{i=1}^{W} p(i) \ln p(i)$$

For uniform distribution $p(i) = \frac{1}{W}$ the entropy is maximal $S = \ln W$

Since partons are introduced as the microscopic constituents that compose the macroscopic state of the proton, it is natural to evaluate the corresponding entropy.

Kharzeev, Levin ‘17

But proton as a whole is a pure state and the von Neuman entropy is 0.
Can we get any nontrivial result?

For pure state (one state) density matrix is

$$\rho = |\psi\rangle\langle\psi|$$

$$S_{VN} = -Tr[\rho \ln \rho] = -1 \ln 1 = 0$$

For mixed state i.e. classical statistical mixture

$$\rho = \sum p(i) |\psi_i\rangle\langle\psi_i|$$

$$S_{VN} \neq 0$$

Entanglement entropy in DIS

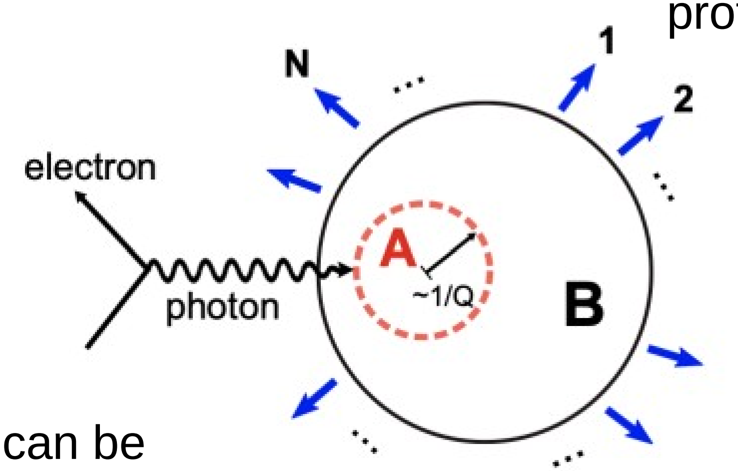
The composite system is described by $\mathcal{H}_A \otimes \mathcal{H}_B$

proton's rest frame

$|\Psi_{AB}\rangle$ in $A \cap B$

physical state in A
physical state in B

$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$



entangled
if the product can not be expressed as separable product state

$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$

separable
if the product can be expressed as separable product state

$$|\Psi_{AB}\rangle = |\varphi^A\rangle \otimes |\varphi^B\rangle$$

$\mathcal{H}_B$ of dimension n_B.

$\mathcal{H}_A$ of dimension n_A

Kharzeev, Levin '17

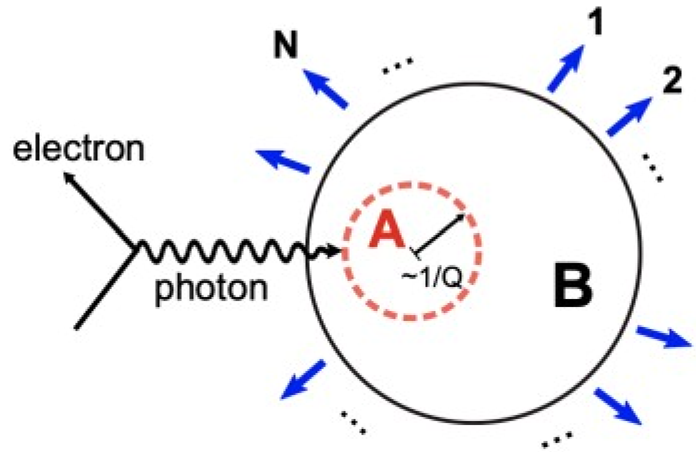
We perform Schmidt decomposition

$$|\Psi_{AB}\rangle = \sum \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$

related to matrix C

Entanglement entropy in DIS

$$|\Psi_{AB}\rangle = \sum_n \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$



proton's rest frame

$$\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

$$\rho_A = \mathrm{tr}_B \rho_{AB} = \sum_n \alpha_n^2 |\Psi_n^A\rangle\langle\Psi_n^A|$$

The density matrix of the mixed state probed in region A can now be written down as

$\alpha_n^2 \equiv p_n$ probability of state with n partons

Kharzeev, Levin '17

$$S = -\sum_n p_n \ln p_n$$

entropy results from the entanglement between the regions A and B, and can thus be interpreted as the entanglement entropy. Entropy of region A is the same as entropy in region B.

