

FeynArtsHelper

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Introduction

In this presentation:

Talk about the motivation behind the project from the:
 physics perspective
 computational perspective

Introduce you to a newly developed Mathematica package:
FeynArtsHelper.

Provide several examples to show how FeynArtsHelper can be used to extract several important features of Feynman diagrams.

Motivation: point of view of Physics

- ⚠ HIERARCHY PROBLEM
- ⚠ ASYMMETRY
- ⚠ GRAVITY
- ⚠ EXPANDING UNIVERSE

- To look for new physics beyond the standard model we use:

The Energy Frontier: *High energy colliders.*

The Intensity Frontier: *intense particle beams.*

The Cosmic Frontier: *underground experiments, ground and space based telescopes.*

Feynman Diagrams

Calculating the matrix element from first principles is cumbersome-not usually a way to go

Using Feynman diagram, we can approximate the matrix-element.

Each Feynman diagram represents a term in perturbation expansion of amplitude which is related to observables, such as the differential cross-section

$$\frac{d}{d\Omega} = \frac{1}{8} \frac{jMj^2}{(E_1 + E_2)^2} \frac{j p_f j^2}{j p_i j^2} \quad (1)$$

A full matrix element contains an **in nite** number of Feynman diagrams sorted by perturbation ordering

$$M_{fi} = M_1 + M_2 + M_3 + \dots \quad (2)$$

Feynman diagram to Observables

Z Standard Model $\left\{ \begin{array}{l} \gamma \\ Z \end{array} \right.$
 $\left. \begin{array}{l} \gamma' \\ Z' \end{array} \right\}$ Beyond the Standard Model

(3)

Figure 1: Feynman diagrams for an scattering experiment.

To access the scale of new physics at TeV level, we need to push one or more experimental parameters to the extreme precision.

We can predict outcomes of Figure (1) by calculating asymmetry

$$A_{LR} = \frac{L}{L+R} - \frac{R}{L+R} = \frac{2\text{Re} \left(M_{\gamma} M_Z^* + M_{Z'} M_{NP}^* + M_Z M_{NP}^* \right)}{L+R} \quad (4)$$

where M_i is the amplitude of the i^{th} diagram and M_{NP} is the amplitude of diagram with Z^0 interaction.^a

^aNP stands for New Physics

Why do we need another package?

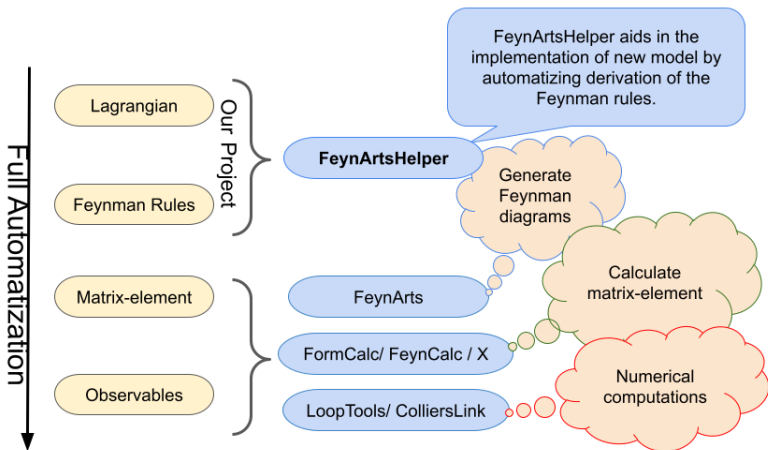


Figure 2: Steps in calculating observables using computer algebra.

Introduction to FeynArtsHelper⁷

This is a Mathematica package which will help you with calculations.

It works in conjunction with existing packages such as FeynArts³, FeynCalc⁸, LoopTools², X⁶, etc.

It will help in reducing some repetitive calculations that the other packages does not already do.

Primarily, it will help us writing the model files for FeynArts.

Automating will streamline the process of going from *Lagrangian* to *observables*.

