

# Investigating saturation effects & the virtual pion in leading neutron events with the color dipole model

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## Abstract

We investigate the Feynman- $x$  spectra of the neutrons produced in the very forward direction in  $ep$  collisions using the impact-parameter dependent color dipole model with and without saturation. Our analysis demonstrate that the  $W$  and  $Q^2$  dependence of the cross-section is independent of the presence of a forward neutron, as predicted by Feynman-scaling. The models prediction are compared with the available HERA data for leading neutrons in  $6 < Q^2 < 100 \text{ GeV}^2$ ,  $70 < W < 245 \text{ GeV}$  and our analysis show that Feynman scaling exists in both the models independent of the value of  $Q^2$ . Thus, we infer that the HERA measurements of semi-inclusive leading neutron production data is insensitive to the non-linear physics and these spectra may not be able to distinguish the gluon saturation effects in future  $ep$  colliders. Additionally, we provide an estimate of the leading neutron structure function at small  $x$  and further show that the observables in exclusive measurements of the leading neutrons with a vector meson in the final state are sensitive to saturation physics at small  $x$ .

## 1 Introduction

The Dipole models of deep-inelastic scattering(DIS) have been very successful in explaining the inclusive DIS and very economically describes the exclusive data at low  $x$ . In some of the DIS events, baryons carrying large fraction of longitudinal momentum ( $x_L > 0.3$ ) of the proton are produced in the far forward direction, commonly called as leading baryons. One can tag this leading baryon in the experiment and perform a semi-inclusive measurement. The H1 collaboration performed the measurements of leading neutron structure function and the Feynman- $x$  spectra in these events in the kinematic regime  $6 < Q^2 < 100 \text{ GeV}^2$  and  $70 < W < 245 \text{ GeV}$  [1, 2] and found the data in agreement with the Feynman scaling. In the events with leading neutrons, one can study the structure of pion employing the one-pion exchange approximation (OPE) [3]. Here in this contribution, we investigate these events using OPE and the impact parameter dependent dipole models: the bSat or IP-Sat model and the linearized version (bNonSat or IP-NonSat) of the applied dipole amplitude, which makes it possible to estimate the magnitude of the saturation effects by comparing the two models. We find that both parametrisation provide a good description of the considered data in all the

kinematic regime and Feynman scaling is not associated with saturation. Further, we show that events the exclusively produced vector meson with leading neutrons are sensitive to non-linear effects and provide a prediction for the  $\hat{t}$  spectrum of the exclusive vector meson production.

## 2 Leading neutrons in the color dipole picture

### 2.1 Leading neutron structure function

In the dipole picture, the virtual photon coming from the incoming electron splits into quark-antiquark pair and forms a color dipole which then interacts strongly with the target. In the case of leading neutrons, the dipole probes the pion cloud of the proton, and the forward neutron comes from the proton as it splits into a neutron and a positive pion. In the one-pion exchange approximation, at high energies, the differential cross section for  $\gamma^* p \rightarrow Xn$  can be written as:

$$\frac{d^2\sigma(W, Q^2, x_L, t)}{dx_L dt} = f_{\pi/p}(x_L, t) \sigma^{\gamma^* \pi^*}(\hat{W}^2, Q^2) \quad (1)$$

where  $f_{\pi/p}(x_L, t)$  is the flux of pions emitted by the proton and  $\sigma^{\gamma^* \pi^*}$  is the cross section of  $\gamma^* \pi^*$  interactions. The leading neutron structure function becomes [1]:

$$F_2^{LN}(W, Q^2, x_L) = \Gamma(x_L, Q^2) F_2^\pi(W, Q^2, x_L) \quad (2)$$

Here  $\Gamma(x_L, Q^2) = \mathcal{K}(Q^2) \int_{t_{\min}}^{t_{\max}} f_{\pi/p}(x_L, t) dt$  is the pion flux factor integrated over the  $t$ -region of the measurement and corrected for the absorptive effects and  $F_2^\pi(W, Q^2, x_L) = \frac{Q^2}{4\pi^2 \alpha_{\text{EM}}} \sigma^{\gamma^* \pi^*}(\hat{W}^2, Q^2)$  is the pion structure function. The flux factor  $f_{\pi/p}(x_L, t)$  describes the splitting of a proton into a  $\pi n$  system and is given by [8]:

$$f_{\pi/p}(x_L, t) = \frac{1}{4\pi} \frac{2g_{p\pi p}^2}{4\pi} \frac{|t|}{(m_\pi^2 + |t|)^2} (1 - x_L)^{1-2\alpha(t)} [F(x_L, t)]^2 \quad (3)$$

where  $m_\pi$  is the pion mass,  $g_{p\pi p}^2/(4\pi) = 14.4$  is the  $\pi^0 pp$  coupling.  $F(x_L, t)$  is the form factor which accounts for the finite size of the vertex and is given by  $F(x_L, t) = \exp\left[-R^2 \frac{|t| + m_\pi^2}{(1-x_L)}\right]$ ,  $\alpha(t) = 0$  where  $R = 0.6 \text{ GeV}^{-1}$  has been determined from HERA data [4]. Using the optical theorem, the total  $\gamma^* \pi^*$  cross section is given by the imaginary part of the forward elastic  $\gamma^* \pi^* \rightarrow \gamma^* \pi^*$  amplitude as following:

$$\sigma_{L,T}^{\gamma^* \pi^*}(\hat{x}, Q^2) = \int d^2\mathbf{b} d^2\mathbf{r} \int_0^1 \frac{dz}{4\pi} |\Psi_{L,T}^f(\mathbf{r}, z, Q^2)|^2 \frac{d\sigma_{q\bar{q}}^{(\pi)}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, \hat{x}) \quad (4)$$

where  $\hat{x} = \frac{Q^2 + m_f^2}{(1-x_L)W^2 + Q^2}$  is the scaled Bjorken variable for the photon-pion system. The photon wavefunctions are well known quantities calculated in [5] and for dipole-pion cross section, we assume that the dipole-pion cross section is related to the dipole-proton cross section by following [6–8],

$$\frac{d\sigma_{q\bar{q}}^{(\pi)}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, \hat{x}) = R_q \frac{d\sigma_{q\bar{q}}^{(p)}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, \hat{x}) \quad (5)$$

we consider two versions of the dipole model for the dipole-proton cross section as discussed in detail in [9]; the saturated bSat model, which tames the growth the gluon density at small- $x$  and large  $r$  by multiple two-gluon scatterings, and its linearised version the bNonSat model

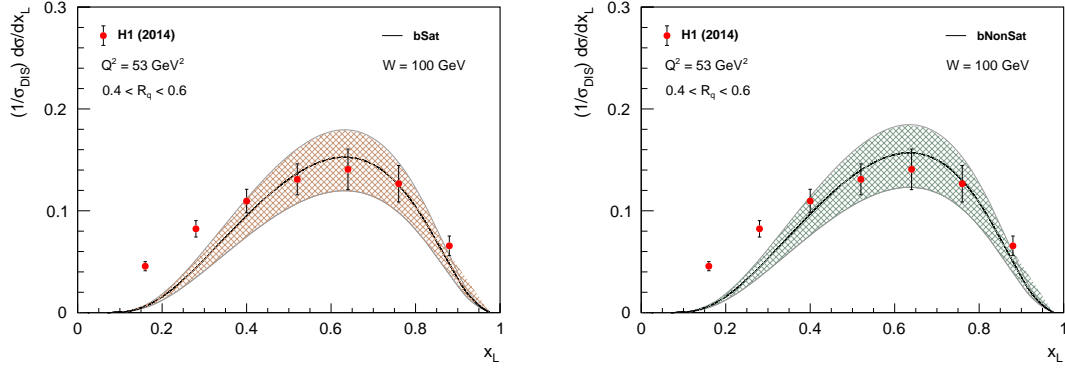


Figure 1: Leading neutron spectrum in bSat model (left) and bNonSat model (right) from [9].

