Maximally entangled low x proton in Deep Inelastic Scattering*

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August 27, 2022

Abstract

We report on our [1, 2] stydy of the proposal by Kharzeev and Levin of a maximally entangled proton wave function in Deep Inelastic Scattering at low x and the proposed relation between parton number and final state hadron multiplicity. We determine partonic entropy from the sum of gluon and quark distribution functions at low x, which we obtain from an unintegrated gluon distribution subject to next-to-leading order Balitsky-Fadin-Kuraev-Lipatov evolution. We find for this framework very good agreement with H1 data.

1 Entanglement entropy

The proton is a coherent quantum state with zero von Neumann entropy. However it has been argued in [3, 4] that when the proton wave function is observed in Deep Inelastic Scattering (DIS) of electrons and protons, this is no longer true. In DIS, the virtual photon, with momentum q and $q^2 = -Q^2$ its virtuality, probes only parts of the proton wave function, which gives rise to entanglement entropy, between observed and unobserved parts of the proton wave function, through tracing out inaccessible degrees of freedom of the density matrix. The resulting entanglement is then a measure of the degree to which the probabilities in the two subsystems are correlated; for other approaches see [5–18]. Based on explicit studies of this entanglement entropy the authors of [3] conclude that DIS probes in the perturbative low x limit a maximally entangled state. With $x = Q^2/2p \cdot q$ and p the proton momentum, the low x limit corresponds to the perturbative high energy limit, where Q^2 defines the hard scale of the reaction and sets the scale of the strong running coupling constant $\alpha_s(Q^2) \ll 1$. The perturbative low x limit of [3] corresponds then to the scenario where parton densities are high, but not yet saturated and non-linear terms in the QCD evolution equations are therefore sub-leading. This is precisely the kinematic regime, where perturbative low x evolution of the proton is described through Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution,

*Presented at DIS2022: XXIX International Workshop on Deep-Inelastic Scattering and Related Subjects, Santiago de Compostela, Spain, May 2-6 2022 which resums terms $[\alpha_s \ln(1/x)]^n$ to all order in α_s; it is this kinematic regime to which the results of [3] are supposed to apply at first.

The proposal that DIS probes in the low x limit a maximally entangled state is closely related to the emergence of an exponentially large number of partonic micro-states which occur with equal probabilities $P_n(Y) = 1/\langle n\rangle$, with $\langle n(Y,Q)\rangle$ the average number of partons at $Y = \ln 1/x$ and photon virtuality Q. Entropy is then directly obtained as

$$S(x, Q^2) = \ln \left\langle n\left(\ln \frac{1}{x}, Q\right)\right\rangle. \tag{1}$$

Assuming that the second law of thermodynamics holds for this entanglement entropy, the above expressions yields a lower bound on the entropy of final states hadrons S_h through $S_h \geq S(x, Q^2)$ [3]. "Local parton-hadron duality" [19] then suggest that partonic entropy coincides with the entropy of final state hadrons in DIS, see also the discussion for hadron-hadron collisions in [5]. The hadronic entropy can be further related to the multiplicity distribution of DIS final state hadrons. The latter has been obtained from HERA data in [20], which allows for a direct comparison of Eq. (1) to experimental data.

Confirmation of Eq. (1) is of high interest, since it links hadron structure to final state multiplicities through entropy. If confirmed, it provides an additional constraint on parton distribution functions (PDFs). Moreover, entropy is defined non-perturbatively and the proposed relation is therefore not necessarily limited to perturbative events, unlike PDFs.

The explicit model calculations of [3] were based on solutions of purely gluonic LO low x evolution, where quarks appear only as a next-to-leading order (NLO) correction; it is therefore natural to assume that at first the total numbers of partons agrees with the number of gluons. In the following we find that for the kinematic regime explored at HERA, quarks are indeed sub-leading, but nevertheless numerically relevant for a correct description of data. We therefore propose in this letter that the average number of partons in Eq. (1) should be interpreted as the sum of the number of all partonic degrees of freedom, *i.e.* of quarks and gluons. Furthermore, since in the experiment only charged hadrons were measured we take only 2/3 of total number of partons.

Our description is based on the NLO BFKL fit [21, 22] (HSS). Initial conditions of the HSS unintegrated gluon distribution have been fitted to HERA data on the proton structure function F_2 and the HSS fit provides therefore a natural framework to verify the validity of Eq. (1) and its conjectured relation to the final state hadron multiplicity. Moreover, the HSS fit is directly subject to NLO BFKL evolution and therefore provides a direct implementation of linear QCD low x evolution.