Deriving a Propagator using FeynArtsHelper I

Feynman rules requires *repetitive, cumbersome* and *lengthy* calculations. The general rules for deriving couplings and propagators are:

Find the kinetic and mass term of the Lagrangian density for **propagator** and interaction terms for **coupling**.

Write down the Lagrangian in momentum space ($\mathcal{L}(p)$) where p is the associated field momentum.

Evaluate the functional derivative

$$B = \frac{\delta^n Z}{\delta v_1(k_1) \cdots \delta v_n(k_n)} d^4x L, \text{ where } v_i \text{ is a field with momentum } k_i: \quad (5)$$

For propagator: $\Pi = iB^{-1}$

For coupling: $\Gamma = iB$

Deriving a Propagator using FeynArtsHelper II

As an example, we derive the the fermion propagator from the Lagrangian

$$L_{qed} = \bar{\psi} (i \not{\partial} - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (6)$$

$$\begin{aligned} S &= \int d^4x L_{\text{fermion}} \\ &= i \int d^4x \bar{\psi}(x) (\not{\partial} - m) \psi(x) \\ &= \int d^4k_1 d^4k_2 d^4x \bar{\psi}(k_1) (i \not{k}_2 - m) \psi(k_2) \\ &= m_f \bar{\psi}(k_1) \psi(k_2) \quad (4) \end{aligned} \quad (7)$$

Deriving a Propagator using FeynArtsHelper III

Then taking the functional derivative (skipping several algebraic simplifications) we get

$$\frac{\delta^2 S}{\delta(p_1) \delta(p_2)} = \int d^4 k_1 d^4 k_2 \frac{h}{m} \text{tr} \left[\gamma^4 (p_1 - k_1) \gamma^i (p_2 - k_2) \gamma^4 (k_2 - k_1) \right] \quad (8)$$

$$\boxed{k_1 = k_2 = p}$$

$$= \frac{p}{m}$$

After inverting $\frac{\delta^2 S}{\delta(p_1) \delta(p_2)}$ we get

$$\Pi_{\text{fermion}} = i \frac{p + m}{p^2 - m^2} \quad (9)$$

Deriving a Propagator using FeynArtsHelper IV

Now, if we consider the spin-3/2 baryon resonances field described by the Rarita-Schwinger Lagrangian

$$L = \bar{\Lambda} \Lambda \quad (10)$$

where

$$\begin{aligned} \Lambda = & \left(i \not{\partial} + M \right) g + iA \left(\not{\partial} + \not{\partial} \right) \\ & + \frac{i}{2} \left(3A^2 + 2A + 1 \right) \not{\partial} + M \left(3A^2 + 3A + 1 \right) \end{aligned} \quad (11)$$

In case $A = 1$ we get:

$$\begin{aligned} D = & \left[\left(\not{k} + m \right) g + \left(\not{k} \not{k} \right) \not{k} m \right] \\ = & \left(\not{k} + m \right) g + \not{k} \not{k} \not{k} m \end{aligned} \quad (12)$$

Deriving a Propagator using FeynArtsHelper V

One form of a generalized second rank tensor which satisfies (all properties*) is given by

$$\Pi = a_1 g + a_2 k + a_3 k + a_4 + a_5 k k \quad (13)$$

such that

$$D : \Pi = g \quad (14)$$

Doing it by hand is tedious.

Using few lines of code on FeynArtsHelper it can be derived.

¹Lagrangian must be invariant under point transformation, so $A' = \frac{A-2x}{1+4x}$ with $a \neq -\frac{1}{4}$

```

In[ ]:= rsLagrangian =
  -QuantumField[ψ1, {α}]
  ((-I * GA[γ] × QuantumField[FCPartialD[γ], ψ2, {β}] × MT[α, β] +
    m MT[α, β] × QuantumField[ψ2, {β}]) -
  I (-GA[α] × QuantumField[FCPartialD[β], ψ2, {β}] -
    QuantumField[FCPartialD[α], ψ2, {β}] × GA[β]) -
  I * GA[α] × GA[γ] × QuantumField[FCPartialD[γ], ψ2, {β}] × GA[β] -
  m GA[α] × GA[β] × QuantumField[ψ2, {β}]) // ExpandAll