which describes a simple two gluon exchange. The resulting leading neutron spectrum with the different values of parameter  $R_q$  is shown in Fig. 1 where we see that the good description of the data in both the saturated and non saturated models at large  $x_L$  values validates the assumption made in eq.(5). In Fig. 2, we plot the leading neutron structure function and obtain a good description of the data in both the models and the curves for the models are on top of each other in all kinematic region implying that no saturation effects are present in this data. Fig. 3 shows the scaling behavior of the leading neutron spectrum. We observe that

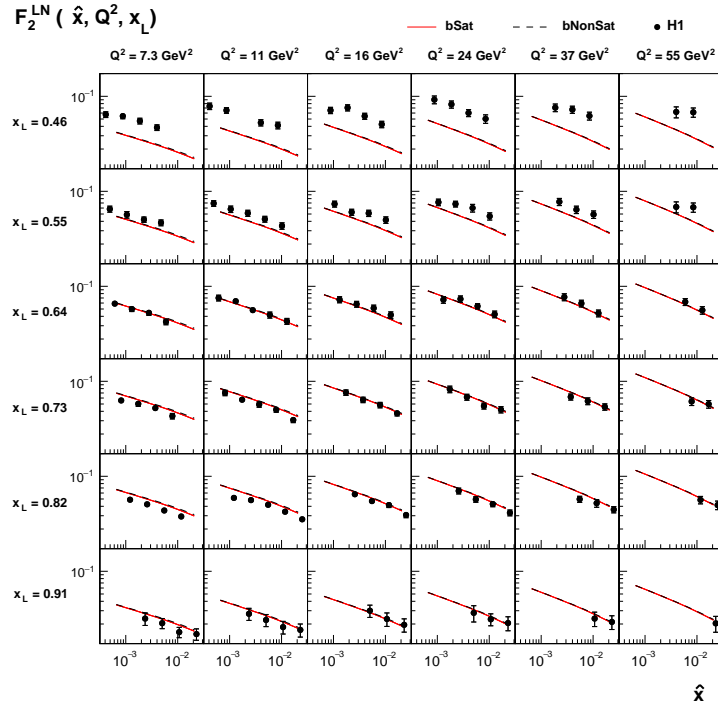


Figure 2: Leading neutron structure function  $F_2^{LN}(\hat{x}, Q^2, x_L)$  as function of  $\hat{x}$ , for different values of  $Q^2$  and  $x_L$ , in the bSat (red solid line) and the bNonSat (black dashed line) dipole models from [9].

this data exhibits Feynman scaling with respect to  $W$ . The band corresponds to the  $W$  values in the range  $100 < W < 1000 \text{ GeV}$ . We show it for two values of  $Q^2 = 6, 53 \text{ GeV}^2$  and observe