Partonic cascade

$$p_n = P_n$$

set of partons is described by set of dipoles with fixed sizes ,Y is rapidity and is related to energy

Lublinsky, Levin '03

$$\frac{dP_n(Y)}{dY} = -\lambda n P_n(Y) + (n-1)\lambda P_{n-1}(Y)$$

depletion of the probability to find n dipoles due to the splitting into (n + 1) dipoles.

the growth due to the splitting of (n − 1) dipoles into n dipoles.

$$P_n(Y) = e^{-\lambda Y}\left(1 - e^{-\lambda Y}\right)^{n-1}$$

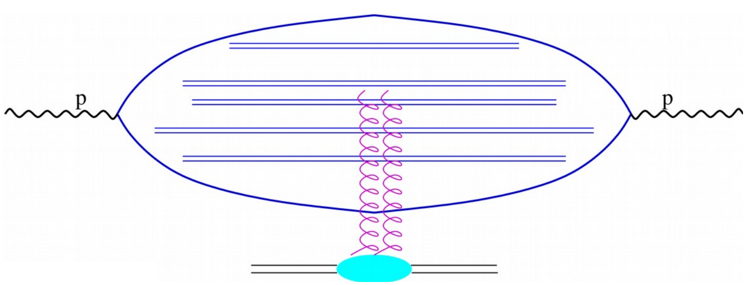
$$S = -\sum_n p_n \ln p_n$$

$$S(Y) = \ln\left(e^{\lambda Y} - 1\right) + e^{\lambda Y}\ln\left(\frac{1}{1 - e^{-\lambda Y}}\right)$$

$S(Y) \approx \lambda Y$ where $Y = \ln 1/x$

$$\langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x}\right)^{\lambda}$$

BFKL intercept



Assumption $xg(x) = \langle n \rangle$

Kharzeev, Levin '17

$$S(x) = \ln(xg(x))$$

$$S(x,Q) = \ln(xg(x,Q))$$

The model can be generalized within 3+1 BK and one can argue how to account for Q dependence

KL entropy formula - interpretation

At low x partonic microstates have equal probabilities $P_n(Y) = e^{-\lambda Y}\left(1 - e^{-\lambda Y}\right)^{n-1}$

In this equipartitioned state the entropy is maximal – the partonic state at small x
is maximally entangled.

In terms of information theory as Shanon entropy:
- equipartitioning in the maximally entangled state means that all “signals” with different number of partons are equally likely
- it is impossible to predict how many partons will be detected in a give event.
- structure function at small x should become universal for all hadrons.

From strict bounds on entanglement entropy (from conformal field theory) one can obtain that at low x (in conformal regime) one has

$$xg(x) \leq \text{const}\, x^{-1/3}$$

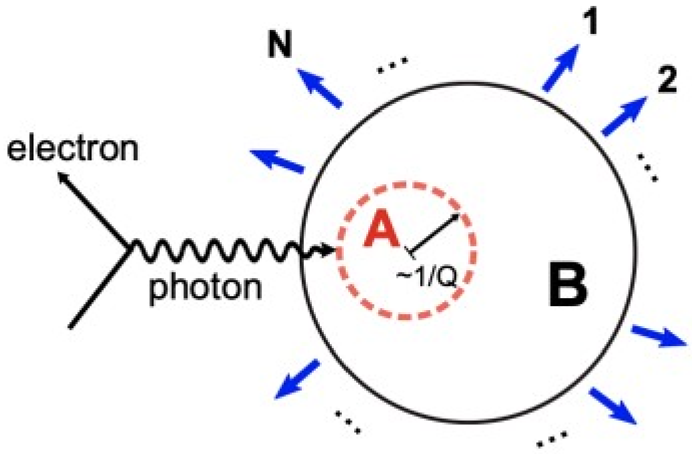
Kharzeev, Levin ‘17

Furthermore entropy of the final state hadrons can not be smaller than entropy of partons.

