

# Phenomenological analysis of the scalar PDF

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**“FORDECYT-PRONACES”**



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# Higher-twist parton distribution functions

## Gluon at low energy, “the glue that binds us all”?

- What are higher-twist distribution functions?
- What information do they encapsulate?
- From low-energy experiments to higher  $Q^2$ .

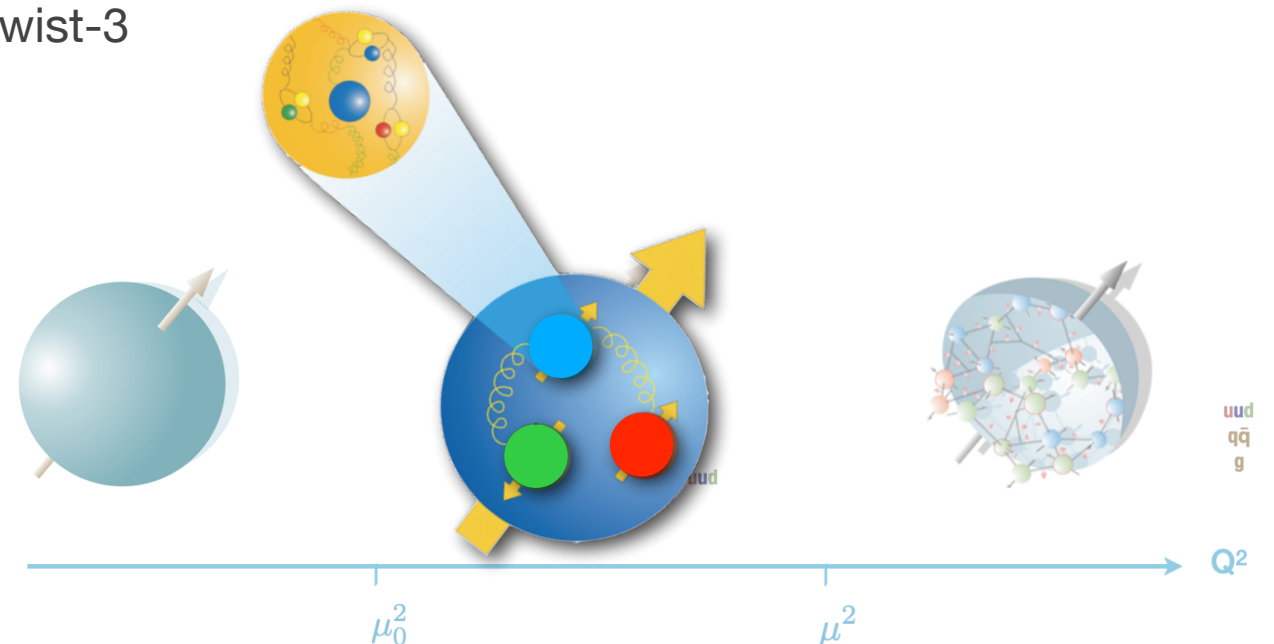
One possible definition for higher-twist contributions: terms effectively suppressed like  $(M/Q)^{t-2}$

Fixed Target **DIS** & **SIDIS**:  $M/Q$  is not so small

- either spurious contaminations
- or spin asymmetries can be defined to be sensitive to twist-3



This talk: extraction of the twist-3 PDF  $e(x)$   
[2203.14975]

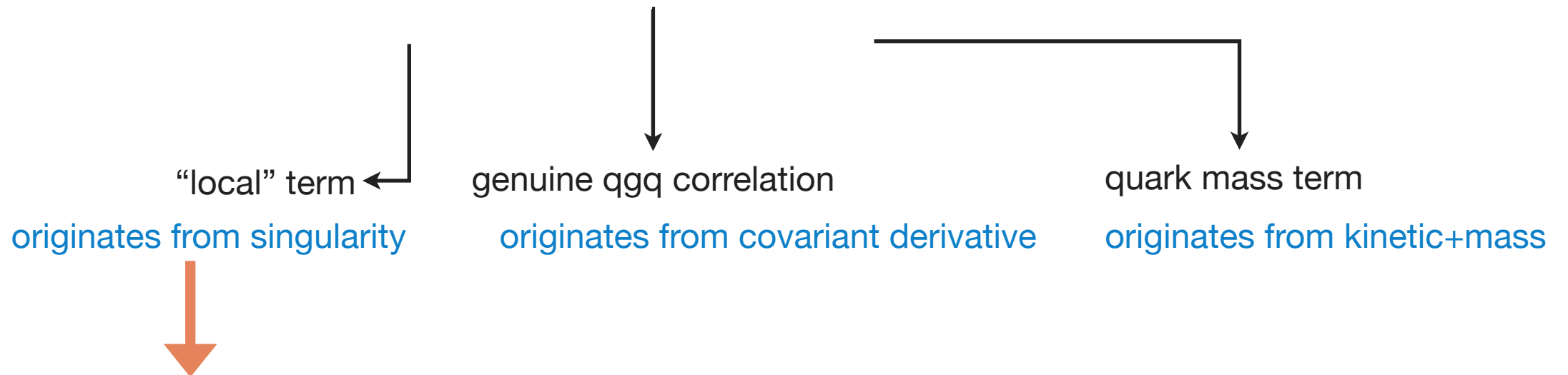




# Scalar PDF

The composition of the scalar PDF is worked out through the EoM of QCD:

$$e^q(x) = e_{\text{loc}}^q(x) + e_{\text{gen}}^q(x) + e_{\text{mass}}^q(x)$$



$$e_{\text{loc}}^q(x) = \frac{1}{2M} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | \bar{\psi}_q(0) \psi_q(0) | P \rangle = \frac{\delta(x)}{2M} \langle P | \bar{\psi}_q(0) \psi_q(0) | P \rangle$$

Only observable-related contribution to the proton mass:

the singularity of  $e(x)$  is proportional to the pion-nucleon sigma

term through sum rules [e.g. Kodaira & Tanaka, PTP, Vol. 101].

[Schweitzer and Efremov, JHEP08006]

[Burkardt & Koike, NPB632]

[Ji, NPB960]

[Lorcé, Pasquini, Schweitzer, JHEP01 (2015)]

[Pasquini & Rodini, PLB788]

[Hatta & Zhao, PRD102]

[Bhattacharya et al., PRD102]

# Scalar PDF and the proton mass

## QCD mass decomposition

$$M_P = M_m + M_a + M_q + M_g$$

quark mass  $M_m = \sum_q \sigma_q$       trace anomaly      quark and gluon energy  $\propto \langle x \rangle_{q,g}$

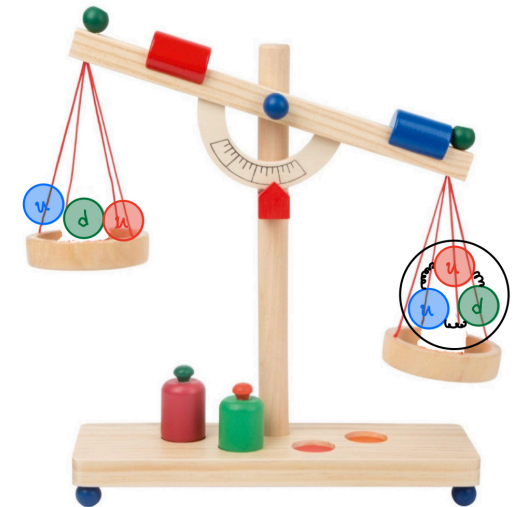
[Ji, PRL 74; Ji, PRD 52]  
[Lorcé, EPJC78; Lorcé et al, 2109.11785]

see talk by Andreas Metz (WG5)

## Sigma terms

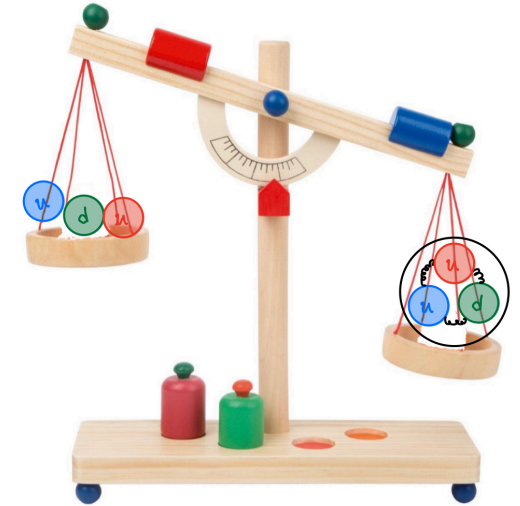
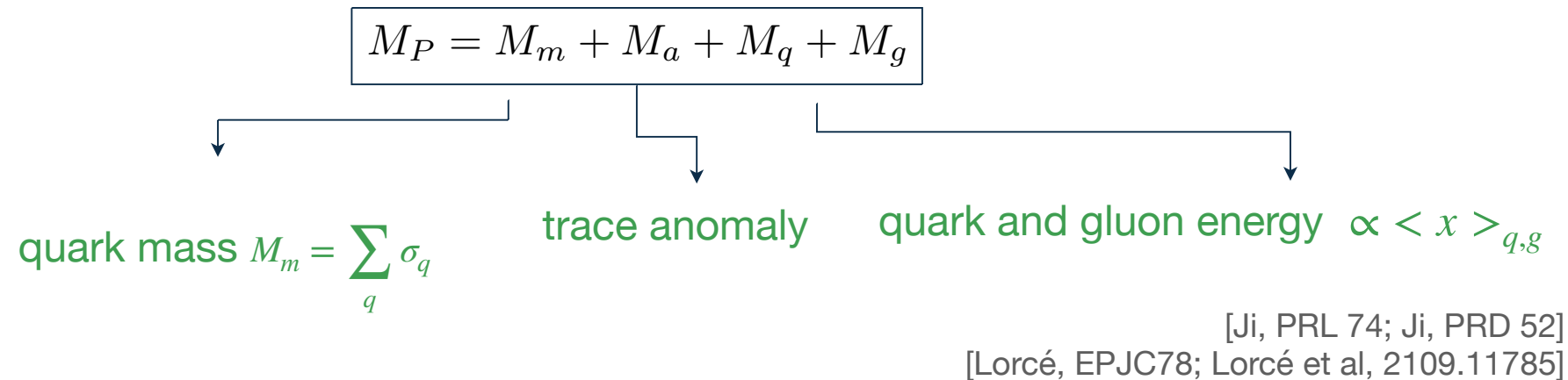
$$\langle P | m_u \bar{u}u + m_d \bar{d}d | P \rangle = \sigma_{\pi N}$$

- have been determined from theoretical analysis of  $\pi N$  data [Meissner et al.]
- have been evaluated on the lattice [Constantinou et al.]
- pheno analysis of  $e(x)$  could pave the way towards another possible determination



# Scalar PDF and the proton mass

## QCD mass decomposition



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## Sigma terms

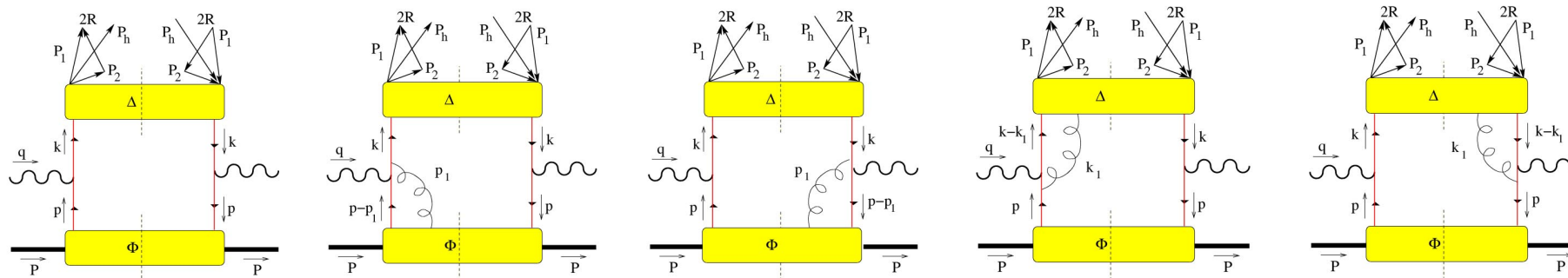
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- can be accessed through single-hadron SIDIS [Efremov et al, PRD67]
- can be accessed through dihadron SIDIS: **this talk**

# Twist-3 in SIDIS dihadron production

[Bacchetta & Radici, PRD69]



● collinear framework — led to collinear transversity extraction [Radici, Jakob & Bianconi, PRD65].