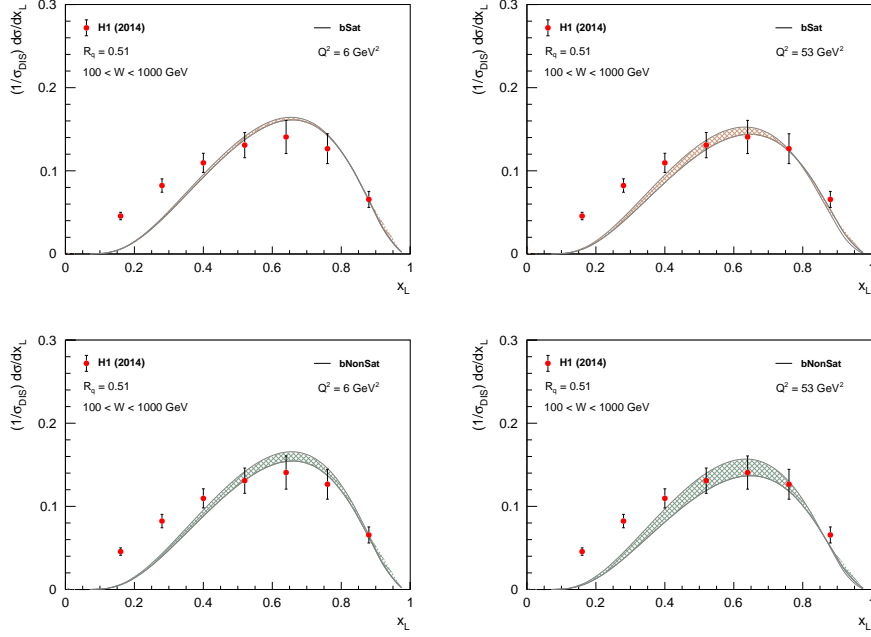


Figure 3: Scaling w.r.t  $W$  in leading neutron spectrum in bSat (first row) and bNonSat (second row) dipole models from [9].

that this scaling is present in both models. This is because the photon-pion cross section has the same energy dependence as the photon-proton cross section in both the models. This also leads us to conclude that saturation is not associated to Feynman-scaling and is present for all  $Q^2$  values in both models.

## 2.2 Exclusive vector meson production with leading neutrons

The total exclusive  $\gamma^* \pi^*$  cross section for  $J/\psi$  production is given by [10]:

$$\sigma^{\gamma^* \pi^* \rightarrow J/\psi \pi} = \sum_{L,T} \int_{-\infty}^0 \frac{d\sigma^{\gamma^* \pi^* \rightarrow J/\psi \pi}}{d\hat{t}} d\hat{t} = \frac{1}{16\pi} \sum_{L,T} \int_{-\infty}^0 |\mathcal{A}_{T,L}^{\gamma^* \pi^* \rightarrow J/\psi \pi}|^2 d\hat{t} \quad (6)$$

where the scattering amplitude is:

$$\begin{aligned} \mathcal{A}_{T,L}^{\gamma^* \pi^* \rightarrow J/\psi \pi}(\hat{x}, Q^2, \Delta) = & i \int d^2\mathbf{r} \int d^2\mathbf{b} \int \frac{dz}{4\pi} (\Psi^* \Psi_V)_{T,L}(Q^2, \mathbf{r}, z) \\ & \times e^{-i[\mathbf{b} - (1-z)\mathbf{r}] \cdot \Delta} \frac{d\sigma_{q\bar{q}}^{(\pi)}}{d^2\mathbf{b}}(\mathbf{b}, \mathbf{r}, \hat{x}). \end{aligned} \quad (7)$$

here  $\hat{t} = -\Delta^2$ ,  $(\Psi^* \Psi_V)$  is the wave-overlap of the photon and the vector-meson wave-functions. We use boosted the Gaussian wavefunction for  $J/\psi$  from [11]. The virtual pion dipole cross section  $d\sigma_{q\bar{q}}^{(\pi)}/d^2\mathbf{b}$  is given in eq.(5). At small  $\hat{t}$ , the spatial resolution is not large enough to resolve the real pion and the dipole interacts with the whole pion cloud hence we assume that the transverse profile of the virtual pion (the entire pion cloud) is given by a 2-dimensional Yukawa function:

$$T_{\pi^*}(b) = \int_{-\infty}^{\infty} dz \rho_{\pi^*}(b, z) \quad (8)$$

where the radial part of the virtual pion wave function is given by Yukawa theory:

$$\rho_{\pi^*}(b, z) = \frac{m_\pi^2}{4\pi} \frac{e^{-m_\pi \sqrt{b^2 + z^2}}}{\sqrt{b^2 + z^2}} \quad (9)$$

