On validity of quasi-static approximation in scalar-tensor theories of Gravity

CAP congress

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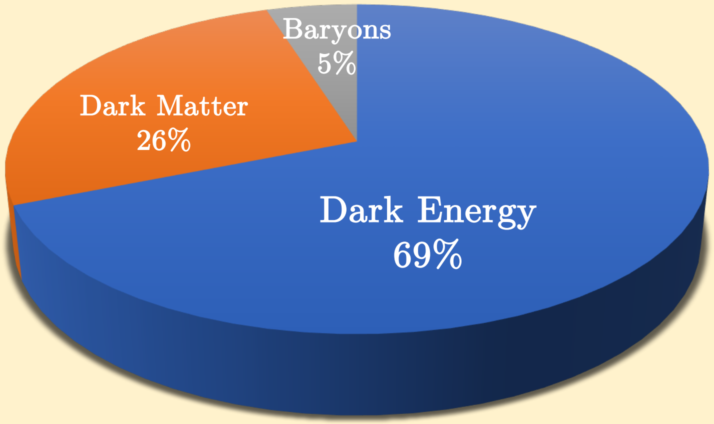
June 6, 2022

The standard ΛCDM model



- In the ΛCDM model, dark energy is represented by the cosmological constant Λ
- Λ and CDM are independent: no interaction between dark energy and dark matter is assumed
  - ➢ Although provides an acceptable fit to all data, we do not understand the physical nature of dark matter and dark energy
  - ➢ It could be that dark matter and dark energy interact
  - ➢ A large tuning is required to reconcile the small observed value of Λ with the large vacuum energy predicted by particle physics

✓ Study models of modified gravity and dynamical dark energy

Scalar-tensor theories

- A well motivated theoretical framework for studying dynamical dark energy: scalar-tensor theories of gravity

$$S = \int d^4x \left\{ \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - V(\phi) \right] + \mathcal{L}_{\rm dm}(\psi_{\rm dm}, \tilde{g}_{\mu\nu}) + \mathcal{L}_{\rm b}(\psi_{\rm b}, g_{\mu\nu}) + \mathcal{L}_{\gamma+\nu}(\psi_{\gamma+\nu}, g_{\mu\nu}) \right\}$$

Only CDM coupled

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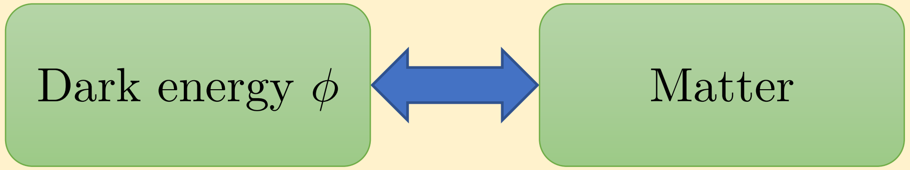
Total matter coupled

$$\tilde{g}_{\mu\nu} = A^2(\phi) g_{\mu\nu}$$



➢ Consider two scenarios: Only CDM coupled, and total matter coupled

➢ When $A \neq 1$, the scalar field is non-minimally coupled to the matter and mediates a $5^{\rm th}$ force between matter particles

✓ We ultimately want to be able to differentiate between these possibilities observationally

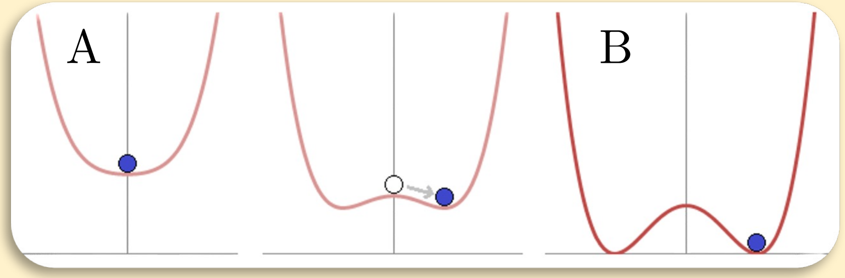
The Symmetron model

- We use the symmetron model as a well-motivated scalar-tensor theory

$$V(\phi) = V_0 - \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4 \qquad\qquad A(\phi) = 1 + \frac{1}{2}\left(\frac{\phi}{M}\right)^2$$

