

# The power of lattice QCD

Phiala Shanahan, MIT

Image Credit: 2018 EIC User's Group Meeting

# Parton physics from lattice QCD

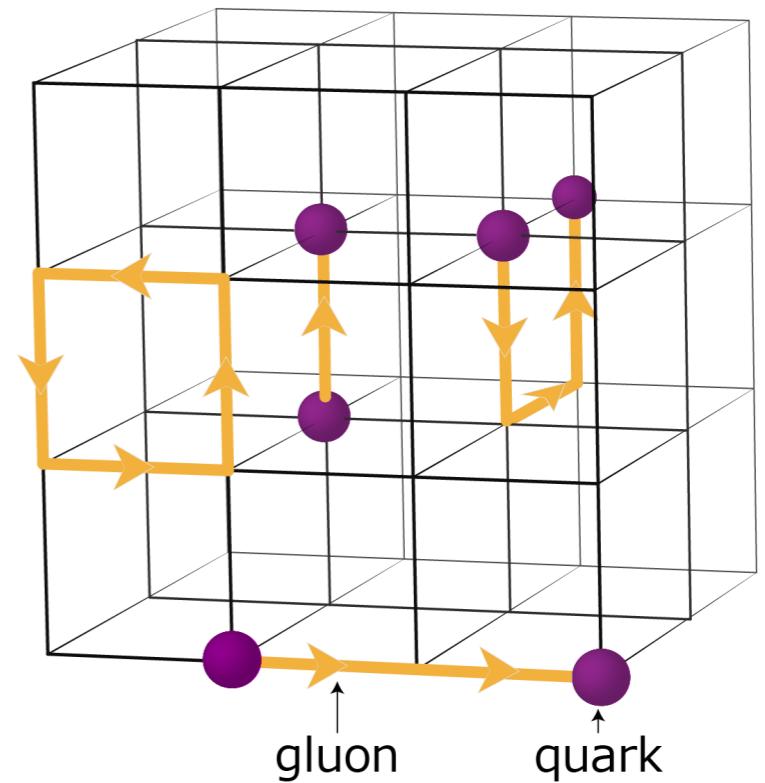
Phiala Shanahan, MIT

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# Lattice QCD

Numerical first-principles approach to  
non-perturbative QCD

- Discretise QCD onto 4D space-time lattice
- Approximate QCD path integral using Monte-Carlo methods and importance sampling
- Run on supercomputers and dedicated clusters
- Take limit of vanishing discretisation, infinite volume, physical quark masses



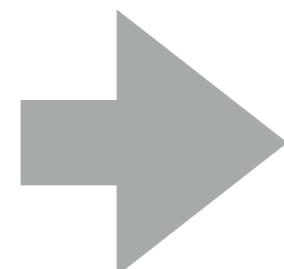
# Lattice QCD

Numerical first-principles approach to  
non-perturbative QCD

## INPUT

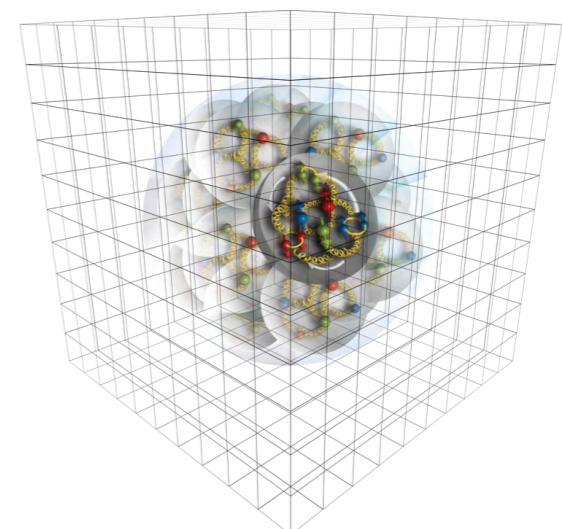
Lattice QCD action has same free  
parameters as QCD: quark masses,  $\alpha_S$

- Fix quark masses by matching to measured hadron masses, e.g.,  $\pi, K, D_s, B_s$  for  $u, d, s, c, b$
- One experimental input to fix lattice spacing in GeV (and also  $\alpha_S$ ), e.g.,  $2S-1S$  splitting in  $\Upsilon$ , or  $f_\pi$  or  $\Omega$  mass



## OUTPUT

Calculations of all other quantities are QCD predictions

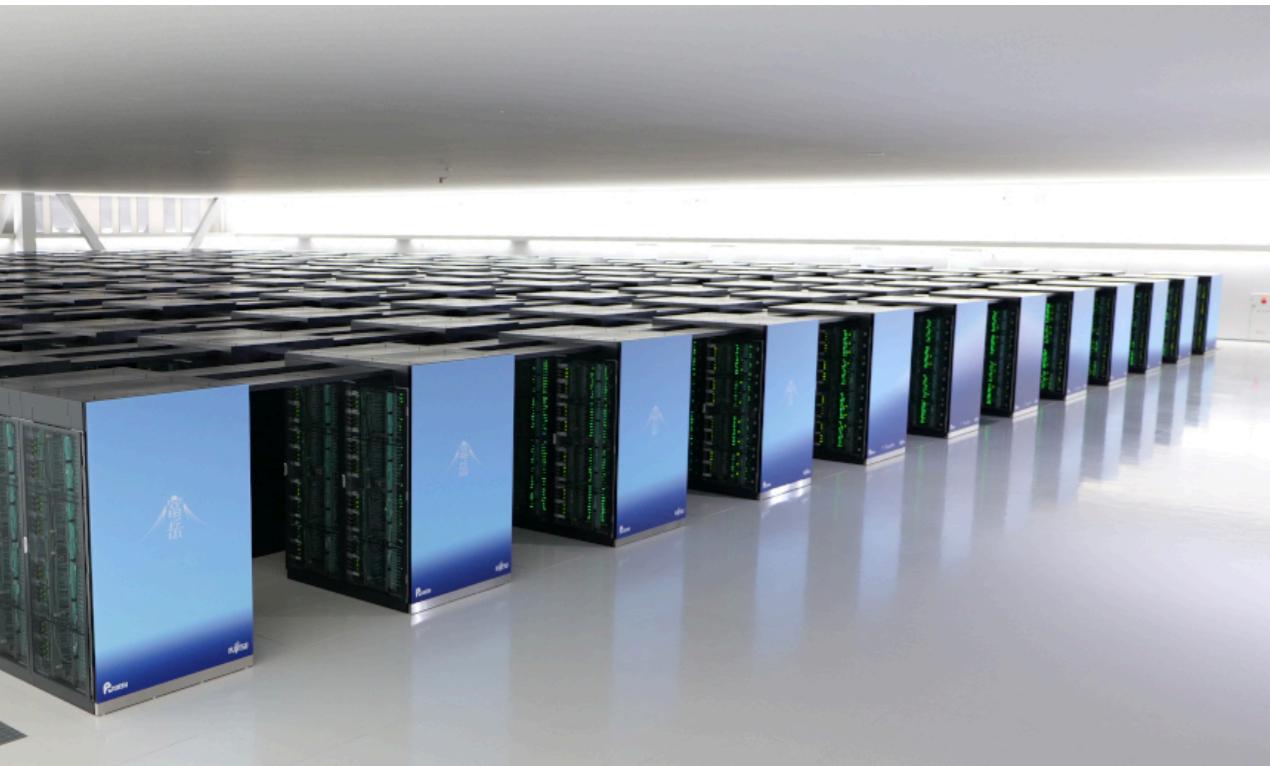


# Lattice QCD

Numerical first-principles approach to  
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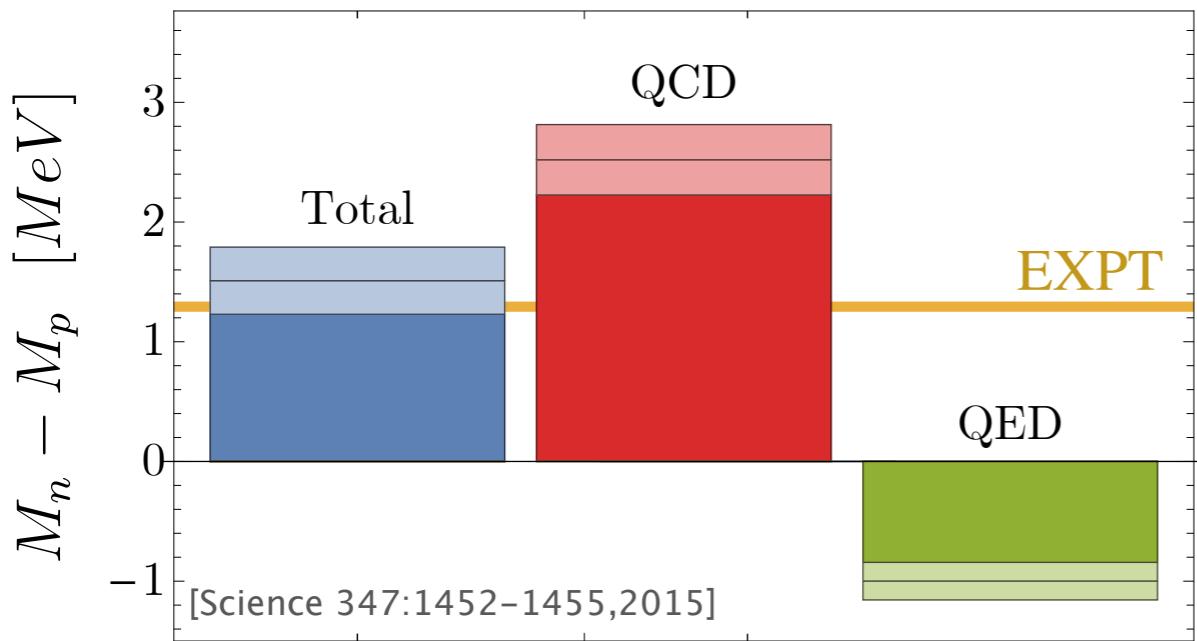
## Calculations use world's largest computers

- Many millions of CPU/GPU hours

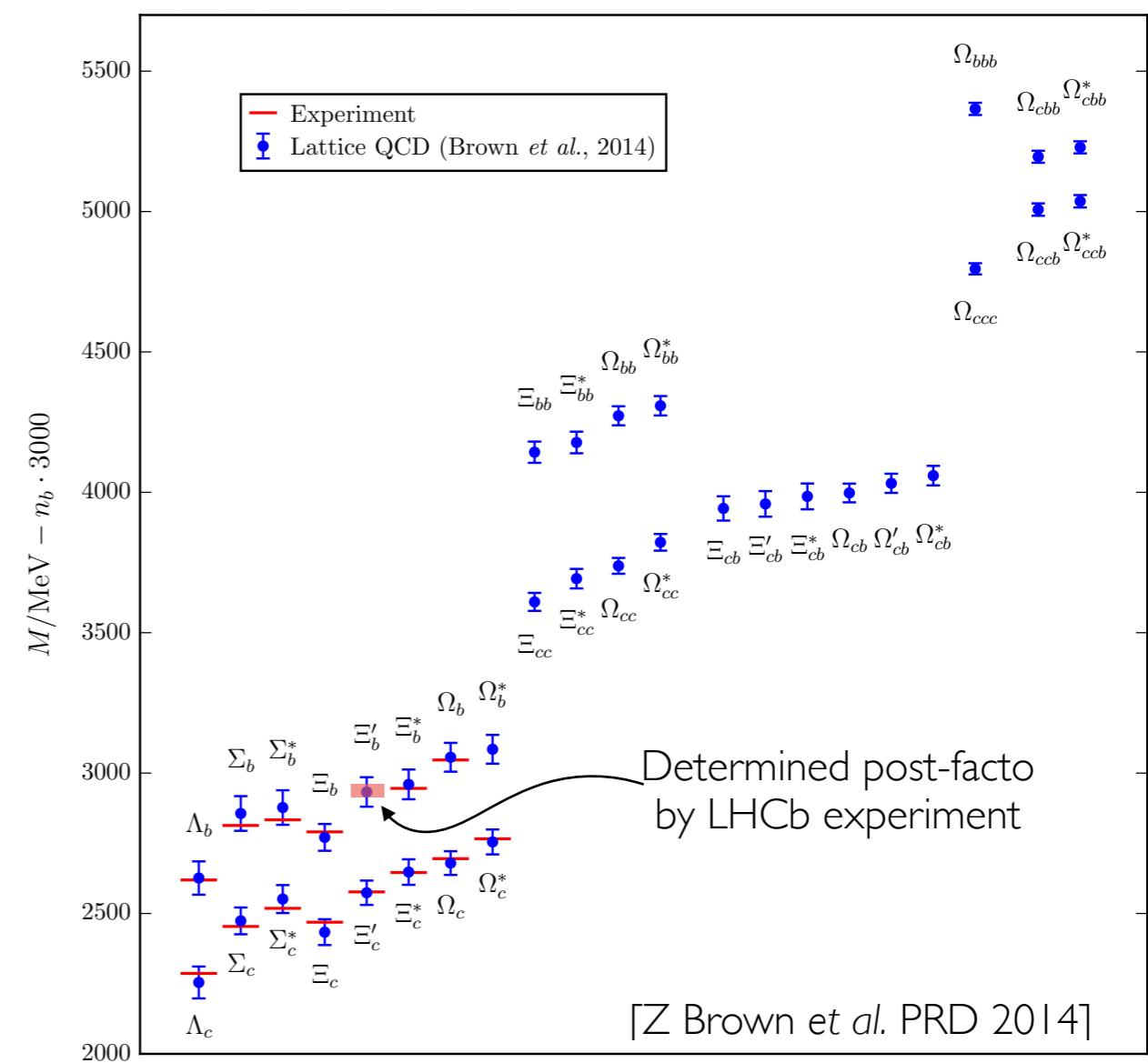


# Lattice QCD works

- Ground state hadron spectrum reproduced
- p-n mass splitting reproduced
- ...

