Non-factorisable QCD corrections to t-channel single-top production

Based on 2204.05770 and 2108.09222 with Kirill Melnikov, Jérémie Quarrroz, Chiara Signorile-Signorile & Chen-Yu Wang

Christian Brønnum-Hansen | 4th of May | DIS2022
Motivation

- Top quark is the heaviest particle of the Standard Model.
  - Better understanding of electroweak symmetry breaking.
  - Hopefully, hints for physics beyond the Standard Model.

- Primarily produced in pairs. However, **single-top** production also occurs frequently

\[
\sigma_{t,\text{single}} \approx \frac{1}{4} \sigma_{t\bar{t}}
\]

- tWb interaction is interesting due to:
  - determination of the CKM matrix element $V_{bt}$
  - indirect determination of $\Gamma_t$ and the top-quark mass $m_t$
  - constraints on bottom-quark PDF $f_b(x_1)$
Single-top production

There are three single-top production modes:

- **t channel:** $qb \rightarrow q't$
  - $u \rightarrow d$
  - $b \rightarrow t$
  - $\mathcal{O}(70\%)$

- **associated production:** $gb \rightarrow Wt$
  - $b \rightarrow W$
  - $g \rightarrow Wt$
  - $\mathcal{O}(25\%)$

- **s channel:** $q\bar{q}' \rightarrow W^* \rightarrow t\bar{b}$
  - $q \rightarrow W^*$
  - $\mathcal{O}(5\%)$

The main production mode is the $t$-channel.
NNLO QCD corrections to t-channel single-top production

- NLO QCD and electroweak corrections have been known for a while. Harris et al. 2002; Campbell, Ellis, et al. 2004; Sullivan 2004; Cao and Yuan 2005; Sullivan 2005; Beccaria et al. 2006; Schwienhorst et al. 2011; Frederix et al. 2019

- NNLO QCD corrections are known except for non-factorisable corrections. Brucherseifer, Caola and Melnikov 2014; Berger, Edmond, Gao, Yuan, Zhu 2016; Campbell, Neumann and Sullivan 2021
Non-factorisable QCD corrections

Non-factorisable corrections vanish at NLO because of colour.

Non-factorisable corrections are colour-suppressed at NNLO and, therefore, expected to be negligible.

\[ \text{tr} \left( t^a t^a \right) \text{tr} \left( t^b t^b \right) = \frac{1}{4} (N_c^2 - 1)^2 \]
Non-factorisable QCD corrections

But it is not obvious that non-factorisable corrections are in fact negligible.

- Factorisable NNLO QCD corrections are **small** (few %).

- **Possible \( \pi^2 \) enhancement** due to the Glauber phase.
  - Virtual effect that, in principle, does not require a scattering to occur. \( p^t_\perp \to 0 \)

\[
\pi^2 \quad \sigma = \sigma_0 + \frac{p^t_\perp}{\sqrt{s}} \sigma_1 + \mathcal{O}\left((p^t_\perp/\sqrt{s})^2\right) \quad p^t_\perp \sim 40 \text{ GeV} \quad \sqrt{s} \sim 300 \text{ GeV}
\]

⇒ Explicitly proved for the non-factorisable corrections to the Higgs production in weak boson fusion in the eikonal approximation. *Liu, Melnikov, et al. 2019*

This factor \( \pi^2 \sim 10 \) could **compensate** the factor 8 from colour suppression.

A better understanding of non-factorisable corrections to single-top production at LHC would be **beneficial**.
Non-factorisable QCD corrections at NNLO

What is needed to compute non-factorisable contribution at NNLO?

\[ d\hat{\sigma}_{\text{n.f.}}^{\text{NNLO}} = d\hat{\sigma}_{\text{RR}} + d\hat{\sigma}_{\text{RV}} + d\hat{\sigma}_{\text{VV}} \]

- \( d\hat{\sigma}_{\text{RR}} : \mathcal{A}^{(0)}_6 \otimes \mathcal{A}^{(0)}_6 = \)

- \( d\hat{\sigma}_{\text{RV}} : \mathcal{A}^{(1)}_5 \otimes \mathcal{A}^{(0)}_5 = \)

Infrared singularities are only of soft origin.

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Non-factorisable QCD corrections at NNLO

- What is needed to compute non-factorisable contributions at NNLO?

\[ d\hat{\sigma}_{\text{n.f.}}^{\text{NNLO}} = d\hat{\sigma}_{\text{RR}} + d\hat{\sigma}_{\text{RV}} + d\hat{\sigma}_{\text{VV}} \]

\[ A_6^{(0)} A_5^{(1)}, A_5^{(0)}, A_4^{(2)}, A_4^{(1)}, A_4^{(0)} \]

\[ d\hat{\sigma}_{\text{VV}} : A_4^{(0)} \otimes A_4^{(2)} = \]

Upon interference, the non-Abelian part of the amplitude disappears and the amplitude is, effectively, Abelian.
Double-virtual contribution

- Only 18 non-vanishing diagrams, but all have maximal topology and four scales ($s$, $t$, $m_t$, $m_W$).

