

CT18QED

Photon PDF in the CTEQ-TEA global analysis

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DIS 2022, May 4, 2022

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Phys.Rev.D 105 (2022) 5, 054006 (2106.10299), also see 2107.13580.

The precision requirements

The precision requirements

- The LHC becomes a precision machine.
- Theoretical cross sections have been achieved at NNLO in QCD, $\mathcal{O}(\alpha_s^2)$, for many processes.
- Due to $\alpha_e \sim \alpha_s^2$, we expect the QED corrections are the same level.
- The photon-initiated processes ($\gamma + \gamma, q, g \rightarrow X$) will have observable effects.

Many applications

The SM processes

- Drell-Yan: $\ell^+ \ell^-$
- $W^\pm H$
- $W^+ W^-$

BSM scenarios

- Heavy leptons: $L^+ L^-$
- Charged Higgs: $H^\pm, H^{\pm\pm}$ [\[2107.13580\]](#)

The existing photon PDFs

The first generation

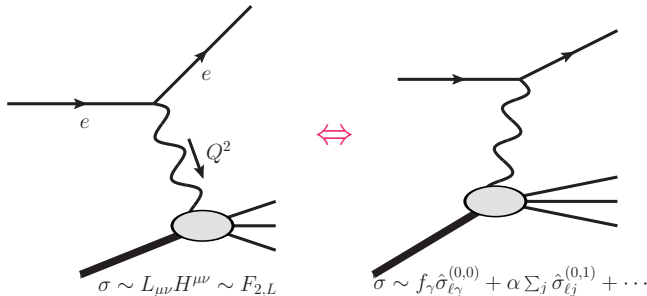
- MRST2004QED [\[0411040\]](#) models the photon PDF with an effective mass scale.
- NNPDF23QED [\[1308.0598\]](#) and NNPDF3.0QED [\[1410.8849\]](#) constrains photon PDF with the LHC Drell-Yan data, $q\bar{q}, \gamma\gamma \rightarrow \ell^+\ell^-$
- CT14qed_inc fits the inelastic ZEUS $ep \rightarrow e\gamma + X$ data [\[1509.02905\]](#), and include elastic component as well.

The second generation

- LUXqed directly takes the structure functions $F_{2,L}(x, Q^2)$ to constrain photon PDF uncertainty down to **percent level** [\[1607.04266,1708.01256\]](#)
- NNPDF3.1luxqed [\[1712.07053\]](#) initializes photon PDF with LUX formula at **$\mu_0 = 100$ GeV** (a high scale) and evolves DGLAP equation both upwardly and downwardly.
- MMHT2015qed [\[1907.02750\]](#) initializes photon at **$\mu_0 = 1$ GeV** (a low scale) and evolve DGLAP upwardly. It's updated as MSHT20qed by the recent fit [\[2111.05357\]](#).
- Our work incorporates the LUX formalism with the CT18 [\[1912.10053\]](#) global analysis.

The LUX formalism [1607.04266,1708.01256]

- The DIS process: $ep \rightarrow e + X$



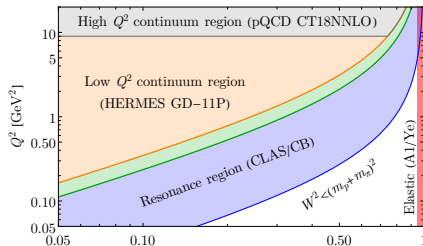
- Matching these two approaches leads to the LUX master formula:

$$x\gamma(x, \mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{dz}{z} \left\{ \int_{\frac{x^2 m_p^2}{1-z}}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha_{\text{ph}}^2(-Q^2) \left[\left(zp_{\gamma q}(z) + \frac{2x^2 m_q^2}{Q^2} \right) \times \right. \right. \\ \left. \left. F_2(x/z, Q^2) - z^2 F_L(x/z, Q^2) \right] - \alpha^2(\mu^2) z^2 F_2(x/z, \mu^2) \right\}.$$

The square bracket term corresponds to the “**physical factorization**” scheme, while the second term is referred as the “ **$\overline{\text{MS}}$ -conversion**” term.

- The structure functions $F_{2,L}$ can be directly measured, or calculated through pQCD in the high-energy regime.