- The effective potential of the scalar field changes its shape depending on the matter density ρ

$$V_{\rm eff} = V_{0\rm eff} + \frac{1}{2}\underbrace{\left(\frac{\rho}{M^2} - \mu^2\right)}_{=0\ at\ SSB}\phi^2 + \frac{1}{4}\lambda\phi^4$$



- At high matter densities, ϕ=0, A=1 : ΛCDM is recovered
  - This is an example of screening, allowing to eliminate the 5th force in the early universe and the solar system
- At low matter densities, $\phi\neq$0, $A\neq$1 : We have a fifth force

Testing the quasi-static approximation

- Given our model, we can solve the differential equations for the evolution of the universe
- To save computational time, many studies instead used the quasi-static approximation (QSA)
- QSA assumes:
  - sub-horizon-size inhomogeneities
  - scalar field is always at the minimum of the effective potential
  - time-derivatives are much smaller than spatial derivatives

$$k^2\Phi \gg \{\ddot{\Phi}, \mathcal{H}\dot{\Phi}\}, \qquad k^2\delta\phi \gg \{\delta\ddot{\phi}, \mathcal{H}\delta\dot{\phi}\}$$

- In QSA, one neglects the oscillations of the scalar field around its expectation value
- We derived analytic expressions for the background field and its perturbation

$$\phi = \phi_* \sqrt{1 - \left(\frac{a_{SSB}}{a}\right)^3} \qquad \delta\phi = -\beta \frac{\rho_m \delta_m}{m^2 + k^2/a^2}$$

- In QSA, the gravity instantly responds to matter
  - ✓ Our study identified the range of parameters for which QSA is a good approximation

Cosmological variables to test

$$\delta_m = \frac{\delta\rho_m}{\rho_m}$$

Matter density fluctuation (density contrast)

Describes how structure formation begins and where there are local enhancement in matter density