● modulations of spin asymmetries single out:

$$\frac{[\text{twist-3 PDF} \times \text{twist-2 FF}] + [\text{twist-2 PDF} \times \text{twist-3 FF}]}{\text{unpolarized}}$$

● Scalar PDF from the beam spin asymmetry

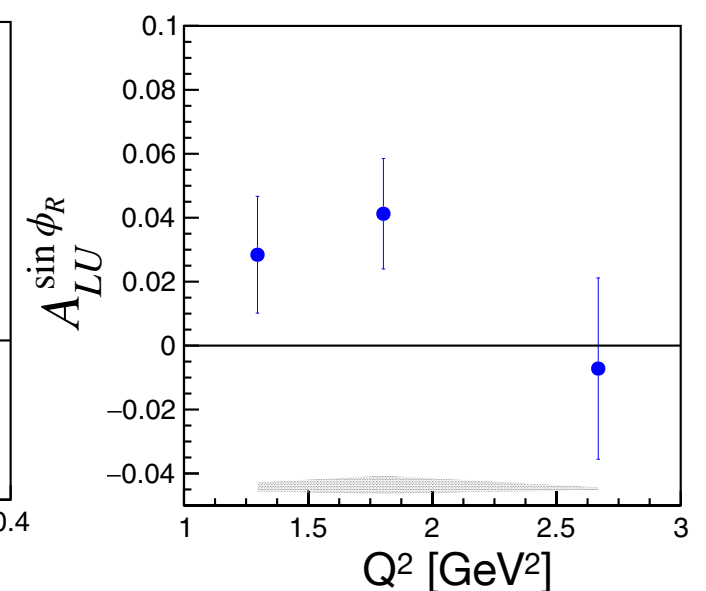
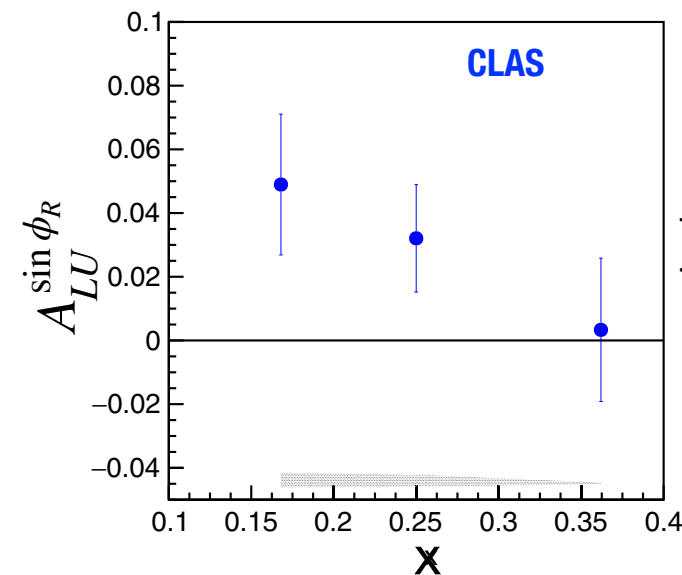
$$A_{LU}^{\sin \phi_R}(x, z, m_{\pi\pi}) \propto \frac{M}{Q} \frac{\sum_q e_q^2 \left[ x e^q(x) H_{1,sp}^{\triangleleft,q}(z, m_{\pi\pi}) + \frac{m_{\pi\pi}}{zM} f_1^q(x) \tilde{G}_{sp}^{\triangleleft,q}(z, m_{\pi\pi}) \right]}{\sum_q e_q^2 f_1^q(x) D_{1,ss+pp}^q(z, m_{\pi\pi})}$$

# Beam spin asymmetry at CLAS and CLAS12

- dihadron SIDIS on proton target is

sensitive to  $e^P \equiv \frac{1}{9}(4 e^{u_v} - e^{d_v});$

[Bacchetta & Radici, *PRD*69 (2004)]  
[Courtoy, 1405.7659]



[CLAS Collaboration, *PRL*126 (2021) 6, 062002]

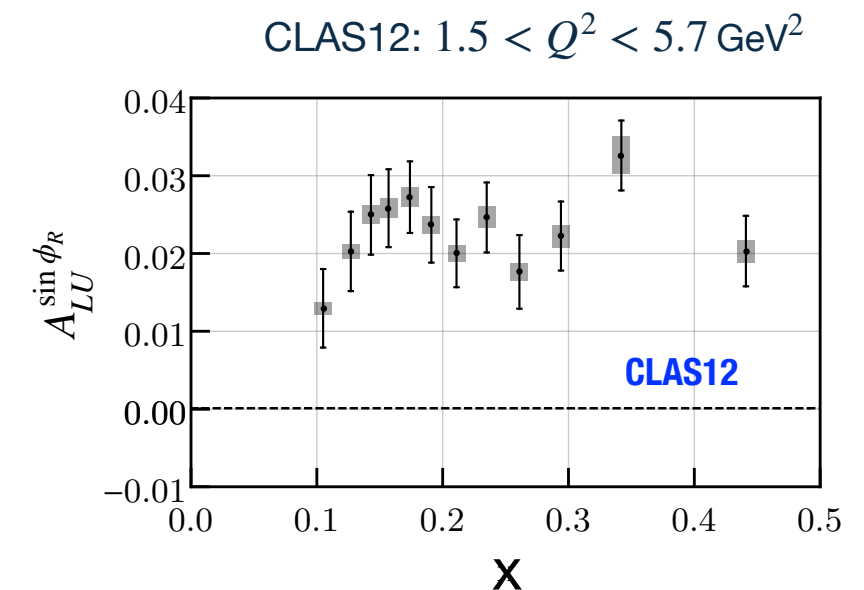
- non-vanishing twist-3 effects at CLAS12;

- projections of beam spin asymmetries on  $(x, z, M_h; Q^2, y)$

$(x, z, M_h)$ -trptychs from the parton distribution and fragmentation function.



Road map for  $e(x)$  extraction and (global) analysis.



[CLAS Collaboration, *PRL*126 (2021) 152501]

# Extraction of $e(x)$ from CLAS data

$$A_{LU}^{\sin \phi_R}(x, z, m_{\pi\pi}) \propto \frac{M}{Q} \frac{\sum_q e_q^2 \left[ x e^q(x) H_{1,sp}^{\triangleleft,q}(z, m_{\pi\pi}) + \frac{m_{\pi\pi}}{zM} f_1^q(x) \tilde{G}_{sp}^{\triangleleft,q}(z, m_{\pi\pi}) \right]}{\sum_q e_q^2 f_1^q(x) D_{1,ss+pp}^q(z, m_{\pi\pi})}$$

↓  
leading-twist DiFFs

## Twist-2 Dihadron Fragmentation Functions

- Phenomenologically tested for the twist-2 transversity PDF [Bacchetta, Courtoy & Radici, PRL107 and follow-ups]
- extracted in  $e^+e^-$  at Belle here: [Radici, Courtoy, Bacchetta, JHEP 05 (2015)]
- we get the ratio  $R$  that is believed to be universal (portable) *up to evolution effects*

$$R(z, M_h) = \frac{|\mathbf{R}|}{M_h} \frac{H_1^{\triangleleft u}(z, M_h; Q_0^2)}{D_1^u(z, M_h; Q_0^2)}$$

→ chiral-odd DiFF

→ unpolarized DiFF

# Extraction of $e(x)$ from CLAS data

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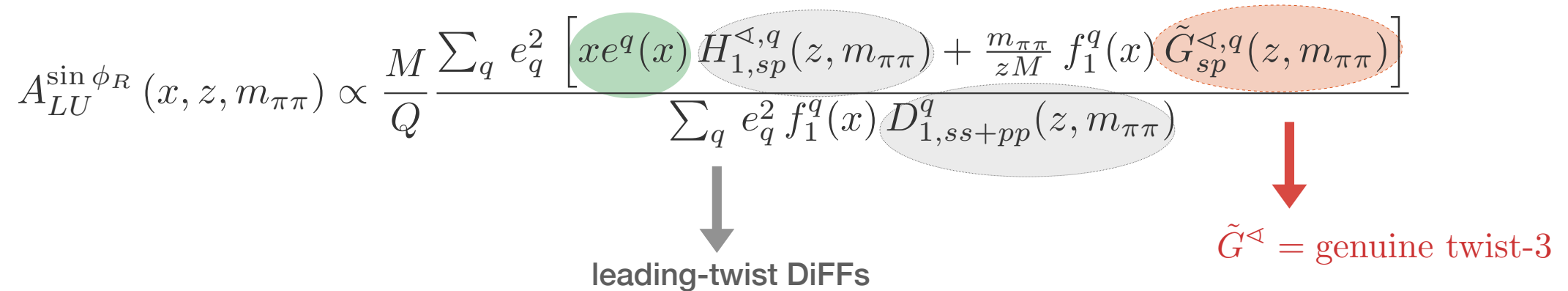
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How to treat twist-3  
Dihadron Fragmentation  
Functions?