- IBP reduction performed **analytically** with KIRA 2.0 *Klappert, Lange, et al. 2020* and FireFly *Klappert and Lange 2020; Klappert, Klein, et al. 2021* in $\mathcal{O}(4)$ days:

\[
A_4^{(0)} \otimes A_4^{(2)} = \sum_{i=1}^{428} c_i(d, s, t, m_t, m_W)I_i
\]
Based on the **auxiliary mass flow method** Liu, Ma, and Wang 2018; Liu, Ma, Tao, et al. 2020; Liu and Ma 2021

\[
I \propto \lim_{\eta \to 0^+} \int \prod_{i=1}^{2} \prod_{a=1}^{d} \frac{1}{[q_a^2 - (m_a^2 - i\eta)]^{\nu_a}} \prod_{i=1}^{9} d^d k_i
\]

- Add an imaginary part to the \( W \) **boson mass**

\[
m_W^2 \to m_W^2 - i\eta.
\]

- Solve differential equations at each kinematic point

\[
\partial_x I = MI, \quad x \propto -i\eta.
\]

with boundary condition \( x \to -i\infty \).

Stepping from the boundary at \( x \to -i\infty \), via regular points, to the **physical** mass. Step size is limited by **singularities** of the equation.
Master integral evaluation

- Expand $I$ around boundary in variable $y = x^{-1} = 0$:

$$I = \sum_{j}^{M} \epsilon^{j} \sum_{k}^{N} \sum_{l}^{c_{jkl}} y^{k} \ln^{l} y + \ldots$$

- Evaluate and expand around regular points:

$$I = \sum_{j}^{M} \epsilon^{j} \sum_{k=0}^{N} c_{jk} x^{k} + \ldots$$

- Evaluate at the physical point. $x = 0 \leftarrow$ regular point
- Path is fixed by singularities and desired precision.
- Expected relative error is $(\frac{\Delta}{R})^{N}$

$m_{W}^{2} \rightarrow m_{W}^{2}(1 + x)$
Some boundary conditions are known analytically: 't Hooft and Veltman 1979; Chetyrkin et al. 1980; Scharf and Tausk 1994; Gehrmann and Remiddi 2000; Gehrmann, Huber, et al. 2005

Some are not available or not known to sufficient $\epsilon$ order:

Apply auxiliary mass flow method for internal $m_t \rightarrow i\eta \rightarrow \infty$ on boundary integrals.

Motivation | Diagrams | Singularities | Results
---|---|---|---
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Double-virtual contribution

- All **428** master integrals evaluated numerically using the **auxiliary mass flow method** to 20 digits in $\sim 30$ minutes on a single core.

- Comparison of poles at a typical phase space point $s \approx 104.337 \text{ GeV}^2$ and $t \approx -5179.68 \text{ GeV}^2$.

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<th>$\epsilon^{-2}$</th>
<th>$\epsilon^{-1}$</th>
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<td>$A_4^{(0)} \otimes A_4^{(2)}$</td>
<td>$-229.0940408654660 - 8.978163333241640i$</td>
</tr>
<tr>
<td>IR poles</td>
<td>$-301.1802988944764 - 264.1773596529505i$</td>
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- 10 sets of $10^4$ points extracted from a grid prepared on the **Born squared amplitude**.

- The 10 different sets give an estimation of the error on $\sigma_{VV}: O(2\%)$.
The non-factorisable correction to the leading-order cross section at 13 TeV and $\mu_F = m_t$

$$\frac{\sigma_{pp \to X+t}}{1 \text{ pb}} = 117.96 + 0.26 \left( \frac{\alpha_s(\mu_R)}{0.108} \right)^2$$

- Non-factorisable correction is $0.22^{−0.04}_{+0.05}$% for $\mu_R = m_t$.

- Non-factorisable corrections appear for the first time at NNLO $\rightarrow$ No indication of a good scale choice.

- At $\mu_R = 40$ GeV, typical transverse momentum of the top quark, corrections become close to 0.35%.

- In comparison, NNLO factorisable correction to NLO cross section are about 0.7% \cite{Campbell2021}.
Top-quark transverse momentum distribution

There is a significant $p_{\perp}^{t}$-dependence of the non-factorisable corrections.

Non-factorisable corrections vanish around 50 GeV. The factorisable corrections vanish around 30 GeV. \textit{Campbell, Neumann, et al. 2021}

At low $p_{\perp}^{t}$, around the peak of the distribution, non-factorisable corrections are dominant compared to factorisable corrections.

Figure: The top quark transverse momentum distribution.
We computed the missing part of NNLO QCD corrections to the $t$-channel single-top production: the non-factorisable corrections.

The auxiliary mass flow method has been used for integral evaluation. It is sufficiently robust to produce results relevant for phenomenology.

Non-factorisable corrections are smaller than, but quite comparable to, the factorisable ones.

If a percent precision in single-top studies can be reached, the non-factorisable effect will have to be taken into account.
Thank you for your attention!