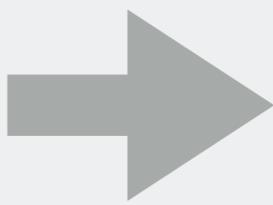


- Predictions for new states with controlled uncertainties



# Lattice QCD works

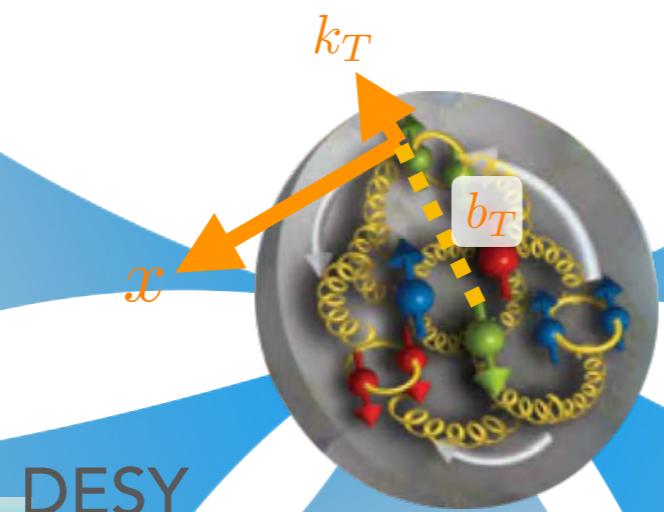
Precision era for  
lattice QCD



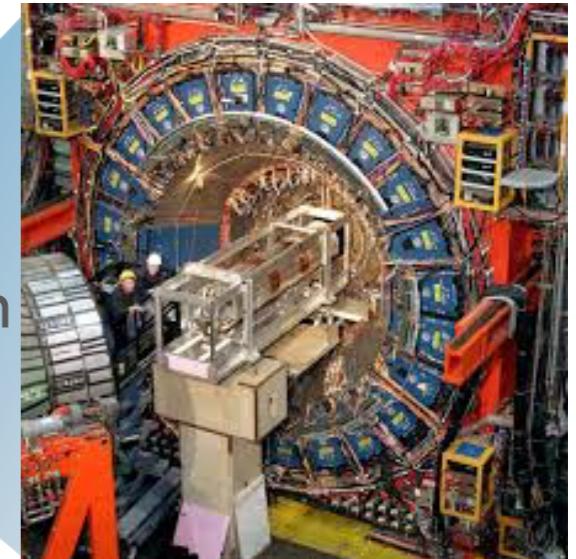
Theory complements  
experiment



JLab



Tevatron



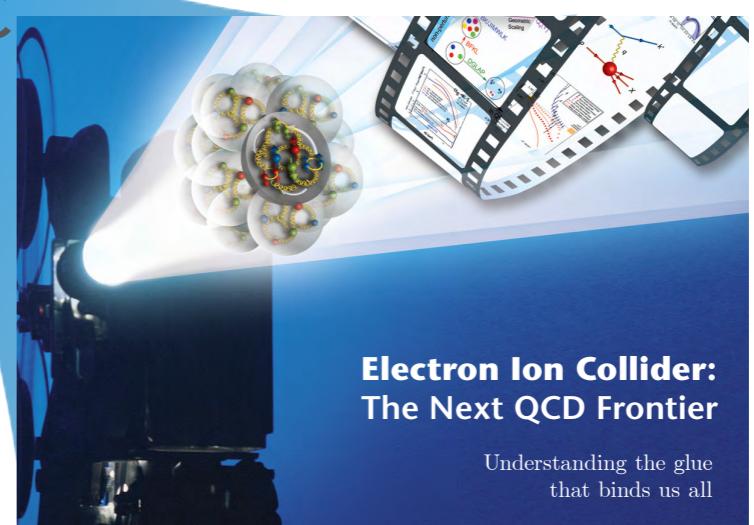
DESY



LHC



EIC

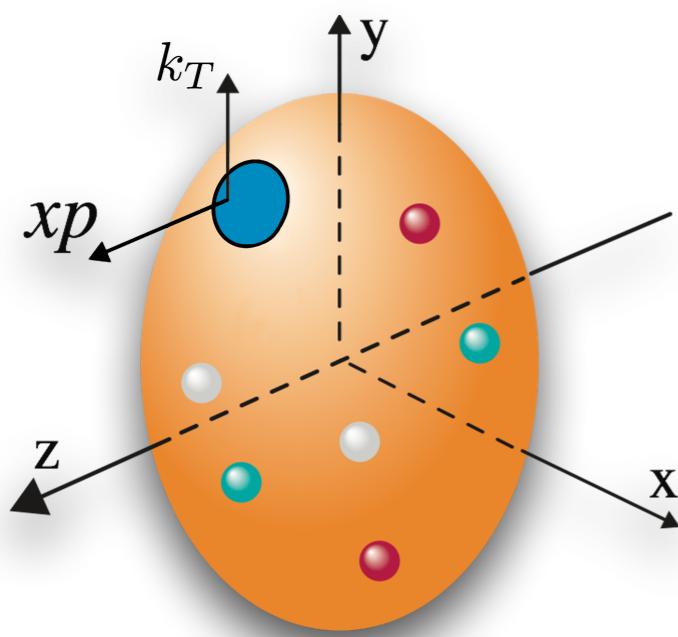


**Electron Ion Collider:**  
The Next QCD Frontier

Understanding the glue  
that binds us all

# Parton physics from Lattice QCD

Understanding the quark and gluon  
structure of matter



Three-dimensional partonic structure  
of the proton

**PDF**  
 $f_{q/H}(x)$   
longitudinal



**TMD**  
 $f_{q/H}(x, k_T)$   
+ transverse

Community overview:  
Constantinou et al.,  
"2020 PDFLattice Report"  
arXiv:2006.08636

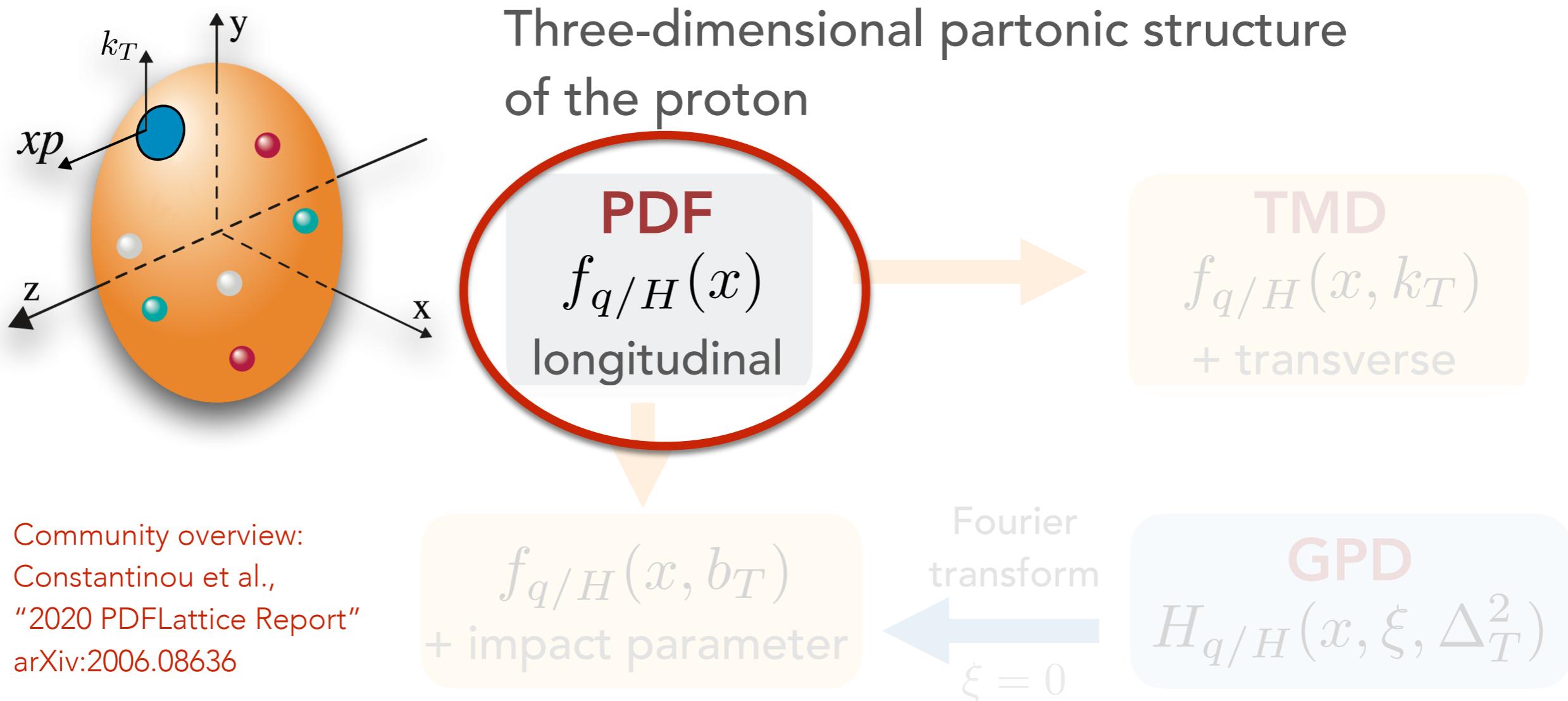
$f_{q/H}(x, b_T)$   
+ impact parameter

Fourier  
transform  
 $\xi = 0$

**GPD**  
 $H_{q/H}(x, \xi, \Delta_T^2)$

# Parton physics from Lattice QCD

Understanding the quark and gluon  
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# Parton distribution functions

Parton distribution  
functions  $f(x, \mu^2)$

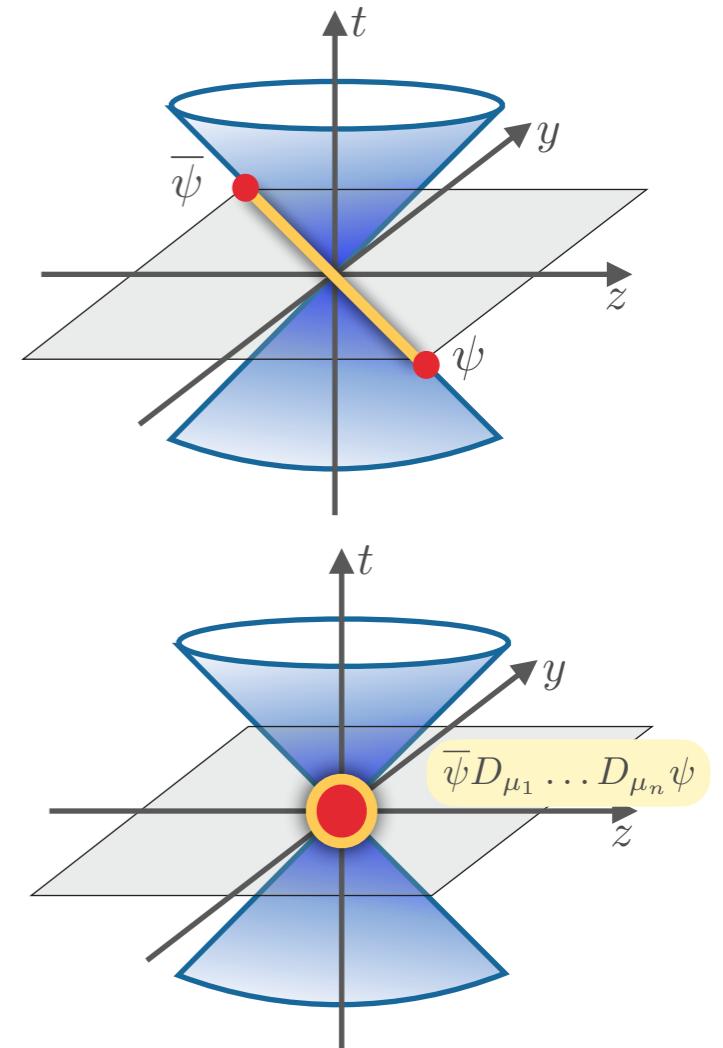
- Non-local light-cone correlations
- Encode non-perturbative physics

- Correlations at light-like separation not directly accessible in Euclidean-space calculations

$$f(x) = \int \frac{d\xi^-}{2\pi} e^{-2i\xi^-(xP^+)} \langle p | \bar{\psi}_f(\xi^-) \gamma^+ W[\xi^-, -\xi^-] \psi_f(-\xi^-) | p \rangle$$

- Operator Product Expansion relates Mellin moments of PDFs to local operators

$$\langle h | \bar{\psi}_f D_{\mu_1} \dots D_{\mu_n} \psi_f | h \rangle \sim \langle x^n \rangle_f^h = \int_0^1 dx x^n f(x)$$



# Decomposition of proton momentum

Moments of PDFs encode key aspects of hadron structure

$$\int_0^1 dx x^n f(x, \mu^2) = \langle x^n \rangle_f(\mu^2)$$

- Lattice QCD can cleanly access low moments of PDFs ( $n \lesssim 3$ )
- [Also, moments of structure functions incl.  $Q^2$ -dependence]

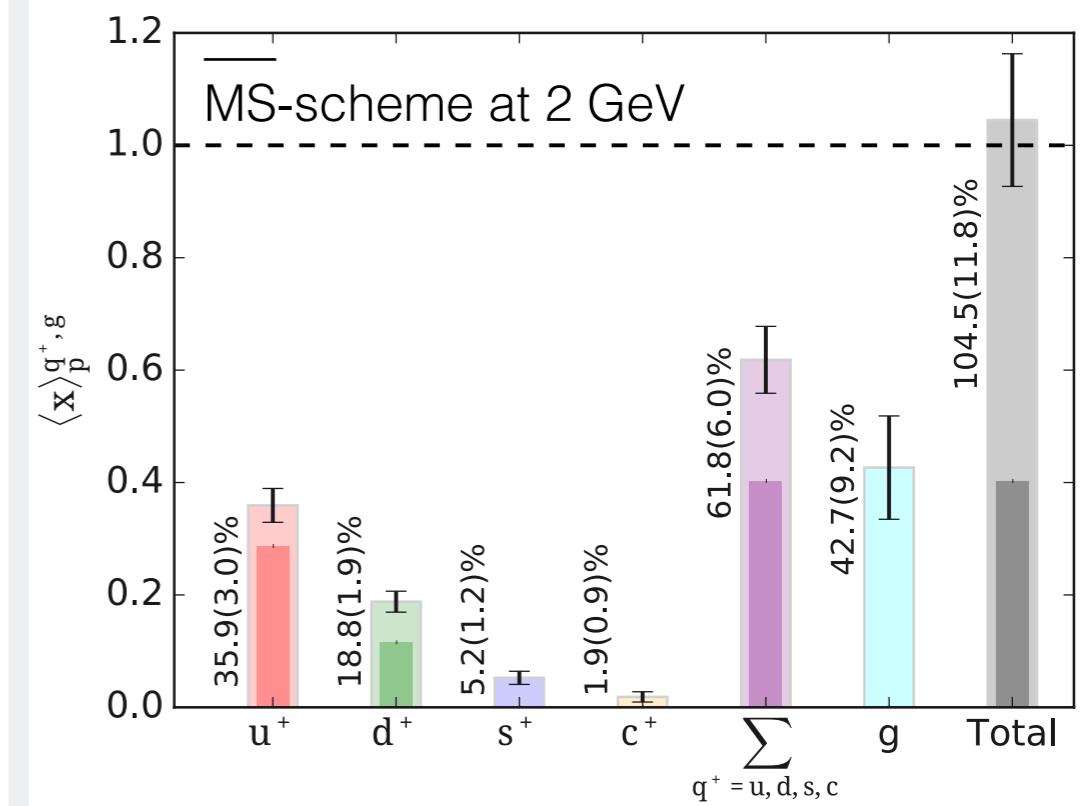
**Thursday WG1 session:**

**Utku Can**

Moments of the longitudinal structure function of the proton from Lattice QCD

**Example:** Lowest moment defines contribution of each type of parton to the hadron momentum

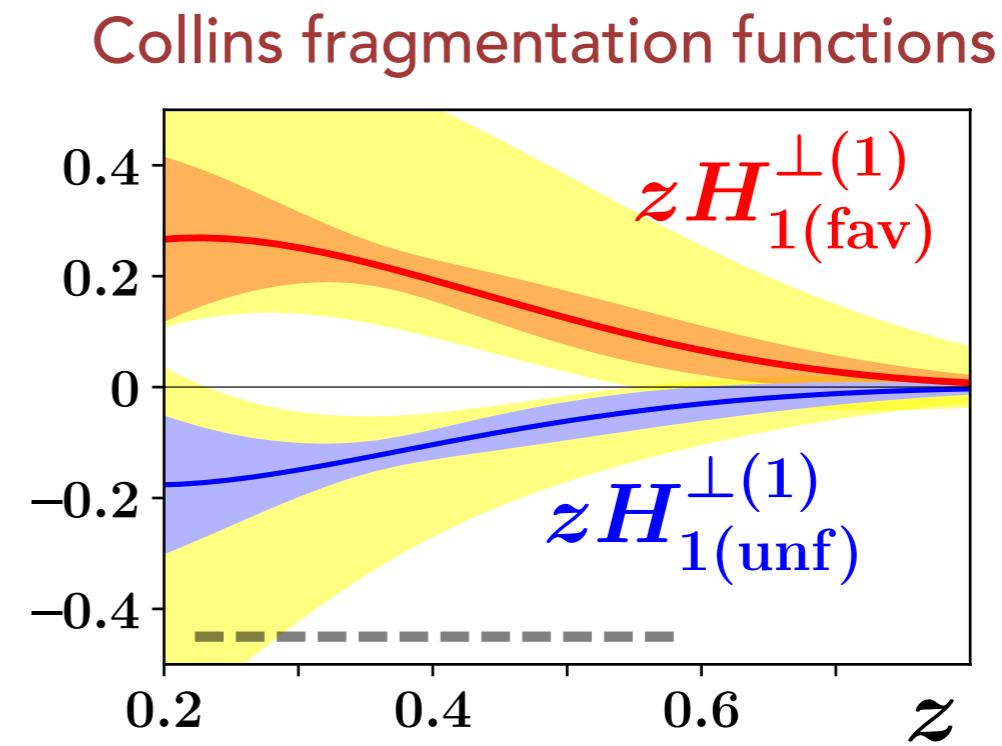
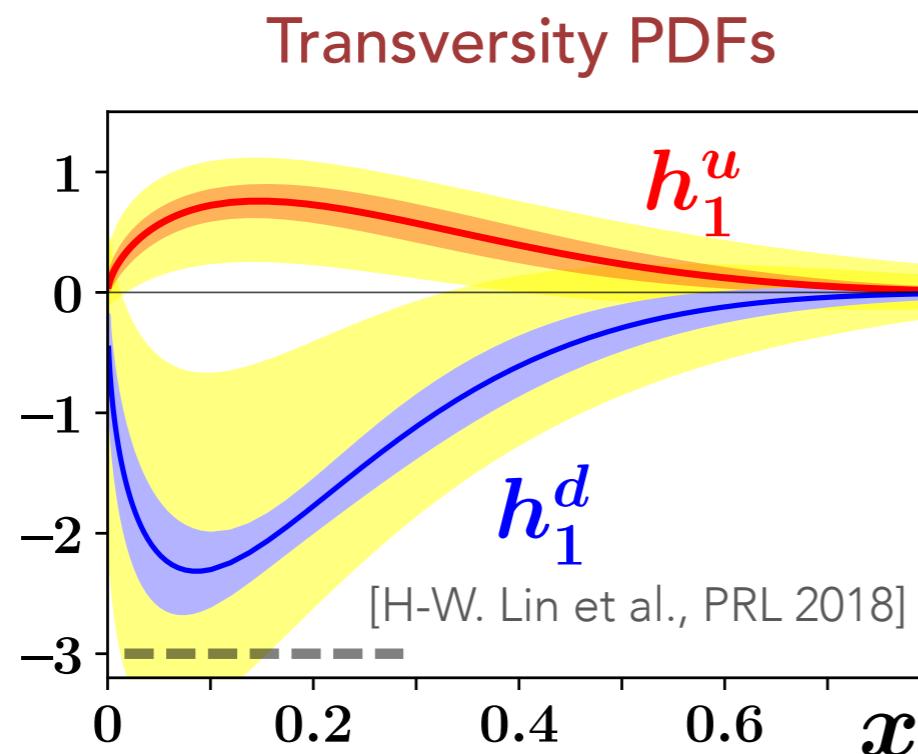
**2020 Highlight:**  
Proton momentum decomposition