```

$$\begin{aligned}
\text{Out[]} = & -i \psi_{1\alpha} \bar{\gamma}^\beta (\partial_\alpha \psi_{2\beta}) - i \psi_{1\alpha} \bar{\gamma}^\alpha (\partial_\beta \psi_{2\beta}) + i \bar{\gamma}^\gamma \psi_{1\alpha} \bar{\gamma}^\alpha \bar{\gamma}^\beta (\partial_\gamma \psi_{2\beta}) + \\
& i \bar{\gamma}^\gamma \psi_{1\alpha} \bar{g}^{\alpha\beta} (\partial_\gamma \psi_{2\beta}) - m \psi_{1\alpha} \psi_{2\beta} \bar{g}^{\alpha\beta} + m \psi_{1\alpha} \psi_{2\beta} \bar{\gamma}^\alpha \bar{\gamma}^\beta
\end{aligned}$$

```

In[ ]:= quadForm2FromLag =
  FCReplaceAll[
    FAHFeynmanRules[rsLagrangian,
      {{QuantumField[ψ1, {ρ}][k1], QuantumField[ψ2, {σ}][k2]}]} //
    DiracSimplify, {k1 → k, k2 → k, ρ → α, σ → β}] [[1]] // FCE

```

$$\text{Out[]} = \bar{g}^{\alpha\beta} \bar{\gamma} \cdot \bar{k} - m \bar{g}^{\alpha\beta} + \bar{\gamma}^\alpha (-\bar{k}^\beta) - \bar{\gamma}^\beta \bar{k}^\alpha + \bar{\gamma}^\alpha \bar{\gamma}^\beta \bar{\gamma} \cdot \bar{k} + m \bar{\gamma}^\alpha \bar{\gamma}^\beta$$

In[*]:= **struc2** = {MT[α , β], GA[α] * FV[k , β], GA[β] * FV[k , α], GA[α] * GA[β],
 FV[k , α] * FV[k , β]}

Out[*]:= $\{\bar{g}^{\alpha\beta}, \bar{\gamma}^{\alpha} \bar{k}^{\beta}, \bar{\gamma}^{\beta} \bar{k}^{\alpha}, \bar{\gamma}^{\alpha} \bar{\gamma}^{\beta}, \bar{k}^{\alpha} \bar{k}^{\beta}\}$

In[*]:= **pg2** = GetPropagator[quadForm2FromLag, **struc2**, { $\alpha \rightarrow \beta$, $\beta \rightarrow \delta$ }] // Simplify

Out[*]:=
$$-\frac{(\bar{\gamma} \cdot \bar{k} + m) (m (-3 m \bar{g}^{\beta\delta} + \bar{\gamma}^{\beta} (-\bar{k}^{\delta}) + m \bar{\gamma}^{\beta} \bar{\gamma}^{\delta}) + \bar{k}^{\beta} (4 \bar{k}^{\delta} - m \bar{\gamma}^{\delta}))}{3 m^2 (\bar{k}^2 - m^2)}$$

In[*]:= **str1** = GenPropInternal[pg1, V[3], λ , { $\beta \rightarrow li1$, $\delta \rightarrow li2$, $k \rightarrow mom$ }]

Out[*]:= -FourVector[mom,li1] FourVector[mom,li2] PropagatorDenomiator[mom,Mass[V[3]]]
 PropagatorDenomiator[mom,Sqrt[λ]Mass[V[3]] (1 - Sqrt[GaugeXi[V[3]])] -
 MetricTensor[li1,li2] PropagatorDenomiator[mom,Mass[V[3]]]

Toy Model: Deriving counter-term Lagrangian I

Suppose we have a Lagrangian with an fermion electric dipole moment extension as follows:

$$L = \bar{\psi} \left(i \not{D} - m \right) \psi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{d_f}{2} \bar{\psi} \gamma_5 \psi F \quad (15)$$

L_{QED} L_{Dipole}

where

$$\not{D} = \not{\partial} + ie \not{A}; \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \text{and} \quad \gamma_5 = \frac{i}{2} [\gamma_1, \gamma_2] \quad (16)$$

Extract Lagrangian and counter-term Lagrangian.