# Extraction of $e(x)$ from CLAS data

$$A_{LU}^{\sin \phi_R}(x, z, m_{\pi\pi}) \propto \frac{M}{Q} \frac{\sum_q e_q^2 \left[ x e^q(x) H_{1,sp}^{\angle,q}(z, m_{\pi\pi}) + \frac{m_{\pi\pi}}{zM} f_1^q(x) \tilde{G}_{sp}^{\angle,q}(z, m_{\pi\pi}) \right]}{\sum_q e_q^2 f_1^q(x) D_{1,ss+pp}^q(z, m_{\pi\pi})}$$



leading-twist DiFFs

$\tilde{G}^{\angle} = \text{genuine twist-3}$

## Twist-3 Dihadron Fragmentation Functions

- Unknown phenomenologically;
- Model evaluations for genuine twist-3 DiFF:  $\tilde{D}^{\angle}$  [Luo et al., PRD100],  $\tilde{G}^{\angle}$  [Yang et al., PRD99]
- Estimate of Interference FF through the asymmetries on longitudinally-polarized target at COMPASS [Sirtl, PhD thesis, 2017]

$$A_{UL}^{\sin(\phi_R)} = -\frac{M}{Q} \frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 \left[ x h_L^q(x) H_1^{\angle,q,sp}(z, M_h^2) + \frac{M_h}{Mz} g_1^q(x) \tilde{G}^{\angle,q,sp}(z, M_h^2) \right]}{\sum_q e_q^2 f_1^q(x) D_1^{q,ss+pp}(z, M_h^2)}$$

$$A_{LL}^{\cos(\phi_R)} = \frac{M}{Q} \frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 \left[ x e_L^q(x) H_1^{\angle,q,sp}(z, M_h^2) - \frac{M_h}{Mz} g_1^q(x) \tilde{D}^{\angle,q,sp}(z, M_h^2) \right]}{\sum_q e_q^2 f_1^q(x) D_1^{q,ss+pp}(z, M_h^2)}.$$

$$A_{UL}^{\sin(\phi_R)} = 0.0050 \pm 0.0010(\text{stat}) \pm 0.0007(\text{sys}).$$

$$A_{LL}^{\cos(\phi_R)} = -0.0135 \pm 0.0064(\text{stat}) \pm 0.0046(\text{sys}).$$

$$\Rightarrow |A_{LL}^{\cos \phi_R}| > > A_{UL}^{\sin \phi_R}$$



# Twist-3 dihadron fragmentation functions: COMPASS

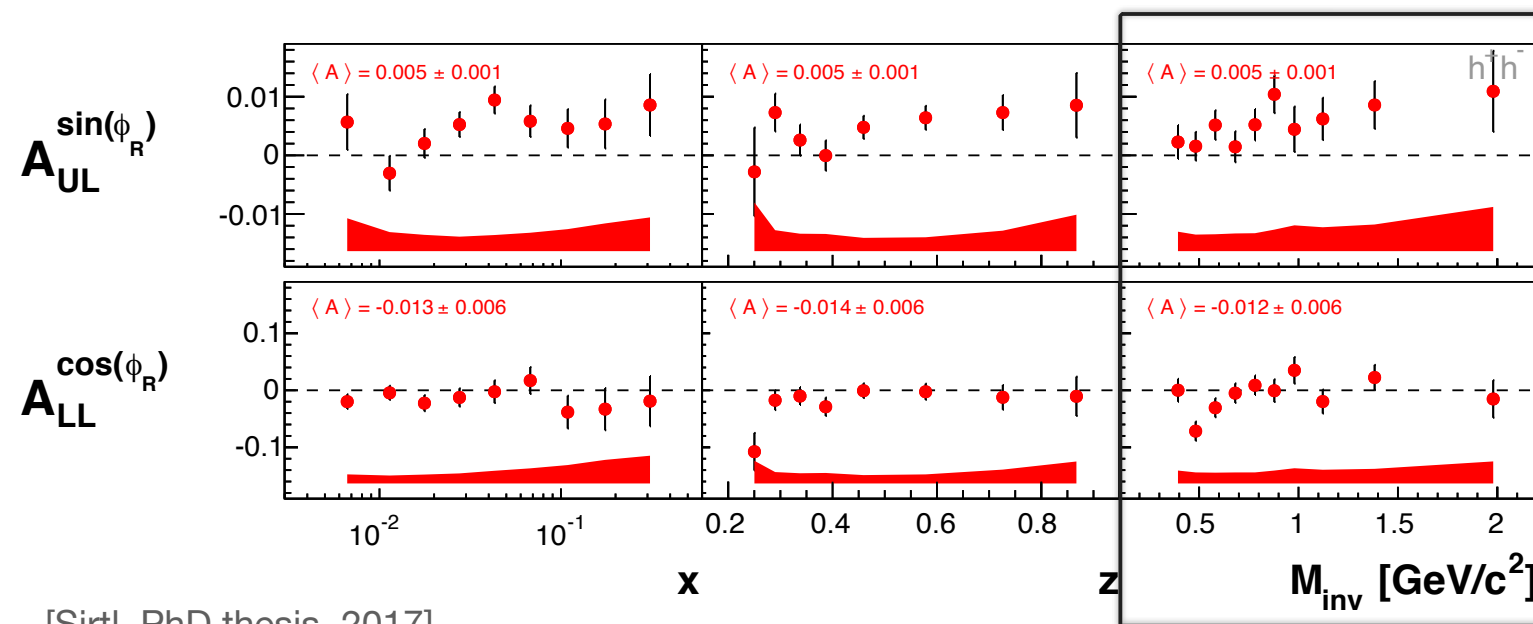
Can we understand  $|A_{LL}^{\cos \phi_R}| \gg A_{UL}^{\sin \phi_R}$  in the usual approximations?

Can we single out twist-3 DiFFs through this approximation?

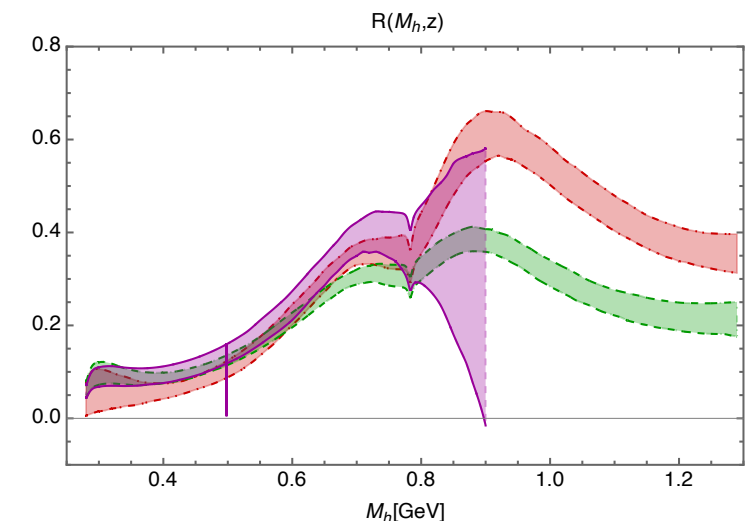
$$A_{LL}^{\cos \phi_R} \stackrel{?}{=} -\frac{M}{Q} \frac{|\mathbf{R}|}{M} \frac{\sum_q e_q^2 g_1(x)/z \tilde{D}^{\triangleleft}}{\sum_q e_q^2 f_1(x) D_1}$$

Our goal here is to find a proportionality factor  $\kappa$  such that

$$\max \left\{ \int \tilde{G}^{\triangleleft} \right\} \equiv \int \tilde{D}^{\triangleleft} = \kappa \int \tilde{H}_1^{\triangleleft}$$



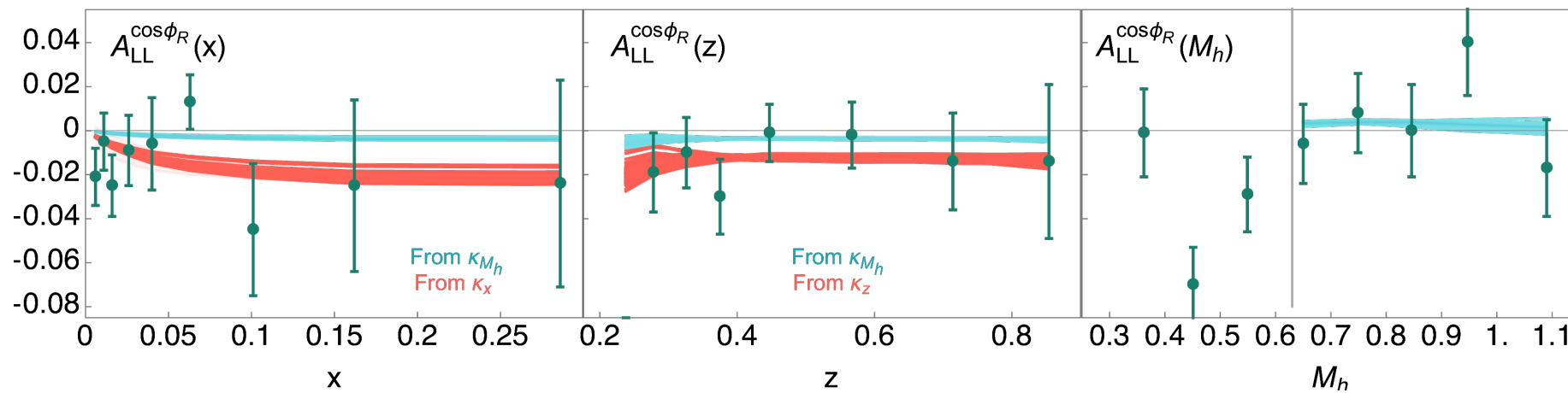
[Sirtl, PhD thesis, 2017]



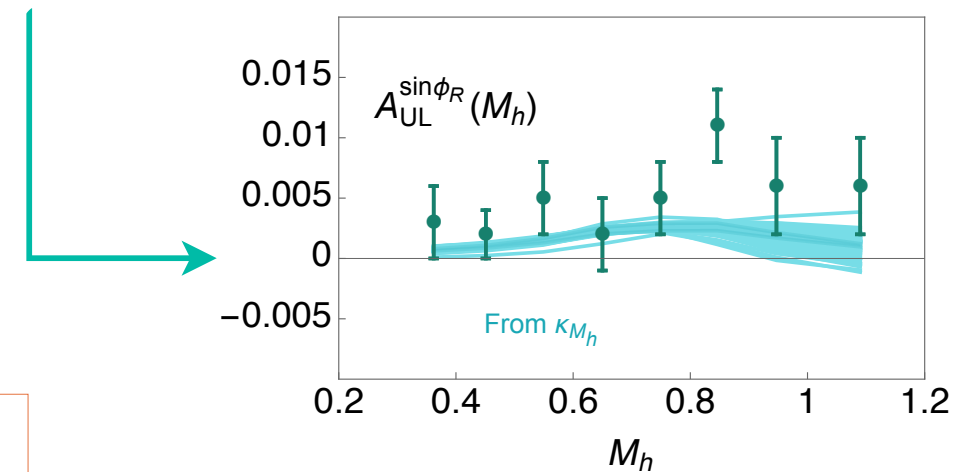
[Radici, AC, Bacchetta, *JHEP* 05 (2015)]

# Our ansatz for the twist-3 DiFF contribution

- CLAS12: split invariant-mass regions  $M_h >$  or  $< 0.63$  GeV to pinpoint vector meson contributions
- We assume the trend of all interference DiFFs in the invariant mass is similar for  $M_h > 0.63$  GeV (up to overall sign)  
 $\Rightarrow$  supported by model evaluation of  $\tilde{D}^{\triangleleft}$  and  $\tilde{G}^{\triangleleft}$
- Reproducing  $A_{LL}^{\cos\phi_R}$  in that range sets our upper bound to  $\kappa \Rightarrow \kappa_{M_h}$



- $\kappa_{M_h}$  reproduces the order of magnitude for  $A_{UL}^{\sin\phi_R}$  adequately



Invariant-mass behavior is key, twist-2 DiFFs alone not enough to interpret all  $M_h$ -projected twist-3 asymmetries.

