DIS in the big picture of HEP

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DIS: a unique microscope
Fundamentals of QCD in a clean environment

- Bjorken scaling
- QCD evolution & the rise of the gluon at HERA
- New regimes of QCD: saturation, CGC...
- Nuclear Theory: from models to first principles

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- QCD evolution & the rise of the gluon at HERA
- New regimes of QCD: saturation, CGC...
- Nuclear Theory: from models to first principles

**DIS: a unique microscope**

**A precise mapping of the proton/nuclei**

- PDFs at high precision → crucial for hadron colliders
- HERA: high-precision at hadron colliders is possible
- Beyond PDFs: TMDs, 3D tomography...
- Mass/spin of the proton
**DIS: a unique microscope**

**Fundamentals of QCD in a clean environment**
- Bjorken scaling
- QCD evolution & the rise of the gluon at HERA
- New regimes of QCD: saturation, CGC...

**A high-energy probe**
- EW physics in DIS
- Precision SM parameters
- Higgs couplings
- BSM models

**A precise mapping of the proton/nuclei**
- PDFs at high precision → crucial for hadron colliders
- HERA: high-precision at hadron colliders is possible
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- Mass/spin of the proton
This talk:

Some **illustrative** examples of the richness of the DIS program, emphasising their connection to a broader HE picture

Caveats

- Examples drawn mostly from topics I am familiar with. Apologies if your favourite subject is not here!
- Mostly focus on DIS now, but with an eye on the future.

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``Future of DIS//new facilities``” → J. d’Hondt, M. d’Onofrio’s talks

In order to study the geometric scaling at HERA, first of all we have to know the kinematic region where the scaling behaviour holds. The goal of this section is just the determination of this kinematic range.

As a warm-up, we want to reproduce the original geometric scaling observation [1]. To this purpose, we have generated a grid of 1685 points in the $(x, Q^2)$ plane, with the condition $x < 0.01$. At HERA this bound implies $x \in [1.14 \cdot 10^{-6}, 0.01]$ and $Q^2 \in [0.05 \text{ GeV}^2, 450 \text{ GeV}^2]$. Since the natural variables in our theories are $x$ and $Q^2$ but rather $\xi$ and $t$, we have chosen our points equally spaced in logarithmic units. Our sample is shown in Fig. 3.3, where we have also depicted two (fixed coupling) geometric lines, $Q^2 \cdot x^{\lambda} = 10$ and $Q^2 \cdot x^{\lambda} = 0.01$. Note that we have followed carefully the contours of the HERA triangle, in order to be absolutely sure about the output of the net.
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As a starting point for our analysis we use the fixed coupling form for the saturation line. For $\lambda$ we use the same value of [1], that is $\lambda = 0.29$. We hence plot $x$ vs $Q^2$.

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June 10, 2016. CERN Main Auditorium
 Speakers
Organizers

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G. Dissertori (ETH Zürich, CH)
I. Masina (Università di Ferrara, IT)
DIS kinematics

Figure 3.2: HERA kinematic range

can safely trust it, provided that we stay in the kinematic region shown in Fig. 3.2.

3.2.2 The geometric scaling window

In order to study the geometric scaling at HERA, first of all we have to know the kinematic region where the scaling behaviour holds. The goal of this section is just the determination of this kinematic range.

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As a starting point for our analysis we use the fixed coupling form for the saturation line. For $\lambda$ we use the same value of [1], that is $\lambda = 0.29$. We hence plot $x$ and $Q^2 = C(\mu^2) \otimes \langle O(\mu^2) \rangle + \frac{Q^2}{\Lambda^2}$.
In order to study the geometric scaling at HERA, first of all we have to know the condition $Q^2 \in \mathcal{J}$ and $\xi \in \mathcal{I}$, but rather than being the output of the net.

To this purpose, we have generated a grid of $1685$ points in the $\xi - \log Q^2$ plane, with $\xi$ at leading log $10^0$ and $Q^2$ from $10^{-6}$ to $10^{10}$. We have followed carefully the contours of the HERA triangle, in order to be absolutely sure about the output of the net.