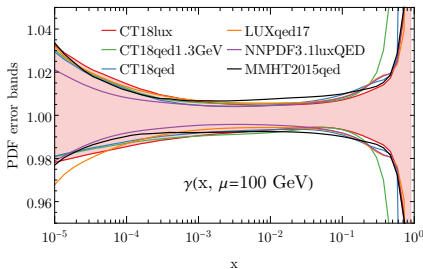
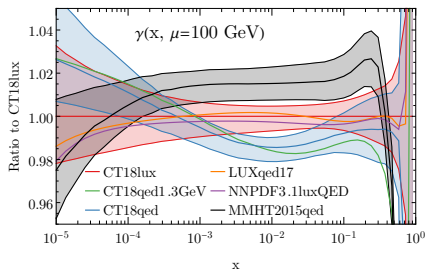
The breakup of (x, Q^2) plane: nonperturbative resources



- In the resonance region $W^2 = m_p^2 + Q^2(1/x - 1) < W_{\text{lo}}^2 = 3 \text{ GeV}^2$, the structure functions are taken from CLAS [\[0301204\]](#) or Christy-Bosted [\[0712.3731\]](#) fits.
- In the low- Q^2 continuum region $W^2 > W_{\text{hi}}^2 = 4 \text{ GeV}^2$, the HERMES GD11-P [\[1103.5704\]](#) fits with ALLM [\[PLB1991\]](#) functional form.
- In the high- Q^2 region ($Q^2 > Q_{\text{PDF}}^2$), $F_{2,L}$ are determined through pQCD.
- The elastic form factors are taken from A1 [\[1307.6227\]](#) or Ye [\[1707.09063\]](#) fits of world data.

Two approaches: LUX vs DGLAP

- CT18lux: directly calculate the photon PDF with the **LUX** formalism
- CT18qed: initialize the inelastic photon PDF with the LUX formalism at low scales, and evolve the $\text{QED}_{\text{NLO}} \otimes \text{QCD}_{\text{NNLO}}$ **DGLAP** equations up to high scales, similar to MMHT2015qed.



The take-home message:

- In the intermediate- x region, all photon PDFs give similar error bands.
- CT18lux photon PDF is **in between** LUXqed (also, NNPDF3.1luxQED) and MMHT2015qed, while CT18qed gives a **smaller** photon PDF.
- In the large- x region, the DGLAP approach (for both MMHT2015qed and CT18qed) gives a smaller photon than the LUX approach.

The difference between LUX and DGLAP

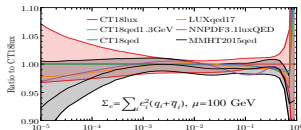
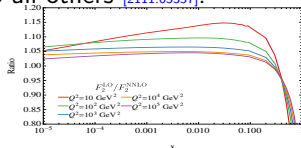
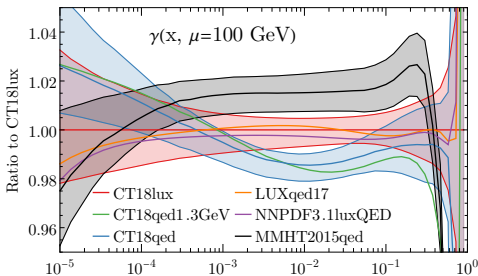
- The DGLAP only evolves the inelastic photon

$$\frac{dx\gamma^{\text{inel}}}{d\log\mu^2} = \frac{\alpha}{2\pi} \left(xP_{\gamma\gamma} \otimes x\gamma^{\text{inel}} + \sum_i e_i^2 xP_{\gamma q} \otimes xq_i \right)$$

- The first-order solution corresponds to the LO F_2 in LUX formalism

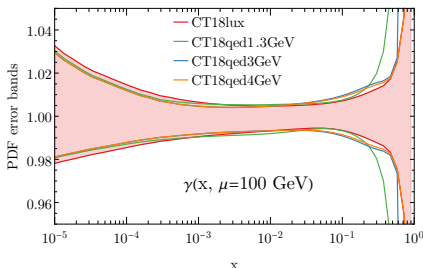
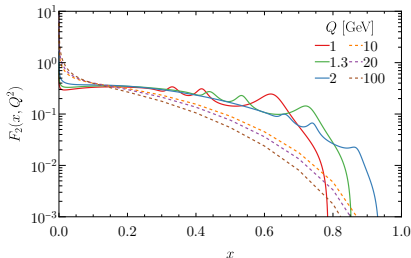
$$x\gamma^{\text{inel}}(x, \mu^2) \sim \int^{\mu^2} d\log Q^2 \frac{\alpha}{2\pi} \sum_i e_i^2 xP_{\gamma q} \otimes \textcolor{red}{x}f_{q_i} \rightarrow F_2^{\text{LO}} \text{ in LUX formula}$$

- It explains CT18qed gives larger photon at small x than CT18lux.
- MMHT2015qed gives smaller photon at small x , because the smaller charge-weighted singlet quark distributions.
- The updated MMHT20qed gets very closed to all others [\[2111.05357\]](#).