# Point-by-point $e(x)$ from CLAS data

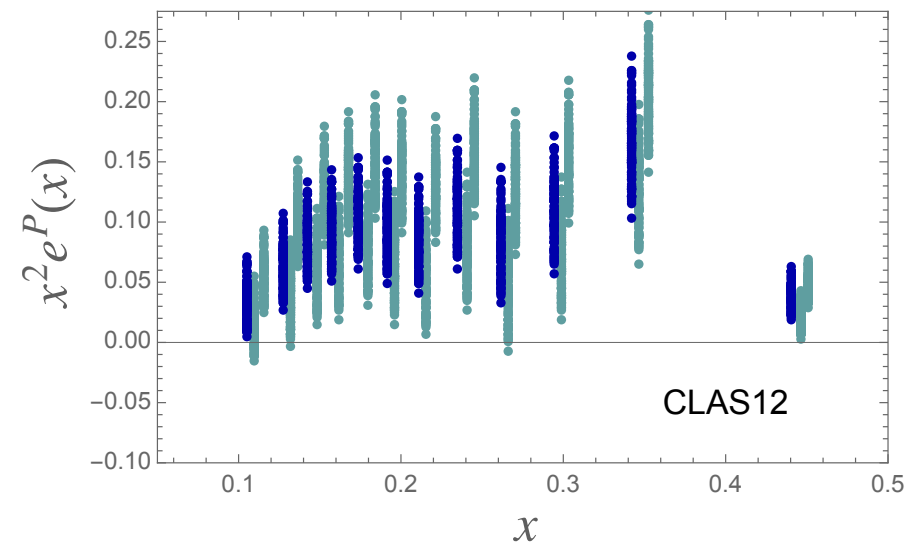
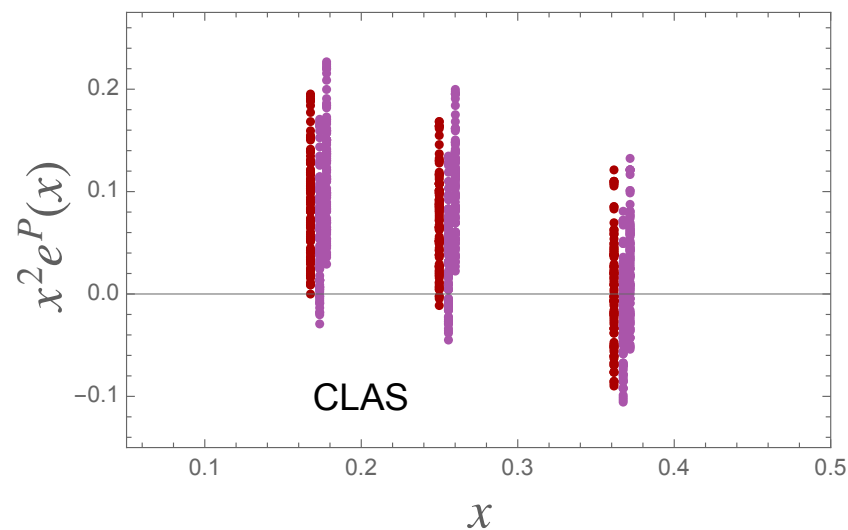
[Courtoy, 1405.7659]  
[Courtoy, Miramontes, Avakian, Mirazita, Pisano, 2203.14975]

● Scenario I: Wandzura-Wilczek approximation

$$\frac{e^V(x)}{f_1^\Sigma(x)} \frac{\tilde{H}_1^{\triangleleft}}{D_1} \propto \frac{Q}{M} A_{LU}^{\sin \phi_R}$$

● Scenario II: beyond WW approximation

$$\frac{e^V(x)}{f_1^\Sigma(x)} \frac{\tilde{H}_1^{\triangleleft}}{D_1} \propto \frac{Q}{M} A_{LU}^{\sin \phi_R} \pm \kappa \frac{f_1^V(x)}{f_1^\Sigma(x)} \frac{\tilde{H}_1^{\triangleleft}}{D_1}$$



Evolution omitted thanks to low- $Q^2$  values —  $Q=1\text{GeV}$

Uncertainty on unpolarized PDF taken into account

Sign of twist-3 DiFFs undetermined

# Point-by-point $e(x)$ from CLAS data

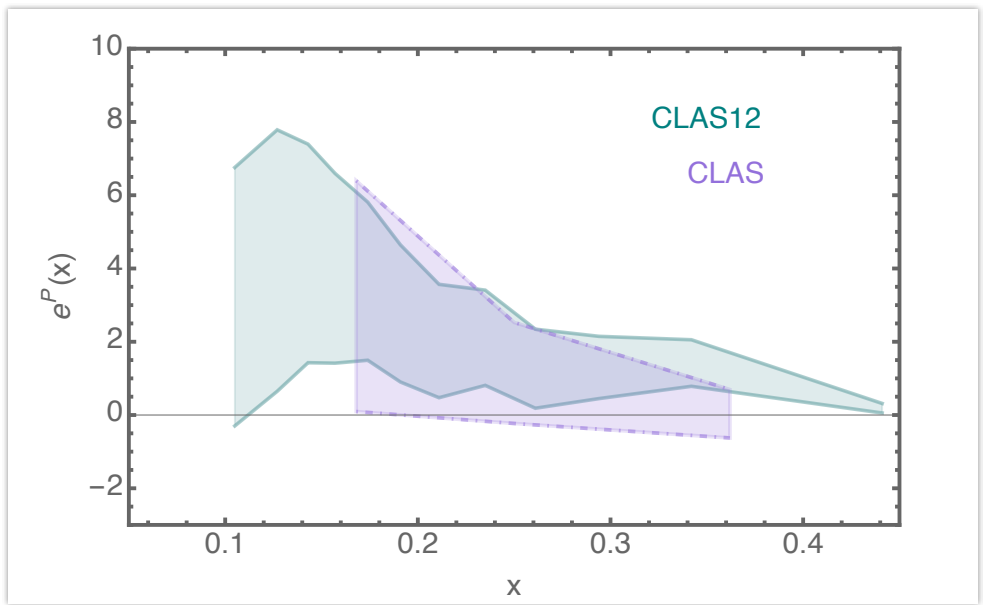
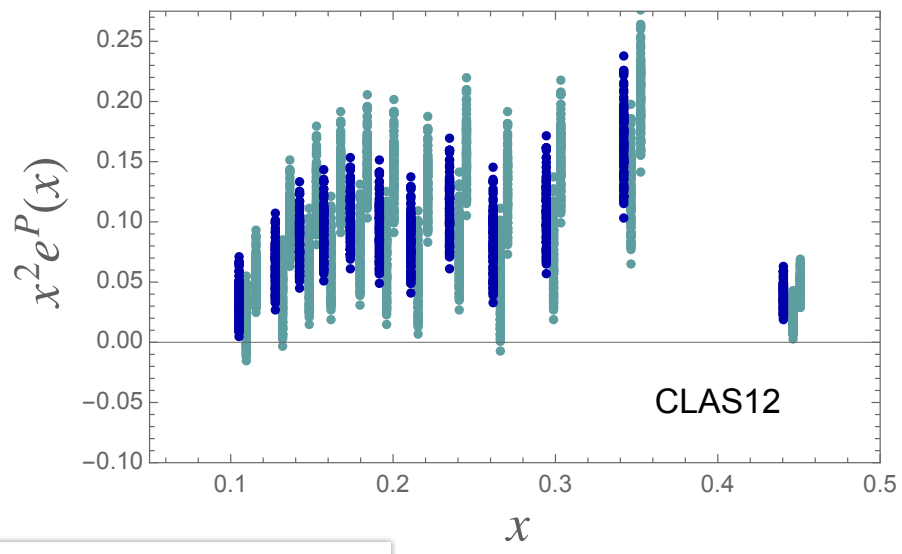
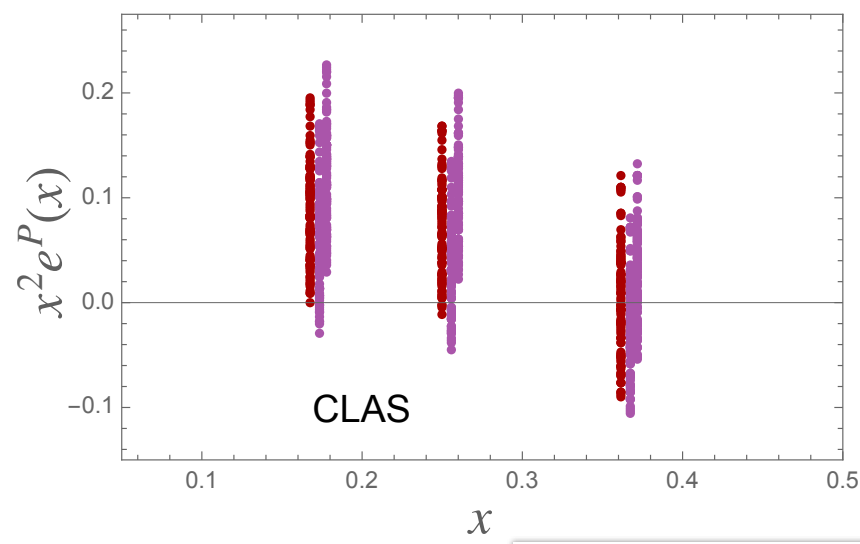
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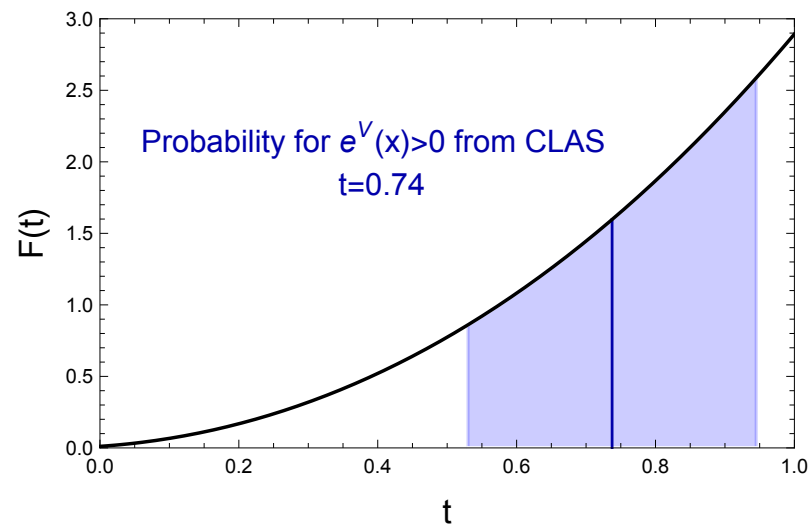
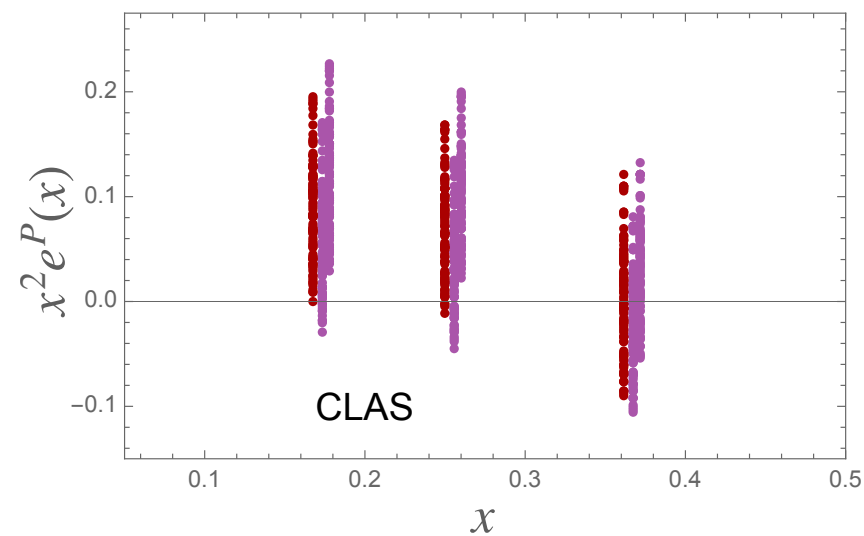
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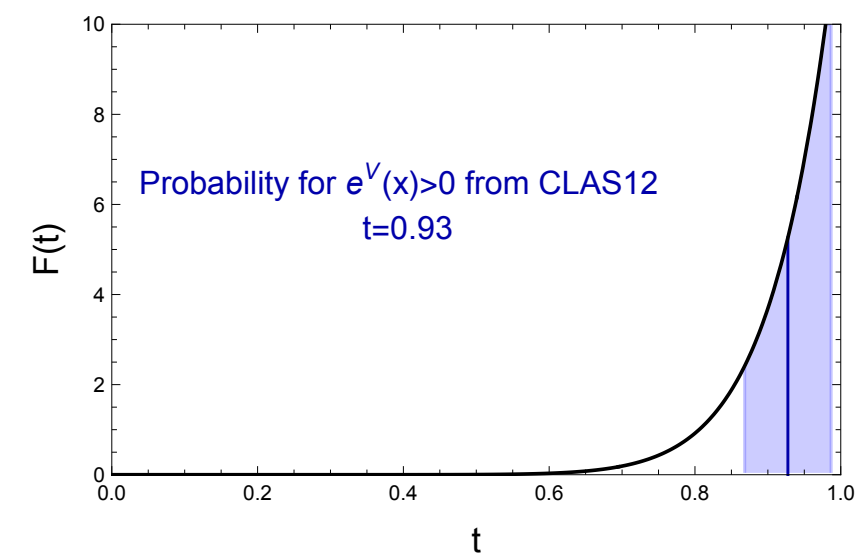
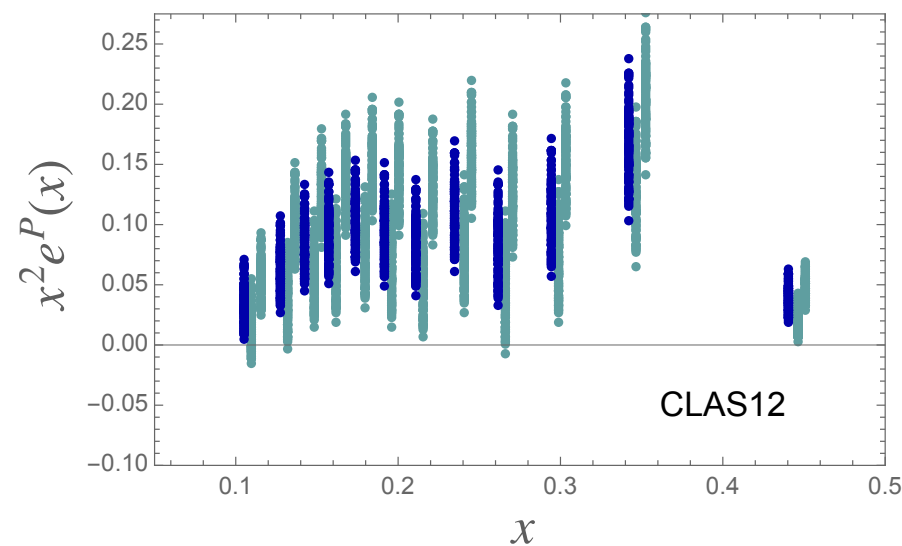
Combined uncertainty at 90% CL