Entanglement entropy – calculation and measurement

For DIS at high energies, this entanglement entropy can be calculated using

$$S(x, Q^2) = \ln \left\langle n\left(\ln \frac{1}{x}, Q\right)\right\rangle$$

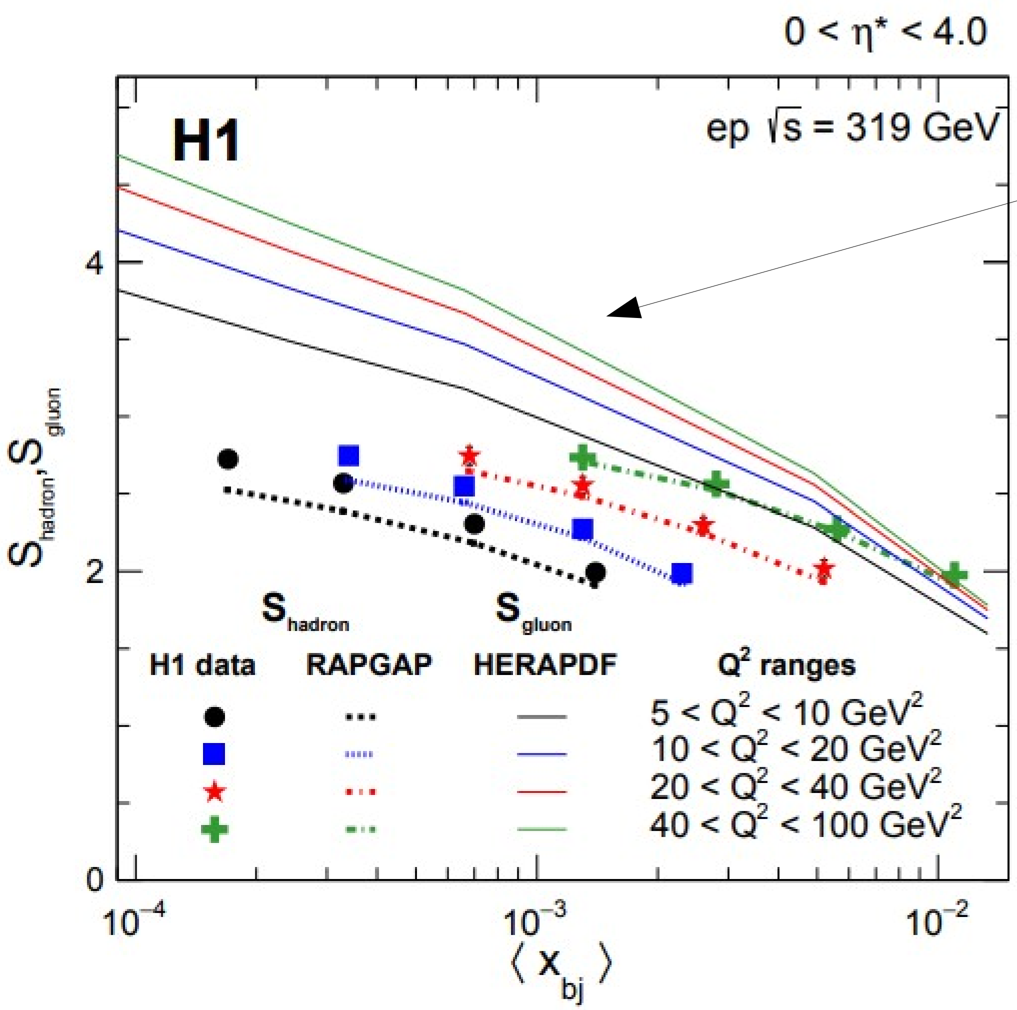


$$S_{hadron} = \sum P(N) \ln P(N)$$

The charged particle multiplicity distribution measured in either the current fragmentation region or the target fragmentation region

Fraction of events with charged hadron

Monte Carlo, KL formula, and data



HERA pdf used

$$S(x,Q) = \ln(xg(x,Q))$$

Also attempt by Kharzeev and Levin to use quarks instead of gluons
Phys. Rev. D 104, 031503 (2021)

$$S(x,Q) = \ln(x\Sigma(x,Q))$$

This argument is however based on incorrect formula...but it is a illuminating mistake

H1

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See also Z. Tu, D. Kharzeev, T. Ulrich ’20 for calculations of ee in p-p.