[C. Alexandrou et al., PRD 101 (2020)]

# Constraints on global PDF fits

- Including lattice QCD results for moments in global PDF fits can yield significant improvements
- Community white paper (LQCD + phenomenologists) assessed potential impacts [Lin et al., Prog. Part. Nucl. Phys 100 (2018), 107]



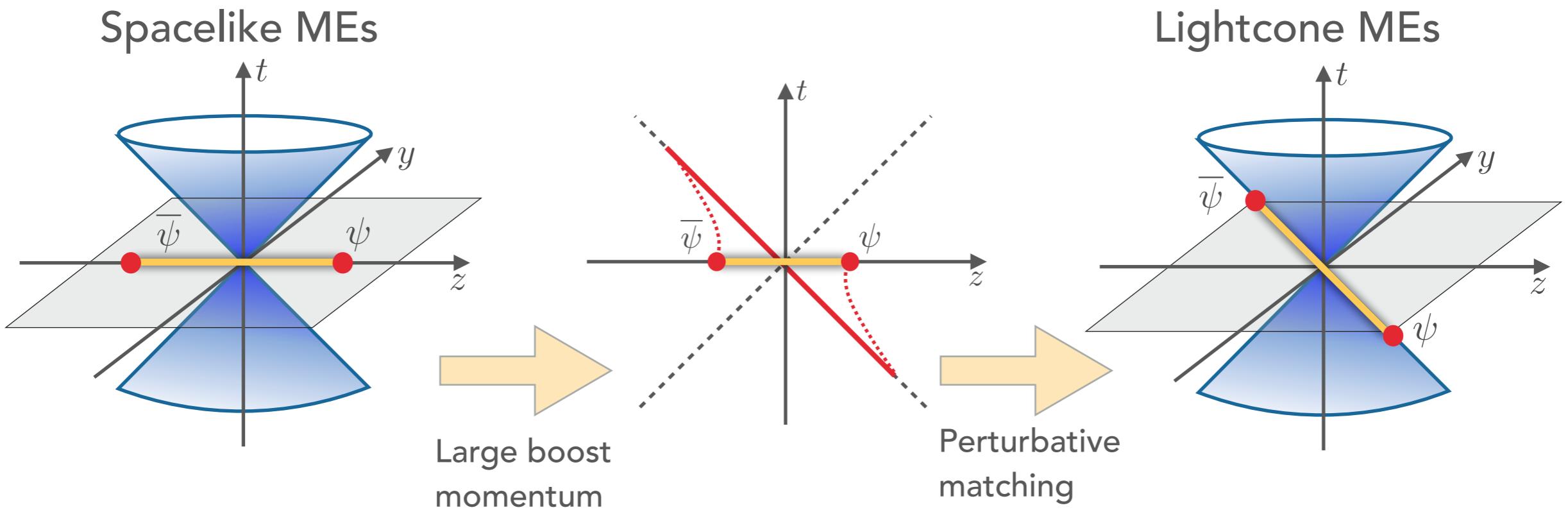
Yellow: SIDIS data only: direct constraints in region indicated by dashes  
Blue/Red: SIDIS + lattice QCD for tensor charge (zeroth moment)

# x-dependence of PDFs/GPDs

Parton distribution functions  $f(x, \mu^2)$

- Non-local light-cone correlations
- Encode non-perturbative physics

- Access x-dependence in Euclidean calculation by relating spacelike non-local operator matrix elements to lightlike

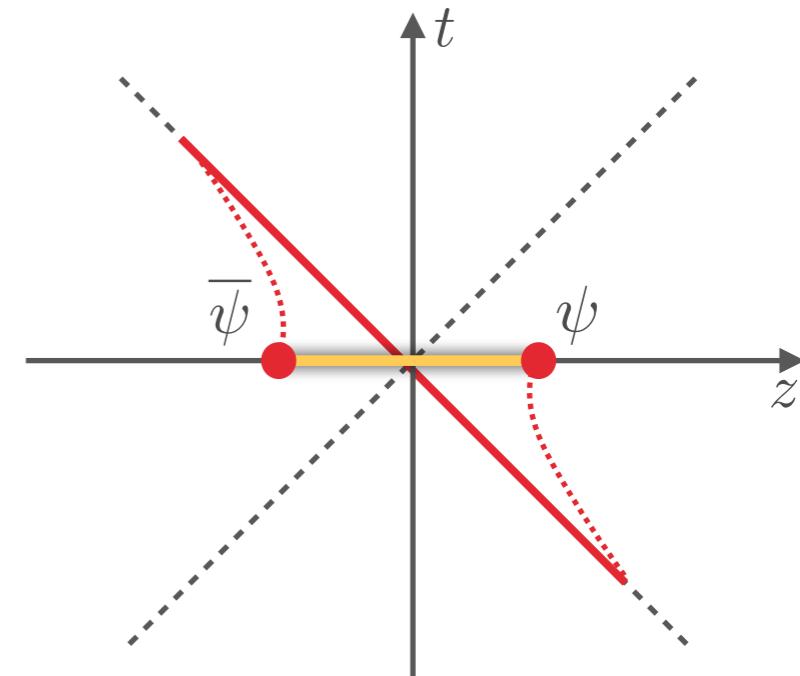


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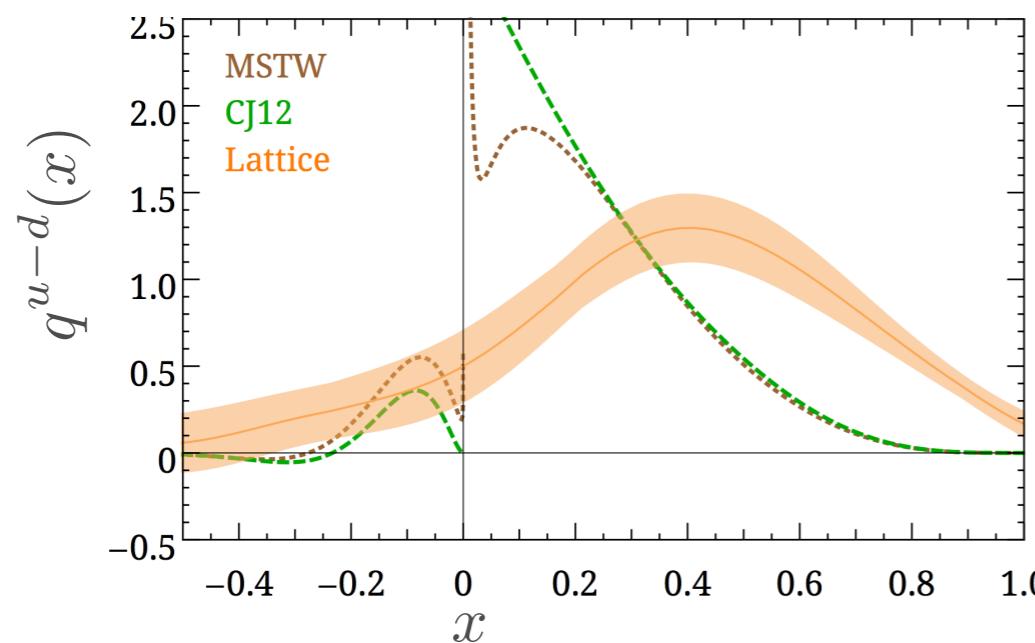
- Access x-dependence in Euclidean calculation by relating spacelike non-local operator matrix elements to lightlike
  - **Quasi-PDFs** [Ji, PRL 110 (2013) 262002]
  - **Pseudo-PDFs** [Radyushkin, PRD 96 (2017) 034025]
  - **Factorisable matrix elements**  
[Ma & Qiu, PRL 120 (2018) 022003]
  - **(Heavy quark) Compton tensor**  
[Braun & Müller, EPJ C55 (2008) 349; Chambers et al., PRL 118 (2017) 242001, Detmold & Lin, PRD 73 (2006) 014501, Liu & Dong, PRL 72 (1994) 1790]



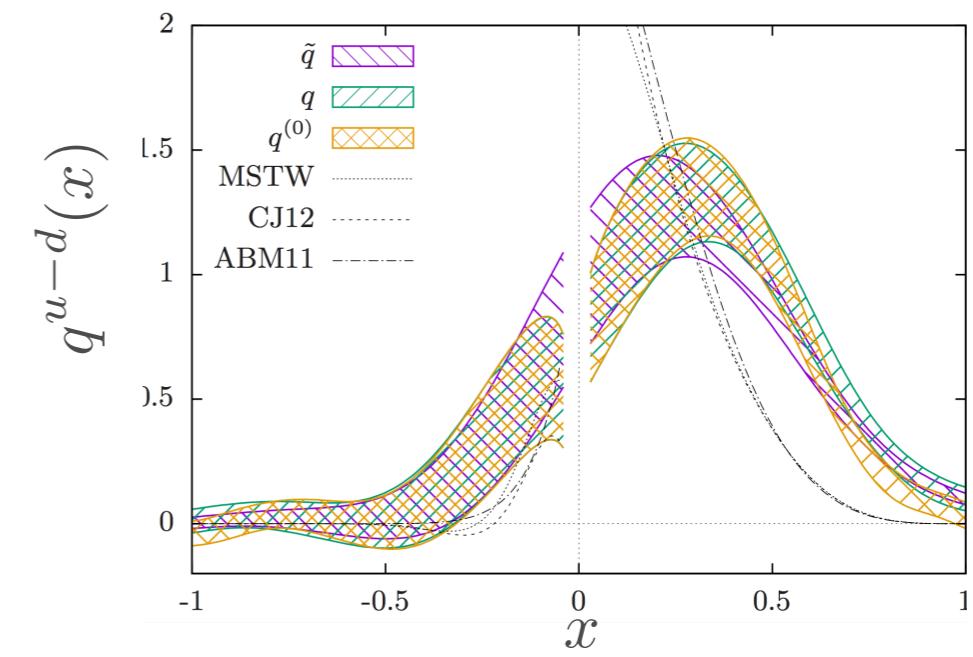
# x-dependence of PDFs

- Rapid progress since first calculations in ~2014 BUT many systematics
- Renormalisation and perturbative matching understood (~5 year effort)
- Low-x, high-x, regions particularly challenging (lattice systematics)
- Flavour separation is relatively straightforward

## Nucleon (u-d) PDFs [quasi-PDF approach] First exploratory attempts 2015



[H.W. Lin et al., Phys. Rev. D 91, 054510 (2015)]



[C. Alexandrou, Phys. Rev. D 92, 014502 (2015)]

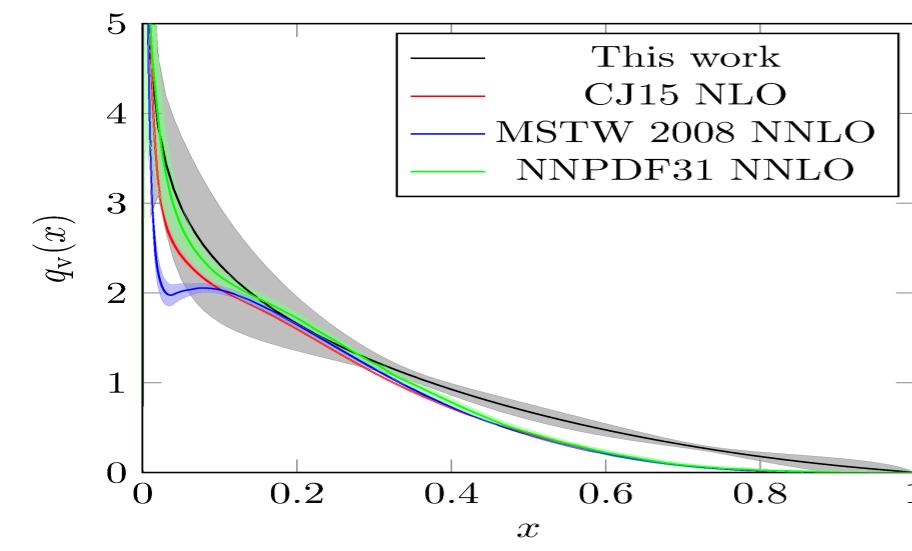
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**State-of-the-art results [2020-2022]**

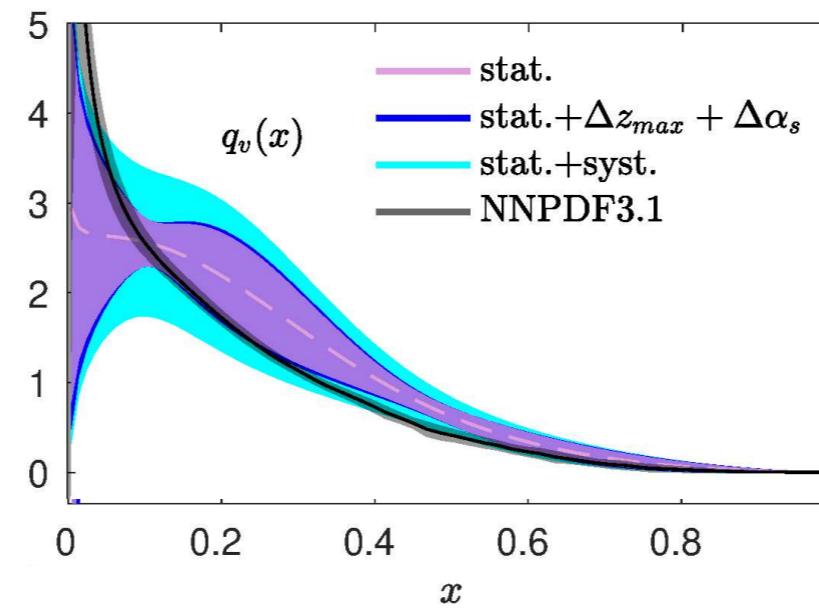
**Qualitative agreement with experiment, systematics challenging**

Unpolarized isovector nucleon PDF

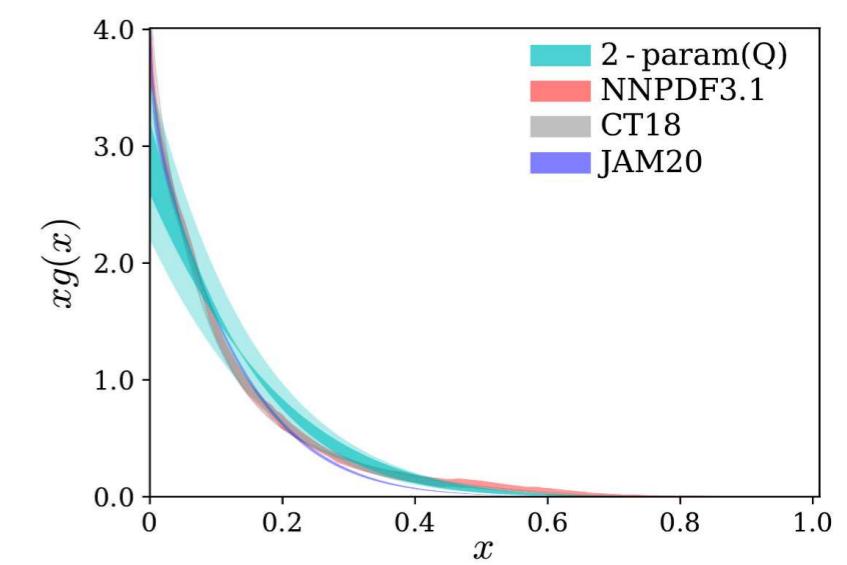


[B. Joó et al., PRL 125, 232003 (2020)]