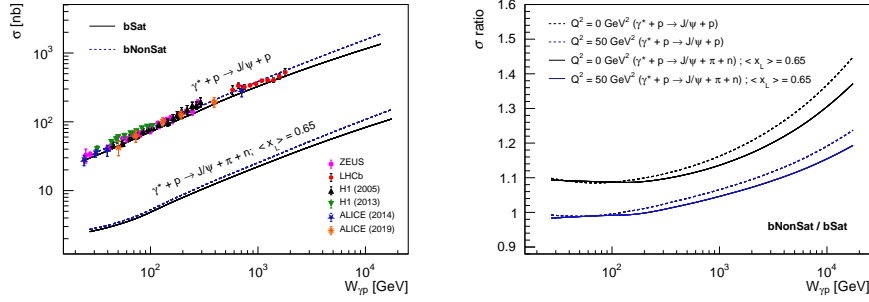


Figure 4: Exclusive  $J/\psi$  production with leading neutrons in ep collisions at future colliders in bSat and bNonSat models from [9].

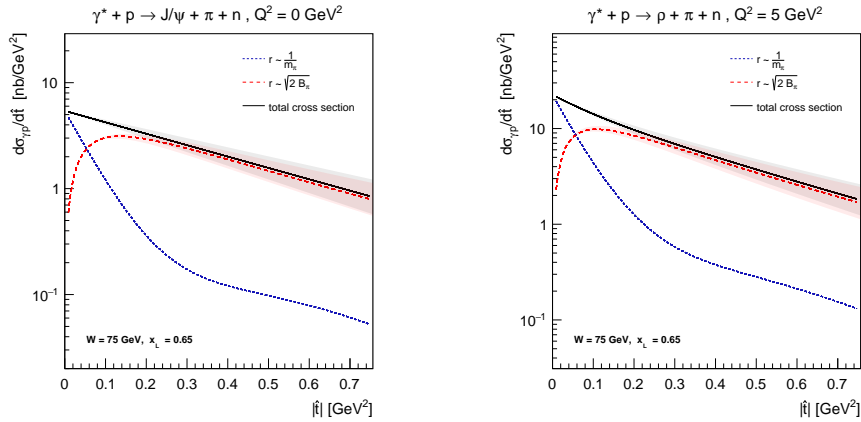


Figure 5:  $\hat{t}$ -dependence of the exclusive  $J/\psi$  (left) and  $\rho$  (right) production with leading neutron in  $\gamma^*p$  scattering in bNonSat model from [9]. The band corresponds to  $B_\pi = 2.0 \pm 0.5 \text{ GeV}^{-2}$ .

For the real pion, we assume a Gaussian profile as  $T_\pi(b) = \frac{1}{2\pi B_\pi} e^{-\frac{b^2}{2B_\pi}}$  similar to the proton case. At larger  $|\hat{t}|$ , the dipole begins to resolve the real pion  $T_\pi$  inside the wave function  $T_{\pi^*}$  and since the transverse position of the pion are unknown within the whole cloud, we have an event-by-event spatial distribution given by eq.(8). For more details, we refer to [9]. In Fig. 4, we plot the total exclusive cross section and the ratio of cross section in bNonSat to bSat model employing eq.(5) to estimate the magnitude of saturation effects and we observe that the ratio of the cross sections for the models with and without non-linear effects shows an effect which may reach several tens of percent at the highest energies. However the non-linear effects seen here are less so than that of the proton. Fig. 5 provides a first prediction of the  $\hat{t}$ -dependence of the differential cross section of vector mesons production in leading neutron events making use of the Yukawa theory for the pion cloud (eq.(8)) and assuming a Gaussian profile for real pion inside the cloud.

### 3 Conclusion

We have shown that making use of a simple assumption that the small  $x$  structure of protons and pions is universal up to a normalization, we could describe the Feynman- $x$  spectrum, the leading neutron structure function data and observed that the leading neutron structure function data is insensitive to non-linear effects. Moreover, Feynman scaling is also not associated

with saturation and is a result of the same asymptotic behavior of pion and proton structure functions at high energies. Though the exclusively produced vector mesons in leading neutron events are sensitive to non-linear effects but lesser than the usual exclusive vector meson production in DIS. Further, we provide a first prediction of the  $\hat{t}$ -dependence of the differential distribution of the exclusive vector meson production in leading neutron events.