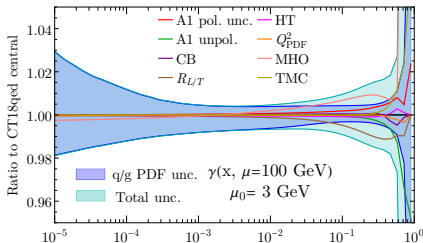
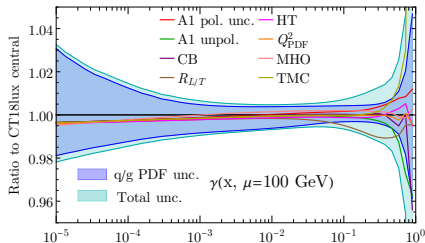


The large x behavior: nonperturbative contribution

- At large x , the LUX approach gives significantly larger PDF than the DGLAP one.
- It is resulted from the non-perturbative F_2 at low energy (resonance and low- Q^2 continuum regions).
- It induces a big uncertainty with the DGLAP low initialization scale approach, because scaling violation is not well behaved in the non-perturbative F_2 .
- It can be rescued with a slightly higher initialization scale above the pQCD matching scale $Q_{\text{PDF}} \sim 3$ GeV, as compared to CT18's 1.3 GeV.

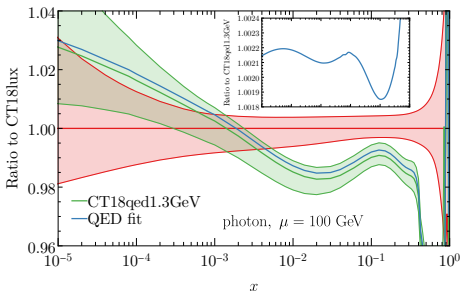
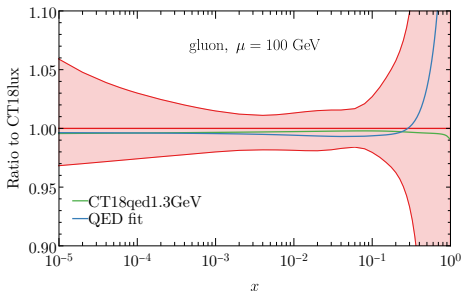
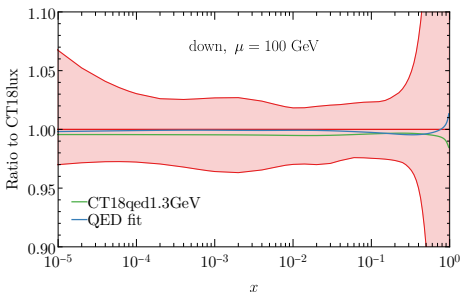
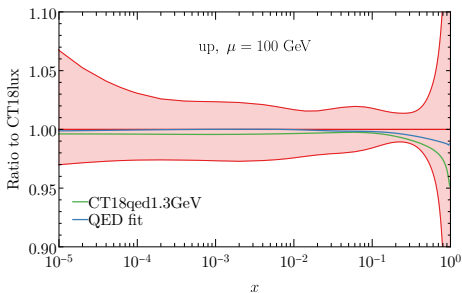


Photon PDF uncertainties



- A1 pol. unc.: the uncertainty of the A1 fit of the world^x polarized data
- A1 unpol.: Switching to A1 fit of the world unpolarized data
- CB: Changing resonance SF from CLAS to Christy-Bosted fit
- Variations of $R_{L/T} = \sigma_L/\sigma_T$ by 50% [1708.01256]
- HT: Adding higher-twist contribution to F_L [1708.01256] and F_2 [1602.03154].
- Q_{PDF}^2 : changing the matching scale $9 \rightarrow 5 \text{ GeV}^2$
- MHO: varying the scale to estimate the missing high-order uncertainty
- TMC: adding the target mass correction to the SFs.

