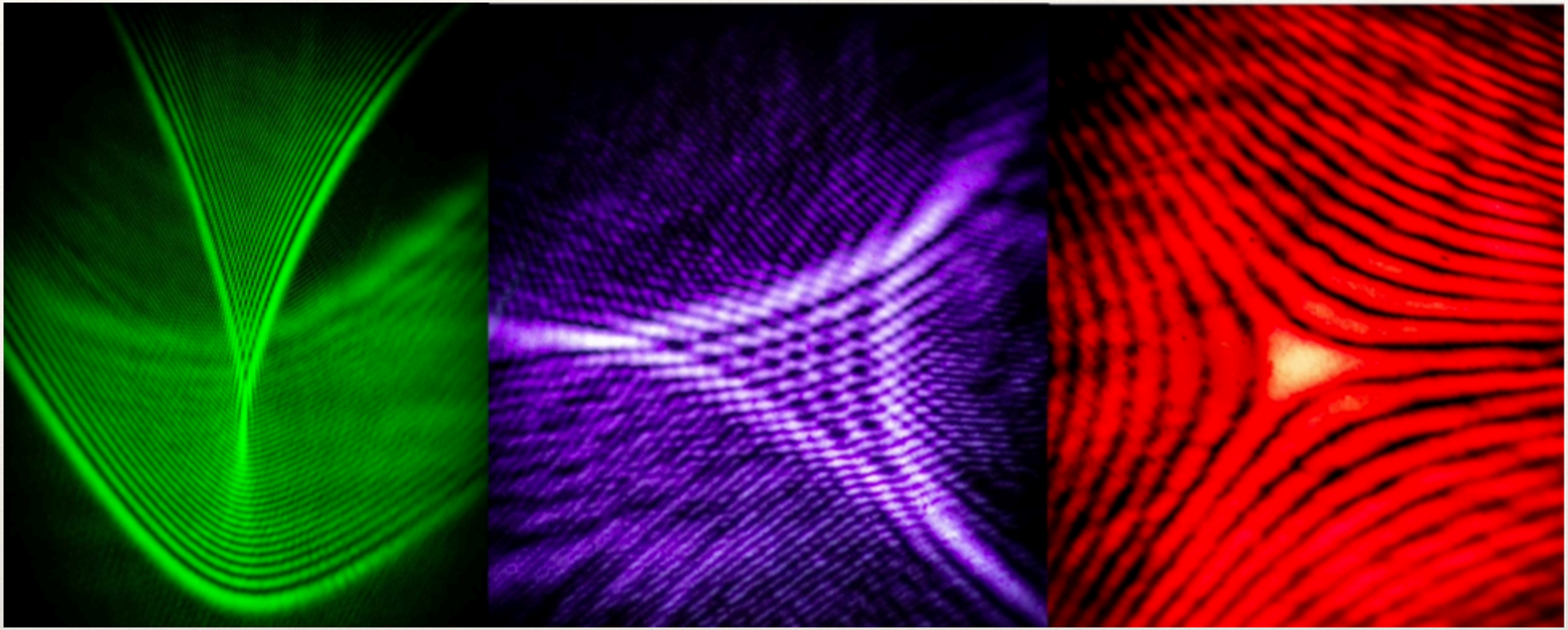


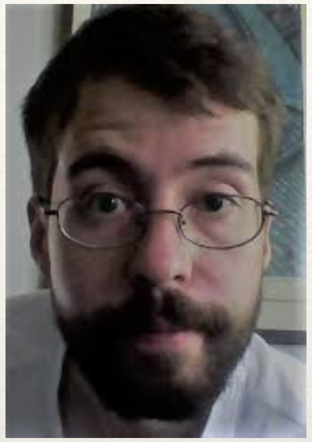
Quantum caustics in many body dynamics

Duncan O'Dell
McMaster University

CAP, 6 June 2022

The Team

Wyatt Kirkby
(—> Innsbruck)



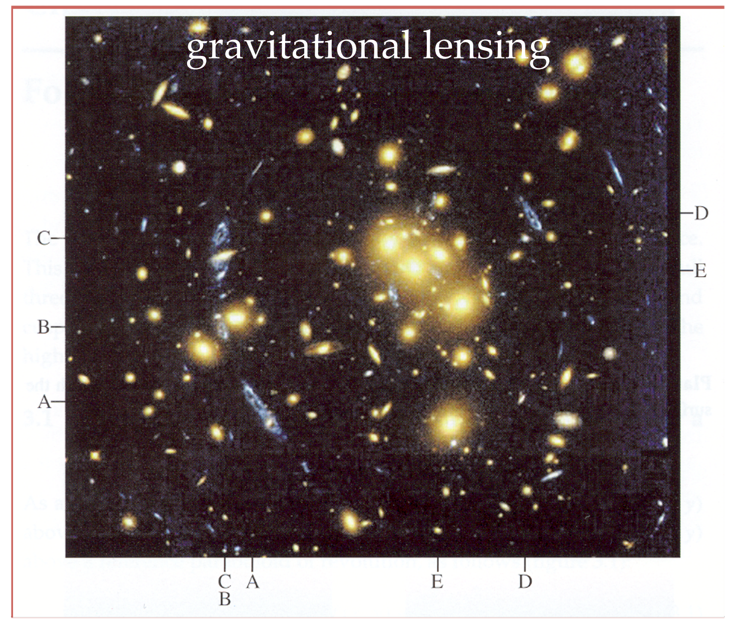
Jesse Mumford
(—> Red Deer College)