Gluon nucleon PDF



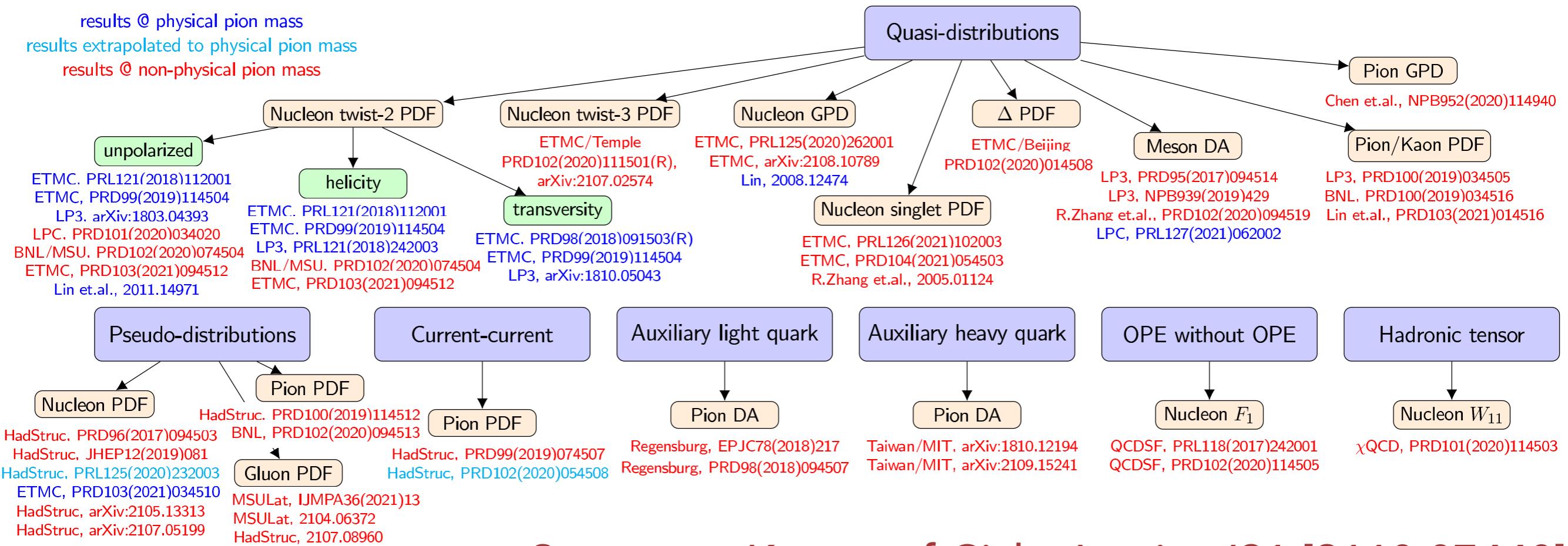
[M. Bhat et al., PRD 103, 034510 (2021)]



[T. Khan et al., PRD 104, 094516 (2021)]

# x-dependence of PDFs

- Rapid progress since first calculations in ~2014 BUT many systematics
- Renormalisation and perturbative matching understood (~5 year effort)
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Summary: Krzysztof Cichy Lattice '21 [2110.07440]

# x-dependence of PDFs

- Rapid progress since first calculations in ~2014 BUT many systematics
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## Thursday WG1 session:

**Huey-Wen Lin**

Recent Lattice-QCD Calculations of Parton Distributions

**Savvas Zafeiropoulos**

The extraction of light cone parton distributions from lattice quantum chromodynamics

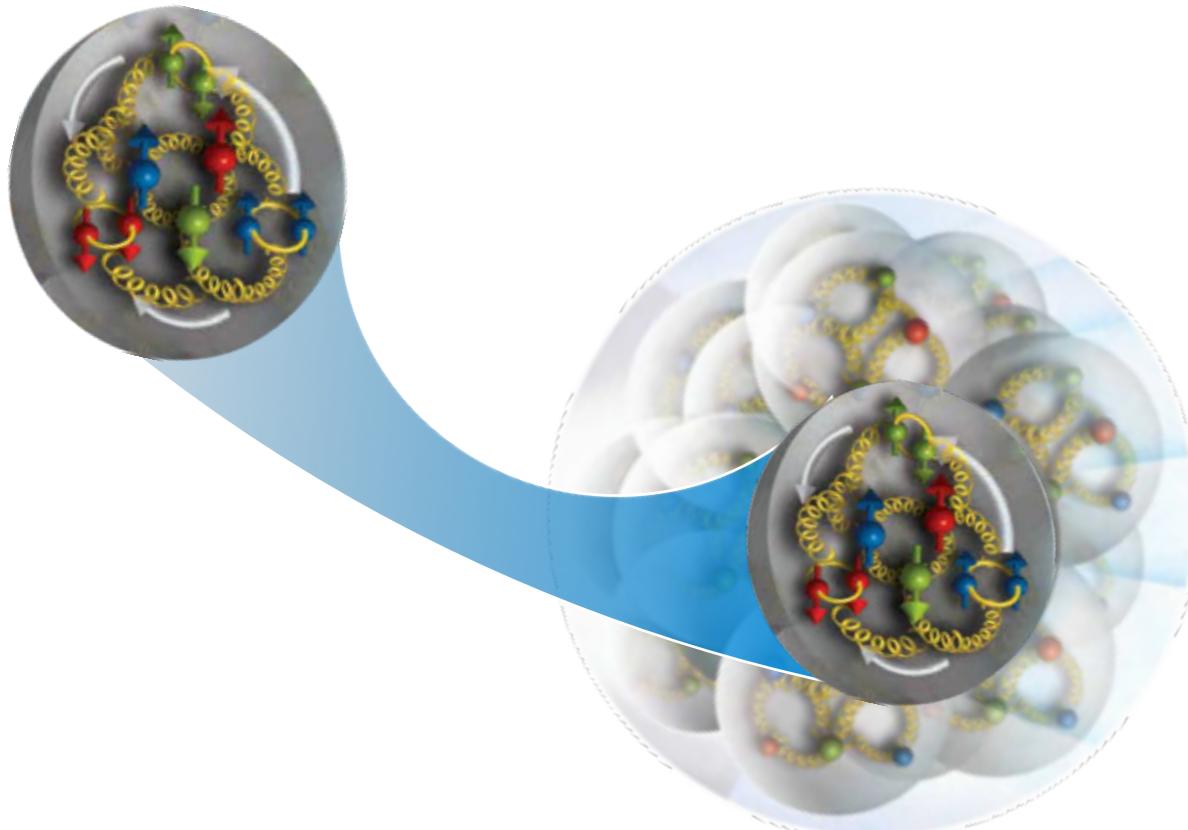
**Patrick Barry**

Complementarity of experimental and lattice QCD data on pion parton distributions

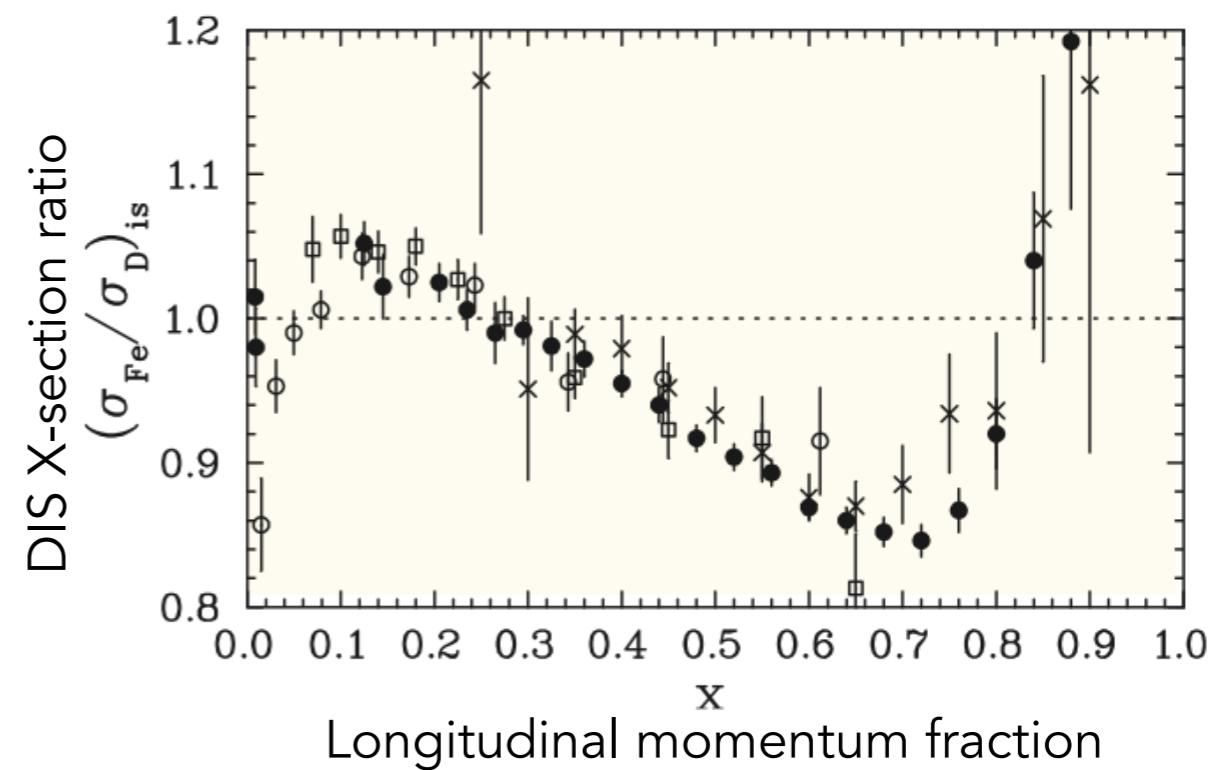
# PDFs of nuclei

Understanding the quark and gluon structure of matter

How is the partonic structure of nuclei different from that of nucleons?



Encoded in EMC-type effects

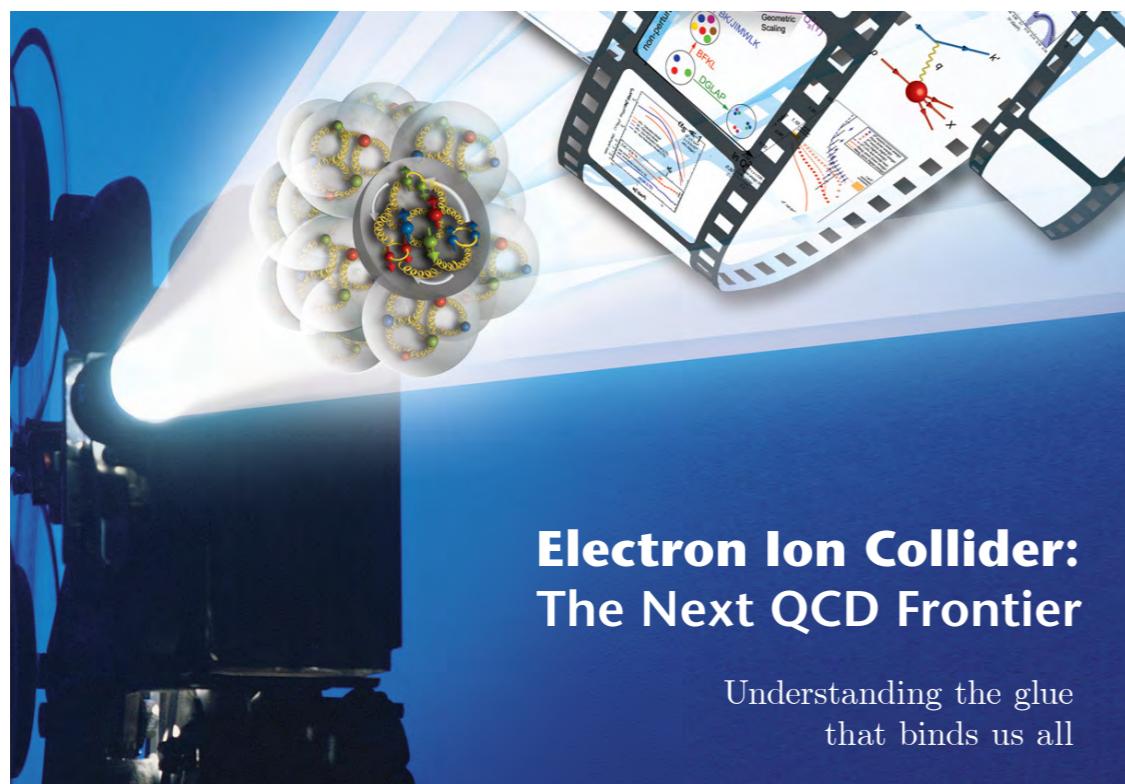


(EMC: Aubert et al., 1983)

# EMC-type effects from Lattice QCD

Understanding the quark and gluon  
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Many aspects of EMC effects will be accessible  
at a future Electron-Ion Collider



Cover image from EIC whitepaper arXiv:1212.1701

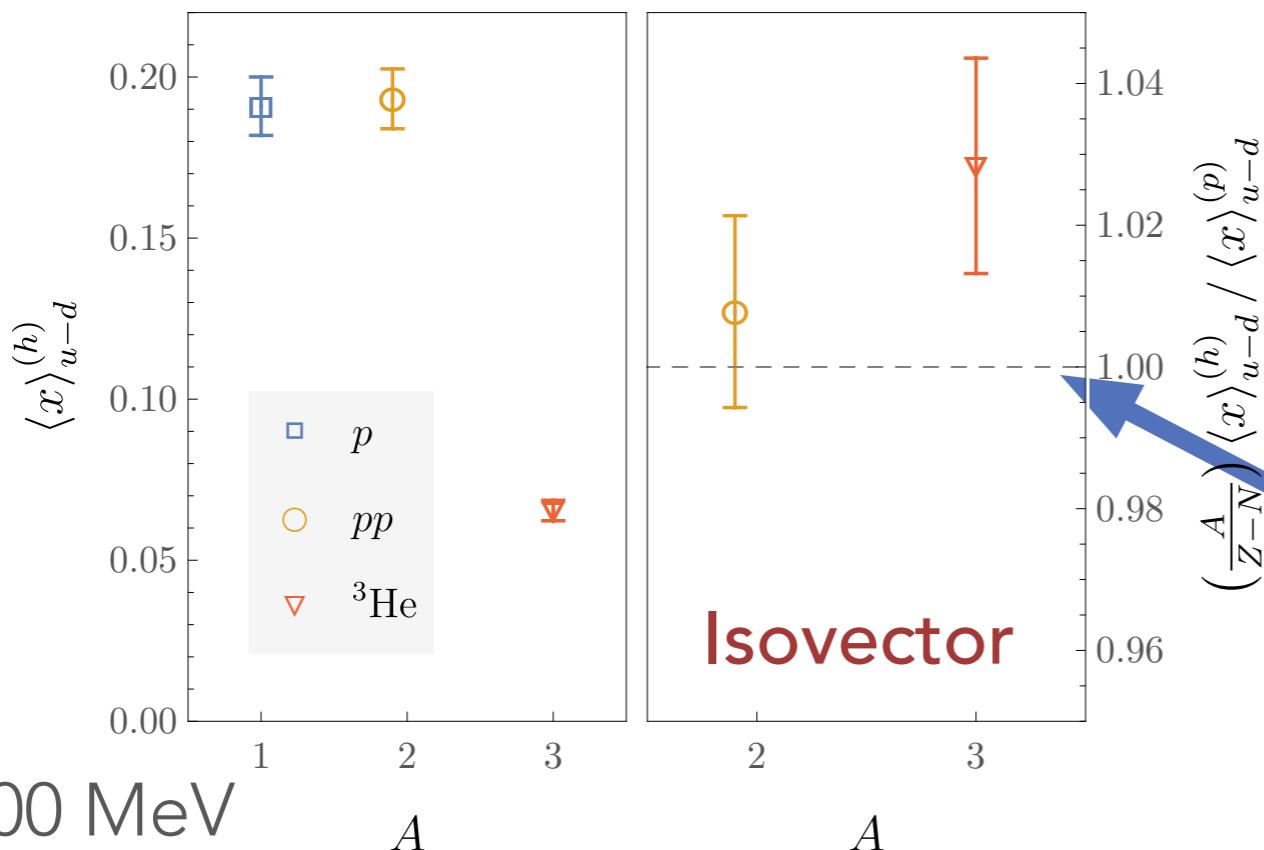
- Polarised EMC (polarised light ions)
- Isovector EMC (SIDIS)
- Gluon EMC (quarkonium production)
- LQCD will make predictions!

# Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei  
→ first QCD determination of momentum fraction of nuclei

- Bounds on EMC effect in moments at ~few percent level, consistent with phenomenology

## Ratio of quark momentum fraction in nucleus to nucleon



- No mixing
- No sum rule constraint

Normalised to  
proton result

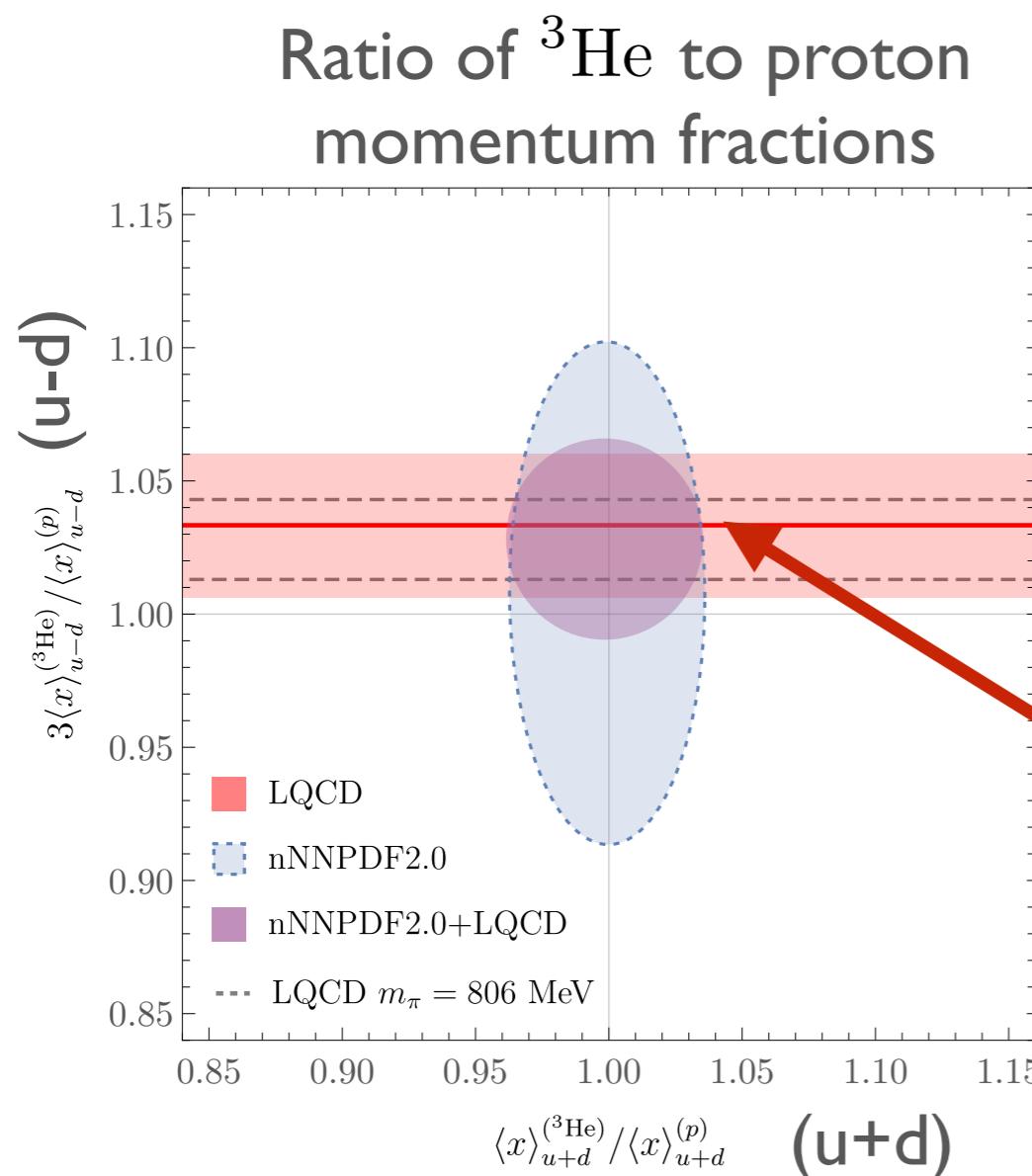


[NPLQCD PRL 126, 202001 (2021) [2009.05522]]

$m_\pi \sim 800 \text{ MeV}$

# Momentum fraction of ${}^3\text{He}$

Matrix elements of the Energy-Momentum Tensor in light nuclei  
→ first QCD determination of momentum fraction of nuclei



- Match isovector (u-d quark combination) momentum fraction to low-energy constants of effective field theory, extrapolate to physical quark masses
- Include into nNNPDF global fits of experimental lepton-nucleus scattering data

Blue → Purple:  
Improvement using theory constraints

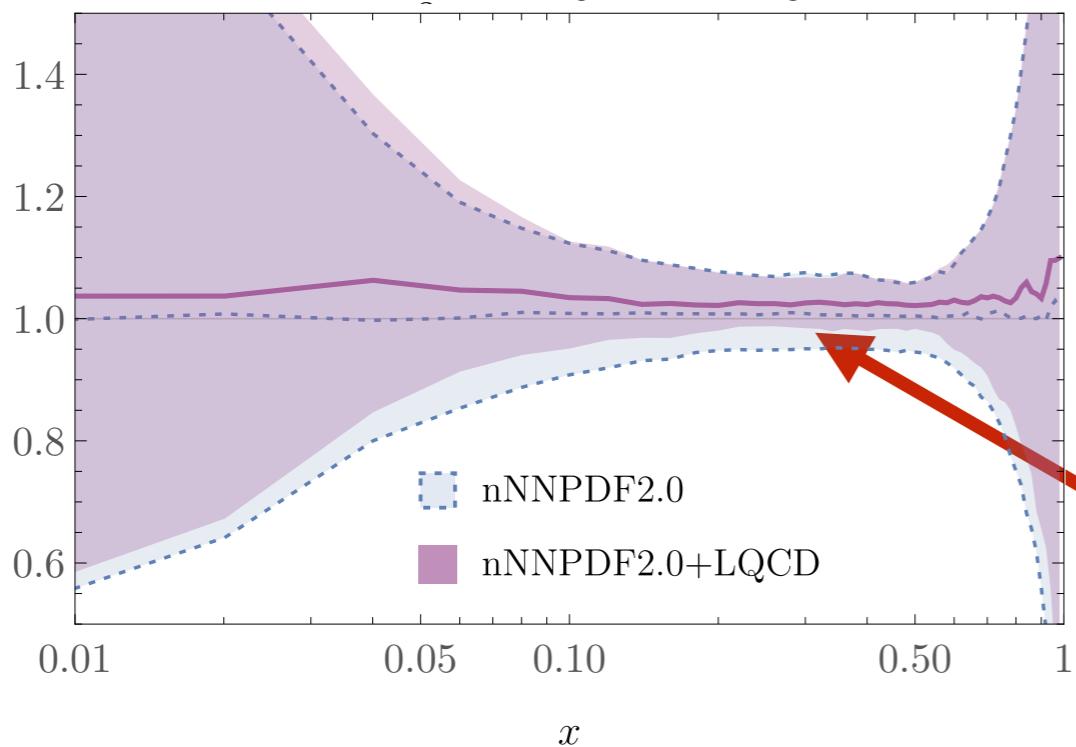
[NPLQCD PRL 126, 202001 (2021) [2009.05522]]

# Momentum fraction of ${}^3\text{He}$

Matrix elements of the Energy-Momentum Tensor in light nuclei  
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Ratio of  ${}^3\text{He}$  to proton parton distributions

$$R^{({}^3\text{He})}(x) = 3q_3^{({}^3\text{He})}(x)/q_3^{(p)}(x)$$



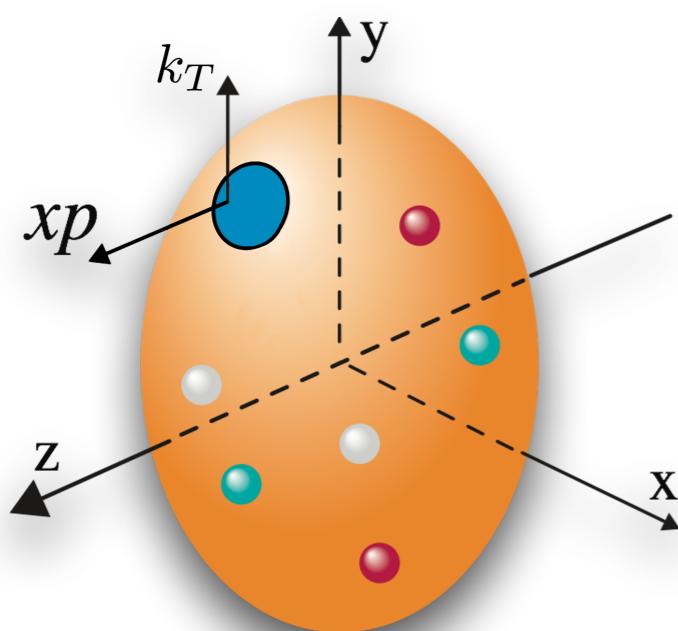
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TMD  
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Community overview:  
Constantinou et al.,  
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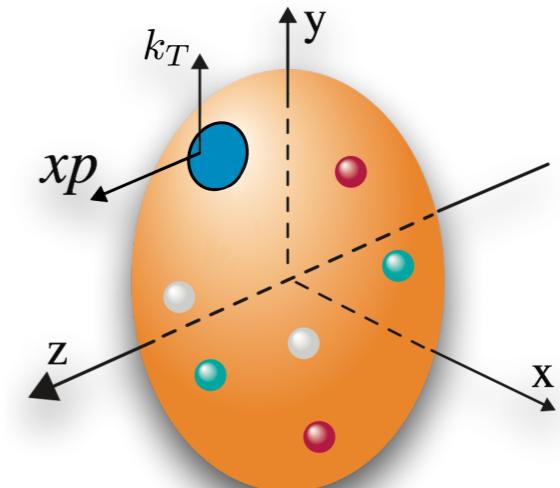
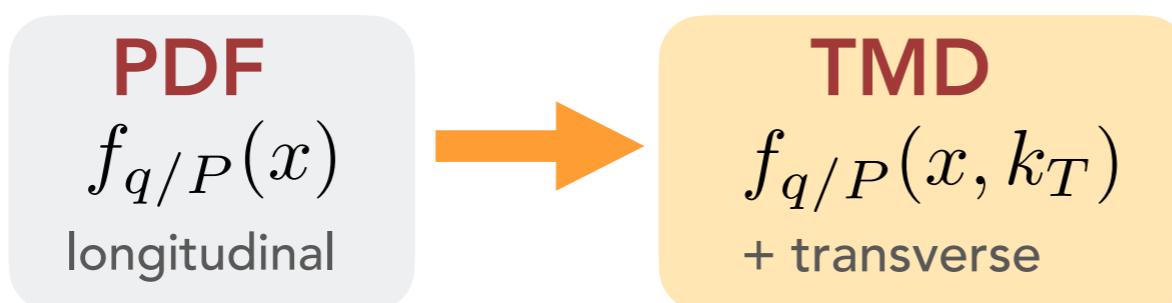
$f_{q/H}(x, b_T)$   
+ impact parameter

Fourier transform  
 $\xi = 0$

GPD  
 $H_{q/H}(x, \xi, \Delta_T^2)$

# Collins-Soper evolution kernel

- More detailed picture of nucleon including transverse structure



## Collins-Soper Evolution Kernel

$$\gamma_\zeta^q(\mu, b_T) = \zeta \frac{d}{d\zeta} \ln f_q(x, \vec{b}_T, \mu, \zeta)$$

- Governs TMD evolution
- Needed to match quasi-TMD (lattice QCD) to physical TMD

- Perturbative at short distances  $\mu, b_T^{-1} \gg \Lambda_{\text{QCD}}$
- Non-perturbative for  $b_T^{-1} \lesssim \Lambda_{\text{QCD}}$  **lattice QCD?**
- Independent of hadron

# Collins-Soper evolution kernel

## Estimates of size of nonperturbative contributions to CS kernel

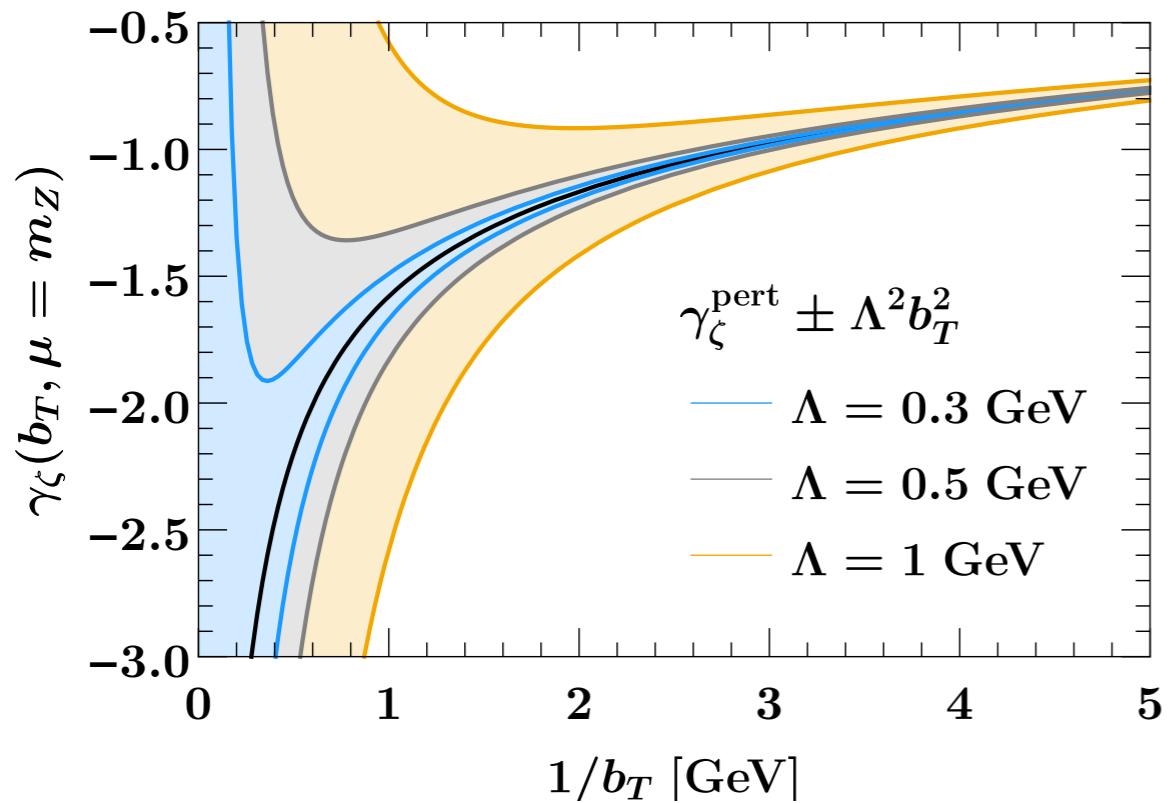


Figure: Iain Stewart

- Large uncertainties in estimates of the size of nonperturbative contributions to the CS kernel