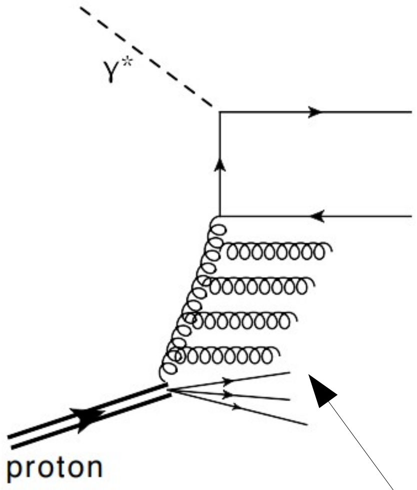
Extension of KL entropy formula

2110.06156 Hentschinski, Kutak

$$\left\langle n\left(\ln\frac{1}{x},Q\right)\right\rangle = xg(x,Q)+x\Sigma(x,Q)$$

To get the entropy of system of partons one needs to account for both quarks and gluons. One can view this as a higher order correction to KL formula. Furthermore it is impossible to isolate quarks from gluons therefore the compete entropy formula should receive contributions from quarks and gluons

Gluon and quark distribution



In the linear regime obeys BFKL Equation. In our calculations we use NLO BFKL with kinematical improvements and running coupling The gluon density has been fitted to F2 data (exact kinematics was used)

Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

We calculate the sea quarks distribution using

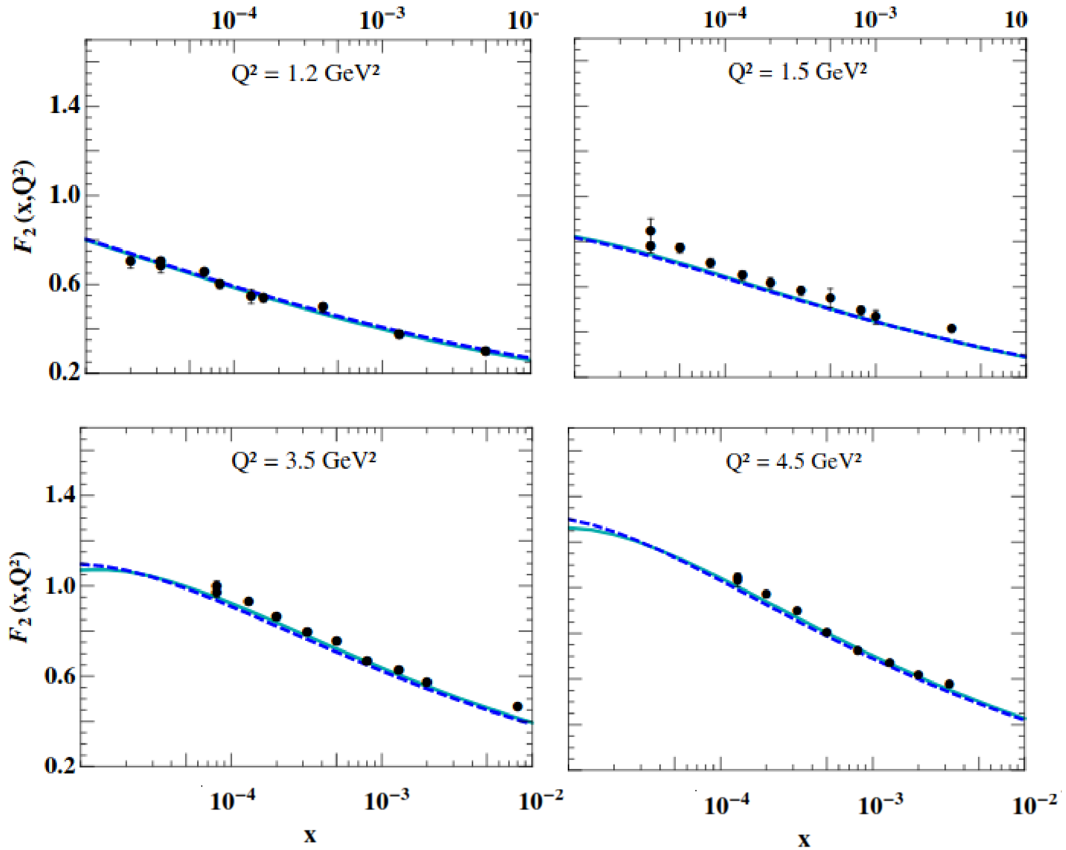
$$x\Sigma(x,Q) = P_{qg}(Q,\mathbf{k}) \otimes \mathcal{F}(x,\mathbf{k}^2)$$