Aman Agarwal
(—> Perimeter Institute)

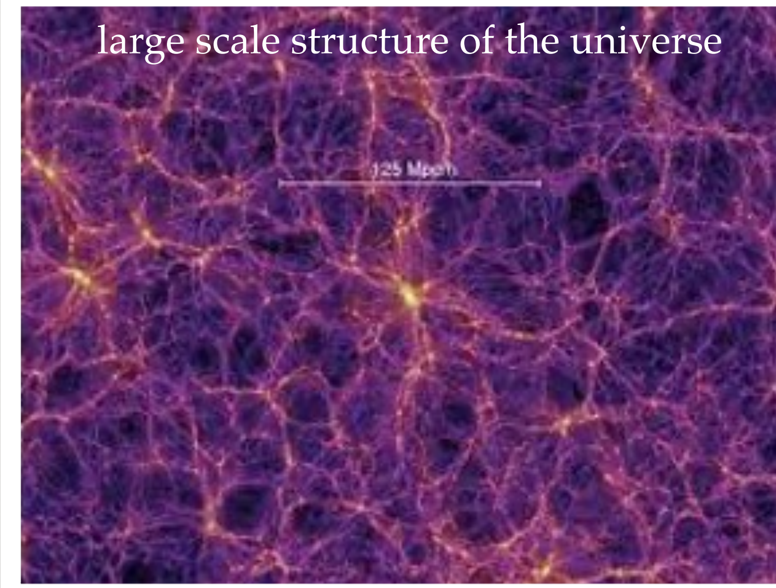
Manas Kulkarni
International Centre for Theoretical Sciences (Bengaluru)