# What is the probability for $e^P(x)$ to be non-zero?

Probability that the proton combination is greater than zero —not exactly “how incompatible with zero is it?” — is a useful information from the point-by-point extraction of a collinear twist-3 PDF with a minimum set of approximations.



Mostly far from zero!



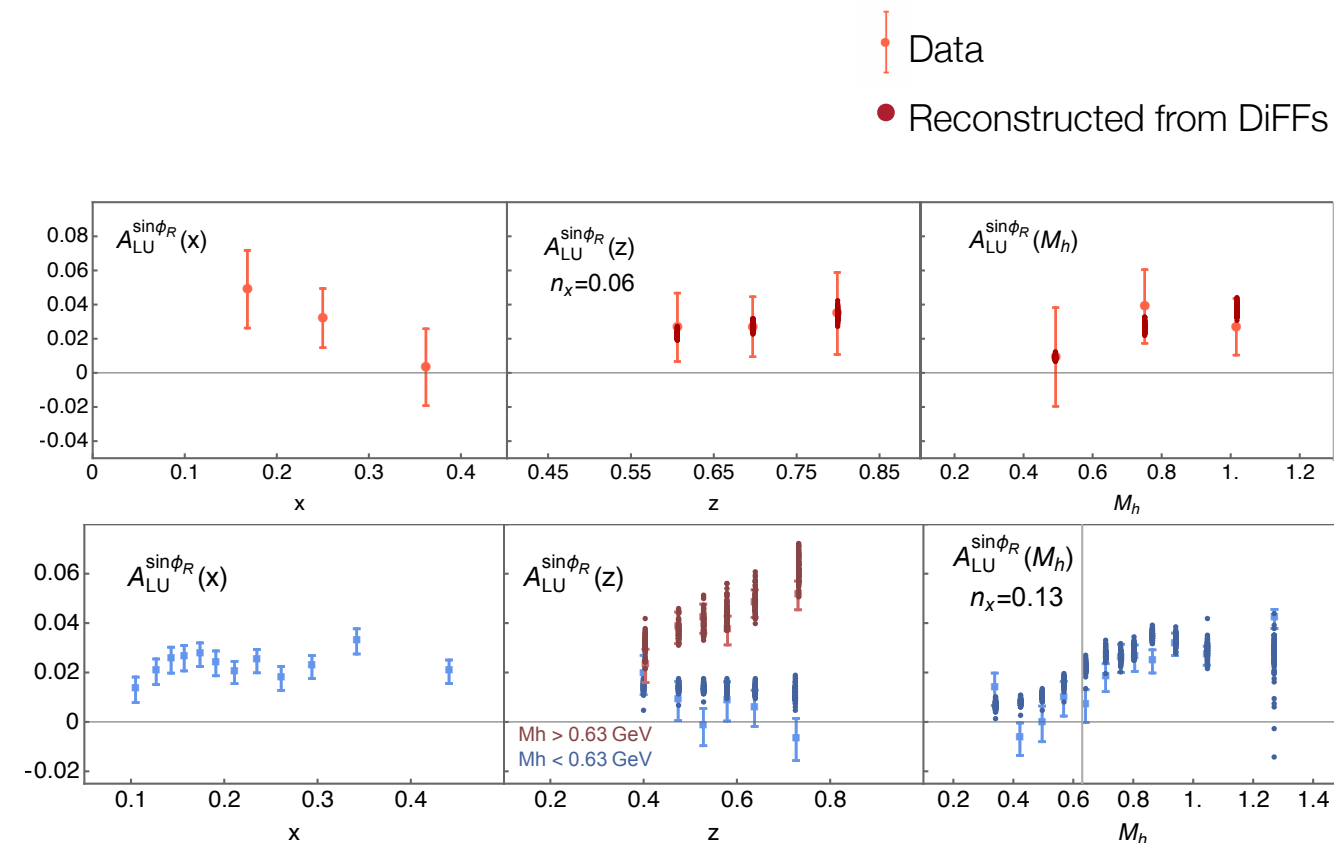
# Universality of non-perturbative functions

## ● Dihadron fragmentation functions

DiFF extracted in  $e^+e^-$ , to be tested against SIDIS multiplicities

Consistency check on SIDIS  $(z, M_h)$  dependence at CLAS & CLAS12

Determination of the integral of  $e^P(x)$  from reconstruction:  $n_x$



## ● Twist-2 and -3 PDFs

- ➔ Universality of transversity in pp and SIDIS [Radici et al, PRD94]
- ➔ Global analysis of the transversity possible [Radici & Bacchetta, PRL120; JAM Coll., PRD102]
- ➔ **Are twist-3 PDFs universal?**

Yet to be answered.

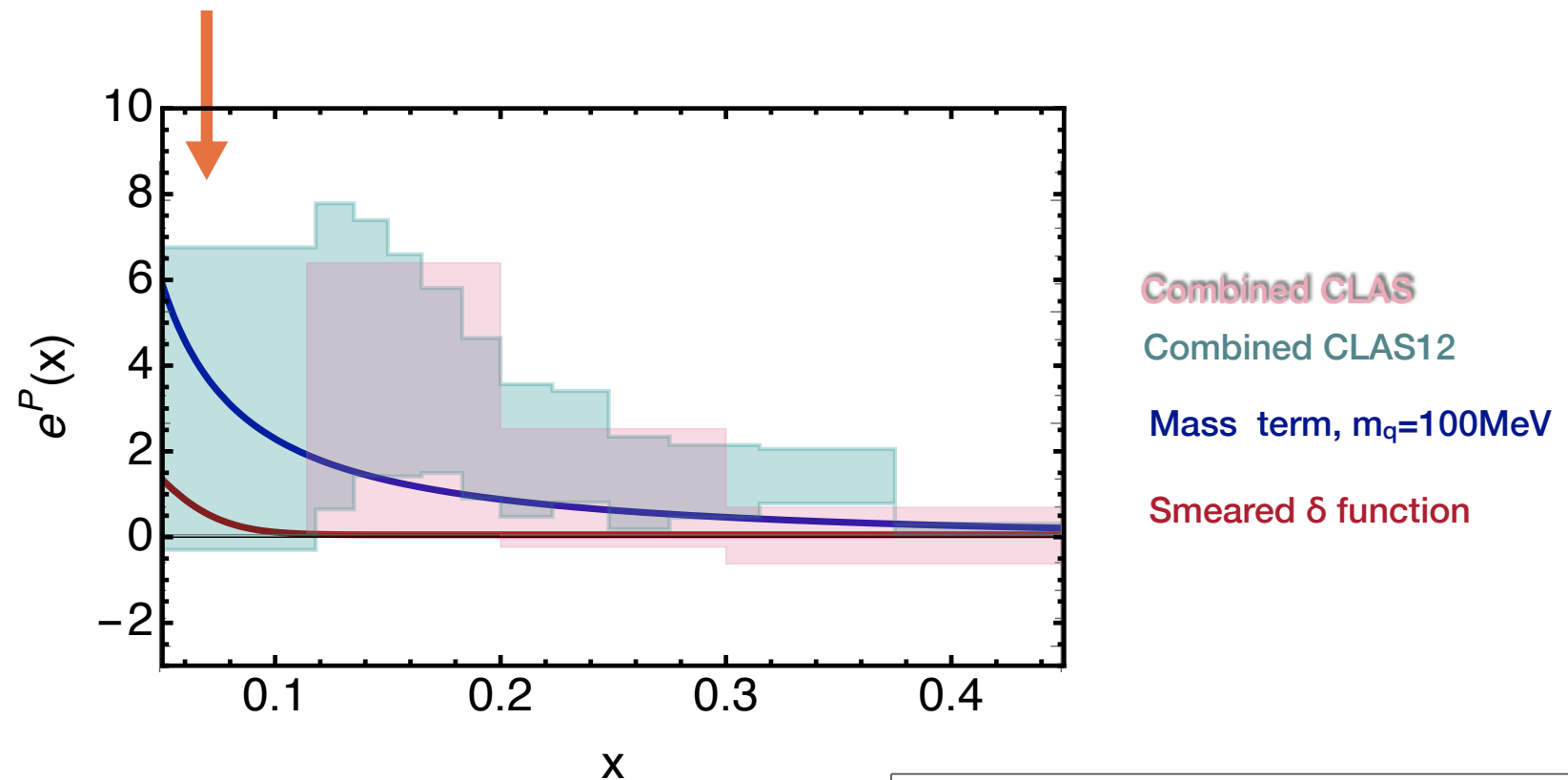
Examples through TMD and dynamical twist-3 relations (e.g. Sivers and Qiu-Sterman)

see talk by Shohini Bhattacharya (WG5)