$$xg(x,Q) = \int_0^{Q^2} d\mathbf{k}^2 \mathcal{F}(x,\mathbf{k}^2)$$

Transverse momentum dependent splitting function
Catani, Hautmann
Nucl.Phys. B427 (1994) 475-524

Gluon and quark distribution

F2 data description



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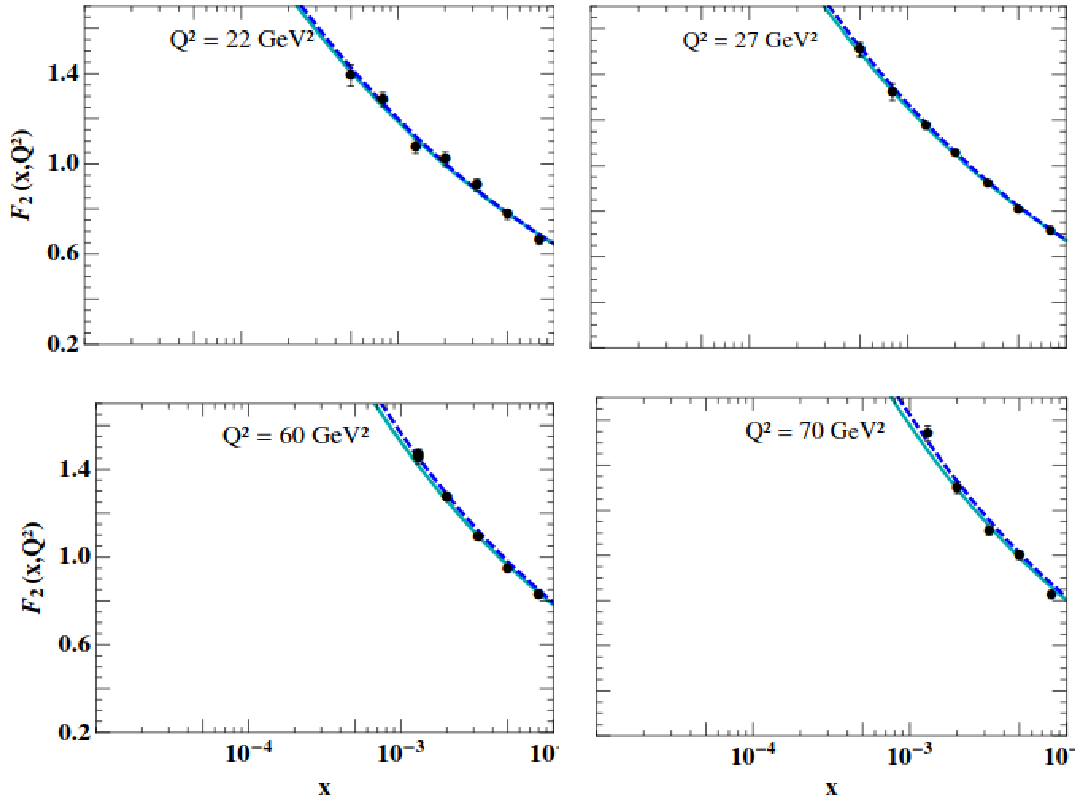