Global fit with QCD+QED evolution (“QEDfit”)



Fitting quality: χ^2

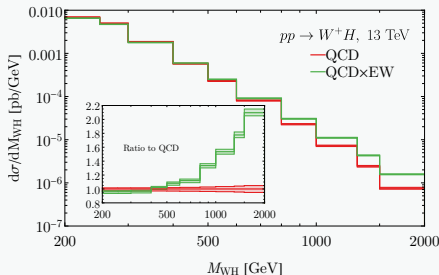
ID	Experimental dataset	References	N_{pt}	CT18lux	CT18qed	QED fit
160	HERA-I + II 1 fb ⁻¹ , H1 and ZEUS combined	[61]	1120	1406	1405	1405
101	BCDMS F_2^p	[60]	337	375	381	377
102	BCDMS F_2^d	[62]	250	281	283	281
104	NMC F_2^p/F_2^d	[63]	123	126	126	126
108	CDHSW F_2^p	[64]	85	85.6	86.6	86.6
109	CDHSW $x_B F_2^p$	[64]	96	86.4	87.1	86.0
110	CCFR F_2^p	[65]	69	78.4	77.6	77.7
111	CCFR $x_B F_2^p$	[66]	86	33.4	32.3	33.9
124	NuTeV $\nu\mu\mu$ SIDIS	[67]	38	18.6	18.8	18.4
125	NuTeV $\nu\mu\mu$ SIDIS	[67]	33	38.4	38.5	37.8
126	CCFR $\nu\mu\mu$ SIDIS	[68]	40	29.8	29.7	29.8
127	CCFR $\nu\mu\mu$ SIDIS	[68]	38	19.8	19.7	19.8
145	H1 e_p^+	[69]	10	6.81	6.81	6.91
147	Combined HERA charm production	[70]	47	58.7	58.7	57.7
169	H1 F_L	[71]	9	17.0	17.0	16.9
201	E866 Drell-Yan $sd^2\sigma/(d\sqrt{s}dy)$	[72]	119	103	104	103
203	E866 Drell-Yan $\sigma_{\text{had}}/(2\sigma_{\text{pp}})$	[73]	15	16.2	16.4	16.6
204	E866 Drell-Yan $Q^2 d^2\sigma_{pp}/(dQ^2 dx_F)$	[74]	184	244	245	246
225	CDF Run-1 lepton A_{th} , $p_{T\ell} > 25$ GeV	[75]	11	9.04	9.30	9.17
227	CDF Run-2 electron A_{th} , $p_{T\ell} > 25$ GeV	[76]	11	13.5	12.8	13.4
234	DØ Run-2 muon A_{th} , $p_{T\ell} > 20$ GeV	[77]	9	8.91	10.2	9.36
260	DØ Run-2 Z rapidity	[78]	28	16.8	16.8	16.8
261	CDF Run-2 Z rapidity	[79]	29	49.1	50.5	49.1
266	CMS 7 TeV 4.7 fb ⁻¹ , muon A_{th} , $p_{T\ell} > 35$ GeV	[80]	11	7.72	8.23	7.92
267	CMS 7 TeV 840 pb ⁻¹ , electron A_{th} , $p_{T\ell} > 35$ GeV	[81]	11	11.0	12.4	12.0
268	ATLAS 7 TeV 35 pb ⁻¹ , W/Z cross sec., A_{th}	[82]	41	44.8	44.1	44.0
281	DØ Run-2 9.7 fb ⁻¹ , electron A_{th} , $p_{T\ell} > 25$ GeV	[83]	13	22.9	23.6	22.4
504	CDF Run-2 inclusive jet production	[84]	72	125	126	124
514	DØ Run-2 inclusive jet production	[85]	110	114	113	114

ID	Experimental dataset	Ref.	N_{pt}	CT18lux	CT18qed	QED fit
245	LHCb 7 TeV 1.0 fb ⁻¹ , forward W/Z	[59]	33	53.4	49.9	53.9
246	LHCb 8 TeV 2.0 fb ⁻¹ , forward $Z \rightarrow e^+e^-$	[86]	17	25.5	23.7	25.5
249	CMS 8 TeV 18.8 fb ⁻¹ , muon A_{th}	[58]	11	12.4	15.5	11.7
250	LHCb 8 TeV 2.0 fb ⁻¹ , forward W/Z	[87]	34	73.2	69.2	72.6
253	ATLAS 8 TeV 20.3 fb ⁻¹ , Z , p_T	[88]	27	30.0	29.4	31.1
542	CMS 7 TeV 5 fb ⁻¹ , single incl. jet $R = 0.7$	[89]	158	195	193	195
544	ATLAS 7 TeV 4.5 fb ⁻¹ , single incl. jet $R = 0.6$	[90]	140	202	200	204
545	CMS 8 TeV 19.7 fb ⁻¹ , single incl. jet $R = 0.7$	[91]	185	213	220	210
573	CMS 8 TeV 19.7 fb ⁻¹ , $\bar{t}t$ ($1/\sigma d^2\sigma/(dp_T^2 dy)$)	[92]	16	18.9	18.8	18.9
580	ATLAS 8 TeV 20.3 fb ⁻¹ , $\bar{t}t$ de/dp_T^2 and $de/dm_{t\bar{t}}$	[93]	15	9.51	9.49	9.70
Total χ^2 for all 39 datasets			3681	4293	4302	4296