## CS kernel from phenomenological fits

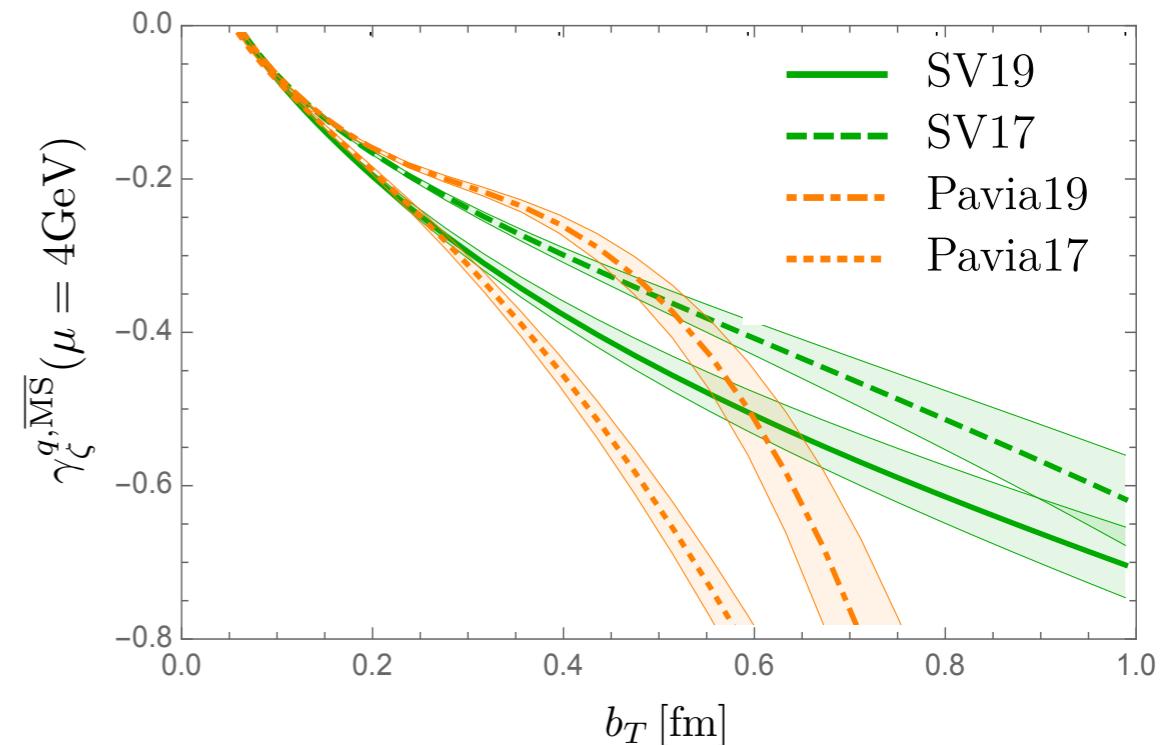


Figure adapted from Vladimirov [2003.02288]  
[see also e.g., Kang, Prokudin, Sun, Yuan Phys.Rev.D 93 (2016), Collins, Rogers Phys.Rev.D 91 (2015), Sun, Yuan Phys.Rev.D 88 (2013)]

- LQCD constraints in nonperturbative region with ~10% uncertainties would contribute to model differentiation

# Collins-Soper kernel from lattice QCD

## Collins-Soper Evolution Kernel

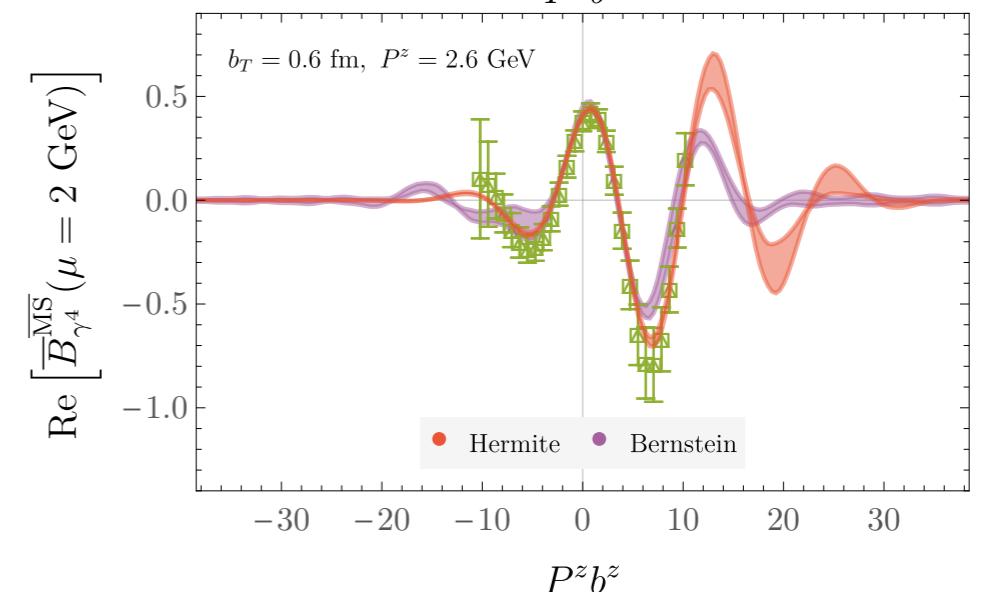
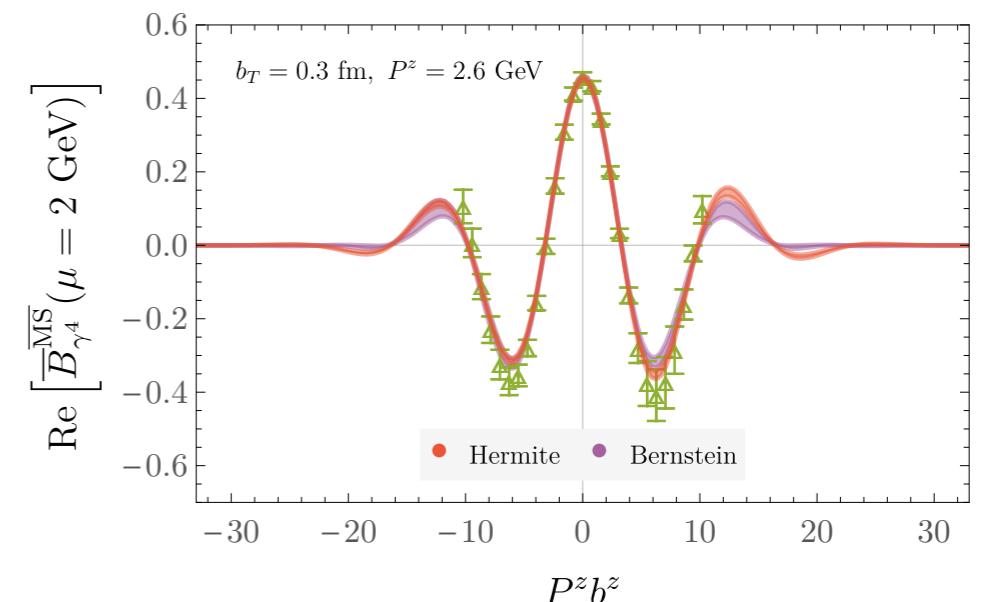
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- Governs TMD evolution
- Needed to match quasi-TMD (lattice QCD) to physical TMD

- Can be accessed via ratio of non-local MEs in LQCD [Ebert, Stewart, Zhao, PRD99 (2019)]

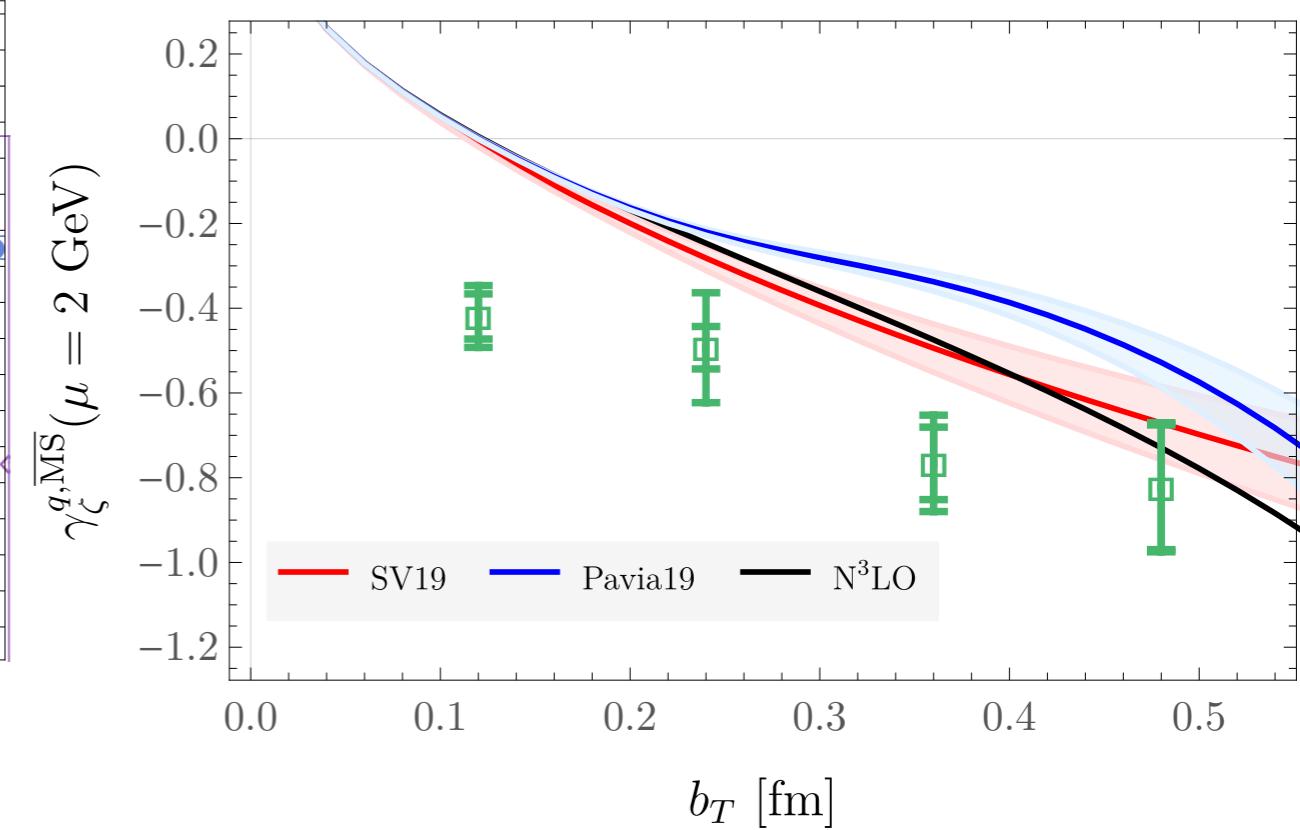
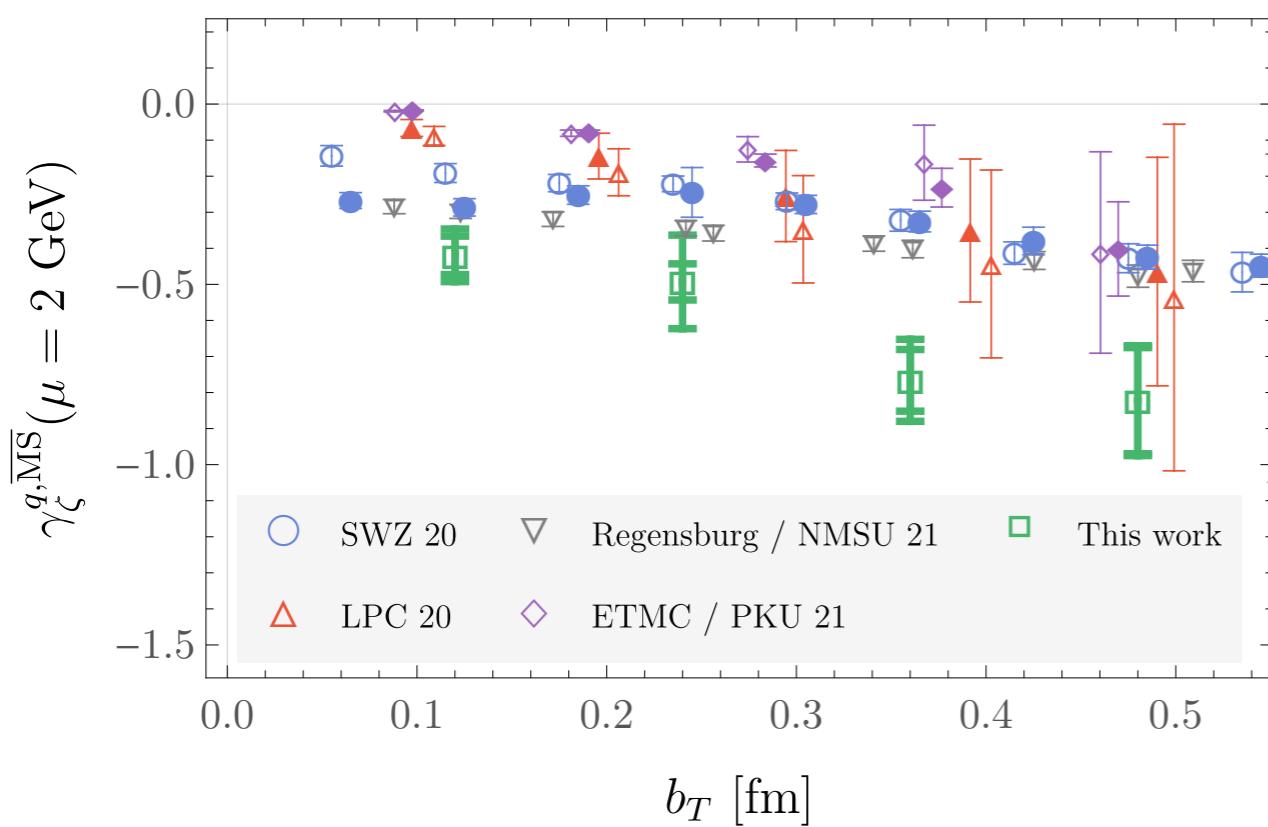
$$\begin{aligned} \gamma_\zeta^q(\mu, b_T) &= \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[ \frac{C_{\text{ns}}^{\text{TMD}}(\mu, xP_2^z)}{C_{\text{ns}}^{\text{TMD}}(\mu, xP_1^z)} \right] \\ &\times \frac{\int db^z e^{-ib^z x P_1^z} P_1^z \lim_{\substack{a \rightarrow 0 \\ \eta \rightarrow \infty}} B_{\gamma^4}^{\overline{\text{MS}}}(\mu, b^z, \vec{b}_T, a, \eta, b_T^R, P_1^z)}{\int db^z e^{-ib^z x P_2^z} P_2^z \lim_{\substack{a \rightarrow 0 \\ \eta \rightarrow \infty}} B_{\gamma^4}^{\overline{\text{MS}}}(\mu, b^z, \vec{b}_T, a, \eta, b_T^R, P_2^z)} \end{aligned}$$

- First (quenched) calculation in 2020 [PES, Wagman, Zhao PRD102, 014511 (2020)]
- CS-kernel independent of state: study unphysically-heavy pion with no systematic bias



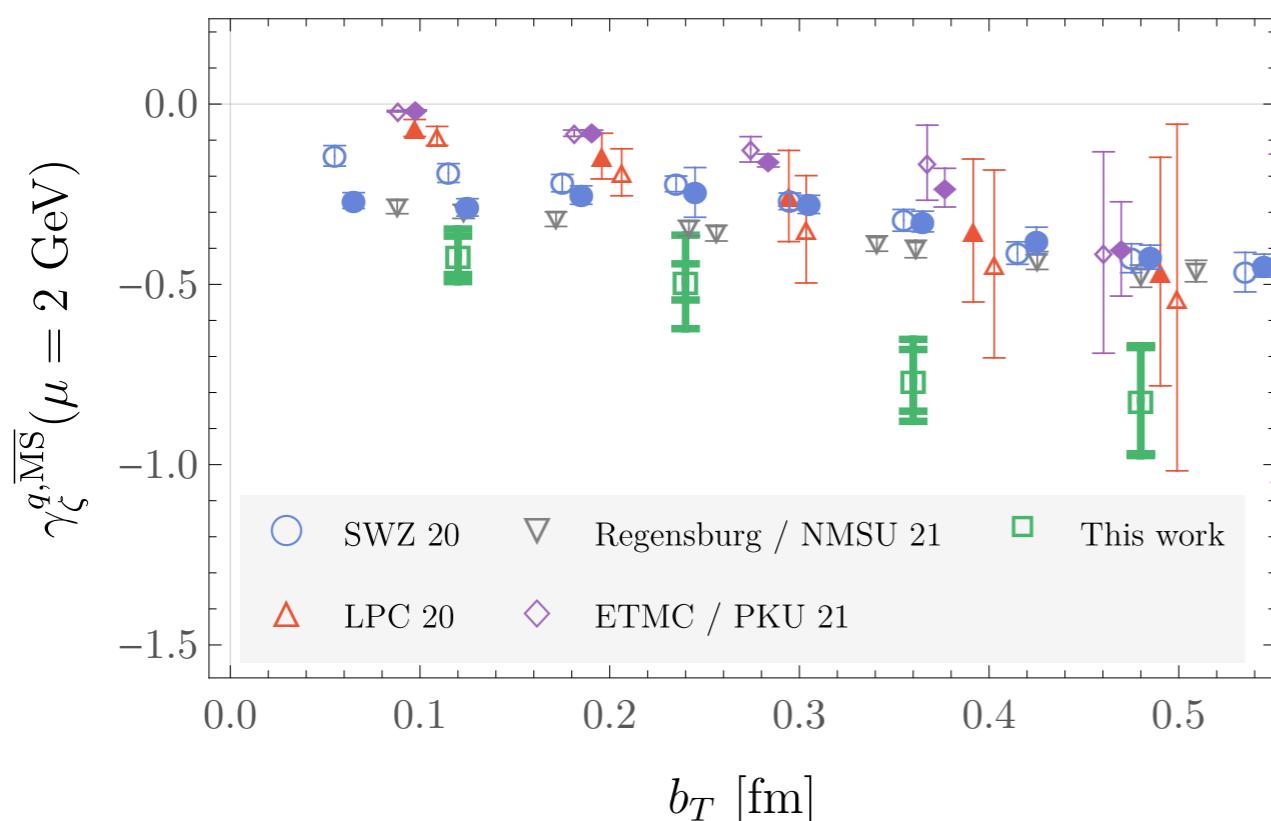
# Collins Soper kernel from lattice QCD

- First dynamical results 2021
- Currently: Significant spread in results from different LQCD studies from analysis systematics (e.g., NLO matching is important)
- Fully-controlled results at similar precision would already impact phenomenology
- Will require more robust treatment of power corrections