An exhaustive presentation of all these derivation goes beyond the scope of this thesis. In the following we will limit ourselves to the Kovchegov theory, which becomes equivalent to the BK equation.

The Balitsky hierarchy decouples and its first term reduces to $O(Q^2)$ approximation and at leading log $14$. Since the natural variables in our theories are not geometrical lines,

For $s < s_0$, we can safely trust it, provided that we stay in the kinematic region shown in Fig. 3.2.

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The Balitsky hierarchy decouples and its first term reduces to $O(Q^2)$ approximation and at leading log $14$. Since the natural variables in our theories are not geometrical lines,
BFKL theory is transparent. To see this, we must derive the BFKL equation in the colour dipole formalism [21]. This part is hence organised as follows. In section 2.2 we introduce the colour dipole approach to DIS and within that context we give a precise definition of geometric scaling. Then in section 2.3 we use the dipole formalism to derive the BFKL equation and we evaluate the leading order kernel.

In order to study the geometric scaling at HERA, first of all we have to know the kinematic region where the scaling behaviour holds. The goal of this section is to find a well-defined geometric region where the scaling behaviour holds.

3.2.2 The geometric scaling window

As a warm-up, we want to reproduce the original geometric scaling observation [1]. To this purpose, we have generated a grid of 1685 points in the large-$x$ and large-$Q^2$ region, with $x$ ranging from $10^{-6}$ to 0.1 and $Q^2$ from $10^0$ to $10^5$ GeV$^2$.

The condition $Q^2/x < 1$ has to be satisfied in order to be sure about the output of the net. For the choice of the net size and the number of points in the grid, we have followed carefully the contours of the HERA triangle, in order to be absolutely sure about the output of the net.

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As a starting point for our analysis we use the fixed coupling form for the DGLAP evolution equation [7]. In this approximation and at leading log $1/x$ and $1/Q^2$, the evolution equation separates in two parts: the first one is a differential equation for the evolution of the parton density, and the second one is a differential equation for the evolution of the parton density.

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Figure 2.1: Schematic picture of parton saturation

This derivation is generalized in section 2.4 to multiple interactions, leading to the JIMWKL hierarchy [6], that describes the change of the correlation functions of the colour charge density in the hadron.
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In section 2.3 we use the dipole approach, where we introduce the colour dipole approach to DIS and within that context we give a precise definition of geometric scaling.

In section 2.2, we introduce the colour dipole approach to DIS and within that context we develop the BFKL theory. This is by far the simplest and it is model independent. Moreover, its link with the Colour Glass Condensate (CGC) approach, an effective theory in which one considers the radiation of soft gluons in a strong background field, is transparent. To see this, we must derive the BFKL equation in the large $N_c$ and $N_f$ limit, where the JIMWKL hierarchy [6], that describes the change of the correlation functions of the colour charge density in the hadron wavefunction, decouples and becomes equivalent to the BK equation.

The equation can be derived in the Color-Glass-Condensate (CGC) approximation and at leading log $1/x$.

As a warm-up, we want to reproduce the original geometric scaling observation [1]. To this purpose, we have generated a grid of 1685 points in the kinematic region shown in Fig. 3.3, where we have also depicted two (fixed $Q^2$) points.

In order to study the geometric scaling at HERA, first of all, we have to know the condition $x < 0.01$. At HERA this bound implies $Q^2 > 0.01$ GeV$^2$ and $Q^2 > 10000$. The condition $x < 0.01$ Note that we have used the condition $x < 0.01$.