Natural focusing: caustics

Descartes 1637



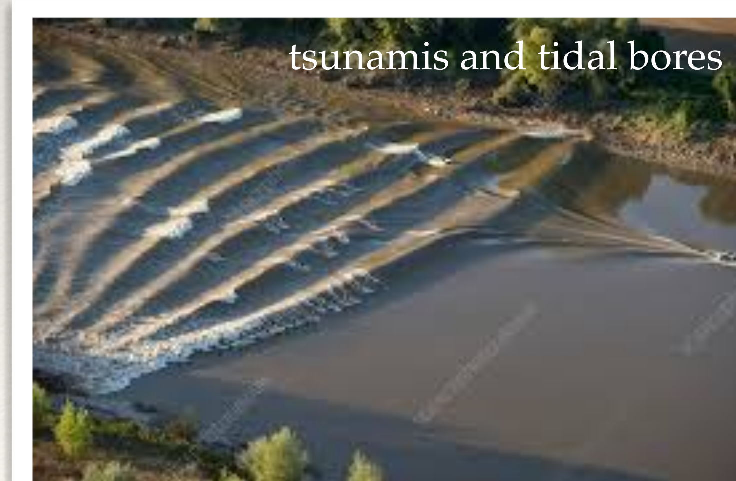
Einstein 1912



Zeldovich 1981

Kelvin 1905

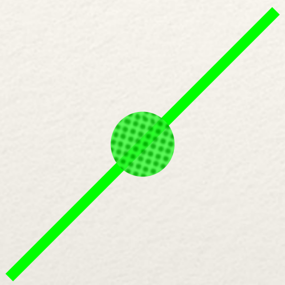
Pelinovsky 2003



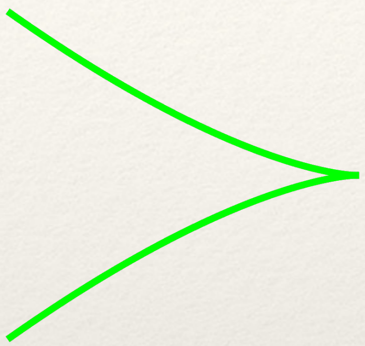
Berry 2005

Catastrophe theory: structurally stable singularities

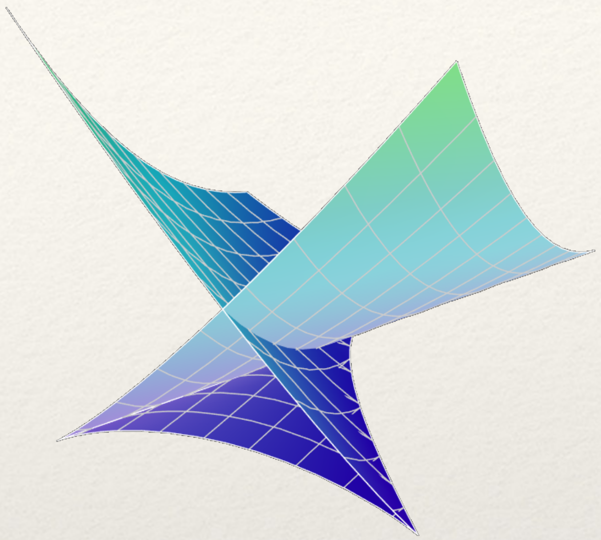
K = codimension



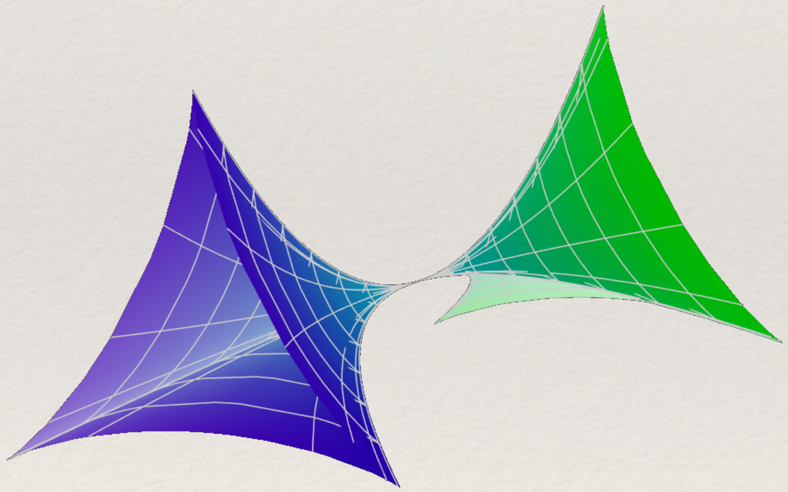
Fold (K=1)



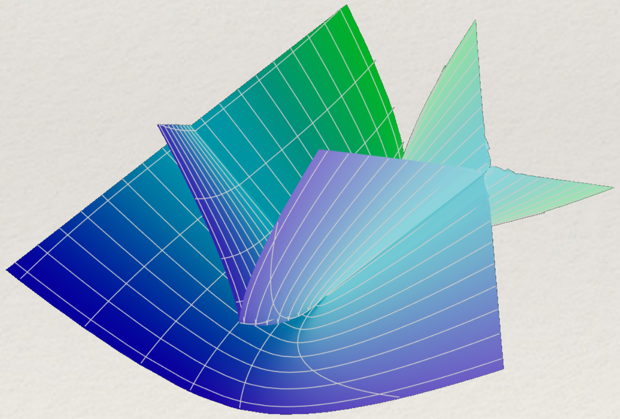
Cusp (K=2)



Swallowtail (K=3)



Elliptic Umbilic (K=3)



Hyperbolic Umbilic (K=3)

Each surface represents a bifurcation where the number of solutions changes (rays are born/die)

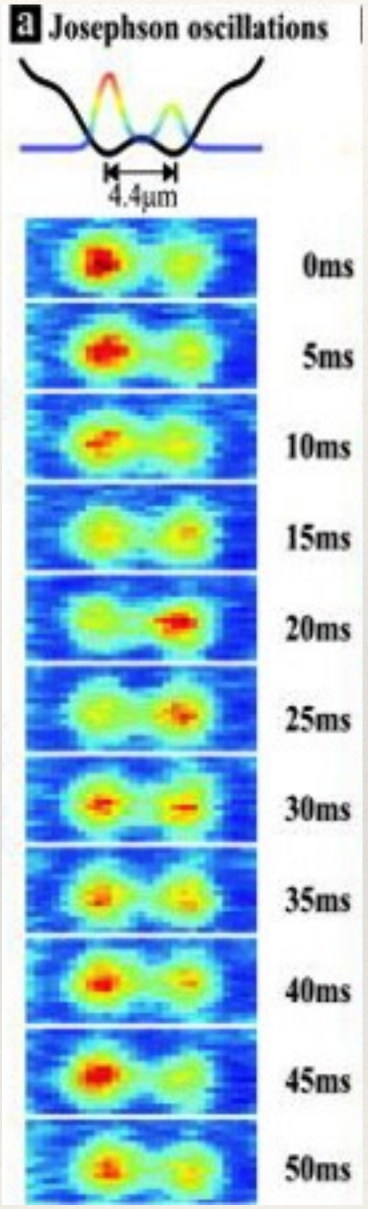
Quantum caustic example 1: Bosonic Josephson junctions

