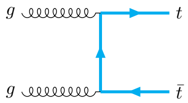
Non-factorisable QCD corrections to t-channel single-top production

Based on 2204.05770 and 2108.09222 with Kirill Melnikov, Jérémie Quarroz, Chiara Signorile-Signorile & Chen-Yu Wang

Christian Brønnum-Hansen | 4th of May | DIS2022

Motivation

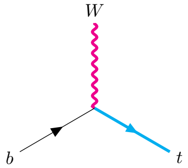
- Top quark is the heaviest particle of the Standard Model.
  - ➥ Better understanding of electroweak symmetry breaking.
  - ➥ Hopefully, hints for physics beyond the Standard Model.



- Primarily produced in pairs. However, **single-top** production also occurs frequently

$$\sigma_{t,\mathrm{single}} \approx \frac{1}{4}\sigma_{t\bar{t}}$$

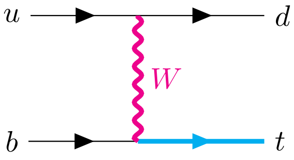
- tWb interaction is interesting due to:
  - ➥ determination of the CKM matrix element V_{bt}
  - ➥ indirect determination of Γ_t and the top-quark mass m_t
  - ➥ constraints on bottom-quark PDF $f_b(x_1)$



Single-top production

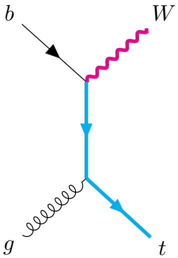
There are three single-top production modes

t channel: $qb \to q' t$



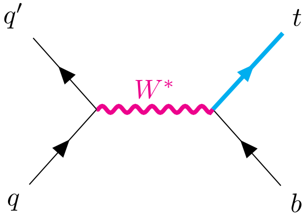
$\mathcal{O}$ (70%)

associated production: $gb \to Wt$



$\mathcal{O}$ (25%)

s channel: $q\bar{q}' \to W^* \to t\bar{b}$



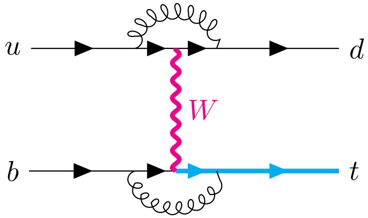
$\mathcal{O}$ (5%)

The main production mode is the t-channel.

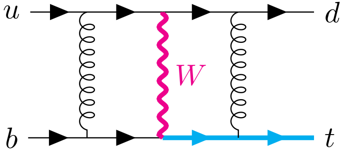
NNLO QCD corrections to t-channel single-top production

- NLO QCD and electroweak corrections have been known for a while. *Harris et al. 2002; Campbell, Ellis, et al. 2004; Sullivan 2004; Cao and Yuan 2005; Sullivan 2005; Beccaria et al. 2006; Schwienhorst et al. 2011; Frederix et al. 2019*
- NNLO QCD corrections are known **except for non-factorisable corrections**. *Brucherseifer, Caola and Melnikov 2014; Berger, Edmond, Gao, Yuan, Zhu 2016; Campbell, Neumann and Sullivan 2021*

Factorisable

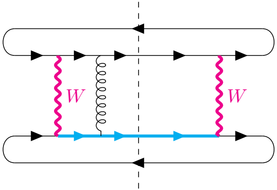


Non-factorisable



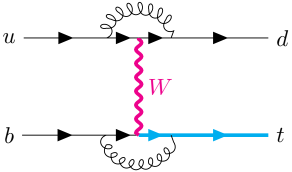
Non-factorisable QCD corrections

Non-factorisable corrections **vanish** at NLO because of colour.

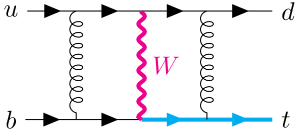


$\propto \mathrm{tr}\left(t^a\right) = 0$

Non-factorisable corrections are **colour-suppressed** at NNLO and, therefore, expected to be **negligible**.



$$\mathrm{tr}\left(t^a t^a\right)\,\mathrm{tr}\left(t^b t^b\right) = \frac{1}{4}(N_c^2 - 1)^2$$



$$\mathrm{tr}\left(t^a t^b\right)\,\mathrm{tr}\left(t^a t^b\right) = \frac{1}{4}(N_c^2 - 1)$$

Non-factorisable QCD corrections

But it is not obvious that *non-factorisable* corrections are in fact negligible.