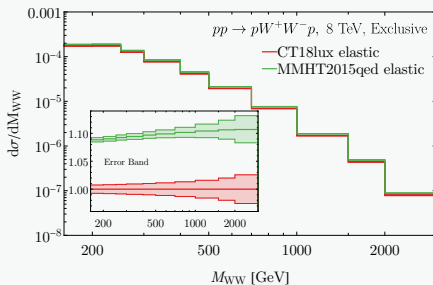
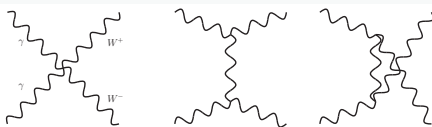
- The CT18lux share the same χ^2 as CT18, as quark and gluon PDFs remain the same.
- CT18QED gives a small corrections to up and down quark PDFs, which increases χ^2 a little.
- Global fit with QCD+QED evolution (“QEDfit”) pull the PDFs and χ^2 back, very closed to CT18lux.

The applications

W^+H production

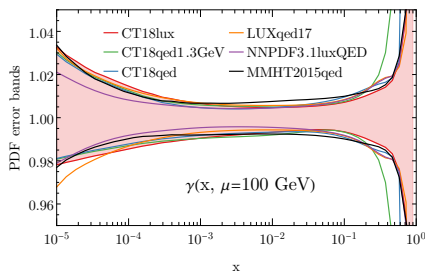
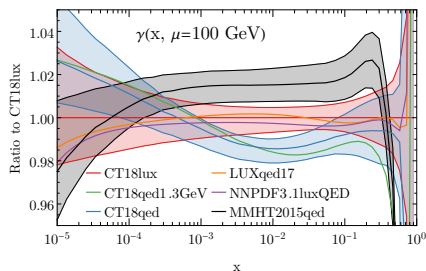


Exclusive $\gamma\gamma \rightarrow W^+W^-$



- At a large invariant mass, the photon initiated processes make a significant contribution
- CT18lux elastic photon (α_e running includes both **quarks** and **leptons**) is smaller than MMHT2015qed one (where only quarks are included).

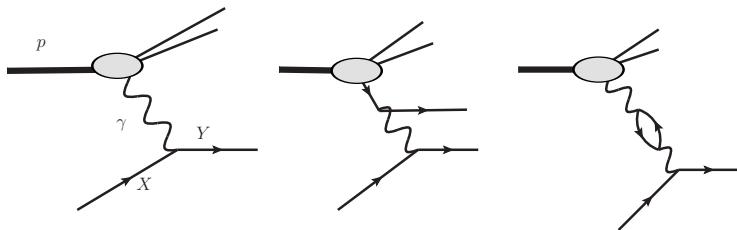
Summary and conclusions



- We published two photon PDF sets, CT18lux and CT18qed, based on the LUX and DGLAP approach, respectively.
- The overall uncertainties agree with the LUXqed (also NNPDF3.1luxQED) and MMHT2015qed.
- In the intermediate- x region, CT18lux is in between the LUXqed (also NNPDF3.1luxQED) and MMHT2015qed, while CT18qed is smaller.
- In the small- x region, the CT18qed is larger than CT18lux, due to the equivalent LO SF. The MMHT2015qed becomes smaller because of the smaller singlet PDFs Σ_e .
- In the large- x region, the DGLAP approach (MMHT2015qed and CT18qed) give smaller PDFs due to the non-perturbative SF contribution included in CT18lux.
- The low- μ_0 DGLAP approach gives larger uncertainty at large x , due to non-perturbative SFs at low scales.

The cancellation in a higher order calculation

- Suppose we want to calculate a process $\gamma + X \rightarrow Y$.