BEC in a double well potential

$$H = \frac{E_c}{2} n^2 - E_J \cos\phi$$

where: $n \equiv \frac{1}{2}(n_l - n_r)$, $\phi \equiv \phi_l - \phi_r$

number difference **phase difference**



$$\dot{\phi} = \frac{E_c}{\hbar} n$$

$$\dot{n} = -\frac{E_J}{\hbar} \sin\phi$$

phase difference, number difference } Josephson's equations

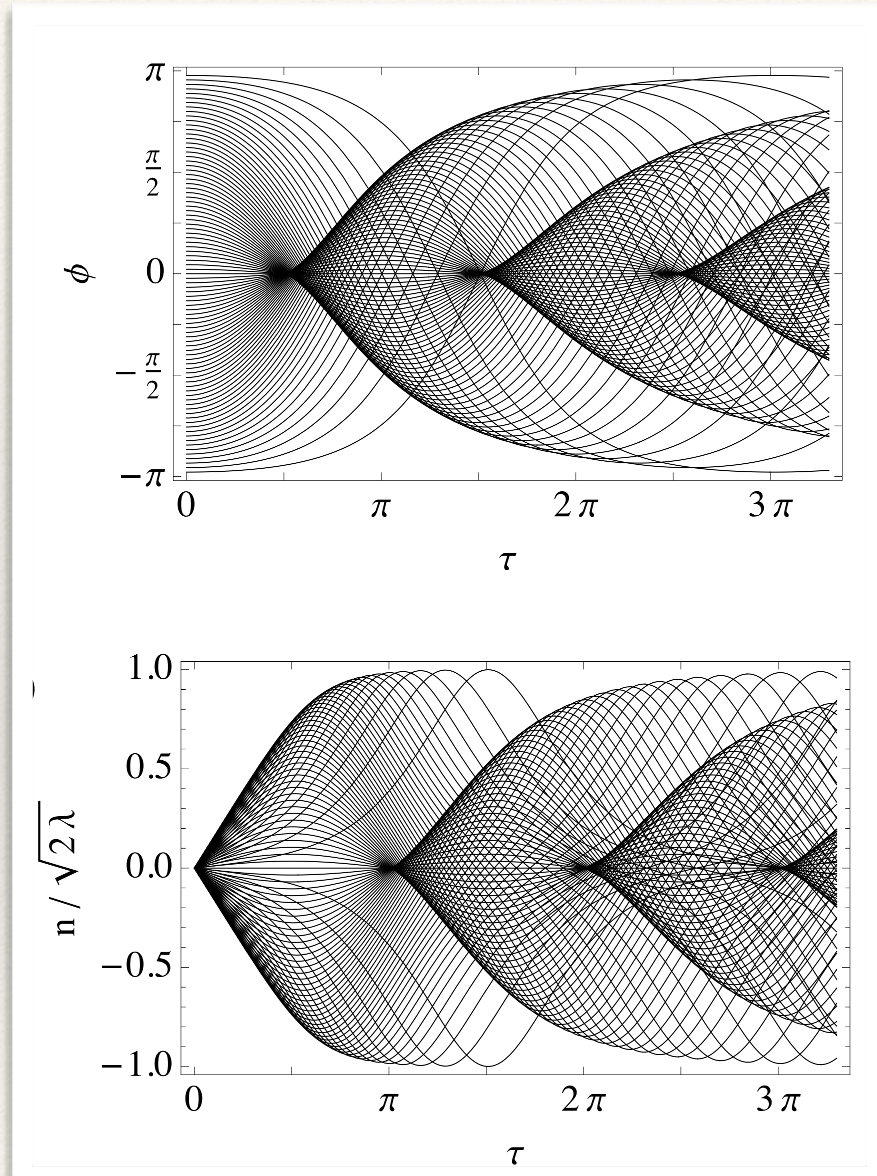
Albiez et al, PRL **95** 010402 (2005)

Classical field dynamics following a quench

Sudden connection of two independent BECs (initial state = Fock state)

phase difference

number difference



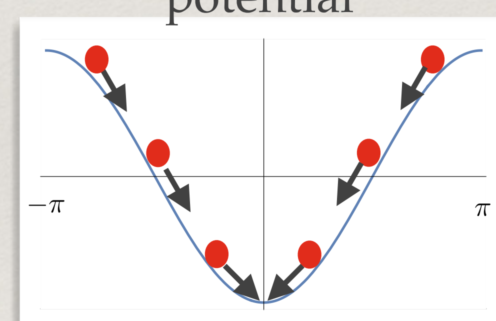
time

$\lambda \equiv 2E_J/E_c$

truncated Wigner approximation:

quantum: $[\hat{\phi}, \hat{n}] \approx i$

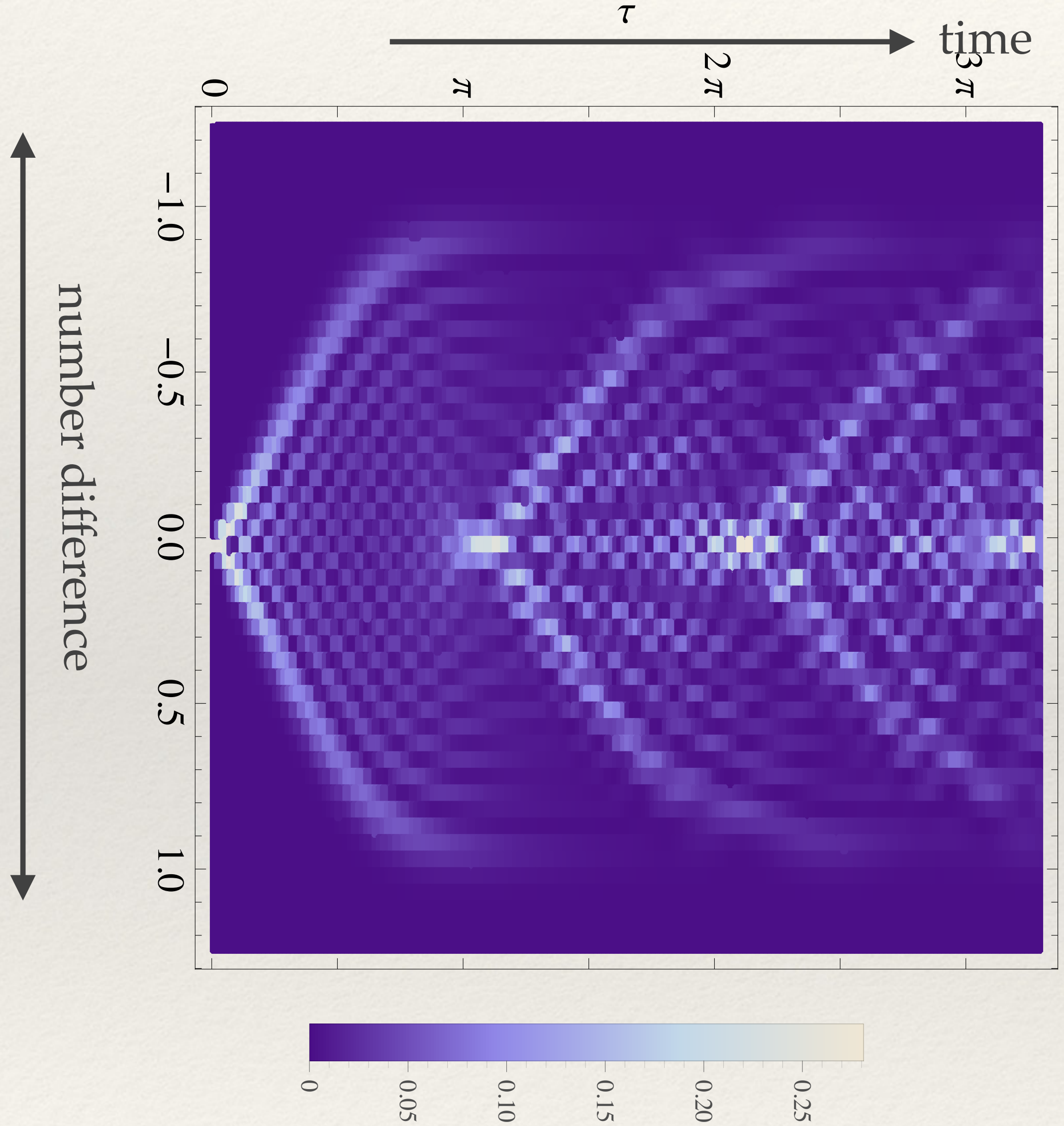
meanfield potential



phase difference

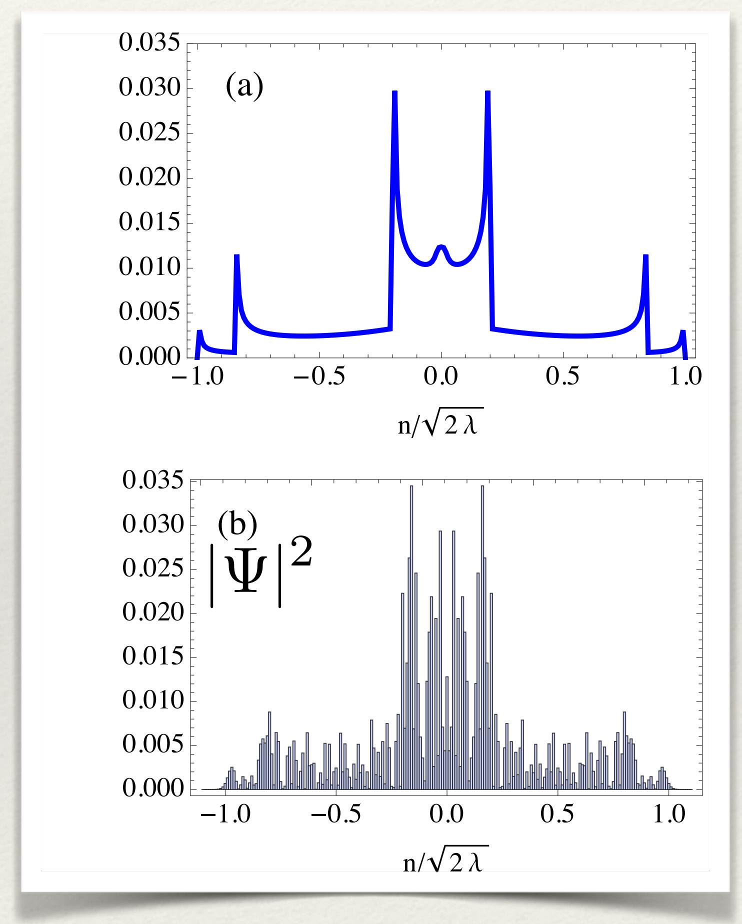
Quantum field dynamics following a quench

$$\lambda \equiv \frac{2E_J}{E_c} = 200$$



$$\lambda \equiv \frac{2E_J}{E_c} = 5000$$

$$\tau = 3.3\pi$$



Classical field
(Josephson equations)