- Factorisable NNLO QCD corrections are **small** (few %).
- **Possible π^2 enhancement** due to the *Glauber phase*.
  - ➥ **Virtual effect** that, in principle, does not require a scattering to occur. $p_\perp^t \to 0$

$$\sigma = \overset{\pi^2}{\sigma_0} + \underbrace{\overbrace{\frac{p_\perp^t}{\sqrt{s}}\,\sigma_1 + \mathcal{O}\left(\left(p_\perp^t/\sqrt{s}\right)^2\right)}^{\text{real emission}}}_{\text{virtual correction}} \qquad p_\perp^t \sim 40\,\mathrm{GeV} \qquad \sqrt{s} \sim 300\,\mathrm{GeV}$$

  - ➥ Explicitly proved for the non-factorisable corrections to the Higgs production in weak boson fusion in the eikonal approximation. *Liu, Melnikov, et al. 2019*

This factor $\pi^2 \sim 10$ could **compensate** the factor 8 from colour suppression.

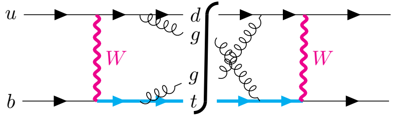
A better understanding of **non-factorisable** corrections to single-top production at LHC would be **beneficial**.

Non-factorisable QCD corrections at NNLO

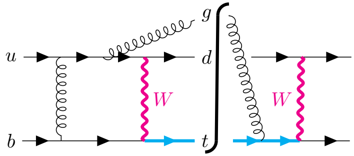
- What is needed to compute non-factorisable contribution at NNLO ?

$$d\hat{\sigma}^{\text{NNLO}}_{\text{n.f.}} = \underbrace{d\hat{\sigma}_{\text{RR}}}_{\mathcal{A}_6^{(0)}} + \underbrace{d\hat{\sigma}_{\text{RV}}}_{\mathcal{A}_5^{(1)},\,\mathcal{A}_5^{(0)}} + \underbrace{d\hat{\sigma}_{\text{VV}}}_{\mathcal{A}_4^{(2)},\,\mathcal{A}_4^{(1)},\,\mathcal{A}_4^{(0)}}$$

$$d\hat{\sigma}_{\text{RR}} : \mathcal{A}_6^{(0)} \otimes \mathcal{A}_6^{(0)} =$$



$$d\hat{\sigma}_{\text{RV}} : \mathcal{A}_5^{(1)} \otimes \mathcal{A}_5^{(0)} =$$



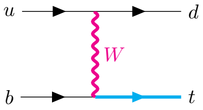
Infrared singularities are only of **soft** origin.

Non-factorisable QCD corrections at NNLO

- What is needed to compute non-factorisable contributions at NNLO ?

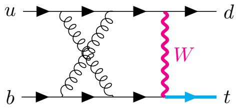
$$d\hat{\sigma}^{\text{NNLO}}_{\text{n.f.}} = \underbrace{d\hat{\sigma}_{\text{RR}}}_{\mathcal{A}_6^{(0)}} + \underbrace{d\hat{\sigma}_{\text{RV}}}_{\mathcal{A}_5^{(1)},\, \mathcal{A}_5^{(0)}} + \underbrace{d\hat{\sigma}_{\text{VV}}}_{\mathcal{A}_4^{(2)},\, \mathcal{A}_4^{(1)},\, \mathcal{A}_4^{(0)}}$$

$d\hat{\sigma}_{\text{VV}} : \mathcal{A}_4^{(0)} \otimes \mathcal{A}_4^{(2)} =$



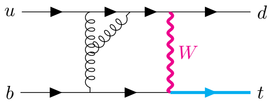
$\delta_{ij}\delta_{kl}$

$\otimes \Big[$



$(t^a t^b)_{ij}(t^b t^a)_{kl} \to \frac{1}{4}(N_c^2 - 1)$

$+$



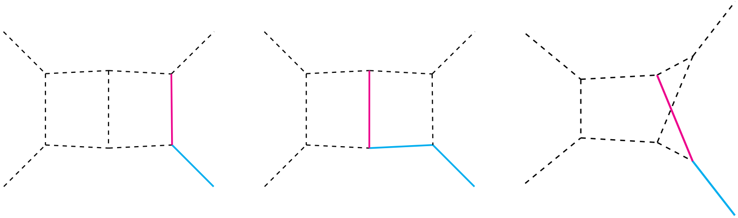
$f^{abc}(t^a t^b)_{ij}(t^c)_{kl} \to 0 \Big]$