Use built-in Mathematica operators to calculate renormalization constants using Ward-Takahashi identities and renormalization conditions.

In[]:= lagDipole =

```
I * QuantumField[ψ1] * GA[μ] * QuantumField[FCPartialD[μ], ψ2] +  
e * QuantumField[ψ1] * GA[μ] * QuantumField[A, {μ}] * QuantumField[ψ2] -  
m * QuantumField[ψ1] * QuantumField[ψ2] -  
1/4 * (QuantumField[FCPartialD[μ], A, {ν}] -  
QuantumField[FCPartialD[ν], A, {μ}]) *  
(QuantumField[FCPartialD[μ], A, {ν}] -  
QuantumField[FCPartialD[ν], A, {μ}]) -  
de/2 *  
( (QuantumField[ψ1] * GA[5] * DiracSigma[GA[μ], GA[ν]] * QuantumField[ψ2]) *  
QuantumField[FCPartialD[μ], A, {ν}] -  
(QuantumField[ψ1] * GA[5] * DiracSigma[GA[μ], GA[ν]] * QuantumField[ψ2]) *  
QuantumField[FCPartialD[ν], A, {μ}]) // Expand // Simplify
```

$$\text{Out[]} = \frac{1}{4} \left(-((\partial_\mu A_\nu) - (\partial_\nu A_\mu))^2 + \psi_1 (\psi_2 (2 de \bar{\gamma}^5 \sigma^{\mu\nu} ((\partial_\nu A_\mu) - (\partial_\mu A_\nu)) + 4 e A_\mu \bar{\gamma}^\mu - 4 m) + 4 i \bar{\gamma}^\mu ((\partial_\mu \psi_2))) \right)$$

In[*]:= `ctlagDipole[0] = GetCounterTermLagrangian[lagDipole, {e, m, de}] // Simplify`

$$\text{Out[*]} = \frac{1}{4} \left(-\text{de} \psi^1 \psi^2 \bar{\gamma}^5 \sigma^{\mu\nu} \left((\partial_\mu A_\nu) - (\partial_\nu A_\mu) \right) \left(\delta Z_2^A + 2 \delta Z_1^{\text{de}} + \delta Z_2^{\psi^1} + \delta Z_2^{\psi^2} \right) + \right. \\ \left. 2 \psi^1 \bar{\gamma}^\mu \left(e \psi^2 A_\mu \left(\delta Z_2^A + 2 \delta Z_1^e + \delta Z_2^{\psi^1} + \delta Z_2^{\psi^2} \right) + i \left((\partial_\mu \psi^2) \left(\delta Z_2^{\psi^1} + \delta Z_2^{\psi^2} \right) \right) - \delta Z_2^A \left((\partial_\mu A_\nu) \right)^2 + \right. \right. \\ \left. \left. 2 \delta Z_2^A \left((\partial_\nu A_\mu) \right) \left((\partial_\mu A_\nu) \right) - \delta Z_2^A \left((\partial_\nu A_\mu) \right)^2 - 4 m \psi^1 \psi^2 \delta Z_1^m - 2 m \psi^1 \psi^2 \delta Z_2^{\psi^1} - 2 m \psi^1 \psi^2 \delta Z_2^{\psi^2} \right) \right)$$

`ctrulesDipole =`

`FeynmanRulesLagrangian[ctlagDipole[0], possibleFieldCombDipole] // Simplify`

$$\text{Out[*]} = \left\{ \frac{1}{4} \left(\text{de} \bar{\gamma}^5 \bar{\gamma}^\alpha \cdot (\bar{\gamma} \cdot \bar{p}) \left(2 \delta Z_1^{\text{de}} + \delta Z_2^\gamma + 2 \delta Z_2^\psi \right) - \text{de} \bar{\gamma}^5 (\bar{\gamma} \cdot \bar{p}) \cdot \bar{\gamma}^\alpha \left(2 \delta Z_1^{\text{de}} + \delta Z_2^\gamma + 2 \delta Z_2^\psi \right) + \right. \right. \\ \left. \left. 2 e \bar{\gamma}^\alpha \left(2 \delta Z_1^e + \delta Z_2^\gamma + 2 \delta Z_2^\psi \right) \right), \delta Z_2^\psi (\bar{\gamma} \cdot \bar{p} - m) - m \delta Z_1^m, \delta Z_2^\gamma (\bar{p}^2 \bar{g}^{\alpha\beta} - \bar{p}^\alpha \bar{p}^\beta) \right\}$$