$$\Phi_+ = \frac{\Phi + \Psi}{2}$$

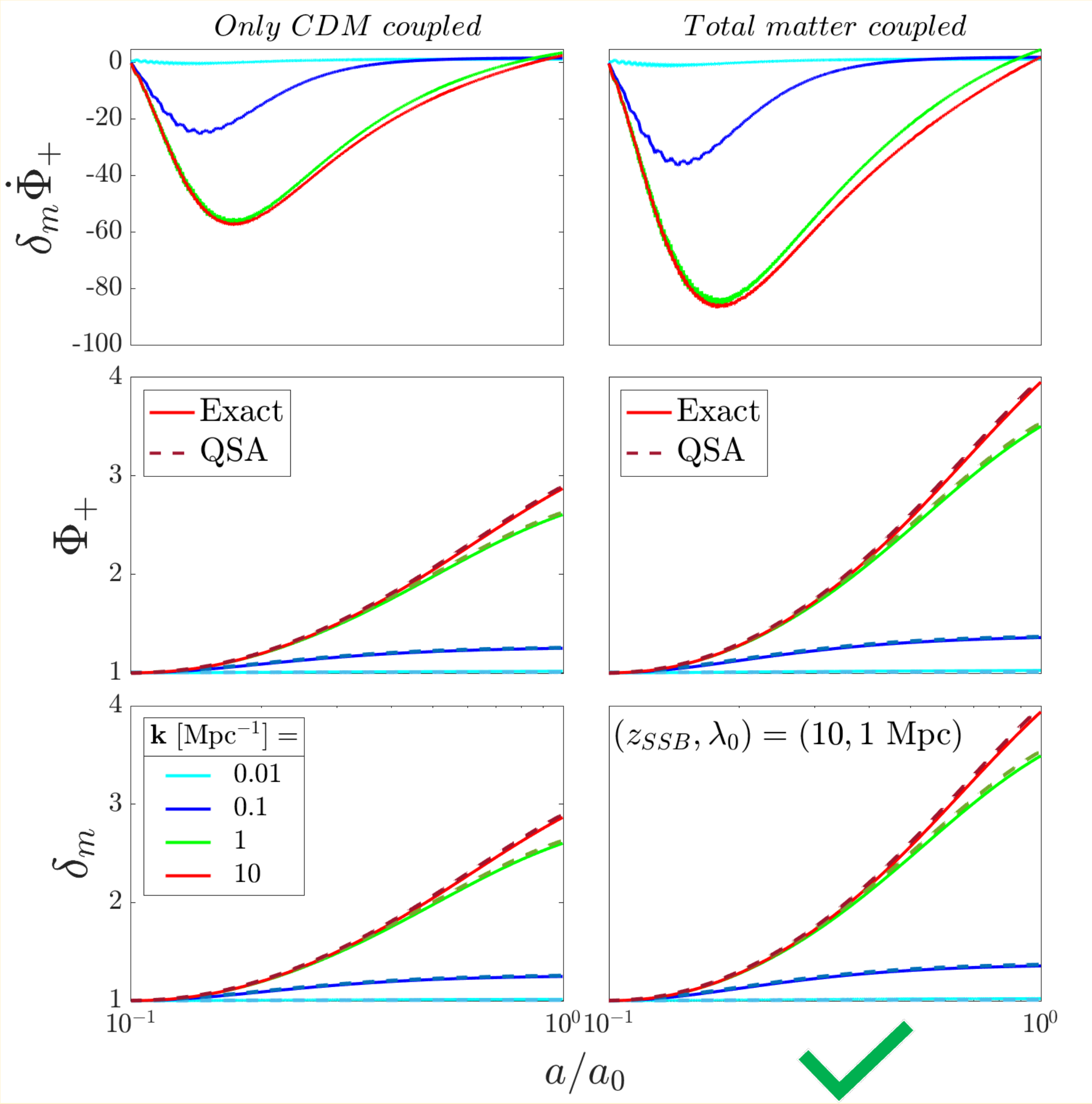
The Weyl potential

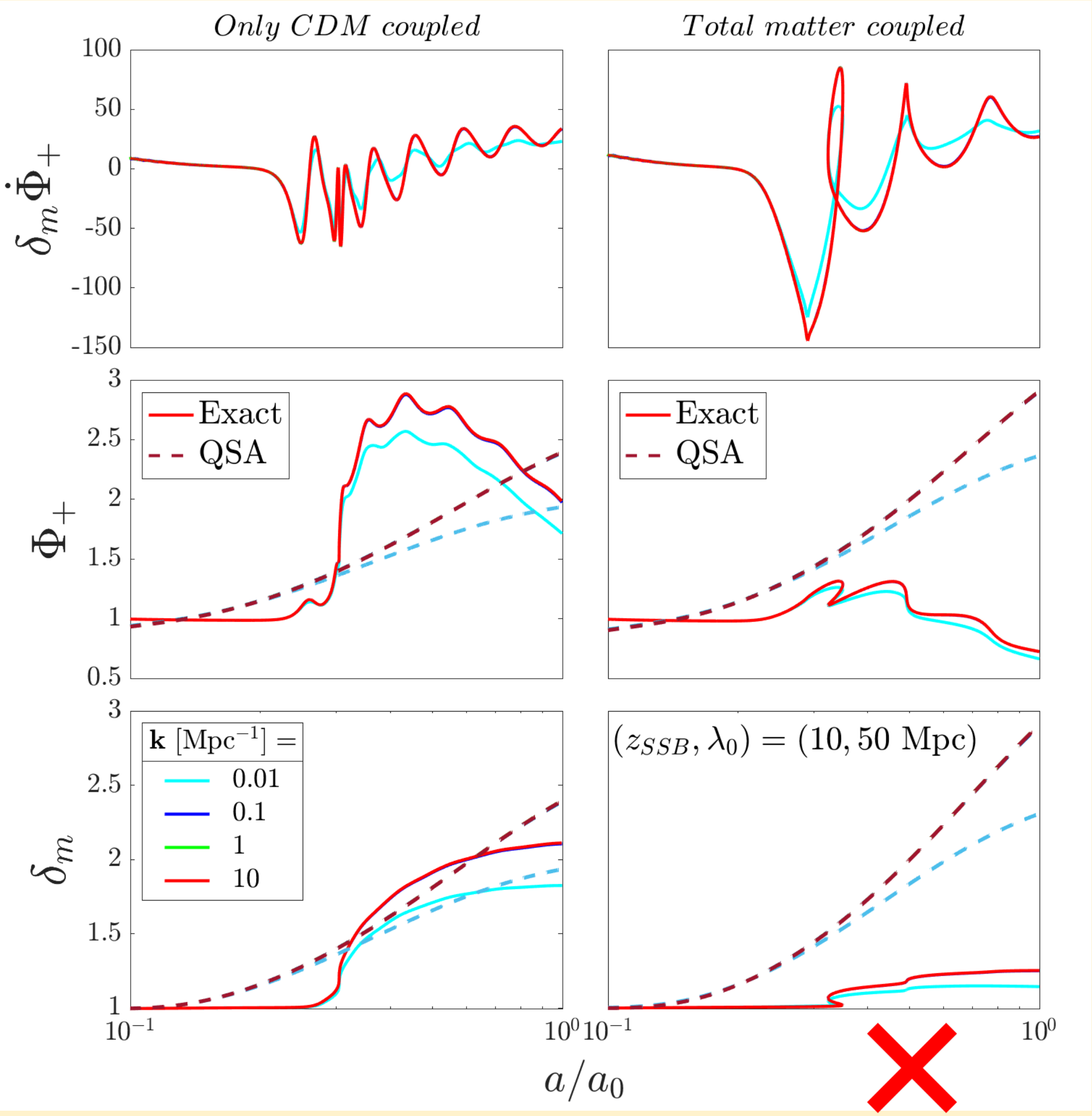
Characterizes the gravitational potential felt by massless particles