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Results 1

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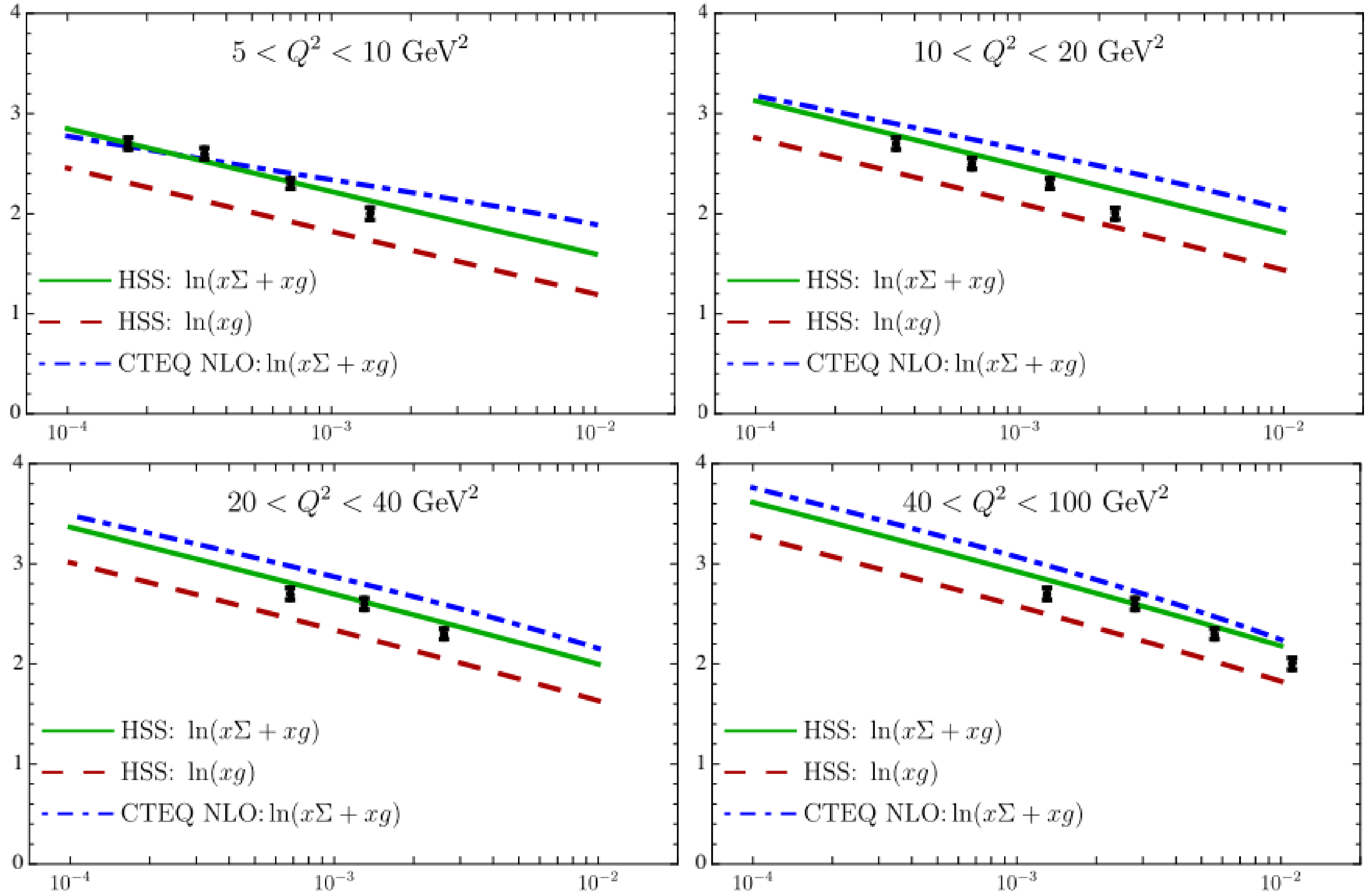

Good description of data. Hint that the general idea works. One definitely has to account both for quarks and gluons.