Toy Model: Deriving Coupling vectors I

As seen in earlier slides, FAHFeynmanRules operator generate the following output

photon-fermion coupling	$\frac{1}{2}d_e$	\not{p}	\not{p}	$+ e$
non-inverted fermion propagator	\not{p}	m		
non-inverted photon propagator ²	$p^2 g$	p	p	

Using GetCouplingVector we can separate the coupling vector into a Lorentz part and kinematic part

$$\begin{aligned}
 \text{5 } \not{p}; \quad \text{5 } \not{p} \quad ; \quad \text{O } \frac{d_l}{2}; \quad \text{A } : \quad \frac{1}{4}d_l \not{z}_1^{d_l} + \not{z}_2 + 2 \not{z}_2 \\
 \text{e}; \quad \text{A } : \quad \frac{1}{4}d_l \not{z}_1^{d_l} + \not{z}_2 + 2 \not{z}_2 \\
 \text{e}; \quad \frac{1}{2}e \not{z}_1^e + \not{z}_2 + 2 \not{z}_2
 \end{aligned}
 \tag{17}$$

z_1^i and z_2^i are the renormalization constants of the associated with input parameters and the field respectively.

From there, we can derive observables which will be shown below.

²without the gauge fixing terms

Electron-Muon scattering cross-section I

The diagrams shown in Figure (3) are used for this calculations.

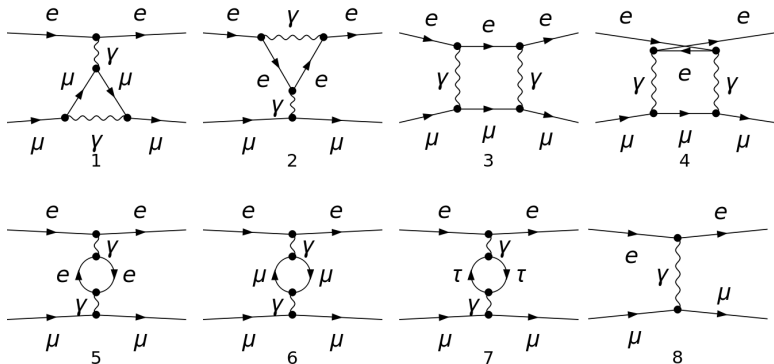


Figure 3: Feynman diagrams for electron-muon scattering.

Electron-Muon scattering cross-section II

Using some arbitrary values for d_I , we calculated the cross-section to the first order of constant d_I , shown in Figure (4).

The differential cross sections are calculated using

$$\frac{d\sigma}{d\Omega} \approx |jM_0 + M_1|^2 = |jM_0|^2 + 2\text{Re}M_0M_1 \quad (18)$$

where M_0 is the amplitude associated with the tree level topology, M_1 is the amplitude associated with the self-energy and triangle topologies.

Higher order contributions in Figure (4) account only for the one-loop topologies.