- At one order higher, both photon and quark parton will participate.
- The PDFs are related with the DGLAP evolution, with divergence properly canceled.
- This can be also achieved in the LUX approach, with proper $\overline{\text{MS}}$ conversion terms order by order.

The scale variation of the $\overline{\text{MS}}$ conversion term

- In the default scale choice $\mu^2/(1-z)$, the $\overline{\text{MS}}$ -conversion term is

$$x\gamma^{\text{con}} \sim (-z^2)F_2(x/z, \mu^2),$$

which is negative

- When varying the scale as μ^2 , the conversion term should be change as well,

$$x\gamma^{\text{con}}([M]) = x\gamma^{\text{con}} + \frac{1}{2\pi\alpha} \int_x^1 \frac{dz}{z} \int_{M^2[z]}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2 z p_{\gamma q}(z) F_2(x/z, Q^2).$$

With $M^2[z] = \mu^2$, we have $\int_{\mu^2}^{\frac{\mu^2}{1-z}} \frac{dQ^2}{Q^2} = \log \frac{1}{1-z}$.

- The central MMHT2015qed corresponds to $M^2[z] = \mu^2$ choice at low scale $\mu_0 = 1 \text{ GeV}$.
- The DGLAP approach at low scale **DOES** give larger uncertainty due to the large non-perturbative contributions to structure functions.
- One method to avoid it is to start γ PDF at a higher scale in the pQCD region, i.e., $\mu_0^2 \geq Q_{\text{PDF}}^2$.

The DGLAP approach gives smaller PDFs at large x

- MMHT2015qed divides the integration into two regions:

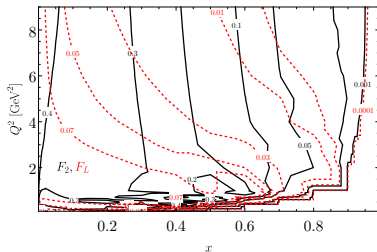
$$\left(\int_{\frac{x^2 m_p^2}{1-z}}^{\mu_0^2} + \int_{\mu_0^2}^{\frac{\mu_0^2}{1-z}} \right) [\dots]$$

The second part is integrated semi-analytically:

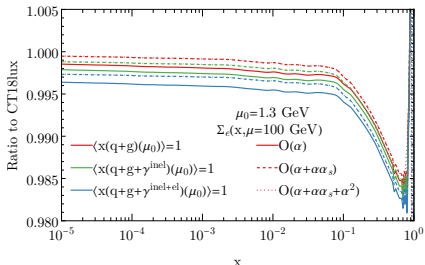
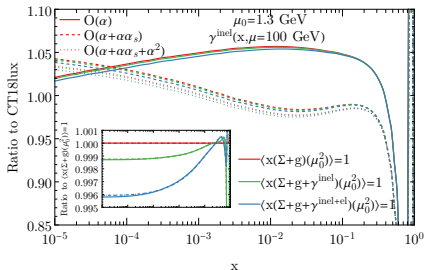
$$\int_{\mu_0^2}^{\frac{\mu_0^2}{1-z}} \frac{dQ^2}{Q^2} \alpha^2 \left(zp_{\gamma q} + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, \mu_0^2) = \alpha^2(\mu_0^2) \left(zp_{\gamma q} \log \frac{1}{1-z} + \frac{2x^2 m_p^2 z}{\mu_0^2} \right) F_2\left(\frac{x}{z}, \mu_0^2\right)$$

The F_L is dropped because $F_L \sim \mathcal{O}(\alpha_s) \ll F_2$.

- In contrast, we integrate over $F_2(x/z, Q^2)$ rather than $F_2(x/z, \mu_0^2)$.
- It explains the MMHT2015qed gives smaller photon at large x than CT18qed.
- MMHT15 does not include the uncertainty induced by μ_0 variation.



The NLO QED evolution and momentum sum rules



- The NLO QED corrections to splitting functions

$$P_{ij} = \frac{\alpha}{2\pi} P_{ij}^{(0,1)} + \frac{\alpha}{2\pi} \frac{\alpha_s}{2\pi} P_{i,j}^{(1,1)} + \left(\frac{\alpha}{2\pi} \right)^2 P_{ij}^{(0,2)} + \dots$$

- The NLO QED correction is negative.
- The momentum sum rules: the impact is $\mathcal{O}(0.1\%)$, negligible compared with higher order QED evolution.

$$\langle x(\Sigma + g + \gamma^{\text{inel+el}}) \rangle = 1$$