# Consequences of the extraction

1. Are twist-3 PDFs non-zero? **Yes, to a certain CL.**
2. Can we access qqg correlations and more non-perturbative information? **Let's take the example of  $e(x)$ .**

Some nonperturbative effects expected in the small(ish)- $x$  region, e.g. [Pasquini & Rodini, PLB 788]



Schematic models for illustration purpose only!

**Moments will matter.**

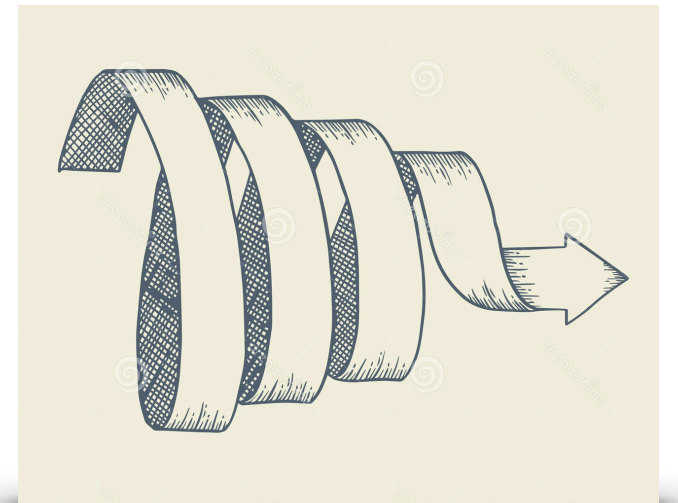
Other nonperturbative effects at not so small  $x$ , e.g. in the MIT bag.

# Conclusions

We have discussed the role of **higher-twist distributions** in the understanding of hadron structure. We have presented a **truly updated extraction of the scalar PDF,  $e(x)$** . It is non-zero to more than 75% probability.

The study of higher-twist PDFs will contribute to, e.g.

- *Precision 3D imaging of nucleons.*
- *Emergence of hadronic mass — from the scalar PDF.*
- *Proton spin puzzle — from GPDs.*



Higher-twist distributions will unveil aspects of hadron dynamics.

Higher-twist distributions are accessible but require more statistics, phenomenological and theoretical developments.

Combine efforts with the lattice QCD?

[e.g. Bhattacharya et al, PRD102; Braun & Vladimirov, JHEP10(2021)087]

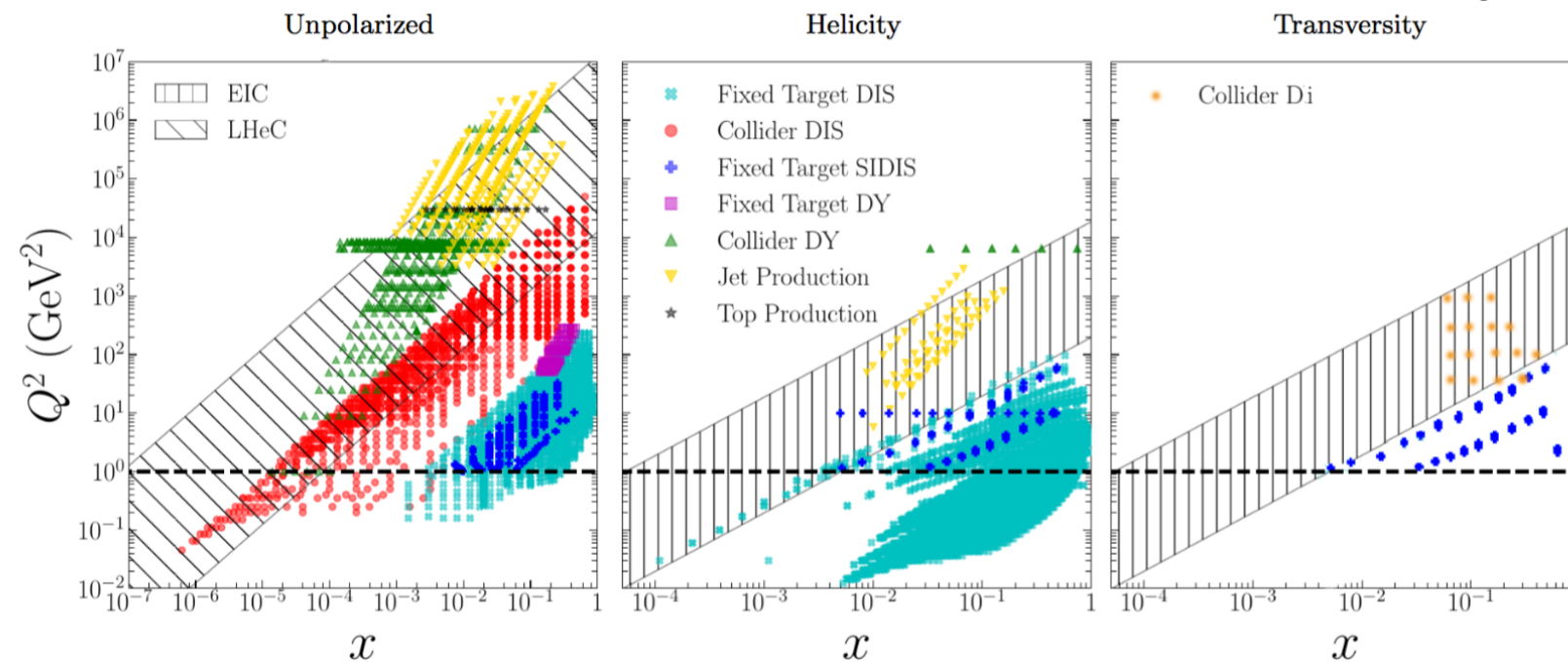


# Backup

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# PDF kinematics coverage: collinear PDFs

[Prog.Part.Nucl.Phys. 121 (2021) 103908]



One possible definition for higher-twist contributions: Fixed Target DIS & SIDIS:  $M/Q$  is not so small  
terms effectively suppressed like  $(M/Q)^{t-2}$

- Spurious contaminations
- Spin asymmetries can be defined to get sensitive to twist-3
- Present data: Hermes, COMPASS, JLab.

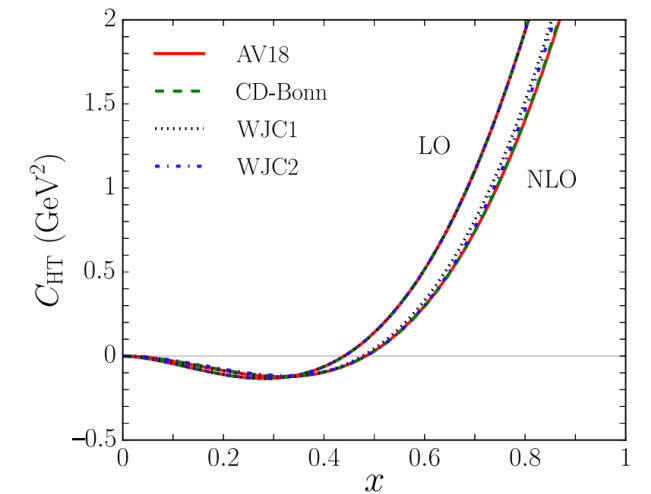
# Higher-twist in observables

## From spurious contaminations...

CJ15 global analysis includes lower cuts on  $W^2$ . [Accardi et al., PRD93]

HT's role in fulfilling duality [e.g. Melnitchouk et al., Phys.Rept.406]

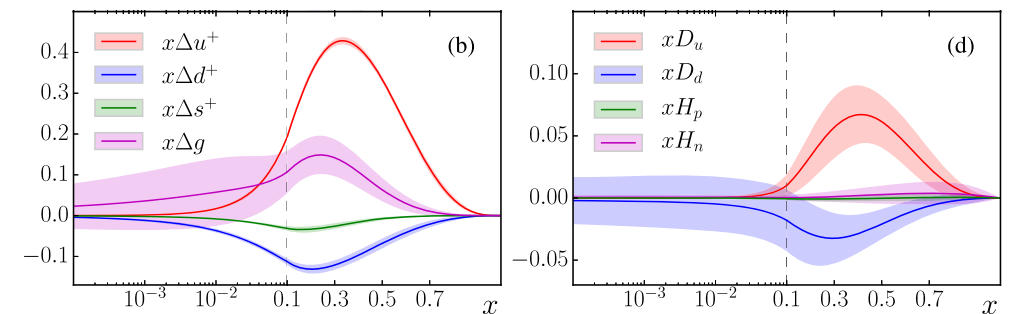
$$F_2(x, Q^2) = F_2^{LT}(x, Q^2) \left( 1 + \frac{C_{HT}(x)}{Q^2} \right)$$



JAM analysis of the helicity PDF  $g_1$  extends to  $g_T$ , with  $g_T = g_1 + g_2$ . [Sato et al., PRD93]