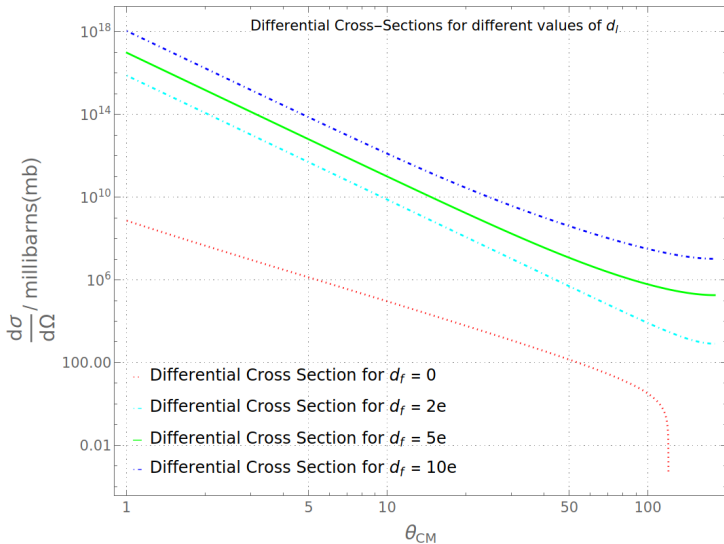


Figure 4: Effects of higher order one-loop correction on cross-section (infrared and uv finite part only) with $e = \sqrt{\frac{p}{4\pi\alpha}}$.

$$\text{In[2]:= } P = \left\{ \left\{ \frac{1}{\sqrt{6}} * \text{QuantumField}[\eta] + \frac{1}{\sqrt{2}} \text{QuantumField}[\pi 0], \text{QuantumField}[\pi p], \right. \right. \\ \left. \left. \text{QuantumField}[Kp] \right\}, \right. \\ \left. \left\{ \text{QuantumField}[\pi m], \frac{1}{\sqrt{6}} * \text{QuantumField}[\eta] - \frac{1}{\sqrt{2}} \text{QuantumField}[\pi 0], \right. \right. \\ \left. \left. \text{QuantumField}[K0] \right\}, \left\{ \text{QuantumField}[Km], \text{QuantumField}[K0b], \frac{-2}{\sqrt{6}} \text{QuantumField}[\eta] \right\} \right\}$$

$$\text{Out[2]= } \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi 0}{\sqrt{2}} & \pi p & Kp \\ \pi m & \frac{\eta}{\sqrt{6}} - \frac{\pi 0}{\sqrt{2}} & K0 \\ Km & K0b & -\sqrt{\frac{2}{3}} \eta \end{pmatrix}$$

$$\text{In[3]:= } dP = P // . \text{QuantumField}[a_] \rightarrow \text{QuantumField}[FCPartialD[\mu], a]$$

$$\text{Out[3]= } \begin{pmatrix} \left(\frac{\partial_\mu \eta}{\sqrt{6}} + \frac{\partial_\mu \pi 0}{\sqrt{2}} \right) & (\partial_\mu \pi p) & (\partial_\mu Kp) \\ (\partial_\mu \pi m) & \frac{\partial_\mu \eta}{\sqrt{6}} - \frac{\partial_\mu \pi 0}{\sqrt{2}} & (\partial_\mu K0) \\ (\partial_\mu Km) & (\partial_\mu K0b) & -\sqrt{\frac{2}{3}} ((\partial_\mu \eta)) \end{pmatrix}$$

$$\text{In[4]:= } \text{LagP} =$$

$$\frac{1}{8 f \pi^2} * \left(-\frac{8}{3} (P \cdot P \cdot dP + P \cdot dP \cdot P + dP \cdot P \cdot P) \cdot dP + 4 (P \cdot dP + dP \cdot P) \cdot (P \cdot dP + dP \cdot P) - \right. \\ \left. \frac{8}{3} dP \cdot (P \cdot P \cdot dP + P \cdot dP \cdot P + dP \cdot P \cdot P) \right) // \text{Flatten}$$

```

In[108]:= Pi0etaK0K0bCoupling[f1_, f2_, f3_, f4_] :=
  Total[
    FAHFeynmanRules[lagP,
      {{QuantumField[f1][k1], QuantumField[f2][k2], QuantumField[f3][k3],
        QuantumField[f4][k4]}}] // Flatten] // Expand;

In[81]:= couplingList = {{Pi0, eta, K0, K0b}, {Pi, Pi, Pi, Pi}};

In[125]:= Print[Pi0etaK0K0bCoupling /@ couplingList];