Upon interference, the non-Abelian part of the amplitude disappears and the amplitude is, effectively, **Abelian**.

Double-virtual contribution

- Only 18 non-vanishing diagrams, but all have maximal topology and four scales (s, t, m_t, m_W).
- IBP reduction performed **analytically** with `KIRA 2.0` *Klappert, Lange, et al. 2020* and `FireFly` *Klappert and Lange 2020; Klappert, Klein, et al. 2021* in $\mathcal{O}(4)$ days:

$$\mathcal{A}_4^{(0)} \otimes \mathcal{A}_4^{(2)} = \sum_{i=1}^{428} c_i(d, s, t, m_t, m_W) I_i$$



Master integral evaluation

- Based on the **auxiliary mass flow method** *Liu, Ma, and Wang 2018; Liu, Ma, Tao, et al. 2020; Liu and Ma 2021*

$$I \propto \lim_{\eta\to 0^+} \int \prod_{i=1}^{2} \mathrm{d}^d k_i \prod_{a=1}^{9} \frac{1}{[q_a^2 - (m_a^2 - i\eta)]^{\nu_a}}$$

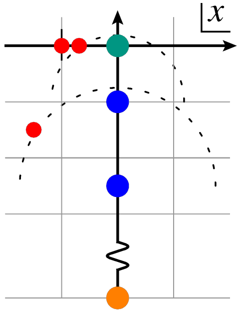
- Add an imaginary part to the W **boson mass**

$$m_W^2 \to m_W^2 - i\eta.$$

- Solve differential equations at each kinematic point

$$\partial_x \boldsymbol{I} = \boldsymbol{M}\boldsymbol{I}, \quad x \propto -i\eta.$$

with boundary condition $x \to -i\infty$.



Stepping from the boundary at $x \to -i\infty$, via regular points, to the physical mass. Step size is limited by singularities of the equation.

Master integral evaluation

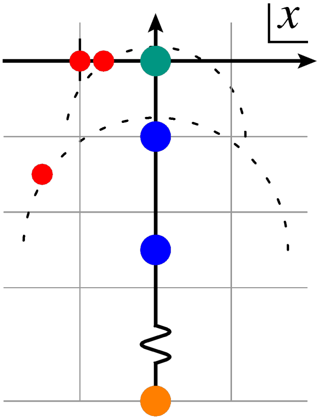
- Expand $\boldsymbol{I}$ around **boundary** in variable $y = x^{-1} = 0$:

$$\boldsymbol{I} = \sum_{j}^{M} \epsilon^{j} \sum_{k}^{N} \sum_{l} \boldsymbol{c}_{jkl} y^{k} \ln^{l} y + \dots$$

- Evaluate and expand around **regular points**:

$$\boldsymbol{I} = \sum_{j}^{M} \epsilon^{j} \sum_{k=0}^{N} \boldsymbol{c}_{jk} x'^{k} + \dots$$

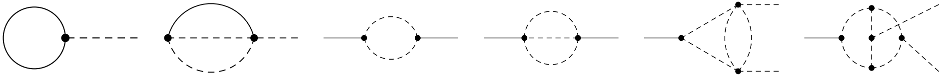
- Evaluate at the **physical point.** $x = 0 \leftarrow$ **regular point**
- **Path** is fixed by **singularities** and desired precision.
- Expected relative error is $\left(\frac{\Delta}{R}\right)^{N}$