Results 2

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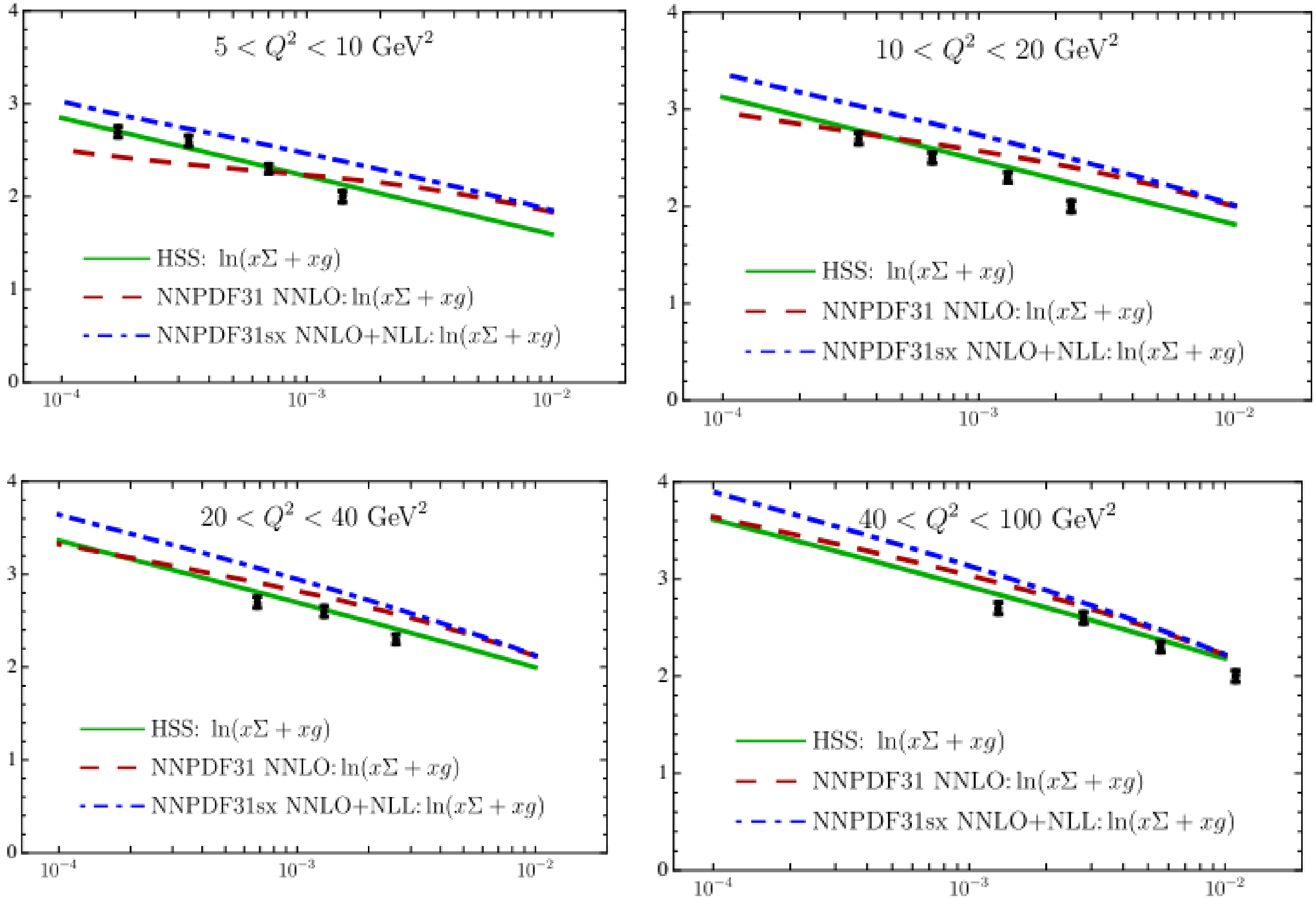