## ...to genuine effects

$$g_2^{(\tau^3)}(x, Q^2) = D(x, Q^2) - \int_x^1 \frac{dz}{z} D(z, Q^2)$$



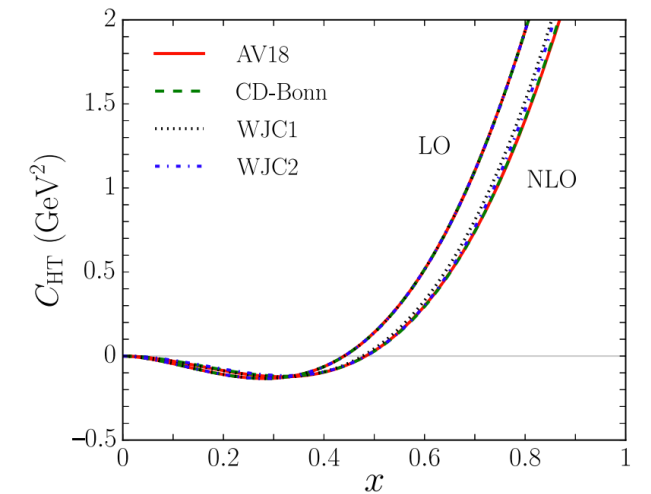
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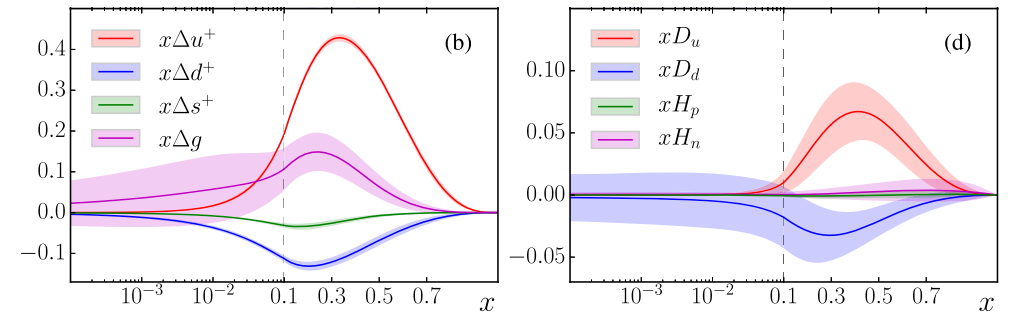
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## JAM analysis of the helicity PDF $g_1$ extends to $g_T$ , with $g_T = g_1 + g_2$ . [Sato et al., PRD93] ...to genuine effects

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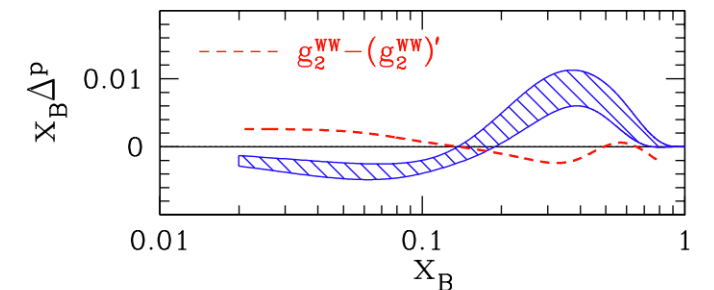


$g_T$  is the only twist-3 PDF accessible through inclusive DIS

Exploratory studies suggest that quark-gluon-quark correlations are non-zero.

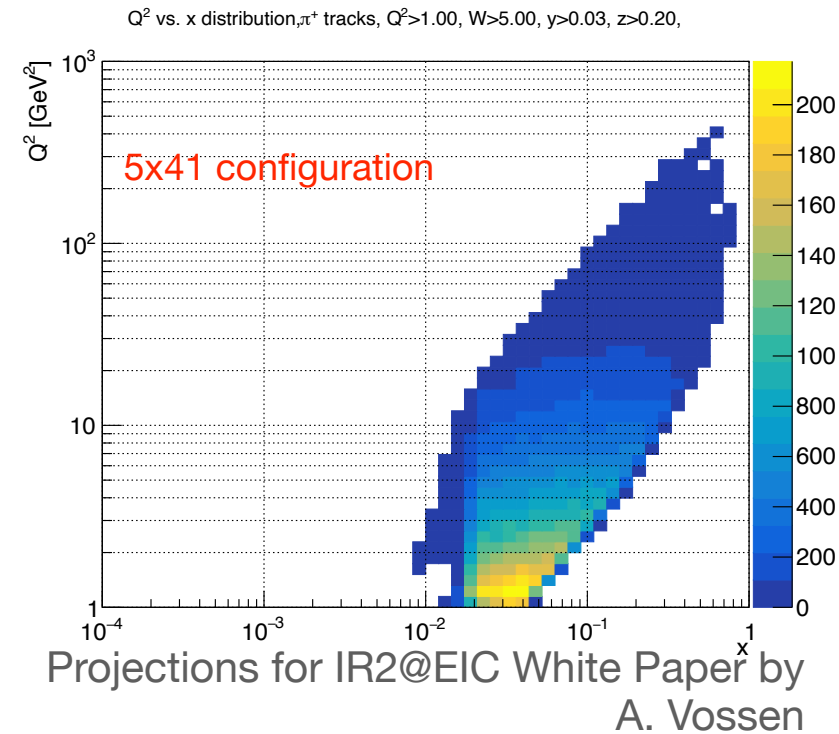
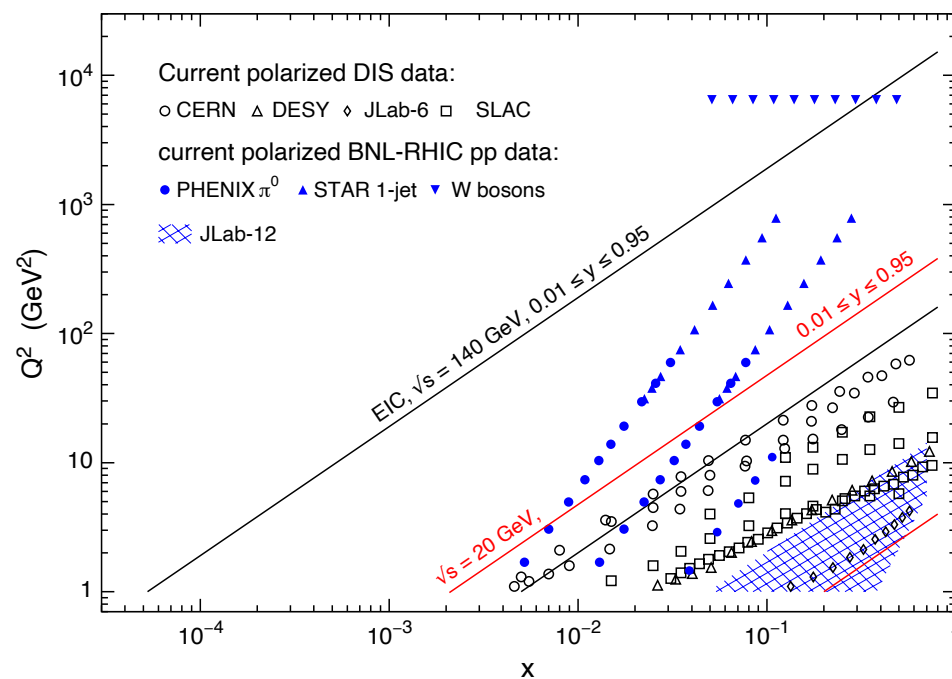
[Accardi et al, JHEP11 (2009)]

$$\Delta_{\text{ex}}(x_B, Q^2) = g_2^{\text{ex}}(x_B, Q^2) - g_2^{\text{WW}}(x_B, Q^2)$$



# Can we study qq̄q correlation at the EIC?

EIC Yellow Report [2103.05419]



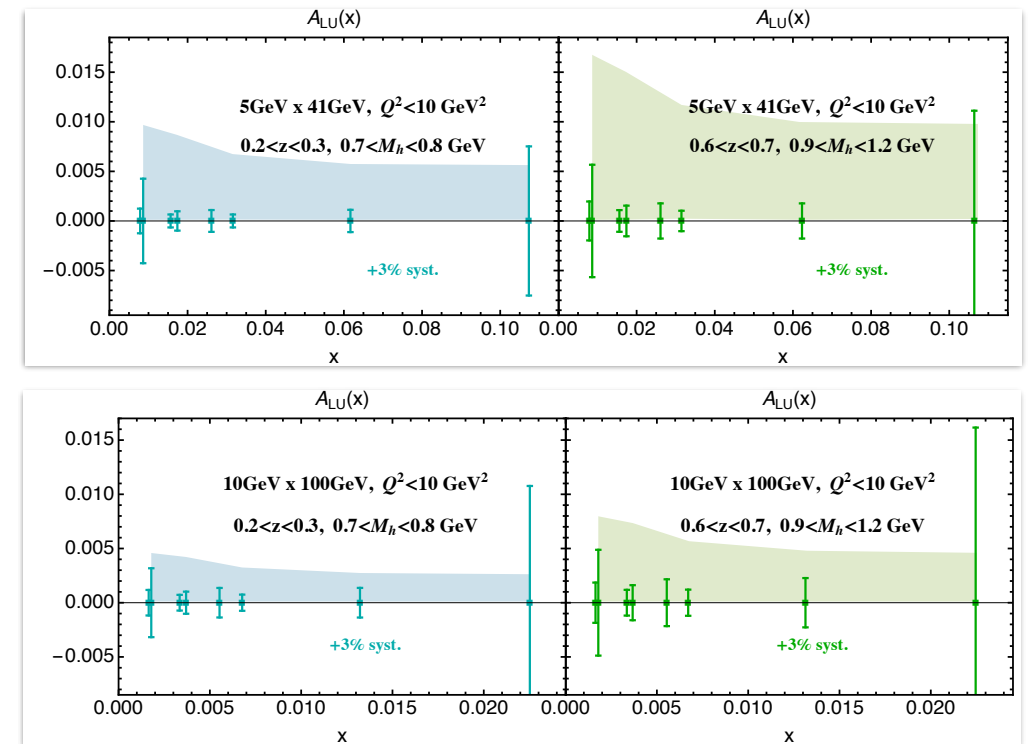
Future: EIC will cover low- to mid- $Q^2$  and smallish  $x$  values

- Yellow Report: access to multiparton correlations.
- Proposal for a 2nd interaction region — IR2@EIC.
- Complementarity with present data.