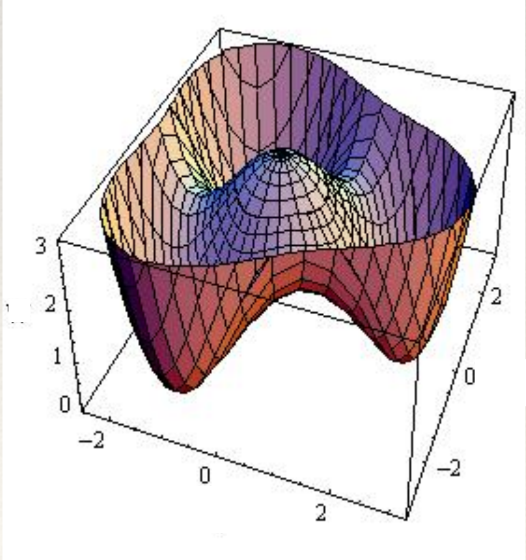
Quantum field
(Bose-Hubbard theory)

Cusps in Fock space: meanfield singularities are regularized by second-quantization

→ **Quantum catastrophes**

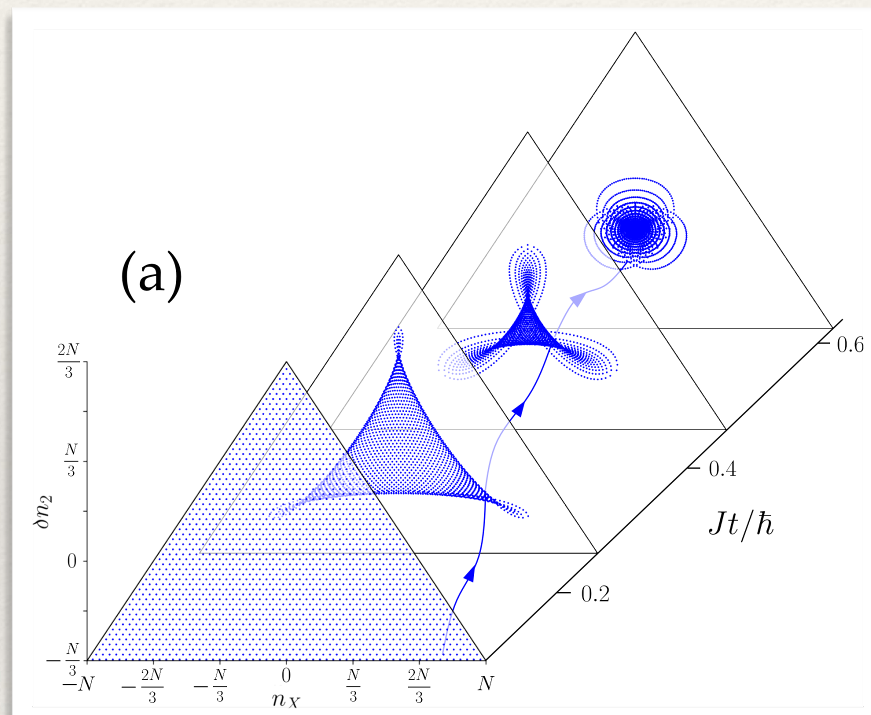
D.O'D, Phys. Rev. Lett. **109**, 150406 (2012)

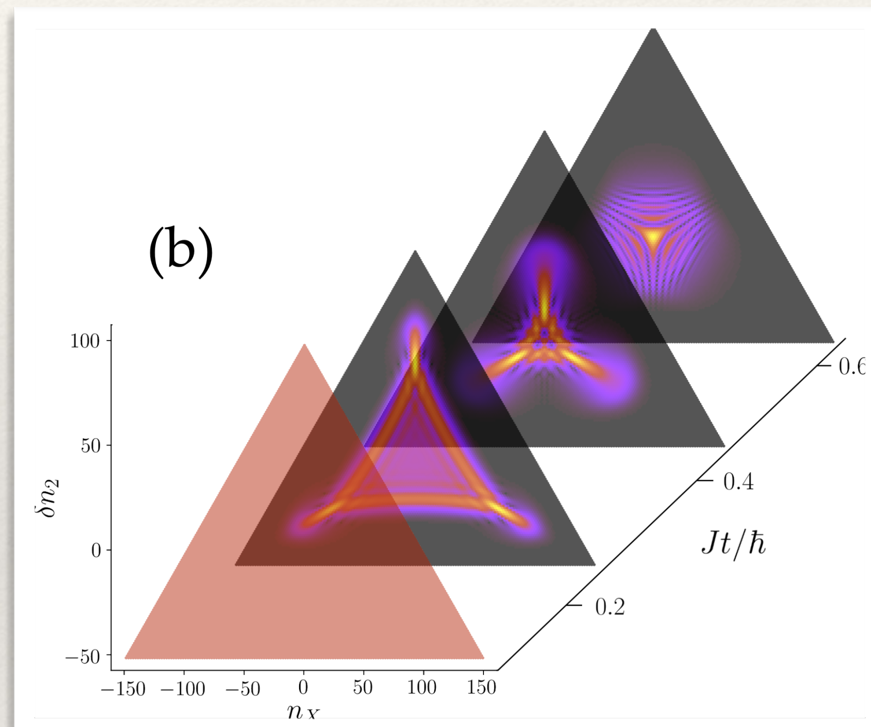
Quantum caustic example 2: BEC in a triple well