# Collins Soper kernel from lattice QCD

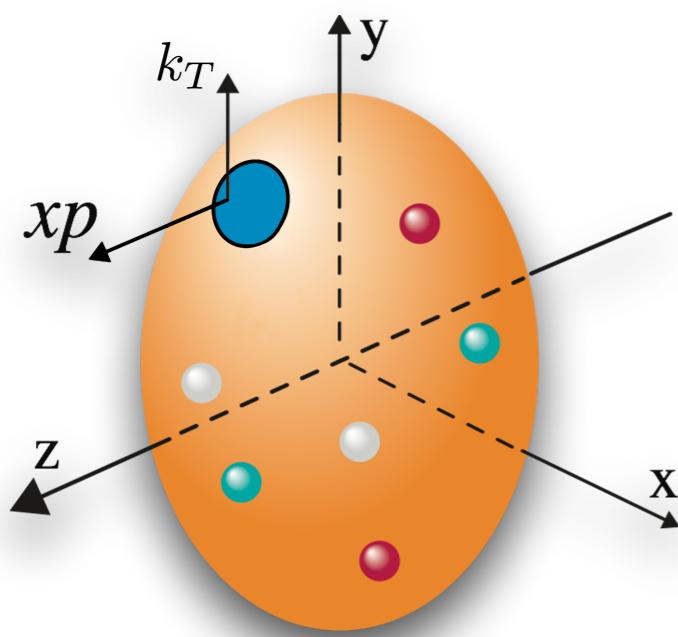
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**Wednesday WG1 session:**  
**Armando Bermudez Martinez**  
Non-perturbative contributions from PB-TMDs and CS kernel determination

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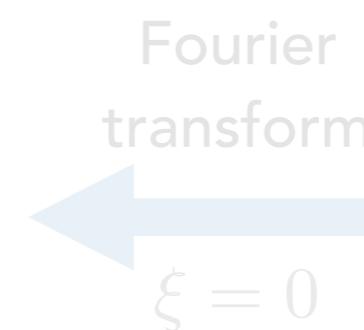
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GPD  
 $H_{q/H}(x, \xi, \Delta_T^2)$

# Moments of GPDs

- Moments of GPDs define generalised form factors which encode e.g., Energy-Momentum Tensor
- Matrix elements of traceless gluon EMT for spin-half nucleon:

$$\langle p', s' | G_{\{\mu\alpha}^a G_{\nu\}}^{a\alpha} | p, s \rangle = \bar{U}(p', s') \left( A_g(t) \gamma_{\{\mu} P_{\nu\}} + B_g(t) \frac{i P_{\{\mu\sigma\nu\}\rho} \Delta^\rho}{2M_N} + D_g(t) \frac{\Delta_{\{\mu} \Delta_{\nu\}}}{4M_N} \right) U(p, s)$$

Gluon field-strength tensor      
 Generalised gluon form factors      
  $\Delta_\mu = p'_\mu - p_\mu \quad P_\mu = (p_\mu + p'_\mu)/2, \quad t = -\Delta^2$

- Sum rules of gluon and quark GFFs in forward limit

- Momentum fraction  $A_a(0) = \langle x \rangle_a \rightarrow \sum_{a=q,g} A_a(0) = 1$
- Spin  $J_a(t) = \frac{1}{2}(A_a(t) + B_a(t)) \rightarrow \sum_{a=q,g} J_a(0) = \frac{1}{2}$

- D-terms  $D_a(0)$  unknown but equally fundamental!
  - $D_a(t)$  GFFs encodes pressure and shear distributions

# D-term from JLab DVCS

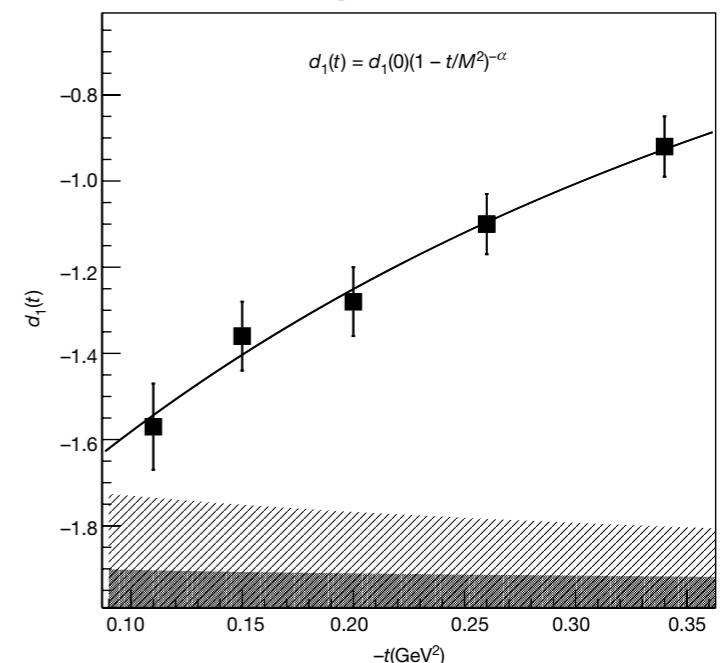
Experimental determination of DVCS D-term  
and extraction of proton pressure distribution  
[Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)]

$$s(r) = -\frac{r}{2} \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}(r), \quad p(r) = \frac{1}{3} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}(r)$$

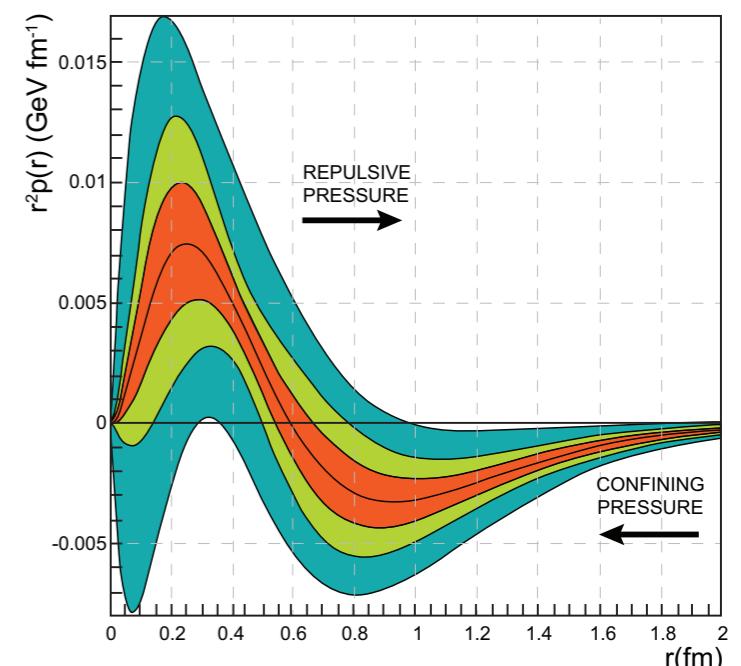
- Peak pressure near centre  $\sim 1035$  Pascal,  
greater than pressure estimated for neutron stars
- Key assumptions: gluon D-term same as quark  
term, tripole form factor model,  $D_u(t, \mu) = D_d(t, \mu)$

**EXP + LQCD**  
**first complete pressure determination**  
[Shanahan, Detmold PRL 122 072003 (2019)]

**DVCS (quark) D-term**



**Radial pressure distribution**



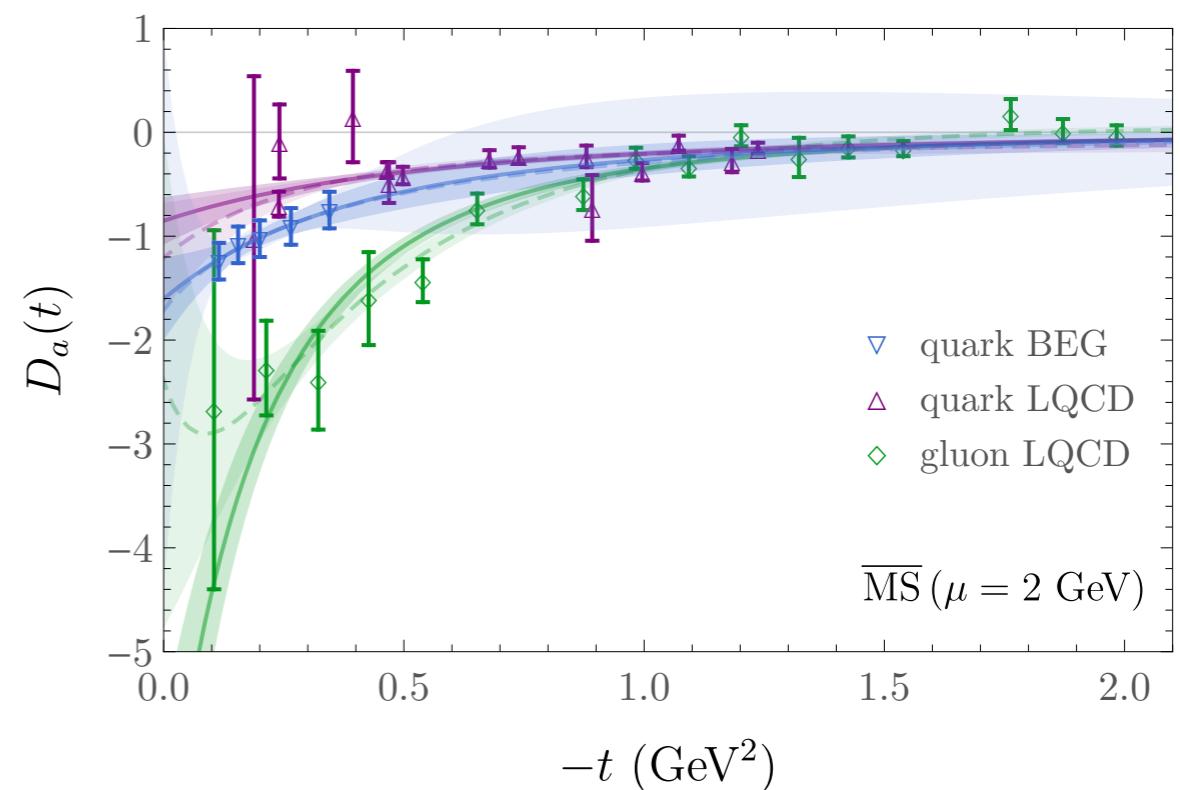
# Nucleon D-term GFFs from LQCD

EXP + LQCD  
first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]

## Key assumptions in pressure extraction from DVCS

- Gluon D-term same as quark term in magnitude and shape  
*Factor of  $\sim 2$  difference in magnitude, somewhat different  $t$ -dependence*
- Tripole form factor model  
*LQCD results consistent with ansatz, but more general form is less well constrained*
- Isovector quark D-term vanishes  
 $D_{u-d}(t) \sim 0$  from other LQCD studies

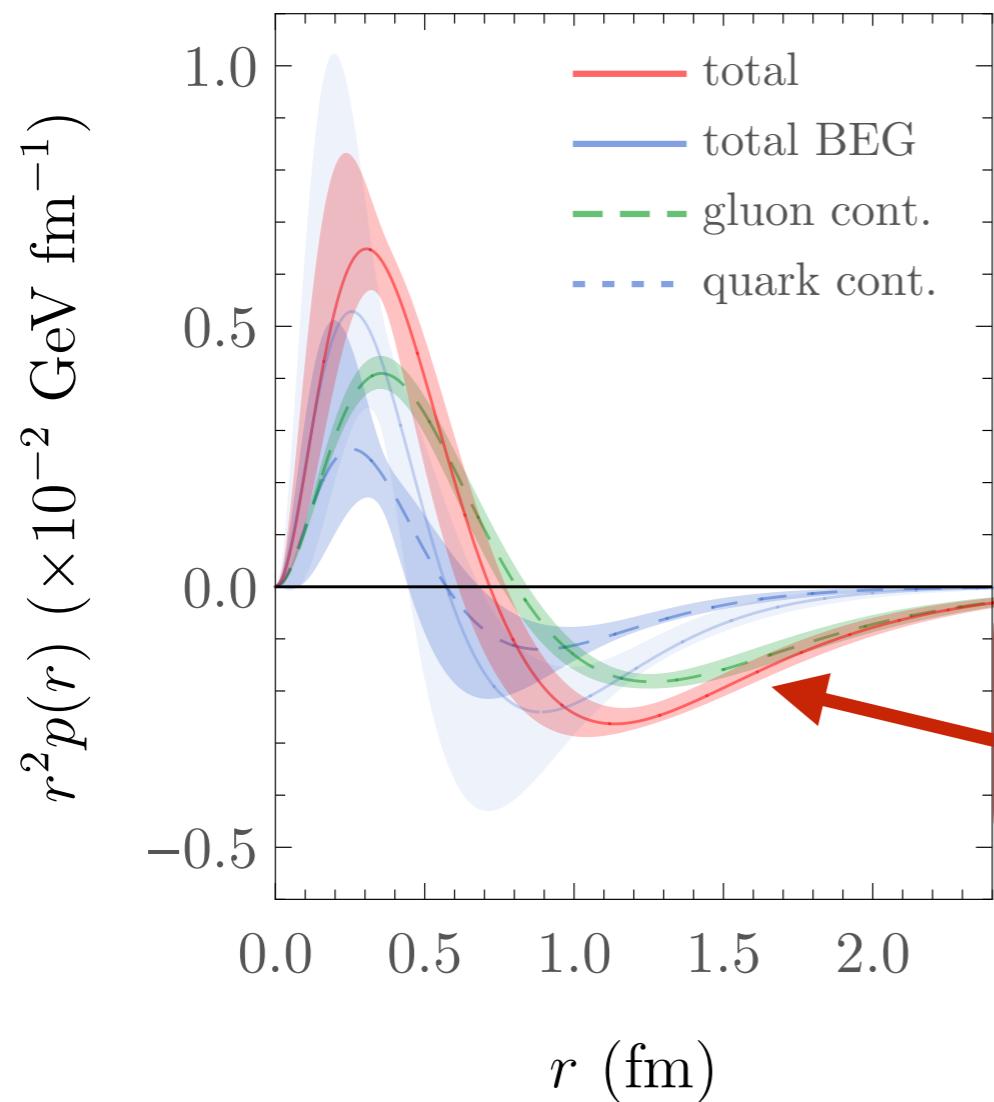


Gluon GFFs: Shanahan, Detmold, PRD 99, 014511  
Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)  
Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