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PDFs: at the core of any hadron collider

From scaling violations to physics at the few percent
PDFs: at the core of any hadron collider

An incredible synergy between DIS and HH

QCD works over many order of magnitudes, in a very precise way. No obvious signs of breakdown

Combining DIS + HH: luminosities with few percent error possible in the bulk of the EW region
PDFs: DIS vs hadron-hadron

LHC bringing in more and more constraining power, but DIS here to stay

HERA legacy dataset:
• very robust, extremely well-understood dataset. Solid backbone
• LHC: often more complex observables/analysis, tensions ($Z p_t$, jets...)
• DIS: QCD theory under better control...
The perturbative expansion in DIS and @LHC

- In general, perturbative expansion much better behaved in DIS
- ggH is an extreme case, but larger K-factors at the LHC [→ M. Bonvini’s talk]
- LHC: more differential, complex observables, often quite delicate
- Understanding of source of large K-factors not yet fully-satisfactory
The perturbative expansion in DIS and @LHC

Consequences for PDFs

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<th>NNLO $\chi^2/N_{pts}$</th>
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<td>LHCb 8 TeV $Z \to ee$ [97]</td>
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<td>1.54 (1.78)</td>
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<td>0.58 (1.30)</td>
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<td>0.86 (0.84)</td>
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<td>1.59 (1.68)</td>
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<td>5.00 (7.62)</td>
<td>1.91 (5.58)</td>
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<td>1.27 (1.32)</td>
<td>1.11 (1.17)</td>
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<td>ATLAS 8 TeV $Z p_T$ [75]</td>
<td>104</td>
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<td>1.81 (1.59)</td>
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<td>CMS 8 TeV jets $R = 0.7$ [101]</td>
<td>174</td>
<td>1.64 (1.73)</td>
<td>1.50 (1.59)</td>
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<td>ATLAS 8 TeV $tt \to l + j$ sd [102]</td>
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<td>0.94 (0.82)</td>
<td>0.68 (1.11)</td>
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<td>ATLAS 8 TeV high-mass DY [73]s</td>
<td>48</td>
<td>1.79 (1.99)</td>
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<td>0.60 (0.57)</td>
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<td>CMS 8 TeV $(d\sigma_{tt}/d\rho_{T,t}dy_{tt})/\sigma_{tt}$ [105]</td>
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<td>1.50 (1.48)</td>
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<tr>
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<td>3.85 (13.9)</td>
<td>2.61 (5.25)</td>
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<td>CMS 2.76 TeV jets [107]</td>
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<td>1.53 (1.59)</td>
<td>1.27 (1.39)</td>
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<td>CMS 8 TeV $\sigma_{it}/dy_{tt}$ [108]</td>
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<td>1.43 (1.02)</td>
<td>1.47 (2.14)</td>
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<td>ATLAS 8 TeV double differential $Z$ [74]</td>
<td>59</td>
<td>2.67 (3.26)</td>
<td>1.45 (5.16)</td>
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<td>Total, LHC data in MSHT20</td>
<td>1328</td>
<td>1.79 (2.18)</td>
<td>1.33 (1.77)</td>
</tr>
<tr>
<td>Total, non-LHC data in MSHT20</td>
<td>3035</td>
<td>1.13 (1.18)</td>
<td>1.10 (1.18)</td>
</tr>
<tr>
<td>Total, all data</td>
<td>4363</td>
<td>1.33 (1.48)</td>
<td>1.17 (1.36)</td>
</tr>
</tbody>
</table>

... but more in general, QCD ``cleaner'' in DIS.
Example of subtleties: CMS jets and NNLO

[Chen, Gehrmann, Glover, Huss, Mo (2022)]

- CMS 8 TeV dijet data, differential in $p_{t,avg}$, $y^* = |\Delta y|/2$ and $y_b = |y_1 + y_2|/2$
- Strongest constraining power among jets, but strong pull for gluon at $x \sim 0.3$
- Tension with legacy DIS/DY $\rightarrow$ discarded

- NNPDF4.0: single-differential CMS 8 TeV. Underlying TH: best prediction available at the time of fitting $\rightarrow$ NNLOQCD+NP, only leading-colour contributions

$$\delta \sigma_{\text{NNLO}} = AN_c^2 + Bn_fN_c + Cn_f^2 + D\frac{N_c^0}{N_c} + E\frac{n_f}{N_c} + \frac{G}{N_c^2}$$