$$\delta_m \dot{\Phi}$$

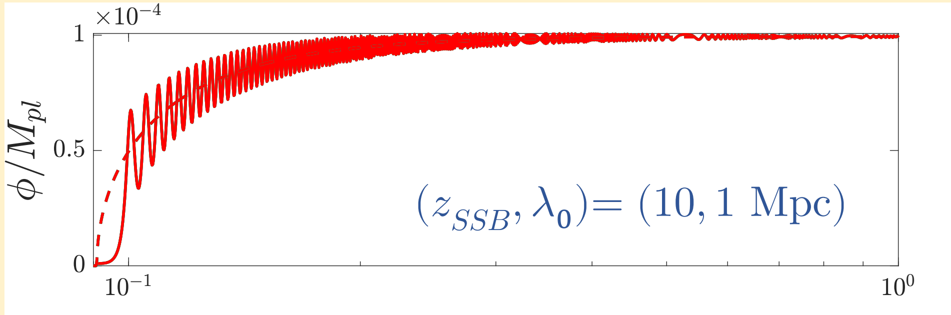
Probing the cross-correlate between galaxy counts and CMB temperature anisotropy to see if there is any integrated Sachs-Wolfe (ISW) effect

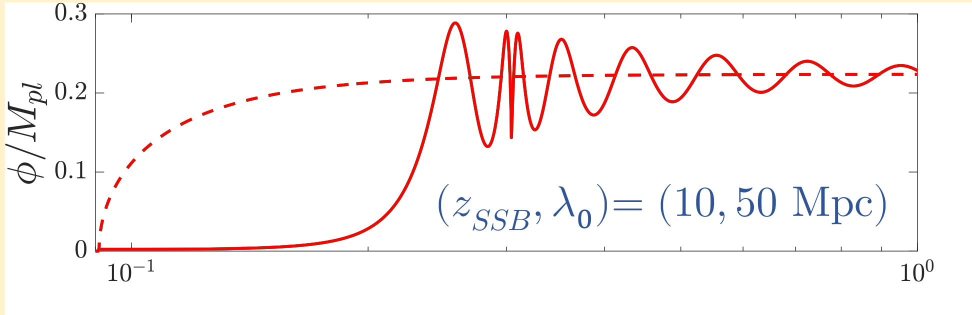
Examples of when QSA works and when it does not





The reason the QSA breaks down





- The agreement between the exact solution and QSA highly depends on the response of the background scalar field to the phase transition and the frequency of oscillations
- A larger Compton wavelength flattens the effective potential and provides more freedom for oscillations around the minimum
- We derived the expression for the critical value for the Compton length at which the QSA fails

$$\lambda_{cr} \sim \frac{1}{\mathcal{H}_0\sqrt{8[(1+z_{SSB})^3-1]}}$$

z_{SSB}	λ_0 [Mpc]
1	500
10	50
100	1

Summary

- The scalar tensor theories of gravity provide a well motivated framework to study the Universe beyond ΛCDM.
- We have examined the range of validity of QSA and identified a range of interaction for when it works.
- This will make it easier to constraint the additional fifth force.