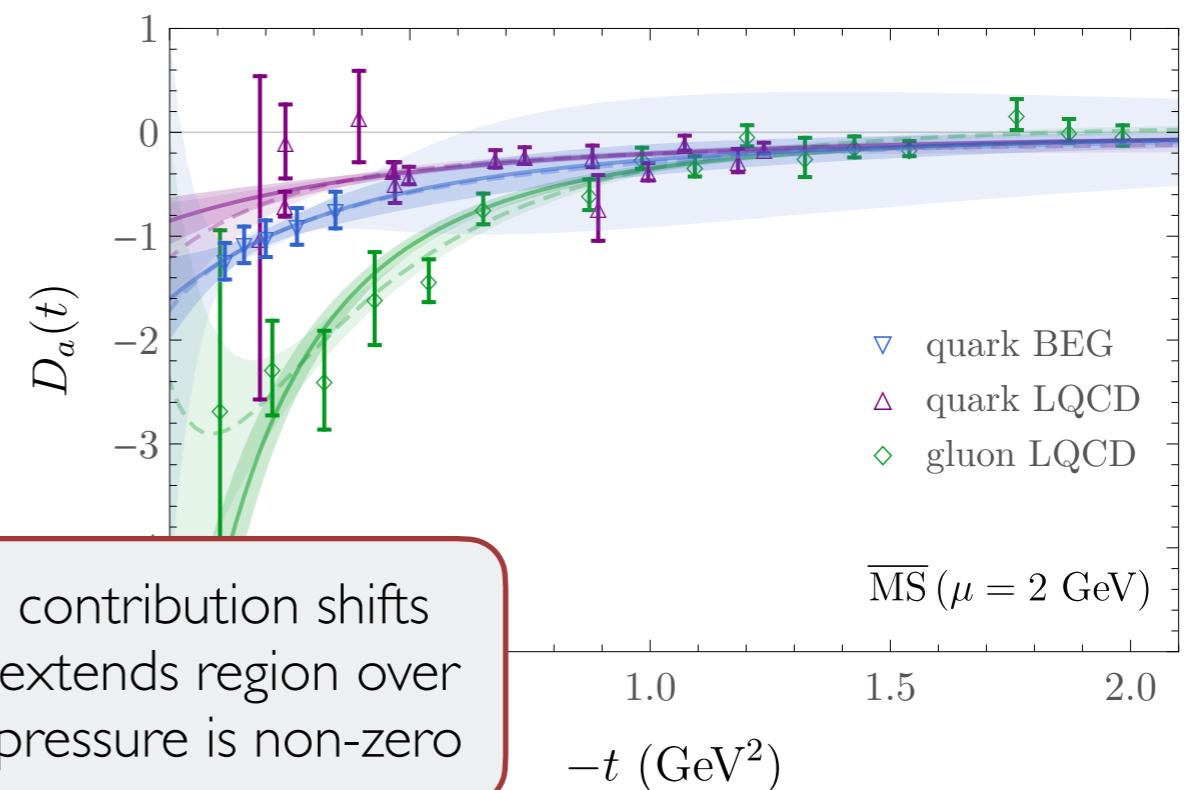
# Proton pressure from LQCD

EXP + LQCD  
first complete pressure determination

[Shanahan, Detmold PRL 122 072003 (2019)]



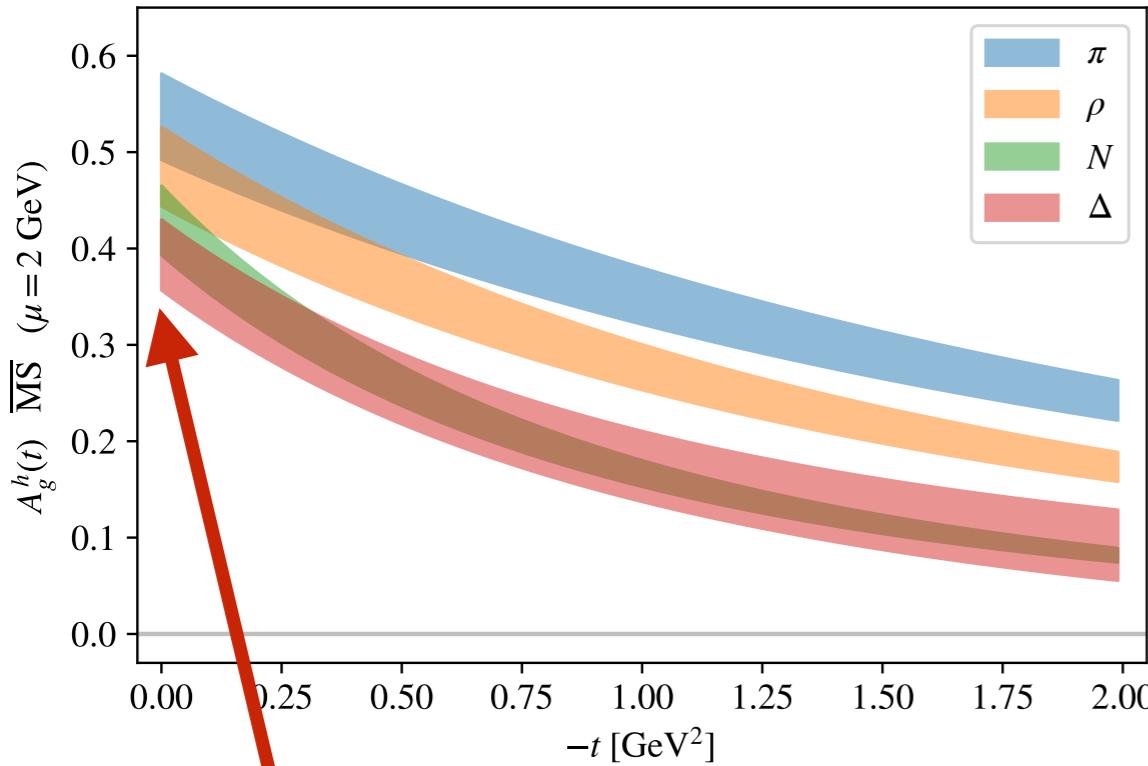
gluon contribution shifts peaks, extends region over which pressure is non-zero



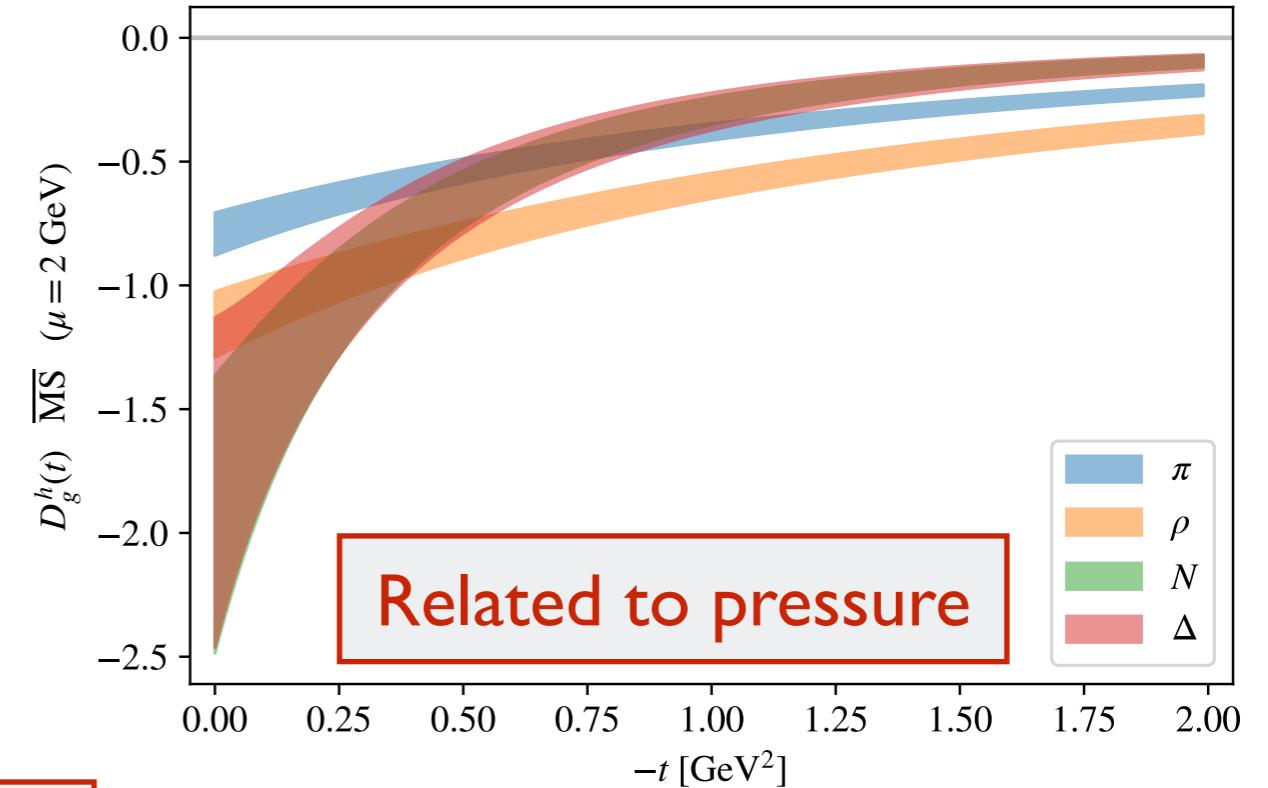
Gluon GFFs: Shanahan, Detmold, PRD 99, 014511  
Quark GFFs: P. Hägler et al. (LHPC), PRD77, 094502 (2008)  
Expt quark GFFs (BEG): Burkert et al, Nature 557, 396 (2018)

# Pressure distribution in other hadrons

- First comparisons of generalised gluon form factors (related to Fourier transforms of pressure distribution) for hadrons of different spin
- Qualitative differences between meson and baryon form factors
- Lattice QCD calculation with unphysically-heavy pion mass



Momentum fraction in forward limit



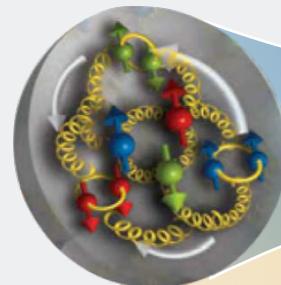
[Pefkou et al., Phys.Rev.D 105 (2022) 5, 054509]

# Parton physics from Lattice QCD

## Precision Era

*Fully-controlled w/  
few-percent errors  
within 0–5y*

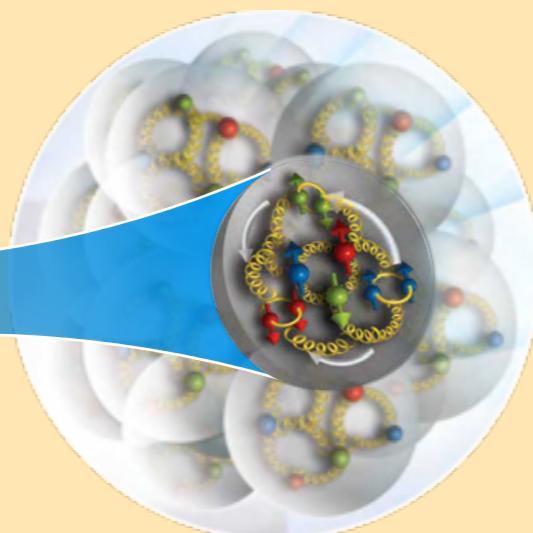
- Static properties of nucleon incl. spin, flavour decomp.
- Mellin moments of PDFs, GPDs



## Early Era

*Fully-controlled w/  
~15-percent errors  
within ~10y*

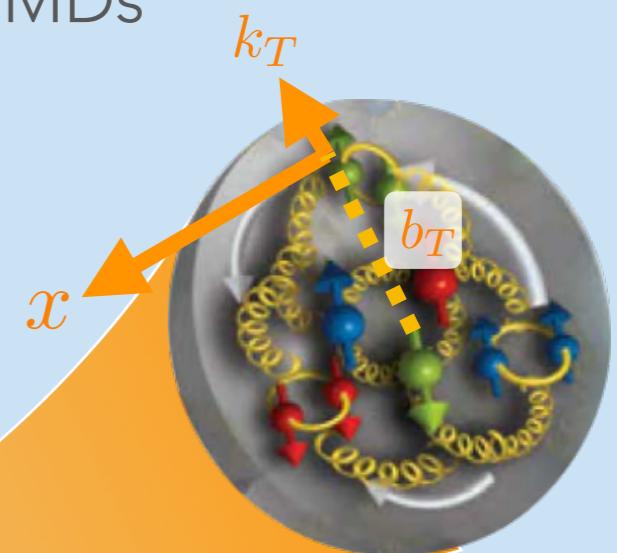
- Nuclear structure  $A < 5$
- Spin, flavour decomp. of EMC-type effects



## Dynamic Era

*Rapid development,  
timeline for  
precision  
calculations unclear*

- $x$ -dependence of PDFs
- TMDs

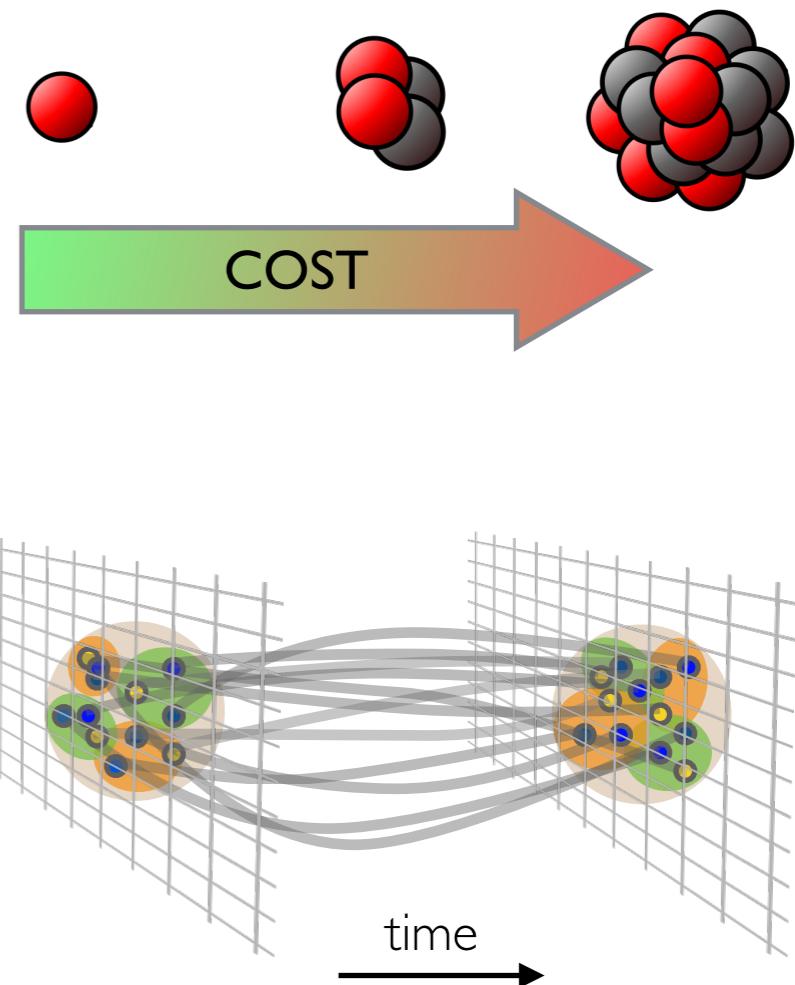




# Nuclear physics from lattice QCD

Nuclei on the lattice are  
**HARD**

- **Noise:**  
Statistical uncertainty grows exponentially with number of nucleons
- **Complexity:**  
Number of contractions grows factorially

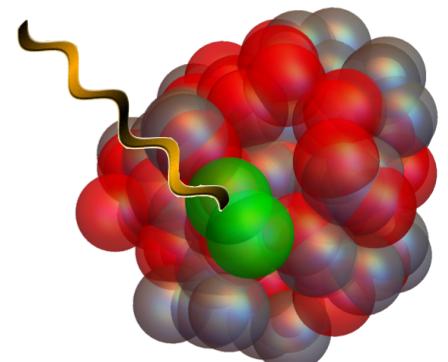
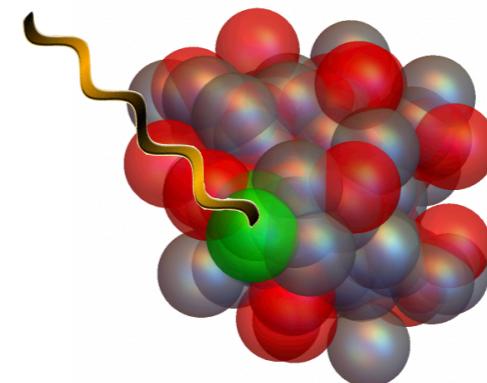


Calculations possible for  $A < 5$

# Larger nuclei

What about larger  
(phenomenologically-relevant) nuclei?

- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading  
*but non-negligible*
- Determine one body contributions from single nucleon
- Determine few-body contributions from  $A=2,3,4\dots$
- Match effective theory and many body methods to lattice results to make predictions for larger nuclei



**Spectrum:** N. Barnea et al, Phys.Rev.Lett. 114 (2015); A. Bansal et al. Phys.Rev.C 98 (2018);  
L. Contessi et al. Phys.Lett.B 772 (2017)

**Interactions:** e.g., J. Kirscher et al, Phys.Rev.C 96 (2017), W. Detmold and PES [2102.04329](#) (2021)