- Recently: full-colour calculations available [Czakon, van Hameren, Mitov, Poncelet (2019); Chen, Gehrmann, Glover, Huss, Mo (2022)]

[→ see talk by J. Mo]
Example of subtleties: CMS jets and NNLO

- Full-colour analysis

\[ \delta \sigma_{\text{NNLO}} = A N_c^2 + B n_f N_c + C n_f^2 + D N_c^0 + E \frac{n_f}{N_c} + \frac{G}{N_c^2} \]

- Sets without CMS8 jet data seem to fit better

- Jury still out, but this example shows that the LHC can be tricky...

- General comment: robust TH uncertainty in the PDFs most welcome

[Chen, Gehrmann, Glover, Huss, Mo (2022)]

[talks by Z. Kassabov, J. McGowan, M. Bonvini]
PDFs: important information still missing

High-mass searches require large-\( x \) PDFs

Interesting ideas for constraining 2\textsuperscript{nd} generation Yukawa require good \( s, c \) and \( \bar{d}/d \) control

UHE neutrinos, prompt \( \nu \) flux \( \rightarrow \) small-\( x \), charm

Any progress in these directions welcome
PDFs: also a theory problem…

\( N^{3}\text{LO} \) PDFs not available \( \rightarrow \) order mismatch

**ggH theory error budget**

![Error Budget Graph](image_url)

- **\( N^{3}\text{LO} \) PDFs not available**
  - Order mismatch

---

**2.2.1.1 Gluon fusion**

In this section we document cross section predictions for a standard model Higgs boson produced through gluon fusion in 27 TeV \( pp \) collisions. To derive predictions we include contributions based on perturbative computations of scattering cross sections as studied in Ref. [47]. We include perturbative QCD corrections through next-to-next-to-next-to-leading order (\( N^{3}\text{LO} \)), electroweak (EW) and approximated mixed QCD-electroweak corrections as well as effects of finite quark masses. The only modification with respect to YR4 [45] is that we now include the exact \( N^{3}\text{LO} \) heavy top effective theory cross section of Ref. [48] instead of its previous approximation. The result of this modification is only a small change in the central values and uncertainties. To derive theoretical uncertainties we follow the prescriptions outlined in Ref. [47]. We use the following inputs:

- \( \sqrt{s} = 27 \text{ TeV} \)
- \( m_t = (162.7 \pm 0.7) \text{ GeV} \)
- \( m_b = (4.18 \pm 0.18) \text{ GeV} \)
- \( m_c = (3.986 \pm 0.086) \text{ GeV} \)
- \( S(m_Z) = 0.118 \)

All quark masses are treated in the MS scheme. To derive numerical predictions we use the program \( iHixs \) [50].

Sources of uncertainty for the inclusive Higgs boson production cross section have been assessed recently in refs. [47, 51, 52, 45]. Several sources of theoretical uncertainties were identified. The figure shows the linear sum of the different sources of relative uncertainties as a function of the collider energy. Each coloured band represents the size of one particular source of uncertainty as described in the text. The component \( \delta(\text{PDF} + \alpha_s) \) corresponds to the uncertainties due to our imprecise knowledge of the strong coupling constant and of parton distribution functions combined in quadrature.
PDFs: also a theory problem...