Low x resummation is essential to get good description of data

HSS gluon density used i.e. NLO BFKL + kinematical Improvements

Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
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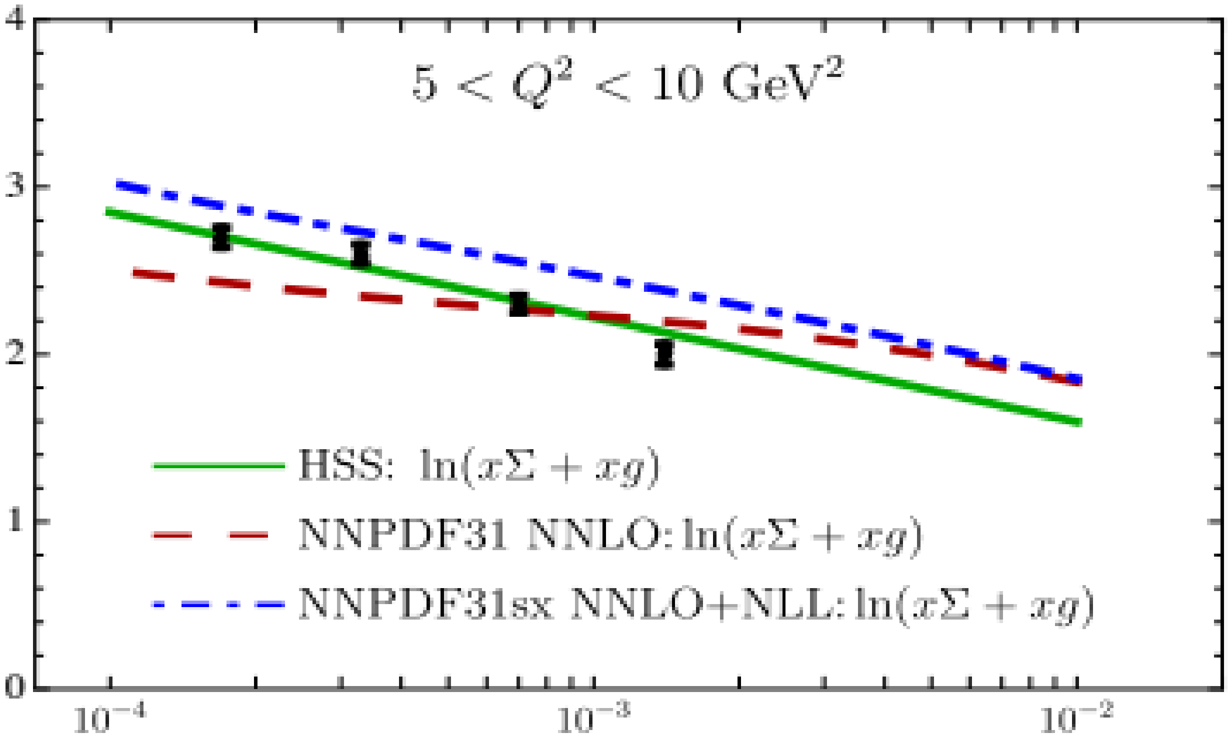
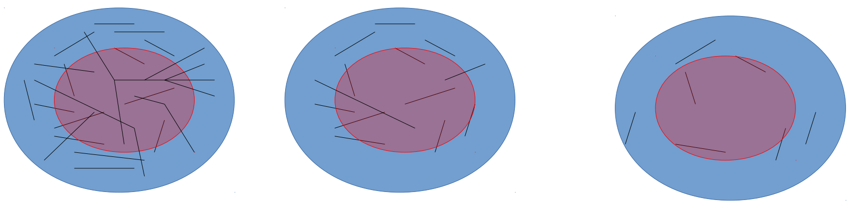
Dipoles and mechanism of entanglement





segments – dipoles, color singlets
maximally entangled states

red circle – resolved area defined by photon

entanglement arises because of dipoles that are partially in the red circle and partially in blue.

They build up pdf and final state hadron multiplicity

If we go to lower x we have more and more dipoles that cross the red line and entanglement grows.

"Entanglement of predictions arises from the fact that the two bodies at some earlier time from in the true sense one system that is were interacting and have left behind choices on each other."

E. Schrodinger

Conclusions and outlook

- We show that the Kharzeev and Levin proposal for low x entanglement entropy can be systematically improved (quark contributions, NLO BFKL) and can describe successfully H1 data.
- We therefore provide phenomenological evidence which is essential for the further development of the field.
- The Q dependence should be derived more directly.
- One can do similar calculations using BK equation. The description of data works very well too. In progress.

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