### Funding Information

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### References

- [1] F. D. Aaron *et al.*, *Measurement of Leading Neutron Production in Deep-Inelastic Scattering at HERA*, Eur. Phys. J. C **68**, 381 (2010), doi:[10.1140/epjc/s10052-010-1369-4](https://doi.org/10.1140/epjc/s10052-010-1369-4), [1001.0532](#).
- [2] V. Andreev *et al.*, *Measurement of Feynman- $x$  Spectra of Photons and Neutrons in the Very Forward Direction in Deep-Inelastic Scattering at HERA*, Eur. Phys. J. C **74**(6), 2915 (2014), doi:[10.1140/epjc/s10052-014-2915-2](https://doi.org/10.1140/epjc/s10052-014-2915-2), [1404.0201](#).
- [3] M. Bishari, *Pion exchange and inclusive spectra*, Physics Letters B **38**(7), 510 (1972), doi:[https://doi.org/10.1016/0370-2693\(72\)90530-8](https://doi.org/10.1016/0370-2693(72)90530-8).
- [4] *How to measure the pion structure function at hera*, Physics Letters B **338**(2), 363 (1994), doi:[https://doi.org/10.1016/0370-2693\(94\)91392-7](https://doi.org/10.1016/0370-2693(94)91392-7).
- [5] H. Kowalski, L. Motyka and G. Watt, *Exclusive diffractive processes at HERA within the dipole picture*, Phys. Rev. D **74**, 074016 (2006), doi:[10.1103/PhysRevD.74.074016](https://doi.org/10.1103/PhysRevD.74.074016), [hep-ph/0606272](#).
- [6] B. Z. Kopeliovich, I. K. Potashnikova, B. Povh and I. Schmidt, *Pion structure function at small  $x$  from DIS data*, Phys. Rev. D **85**, 114025 (2012), doi:[10.1103/PhysRevD.85.114025](https://doi.org/10.1103/PhysRevD.85.114025), [1205.0067](#).
- [7] A. B. Kaidalov, V. A. Khoze, A. D. Martin and M. G. Ryskin, *Leading neutron spectra*, Eur. Phys. J. C **47**, 385 (2006), doi:[10.1140/epjc/s2006-02572-0](https://doi.org/10.1140/epjc/s2006-02572-0), [hep-ph/0602215](#).
- [8] F. Carvalho, V. P. Gonçalves, D. Spiering and F. S. Navarra, *Gluon saturation and Feynman scaling in leading neutron production*, Phys. Lett. B **752**, 76 (2016), doi:[10.1016/j.physletb.2015.11.017](https://doi.org/10.1016/j.physletb.2015.11.017), [1507.07758](#).
- [9] A. Kumar and T. Toll, *Investigating saturation effects and the virtual pion in leading neutron events at hera with the dipole model*, Phys. Rev. D **105**, 114045 (2022), doi:[10.1103/PhysRevD.105.114045](https://doi.org/10.1103/PhysRevD.105.114045).
- [10] V. P. Goncalves, F. S. Navarra and D. Spiering, *Exclusive processes with a leading neutron in ep collisions*, Phys. Rev. D **93**(5), 054025 (2016), doi:[10.1103/PhysRevD.93.054025](https://doi.org/10.1103/PhysRevD.93.054025), [1512.06594](#).
- [11] H. Mäntysaari and P. Zurita, *In depth analysis of the combined HERA data in the dipole models with and without saturation*, Phys. Rev. D **98**, 036002 (2018), doi:[10.1103/PhysRevD.98.036002](https://doi.org/10.1103/PhysRevD.98.036002), [1804.05311](#).