$N^3$LO PDFs not available $\rightarrow$ order mismatch

ggH theory error budget

[Recent progress towards $N^3$LO $\rightarrow$ talks by J. McGowan, K. Schönwald]
Inclusive Drell-Yan at $N^3LO$

In the EW region $Q \sim 100$ GeV: $\sim 2-3\%$ $N^3LO$ vs per-mill $NNLO$

[Duhr, Dulat, Mistlberger (2020–21)]

Band only overlap at large $Q^2$ $\rightarrow$ trouble in the high-precision region?
Neutral-current DY: flavour decomposition

Per-mille NNLO: unnaturally small. Very large cancellations

- Individual channels (μ=Q) much larger than final result, delicate cancellation pattern
- Individual channels: perturbative convergence
- N^3LO "natural", tiny PDFs changes can significantly affect this picture
N\textsuperscript{3}LO PDFs issues: evolution

N\textsuperscript{3}LO: evolution and the problems of small-x

NNLO: an issue at low-mass, not quite so at the EW scale. N\textsuperscript{3}LO?

\[
\chi_0(M) = \frac{C_A}{\pi} \left[ 2\psi(1) - \psi(M) - \psi(1 - M) \right] \rightarrow
\]

\[
\gamma_{LL}(N) = \frac{\bar{\alpha}_s}{N} + 0 \cdot \alpha_s^2 + 0 \cdot \alpha_s^3 + 2\zeta_3 \frac{\bar{\alpha}_s^4}{N^4}, \quad \bar{\alpha}_s = \alpha_s C_A/\pi
\]

Spurious leading pole in 0, starting at N\textsuperscript{3}LO (vs pole at N\~{}0.3).

Is this an issue for precision physics (at the EW scale and beyond)?

- How dangerous is the spurious N\textsuperscript{3}LO growth?
- Are subleading terms under control?
- To which extent DGLAP evolution washes out small-x effects?
- Control-sample with effectively no evolution (i.e. DIS vs LHC-only fits)?

[Resummed evolution → talk by A. Stasto]
Small-x physics and high-energy colliders

Proper understanding of small-x crucial for precision

EW physics at future hadron colliders

ggH production cross section --- effect of small-x resummation

What is the impact of sub-leading terms?

How robust is this picture?

[Bonvini, Marzani (2018)]
Small-x physics: beyond standard evolution

Small-x physics extremely interesting in its own merit.

QCD in a new regime

- A lot of recent progress towards making predictions more precise and accurate → see B. Xiao’s talk
- Effects larger in pA, $A^{1/3}$ enhancement of the saturation scale
- Also can be studied from diffraction, in a relative clean fashion [see E. Iancu’s talk]

Can we study the onset of saturation and its connection to (resummed) DGLAP with as little modelling as possible, in a clean (=protons, perturbative) setting?
Beyond PDFs: TMDs

A lot of progress... In a nutshell

• better determinations

• better theoretical understanding (from phenomenological models to first principles)

Understanding the intrinsic transverse momentum of partons plays an important role for highest-precision LHC studies...
Beyond PDFs: TMDs and the W mass

Legacy LHC measurement: W mass.
- Best handle at the LHC: W transverse momentum distribution
- Require (sub) per-mill control over $p_T$ spectra $\rightarrow$ impossible theoretically
- Idea: calibrate Z using data, only need to control differences between Z and W $\rightarrow$ PDFs, EW
- Few per-mill distortion of spectra $\leftrightarrow O(10 \text{ MeV})$ shift in $m_W$
- Right now: $\Delta m_{W,ATLAS} = \sim 19 \text{ MeV} [\text{CDF: } \sim 10 \text{ MeV}, \text{EW precision: } \sim 8 \text{ MeV}]

Good control of flavour-dependence of intrinsic $k_t$ crucial $\rightarrow$ TMD

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</table>

- Shifts uncomfortably large...
- Better control would be very welcome
- ... especially after the CDF new measurement
Towards a 3D image of the proton

Eventually, we want to map the full 3D structure of the proton → Wigner’s functions

Interesting in its own merit, but in the long run may be important for general studies at hadron colliders, beyond QCD

- PS, bread and butter of colliders, are getting better and better = under quantifiable theoretical control [→ see M. Dasgupta’s talk]
- Non-perturbative bit still from phenomenological models. Ideally, rely less and less on models, and more and more on data
- Example: MPI. VBF Higgs: +5% at low $p_t$ [NNLO QCD: 4–7%]
- Still very far from tomography-informed MPI, but a lot of progress on tomography expected from the EIC...
**DIS and HEP theory**

