

# Azimuthal spectra of the Drell-Yan processes and the top quark-antiquark pair production

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We report our recent calculations on the 1) the double differential spectra  $d^2\sigma_{DY}/(dq_T d\Delta\phi_{\ell\bar{\ell}})$  of the Drell-Yan processes at the  $N^3LL' + N^2LO$  accuracy with the inclusion of the axial-vector and vector singlet contributions [1], as well as 2) the single-differential distribution  $d\sigma_{t\bar{t}}/d\Delta\phi_{t\bar{t}}$  of the  $t\bar{t}$  production up to the approximate  $N^2LL' + N^2LO$  [2].

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# 1 Introduction

In this proceeding, we will discuss the recent resummation improved results of the azimuthal opening angles between the final resolved particles, i.e.,  $\Delta\phi_{\ell\bar{\ell}}$  in the Drell-Yan (DY) processes [1] and  $\Delta\phi_{t\bar{t}}$  for the top anti-top quark pair production [2]. Akin to the transverse momenta, the azimuthal spectra are also strongly affected by soft and collinear radiation in the regime  $\Delta\phi_{\ell\bar{\ell}} \rightarrow \pi$  or  $\Delta\phi_{t\bar{t}} \rightarrow \pi$ , where the fixed-order calculations rarely converge at finite order, thereby necessitating the resummation to reduce the theoretical uncertainties.

The resummation on the DY processes usually starts with the Fourier transformation with respect to  $\vec{q}_T$  [3–11]. Owing to the absence of the azimuthal asymmetric terms (i.e.,  $c(\phi_b)$ ) [12, 13], the logarithmic structures in the impact space,  $L_T$ , are capable of capturing the complete asymptotic behaviour of the momentum space (MS). This permits us to address the resummed spectrum  $d\sigma_{\text{DY}}/d\vec{q}_T$  and in turn the double differential distribution  $d^2\sigma_{\text{DY}}/(dq_T d\Delta\phi_{\ell\bar{\ell}})$ .

However, the presence of the azimuthal asymmetric terms, which are induced by the collinear-gluon polarization and the soft-gluon correlation and can also generate asymptotic terms in MS, makes it subtle to apply the same tactics onto the  $t\bar{t}$  production, e.g.,  $d\sigma_{t\bar{t}}/d\vec{q}_T$ . The investigations on how to consistently handle both the terms  $c(\phi_b)$  and  $L_T$  have been delivered in Refs. [12, 13]. Conversely, it is also observed in the previous literatures that there exists observable free of  $c(\phi_b)$  for all the perturbative orders, such as  $d\sigma_{\text{DY}}/dq_T$  [14–16]. In this work, we will follow the latter strategy and propose another observable independent of  $c(\phi_b)$ , that is,  $d\sigma_{t\bar{t}}/d\Delta\phi_{t\bar{t}}$ .

## 2 Double differential spectrum $d^2\sigma_{\text{DY}}/(dq_T d\Delta\phi_{\ell\bar{\ell}})$ of the DY processes

Within the context of SCET<sub>II</sub> [17–19], the master formula governing the factorisation and resummation in the DY processes reads [8–11],

$$\frac{d^4\sigma_{\text{DY}}}{d^2\vec{q}_T dY_L dM_L^2 d\Omega_L} \sim \sum_{i,j} \int d^2\vec{b}_T e^{i\vec{b}_T \cdot \vec{q}_T} \Sigma_{\text{DY}}^{ij}(M_L, Y_L, \Omega_L, b_T) \quad (1)$$

where  $Y_L$  and  $M_L$  denote the rapidity and the invariant mass of the final state lepton pair, respectively.  $\Omega_L$  represents the solid angle of one of the final leptons in the lepton-pair rest system.  $\Sigma_{\text{DY}}^{ij}$  encodes the contributions from the partonic processes, admitting the factorization pattern

$$\mathcal{H} \otimes \mathcal{S} \otimes \mathcal{B}_n \otimes \mathcal{B}_{\bar{n}},$$

as a result of the decoupling properties of SCET<sub>II</sub>. During the calculation, we have taken into account both the axial-vector and vector singlet contributions [20–26]. To