```

$$\left\{ -\frac{\bar{k}_1 \cdot \bar{k}_2}{\sqrt{3} \text{fr}^2} + \frac{\bar{k}_1 \cdot \bar{k}_3}{2 \sqrt{3} \text{fr}^2} + \frac{\bar{k}_1 \cdot \bar{k}_4}{2 \sqrt{3} \text{fr}^2} + \frac{\bar{k}_2 \cdot \bar{k}_3}{2 \sqrt{3} \text{fr}^2} + \frac{\bar{k}_2 \cdot \bar{k}_4}{2 \sqrt{3} \text{fr}^2} - \frac{\bar{k}_3 \cdot \bar{k}_4}{\sqrt{3} \text{fr}^2}, \right. \\
 \left. \frac{2(\bar{k}_1 \cdot \bar{k}_2)}{3 \text{fr}^2} - \frac{4(\bar{k}_1 \cdot \bar{k}_3)}{3 \text{fr}^2} + \frac{2(\bar{k}_1 \cdot \bar{k}_4)}{3 \text{fr}^2} + \frac{2(\bar{k}_2 \cdot \bar{k}_3)}{3 \text{fr}^2} - \frac{4(\bar{k}_2 \cdot \bar{k}_4)}{3 \text{fr}^2} + \frac{2(\bar{k}_3 \cdot \bar{k}_4)}{3 \text{fr}^2} \right\}$$

Future Plan: Main Goals

Design and write code for necessary Operators to perform

Calculations of theories similar to Electroweak theory.

Automate the process of spontaneous symmetry breaking^{5;9}.

In order to do this we will use multiplicative renormalization scheme.⁴.

Operators of FeynArtsHelper

Operators	Description
GetCounterTermLagrangian	Gives us the counter-term lagrangian.
FAHReplace	Help us make the expression more readable.
FAHFeynmanRules	Gives us couplings and non-inverted propagator from a Lagrangian.
FAHCTFeynmanRules	Gives us couplings and non-inverted propagator from a counter-term Lagrangian.
GetCouplingVector	Help us extract couplings from a Lagrangian and counter-term Lagrangian.
WriteCV	Write coupling vector to FeynArts friendly text file.
GetPropagator	Invert the non-inverted propagators.
GenPropInternal	Replaces the output into a FeynArts friendly output.
WriteGenProp	Write Analytical Propagator to FeynArts friendly text file.
WriteGENOutput	Write Lorentz part coupling vector to FeynArts friendly text file.

Conclusion

The package, FeynArtsHelper works for $U(1)$ and $SO(3)$ gauge groups and their extensions.

The automation of the electroweak processes will be another milestone for the package, since we will be make sure that the package works for a well-established spontaneous symmetry breaking model.

Once the code is written, we will be able to include this mechanism in our subsequent models to search for New Physics (NP).

We will reproduce known results to show the package works.

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Bibliography I

- [1] Aleksejevs, A. and Barkanova, S. (2019). Dynamic Structure of Hadrons in ChPT. *PoS, LeptonPhoton2019*:125.
- [2] Hahn, T. (2000). Automatic loop calculations with feynarts, formcalc, and looptools. *Nuclear Physics B - Proceedings Supplements*, 89(1-3):231–236.
- [3] Hann, T. (2001). Generating feynman diagrams and amplitudes with feynarts 3. *Computer Physics Communications*, 140(3):418–431.
- [4] Hollick, W. F. (1988). Radiative corrections in the standard model and their role for precision tests of the electroweak theory. *Deutsches Elektronen-Synchrotron (DESY)*, pages 16–31.
- [5] Michael E. Peskin, D. V. S. (1995). *An Introduction to Quantum Field Theory*. Cambridge University Press.
- [6] Patel, H. H. (2015). Package-x: A mathematica package for the analytic calculation of one-loop integrals. *Computer Physics Communications*, 197:276–290.

Bibliography II

- [7] Reefat (2021). Documentation of the package: FeynArtsHelper. <https://reefat96.github.io/FeynArtsHelper/>.
- [8] R.Mertig, M.Bohm, A. (1991). Feyn calc - computer-algebraic calculation of feynman amplitudes. *Computer Physics Communications*, 64(3):345–359.
- [9] Weinberg, S. (1995). *The Quantum Theory of Fields*. Cambridge University Press.