- The simplest, yet non-trivial example of hadron collider
- Many techniques developed for DIS then successfully applied to the LHC
- Crucial results in pQCD [QCD evolution...]
- Tools [nested sums, iterated integrals...] widely used for state-of-the-art calculations
- Clean settings for NP studies, solid grounds in QFT (this is less the case for hadron-hadron colliders...)
DIS: the first N^3LO inclusive calculation…

- Good perturbative convergence (away from small-x)
- Naive $\alpha_s$ power counting works well
- Crucial for high-precision fits
DIS: ... and the first fully-exclusive one

- Also in this case good convergence
- Testing grounds for similar calculations at hadron colliders
DIS in disguise: VBF@LHC

\( \mathcal{O}(\alpha_s) \) + \( \mathcal{O}(\alpha_s) \) + \( \mathcal{O}(\alpha_s^2/N_c^2) \)

- Double-DIS approximation very good
DIS in disguise: VBF@LHC

Using DIS in a clever way:

**NNLO exclusive**

- Double-DIS approximation very good... although careful at Glauber phases, $\pi^2/N_c^2$ is not small [Melnikov, Penin (2019)]
DIS in disguise: t-channel single top

A similar argument holds for t-channel single-top production.

- Also requires massless $\rightarrow$ massive DIS transitions [Berger, Gao, Li, Liu, Zhu (2016)]
- Double-DIS approximation very good... although careful at Glauber phases, $\pi^2/N_c^2$ is not small [→ see C. Brønnum-Hansen’s talk]
Non-linear evolution in disguise

- Unitarity: parton evolution ↔ forward scattering of elastic amplitudes

- High-enough logarithmic order: sensitive to full Balitsky-JIMWLK evolution

Non-linear evolution in disguise

- 2→2 QCD scattering amplitudes@3L recently computed [Chakraborty, Gambuti, von Manteuffel, Tancredi, FC (2021)]

Can test Regge factorisation at NNLL

\[ \mathcal{H}_{\text{ren}, \pm} = Z_g^2 e^{L T_i^2 T_g} \sum_{n=0}^{3} \bar{\alpha}_s^n \sum_{k=0}^{n} L^k \mathcal{O}_k^{\pm, (n)} \mathcal{H}_{\text{ren}, 0} \]

Regge trajectory

Multi-Reggeon interactions (SLC)

\[ \mathcal{O}_0^{-, (0)} = 1, \quad \mathcal{O}_0^{-, (1)} = 2 T_1^g, \]
\[ \mathcal{O}_0^{-, (2)} = \left[ 2 T_2^g + (I_1^g)^2 \right] + C^{-, (2)} [(T_{s-u}^2)^2 - \frac{N_c^2}{4}], \]
\[ \mathcal{O}_1^{-, (3)} = C_1^{-, (3)} T_{s-u}^2 [T_t^2, T_{s-u}^2] + C_2^{-, (3)} [T_t^2, T_{s-u}^2] T_{s-u}^2, \]
LHC: almost DIS2, but not always…

QCD with intrinsic heavy quarks:
- collinear factorisation violated at NNLO in hadron-hadron (i.e. $R + V + \text{ren} = \infty$)
- no problem in DIS

[Doria, Frenkel, Taylor (1980); many subsequent studies. See e.g. Melnikov, Napoletano, Tancredi, FC (2020) for a modern derivation and discussion]
A new twist to an old story: intrinsic charm!

Evidence for intrinsic charm in DIS + LHC data

• How to properly deal with it at NNLO at the LHC unclear

• DIS: solid foundation → guide and benchmark

[→ see K. Kudashkin’s talk and G. Magni’s poster]
DIS as a high-energy probe

DIS in the past did probe EW interactions (NC vs CC, γ/Z interference...)