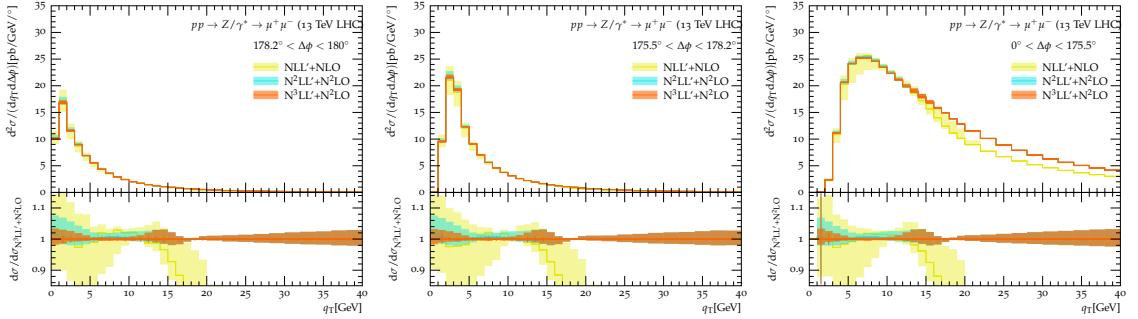


Figure 1: Double-differential cross sections in  $q_T$  and three slices of  $\Delta\phi_{\ell\bar{\ell}}$  [1].

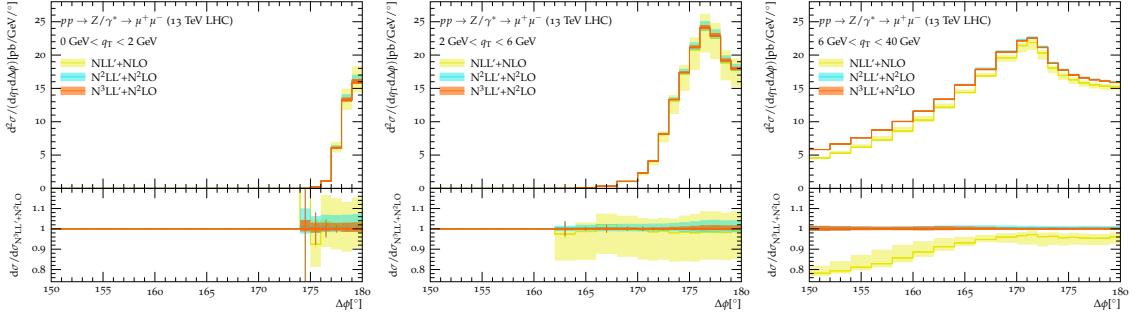


Figure 2: Double-differential cross sections in  $\Delta\phi_{\ell\bar{\ell}}$  and three slices of  $q_T$  [1].

bridge the intrinsic scales amongst those sectors and thereby achieve the resummation, we solve the (rapidity) renormalisation equation [8, 9] of each ingredient under the exponential rapidity regularization prescription [10, 11], and then plug the solutions back into the factorization formula. To extrapolate to the whole phase space, we match the resummed results onto the fixed-order results, that are obtained from SHERPA [27, 28] in combination with OPENLOOPS [29, 30], with the aid of the transition functions. Finally, the spectra  $d^2\sigma_{DY}/(dq_T d\Delta\phi_{\ell\bar{\ell}})$  are calculated by multiplying eq. (1) by the definition function of  $\Delta\phi_{\ell\bar{\ell}}$  and completing the integral over the phase space afterwards.

The numeric outcomes are presented in Figs. 1 and 2. In the low  $q_T$  domain, where the predictions are dominated by the resummed results, it is observed that the central values are nearly independent of the order they are calculated at, and the theoretical uncertainty progressively decreases when going to higher orders. Nevertheless, departing from the Sudakov peak regions, the transition function comes into play for matching the resummation to the fixed-order expressions, such that starting around the matching scale of  $\mu_Q = 16$  GeV the central values start to diverge onwards.

### 3 Azimuthal spectrum $d\sigma_{t\bar{t}}/d\Delta\phi_{t\bar{t}}$ in the $t\bar{t}$ production

We make use of SCET<sub>II</sub> [17–19] and HQET [31–33] to derive the master formula for  $d\sigma_{t\bar{t}}/d\Delta\phi_{t\bar{t}}$ ,

$$\frac{d\sigma_{t\bar{t}}}{dM_{t\bar{t}}^2 d^2 \vec{P}_t^\perp dY_{t\bar{t}} d\delta\phi_{t\bar{t}}} \sim \sum_\kappa \int_{-\infty}^\infty db_\perp \cos(b_\perp |\vec{P}_t^\perp| \delta\phi_{t\bar{t}}) \tilde{\Sigma}_{t\bar{t}}^{[\kappa]}(\vec{b}_\perp, m_t, \beta_{t\bar{t}}, x_t, Y_{t\bar{t}}), \quad (2)$$