Triple well: elliptic umbilic

$$\hat{H} = -K_L(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) - K_R(\hat{a}_2^\dagger \hat{a}_3 + \hat{a}_3^\dagger \hat{a}_2) - K_X(\hat{a}_3^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_3) + \frac{U}{2}\sum_{i=1}^{3} \hat{n}_i(\hat{n}_i - 1) + \sum_{i=1}^{3} \epsilon_i \hat{n}_i$$





initial condition: uniform distribution in Fock space

N=150

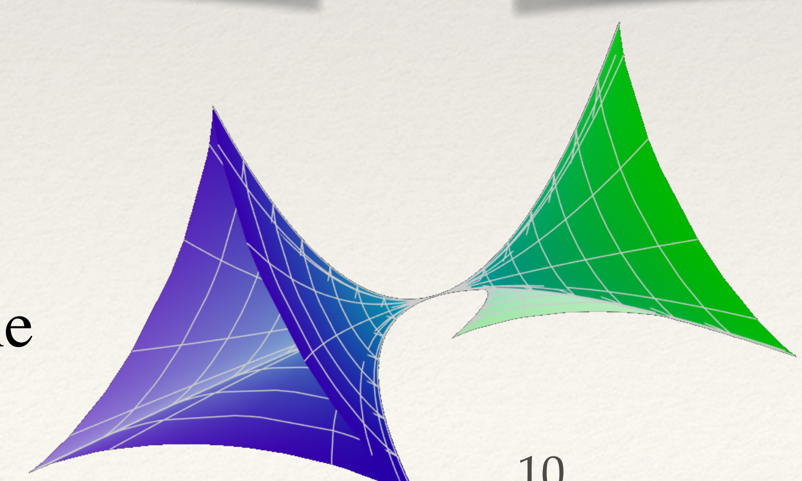
$K_L = K_R = K_X = J$

$J = 0.01U$

$\delta n_2 = n_2 - N/3$

$n_X = n_1 - n_3$

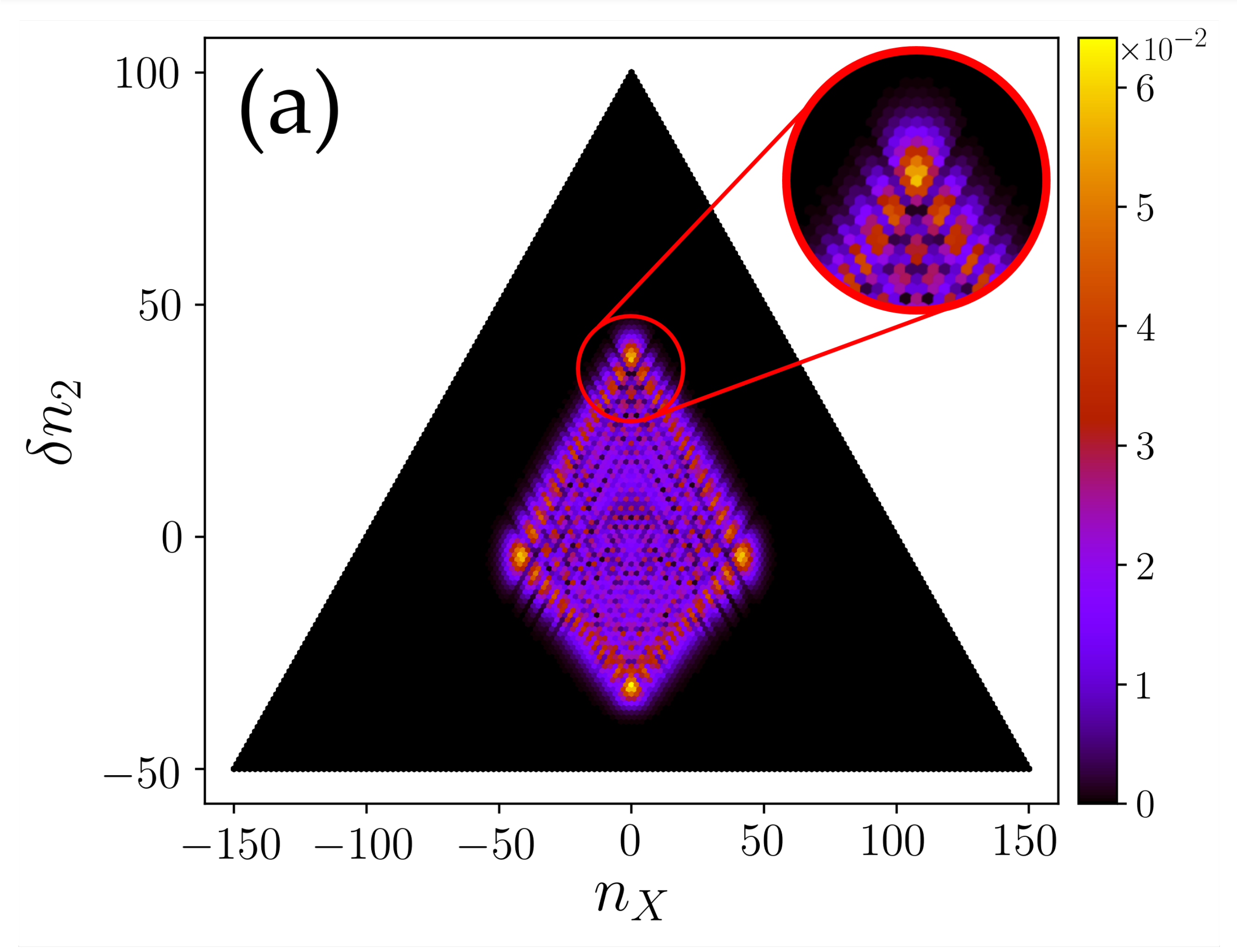
Elliptic Umbilic Catastrophe
(K=3 catastrophe)