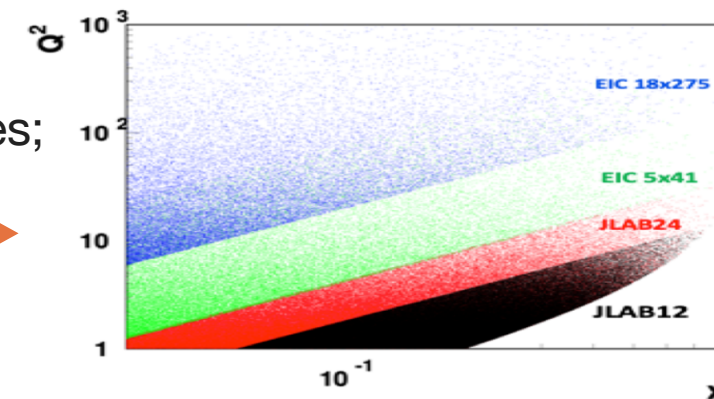
# Expectations for the EIC

EIC Yellow Report [2103.05419]

- EIC error projections (from transversity studies)
- Proton target shown, but need for neutron
- Models × DiFFs predictions
  - ➔ LC model [Pasquini & Rodini, PLB 788]
  - ➔ made-up mass-term contribution with  $m_q=300\text{MeV}$
- Non-negligible for lowest beam configurations
  - ↓
  - Archetype of observables for IR2@EIC



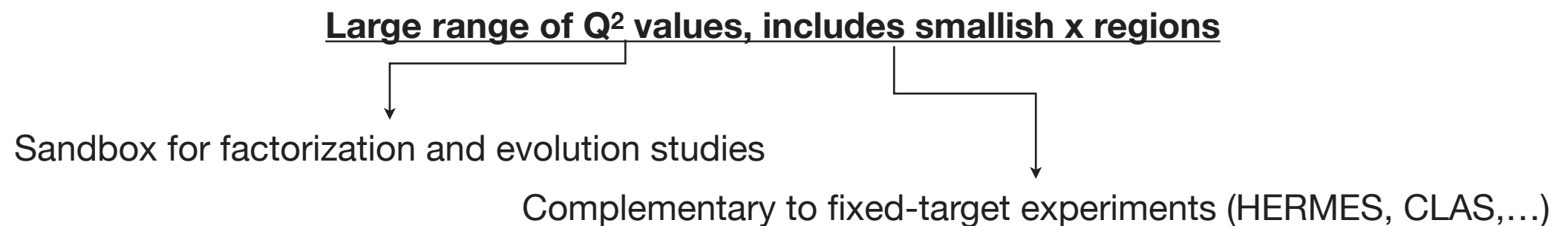
- Evolution equations for genuine qgq twist-3 known in most cases;
- Understanding of the various contributions to twist-3 PDFs;
- Especially “hot” for TMD studies.
- Require a second interaction region @EIC.



from H. Avakian  
the Paper for IR2@EIC.

# Multi-parton distributions at the EIC

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## Golden channel

fully inclusive DIS, access to  $g_T$

## Silver channel

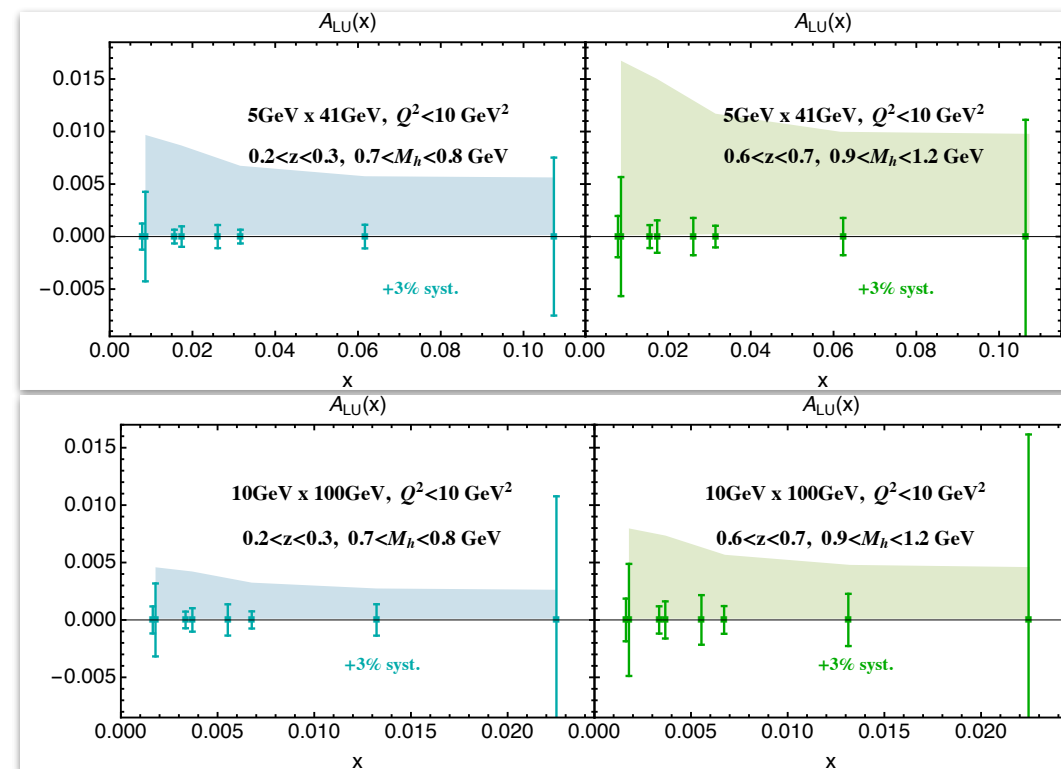
semi-inclusive DIS, access to  $e(x)$

- **Collinear observables.**
- Plethora of interesting TMD, GPD higher-twist observables to be considered too
- subWG: Avakian, Burkardt, AC, Gamberg, Pitonyak, Sato, Schweitzer, Vossen

# EIC coverage

EIC Yellow Report [2103.05419]

- EIC error projections (from transversity studies)
- Proton target shown, but need for neutron
- Models × DiFFs predictions
  - LC model [Pasquini & Rodini, PLB 788]
  - made-up mass-term contribution with  $m_q=300\text{MeV}$
- Non-negligible for lowest beam configurations



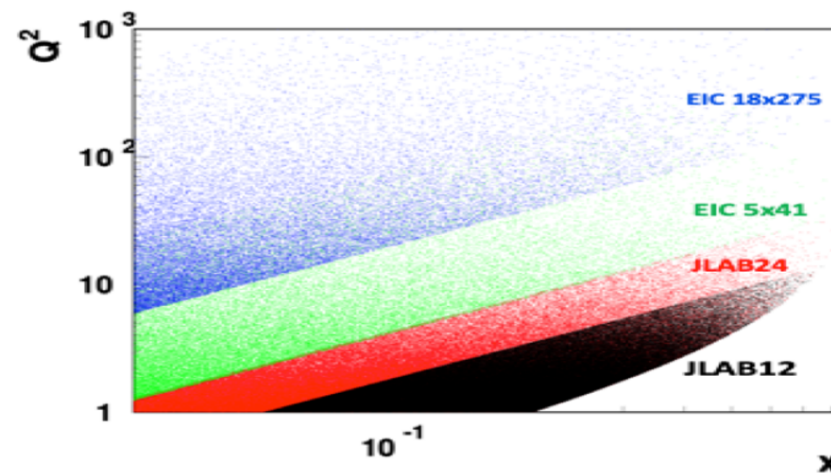
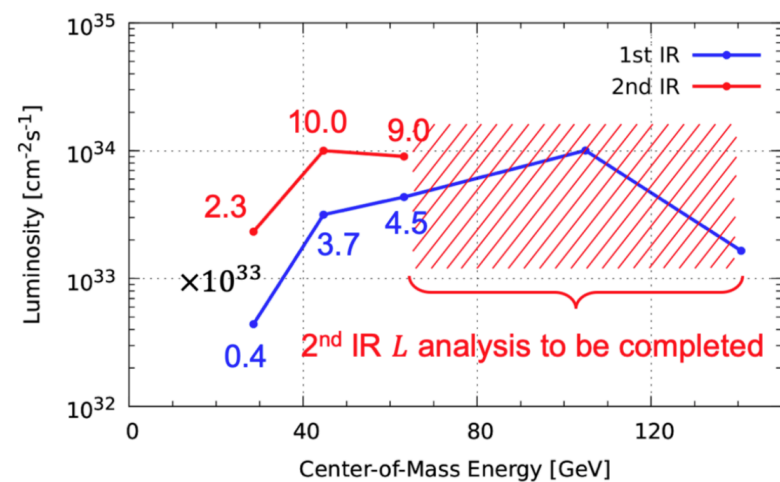
Archetype of observables for IR2@EIC



# QCD and twist-3 PDFs

Underlying and omitted in all this presentation:  $Q^2$ -evolution!

from H. Avakian  
& White Paper for IR2@EIC.



- Evolution equations for genuine qgq twist-3 known in most cases;
- Understanding of the various contributions to twist-3 PDFs;
- Especially “hot” for TMD studies.
- Require a second interaction region @EIC.

# Multi-parton distributions at the EIC

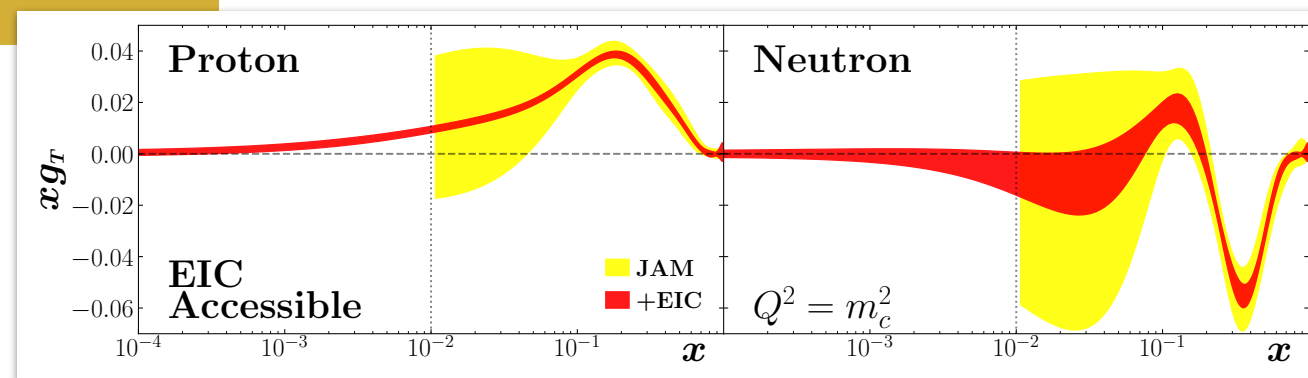
Large range of  $Q^2$  values, includes smallish  $x$  regions

Sandbox for factorization and evolution studies

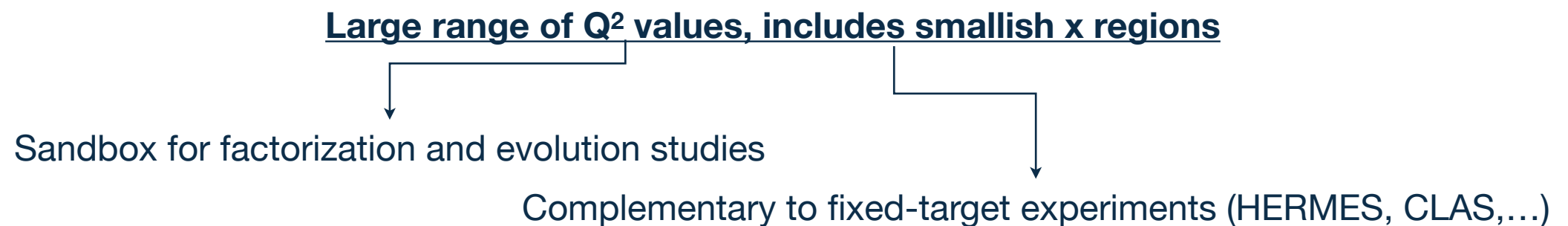
Complementary to fixed-target experiments (HERMES, CLAS,...)

Golden channel

fully inclusive DIS, access to  $g_T$



# Multi-parton distributions at the EIC



## Golden channel

fully inclusive DIS, access to  $g_T$

## Silver channel

semi-inclusive DIS, access to  $e(x)$

- **Collinear observables.**
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- subWG: Avakian, Burkardt, AC, Gamberg, Pitonyak, Sato, Schweitzer, Vossen