Future DIS facilities: clean environment (low pile-up, controlled bkgd...) for precision EW studies

• A famous example: b/c Higgs Yukawa

  • S/B ~ 3!
  • Constrain signal strength to 0.8% (bb) and 7.4% (cc)

• Not the only one! W-mass in the t-channel, top polarisation, radiation zeros, hidden sectors, axions... rich program at future facilities

→ see J. d’Hondt & M. d’Onofrio’s talks
... as any physicist not working on particle physics would tell you

- If a collider can deliver new discoveries, that’s of course great
- Looking at the future: the era of “guaranteed new physics deliveries” (like the Higgs for the LHC) may well be over
- But there is a rich set of unexplored areas in the SM that are worth pursuing

Many interesting open questions in QCD. For example

- Mass/spin proton/nuclei
- The structure of the proton [PDFs, TMDs, tomography...]
- Nuclear physics: from models to first principles
- QCD evolution and new phases of QCD (saturation, QGP...)
- ...

Future DIS facilities (EIC, LHeC, FCC-eh) would shed light on these issues
Conclusion II

DIS: the simplest hadron collider machine

• In this case: simple ↔ powerful (clean, well-understood)

• Hard to overstate the importance of accurate, precise and reliable determinations of the structure of the proton for the HE program at hadron colliders → legacy DIS data augmented with LHC information, interesting cross-talks

  Extreme regions (small/large-x) and individual quarks remain elusive → limiting factor for different physics programs

• HE DIS: clean probe of EW scale and beyond

• Interesting synergies with other experiments

• Many interesting QCD questions

• Techniques developed for DIS have much broader applications
DIS: very interesting and important role in the HEP landscape

Thank you very much for your attention!
Backup
**N³LO: inclusive results**

To a large extent, inclusive N³LO for 2 → 1 processes has been solved


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Fig. 3: The gluon fusion cross-section at all perturbative orders, showing the dependence of the cross section on the invariant mass of the hadron collider. It would be interesting to compare these results with other theoretical predictions for large classes of inclusive processes.

Fig. 4: Drell-Yan process, showing the relative contribution of different channels, and the impact of scale variation on the cross section. The dashed magenta line indicates the central value, while the bands show the scale variation uncertainties.
**N³LO: PDFs**

N³LO PDFs not available → order mismatch

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<th>Q/GeV</th>
<th>K_{QCD}^{N³LO}</th>
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</table>

Error: estimate from previous orders

\[ \delta(\text{PDF-TH}) = \frac{1}{2} \left| \frac{\sigma^{(2)}_{W\pm}, \text{NNLO-PDFs} - \sigma^{(2)}_{W\pm}, \text{NLO-PDFs}}{\sigma^{(2)}_{W\pm}, \text{NNLO-PDFs}} \right| . \]

- ~ 2% PDF-TH error in the EW region
- significant fraction of the error budget
- same order of "standard" PDF+\(\alpha_S\)
N\textsuperscript{3}LO PDFs issues: evolution

N\textsuperscript{3}LO: evolution and the problems of small-\textit{x}

- N\textsuperscript{3}LO calculation underway [Herzog, Moch, Ruijl, Ueda, Vermaseren, Vogt, in progress]
- N\textsuperscript{3}LO: rapid small-\textit{x} growth $\rightarrow$ perturbative instabilities@N\textsuperscript{3}LO
- NLL resummation known, but large subleading effects [Bonvini, Marzani (2018)]

NNLO: an issue at low-mass, not quite so at the EW scale
N³LO PDFs issues: data

- Collider data crucial to reduce perturbative uncertainty → fully-consistent N³LO fit would require top, Z p_t, jets @ N³LO

N³LO for PDFs: status and prospects

- DIS ✔
- DY ✔
- Z p_t: ~ (unknown, but should be possible)
- Top: ~ (unknown, but should be possible given current understanding)
- Jets: ✘ (unknown, and there may be serious problems...)

Multiple independent handles on the gluon PDF

NNPDF3.1, Q=100 GeV