where the index  $\kappa$  runs over  $\{g_n g_{\bar{n}}, q_n^i \bar{q}_{\bar{n}}^j, q_{\bar{n}}^i \bar{q}_n^j\}$  indicating the respective incoming partons.  $Y_{t\bar{t}}$  and  $M_{t\bar{t}}$  characterize the rapidity and the invariant mass of the  $t\bar{t}$  pair, respectively.  $\delta\phi_{t\bar{t}}$  is defined as

$$\delta\phi_{t\bar{t}} \equiv \pi - \Delta\phi_{t\bar{t}}.$$

$\vec{P}_t^\perp$  signifies the transverse momentum of the top quark. To circumvent the Coulomb singularities and the ensuing factorization breaking effects [34–37], we impose the constrain  $M_{t\bar{t}} \geq 400$  GeV, or equivalently,  $\beta_{t\bar{t}} \gtrsim 0.5$  in deriving eq. (2) and the following numeric implementation. Confronting with eq. (1), the radial component, i.e.,  $|\vec{q}_T|$ , has been integrated out in eq. (2) so as to curb the contributions from the azimuthal asymmetric terms. Even though there still exist the residual asymmetric dependences in  $\tilde{\Sigma}_{t\bar{t}}^{[\kappa]}$ , e.g.,  $c(\text{sign}[b_\perp])$ , the reduction of the Fourier bases

$$\exp(i\vec{b} \cdot \vec{q}_T) \rightarrow \cos(b_\perp |\vec{P}_t^\perp| \delta\phi_{t\bar{t}}) \quad (3)$$

renders themselves becoming relevant only at the point  $\delta\phi_{t\bar{t}} = 0$  and therefore no longer sabotaging the  $L_T$  exponentiation.

The numerical outputs are exhibited in Fig. (3) together with the uncertainties estimated by varying the intrinsic scales and focal points of the transition function. As before, the manifest convergent behavior in the low  $\delta\phi_{t\bar{t}}$  region has been observed with the increase in accuracy. However, as the value  $\delta\phi_{t\bar{t}}$  grows, the fixed-order results gradually take over the contributions and the enlarging theoretical uncertainties are thus found afterwards, in particular around the extremity  $\delta\phi_{t\bar{t}} \sim 0.5$ . In comparison with those in Figs. 2, it merits noting that for lack of the fiducial selections and the  $q_T$  constraints, there is no peak formed in Fig. (3).

### 4 Conclusions

To summarize our main observations:

- (1) **DY process:** With the inclusion of the vector and axial-vector singlet contributions, the double differential spectra  $d^2\sigma_{\text{DY}}/(dq_T d\Delta\phi_{\ell\bar{\ell}})$  have been calculated in the context

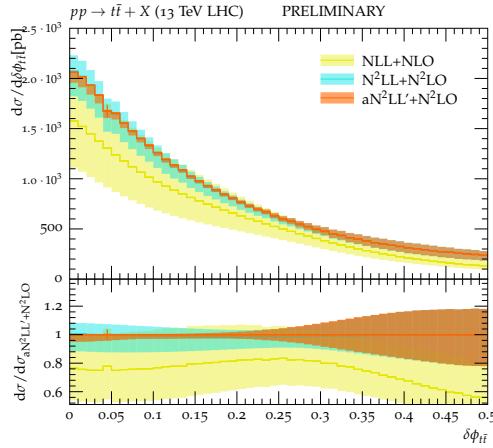


Figure 3: The azimuthal separations in the process  $pp \rightarrow t\bar{t}X$  [2].

of SCET<sub>II</sub> up to the  $N^3LL' + N^2LO$  accuracy. Obvious reductions in the theoretical uncertainties have been unveiled in the asymptotic domain. Those error-controlled outcomes are promising to facilitate the modelling on the DY processes and the measurements on the  $M_W$ .

**(2)  $t\bar{t}$  production:** We demonstrate that, akin to the azimuthally averaged spectra  $d\sigma_{t\bar{t}}$ , the observable  $d\sigma_{t\bar{t}}/d\Delta\phi_{t\bar{t}}$  is an alternative choice to combat the azimuthal asymmetric divergences. Equipped with SCET<sub>II</sub> and HQET, we derive the master formulae in the factorization and resummation procedures, by going through the possible configurations contributing to the regime  $\Delta\phi_{t\bar{t}} \rightarrow \pi$  or  $\delta\phi_{t\bar{t}} \rightarrow 0$ . During the numeric evaluations, we provide the results up to  $N^2LL + N^2LO$  and the approximate  $N^2LL' + N^2LO$ , where the manifest convergent performance has also been found in the low  $\delta\phi_{t\bar{t}}$  region.

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