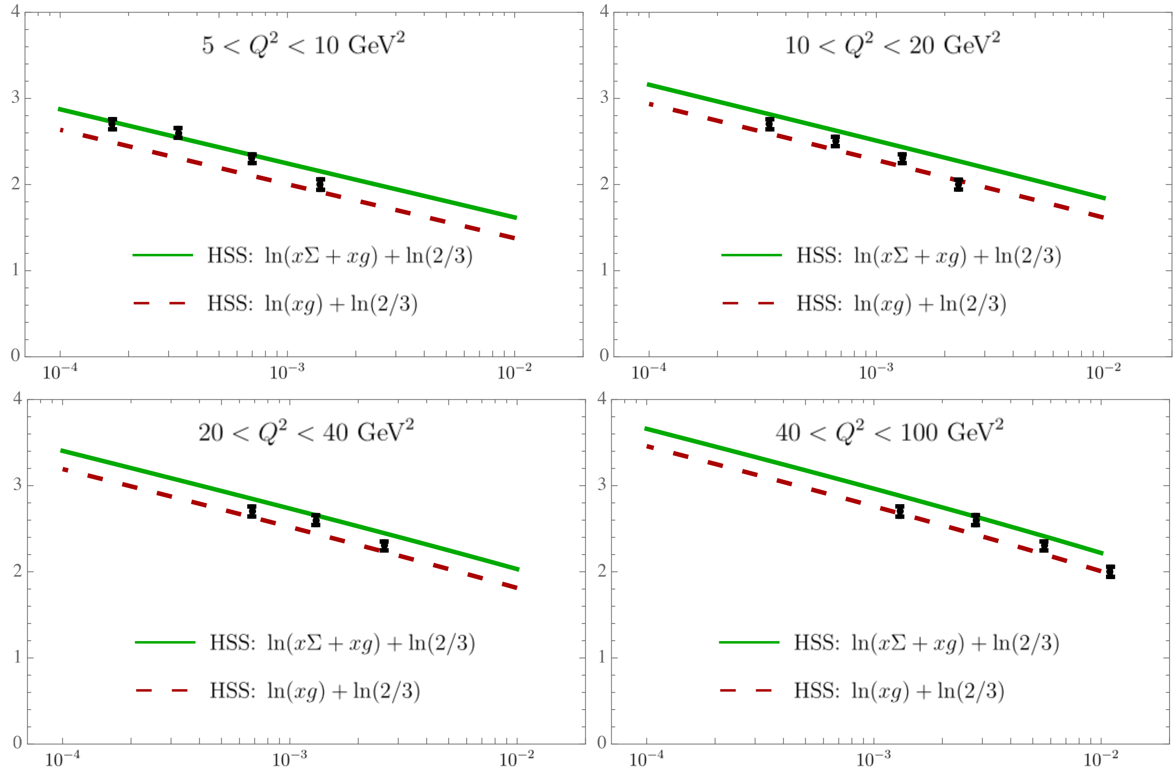

Figure 1: *Partonic entropy versus Bjorken x, as given by Eq. (1) and Eq. (2). We furter show results based on the gluon distribution only as well as on quarks and gluons together. Results are compared to the final state hadron entropy derived from the multiplicity distributions measured at H1 [20]*

2 Results

To compare the HSS unintegrated gluon distribution to data, we need to determine first PDFs, which will yield the total number of partons through [1, 2]

$$\left\langle n\left(\ln\frac{1}{x}, Q\right)\right\rangle = \frac{2}{3}\left[xg(x,Q) + x\Sigma(x,Q)\right], \tag{2}$$

where $g(x, \mu_F)$ ($\Sigma(x, Q)$) denotes the gluon (seaquark) distribution function at the factorization scale μ_F. To this end we use the Catani-Hautmann procedure [23] for the determination of high energy resummed PDFs. At leading order, the prescription is straightforward for the gluon distribution function, which is obtained as

$$xg(x,\mu_F) = \int_0^{\mu_F^2} d\boldsymbol{k}^2 \mathcal{F}(x,\boldsymbol{k}^2), \tag{3}$$

where μ_F denotes the factorization scale which we identify for the current study with the photon virtuality Q, and $\mathcal{F}(x, \boldsymbol{k}^2)$ the unintegrated gluon distribution, subject to BFKL evolution. To obtain the seaquark distribution, we require a transverse momentum dependent splitting function [23] , The integrated seaquark distribution is then obtained as [23]

$$x\Sigma(x,Q) = \int_0^\infty \frac{d\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2} \int_0^\infty d\boldsymbol{k}^2 \int_0^1 dz \Theta\left(Q^2 - \frac{\boldsymbol{\Delta}^2}{1-z} - z\boldsymbol{k}^2\right) \tilde{P}_{qg}\left(z, \frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right) \mathcal{F}(x,\boldsymbol{k}^2). \tag{4}$$

Eq. (3) and Eq. (4) is now used to calculate through Eq. (2) the partonic entropy Eq. (1); the result is then compared to H1 data [20]. To calculate entropy for the H1 Q^2 bins, we employ the following averaging procedure,

$$\bar{S}(x)_{Q_2^2,Q_1^2} = \ln \frac{1}{Q_2^2 - Q_1^2} \int_{Q_1^2}^{Q_2^2} dQ^2 \left\langle n\left(\ln \frac{1}{x}, Q\right)\right\rangle . \tag{5}$$

The results of our study are shown in Fig. 1, where we evaluate all expressions for $n_f = 4$ flavors. We find that the partonic entropy obtained from the total number of partons gives a very good description of H1 data [20]. As anticipated in [3], the purely gluonic contribution is clearly dominant. Given the approximations taken in the derivation of Eq. (2) as well as the possibility that sub-leading corrections are relevant for the determination of hadronic entropy form the multiplicity distribution, we believe that the above result provides an impressive confirmation of Eq. (2) and the results of [3] in general. We obtained similarly good description in [2] using rcBK as well as LOHERA pdfs.
A different approaches have been provided in [25] which uses the sea quark distribution only and [27] which uses Page entropy definition which however has very similar form to KL formula.and difers by additive constant.

Acknowledgments

MH is grateful for support by Consejo Nacional de Ciencia y Tecnología grant number A1 S-43940 (CONACYT-SEP Ciencias Básicas). KK acknowledges the European Union's Horizon 2020 research and innovation programme under grant agreement No. 824093.

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