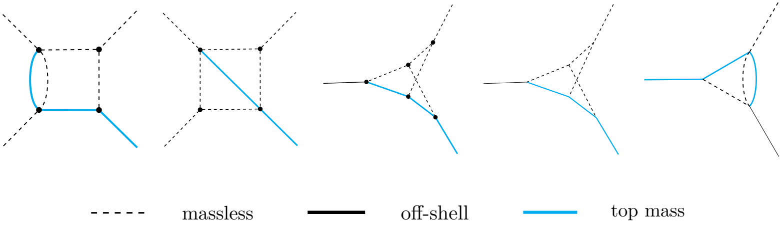
$m_W^2 \to m_W^2(1+x)$

Master integral evaluation

- Some boundary conditions are known analytically *'t Hooft and Veltman 1979; Chetyrkin et al. 1980; Scharf and Tausk 1994; Gehrmann and Remiddi 2000; Gehrmann, Huber, et al. 2005*



- Some are not available or not known to sufficient ϵ order:



- Apply auxiliary mass flow method for *internal* $m_t - i\eta \to \infty$ on boundary integrals.

Double-virtual contribution

- All **428** master integrals evaluated numerically using **the auxiliary mass flow method** to 20 digits in ~ 30 minutes on a single core.
- Comparison of poles at a typical phase space point $s \approx 104.337$ GeV^2 and $t \approx -5179.68$ GeV^2.

	ϵ^{-2}	ϵ^{-1}
$\mathcal{A}_4^{(0)} \otimes \mathcal{A}_4^{(2)}$	$-229.0940408654660 - 8.978163333241640i$	$-301.1802988944764 - 264.1773596529505i$
IR poles	$-229.0940408654665 - 8.978163333241973i$	$-301.1802988944791 - 264.1773596529535i$

- 10 sets of 10^4 points extracted from a grid prepared **on the Born squared amplitude.**
- The 10 different sets give an estimation of the error on σ_{VV}: $\mathcal{O}(2\%)$

Results

- The non-factorisable correction to the leading-order cross section at 13 TeV and $\mu_F = m_t$

$$\frac{\sigma_{pp\to X+t}}{1\,\mathrm{pb}} = 117.96 + 0.26\left(\frac{\alpha_s(\mu_R)}{0.108}\right)^2$$

- *Non-factorisable* correction is $\mathbf{0.22^{-0.04}_{+0.05}}$ % for $\boldsymbol{\mu_R = m_t}$.
- *Non-factorisable* corrections **appear for the first time** at NNLO $\rightarrow$ No indication of a good scale choice.
- At $\mu_R = 40$ GeV, typical transverse momentum of the top quark, corrections become **close to 0.35%.**
- In comparison, NNLO **factorisable correction** to NLO cross section are about **0.7%** *Campbell, Neumann, et al. 2021*

Top-quark transverse momentum distribution

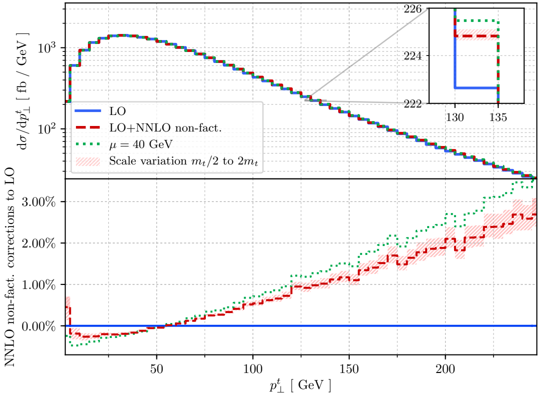


Figure: The top quark transverse momentum distribution.

- There is a **significant** $p_\perp^t$-dependence of the non-factorisable corrections.
- Non-factorisable corrections vanish around 50 GeV. The factorisable corrections vanish around 30 GeV. *Campbell, Neumann, et al. 2021*
- At low $p_\perp^t$, around the peak of the distribution, non-factorisable corrections are **dominant** compared to factorisable corrections.

Conclusion

- We computed **the missing part** of NNLO QCD corrections to the t-channel single-top production: **the non-factorisable corrections**.

- **The auxiliary mass flow method** has been used for integral evaluation. It is sufficiently **robust** to produce results relevant for phenomenology.

- Non-factorisable corrections are smaller than, but **quite comparable** to, the factorisable ones.

- If a percent precision in single-top studies can be reached, **the non-factorisable effect will have to be taken into account.**

Thank you for your attention !