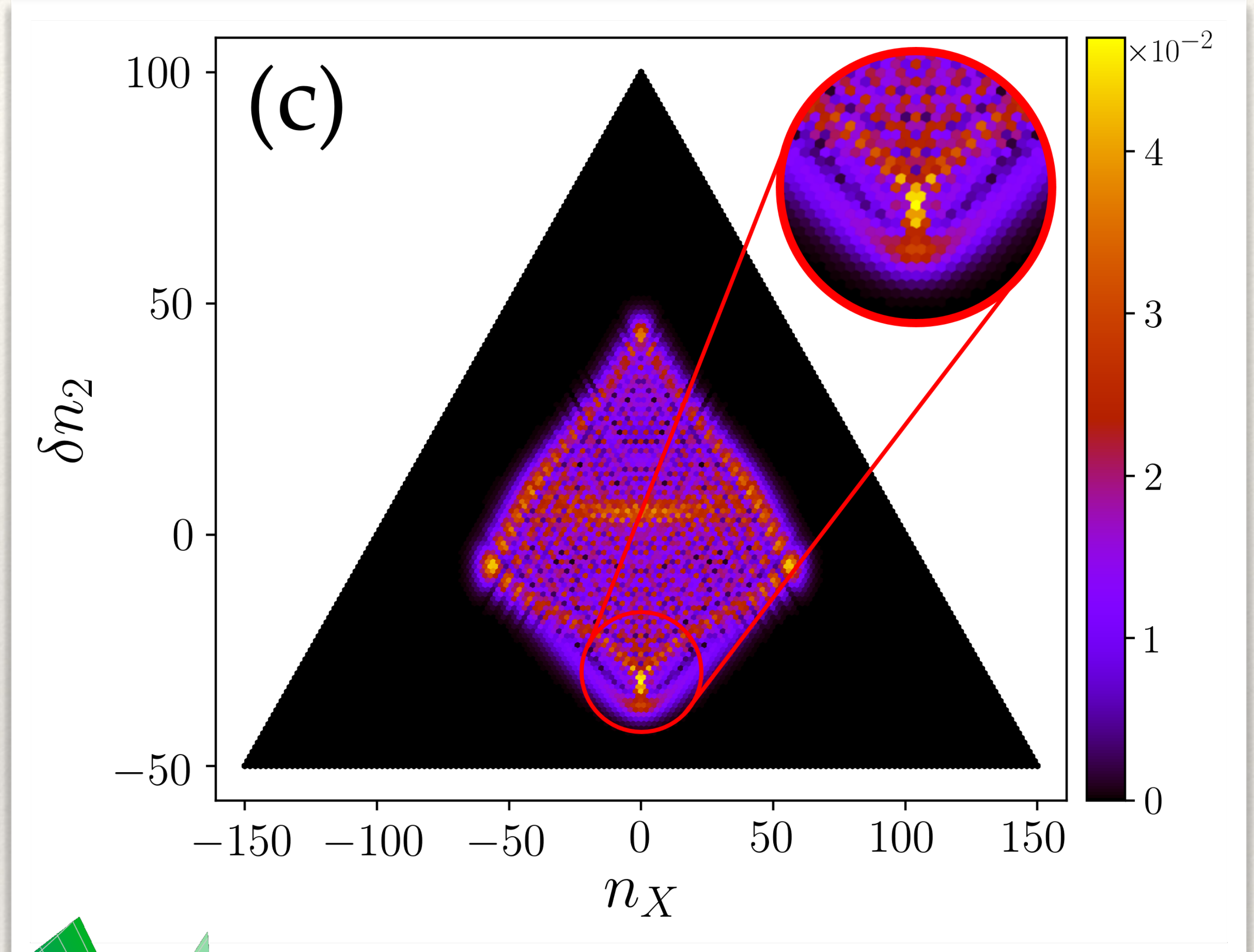
2D Fock space allows higher catastrophes!

W. Kirkby, Y. Yee, K. Shi and D. O'D Phys. Rev. Research **4**, 013105 (2022)

Triple well: hyperbolic umbilic



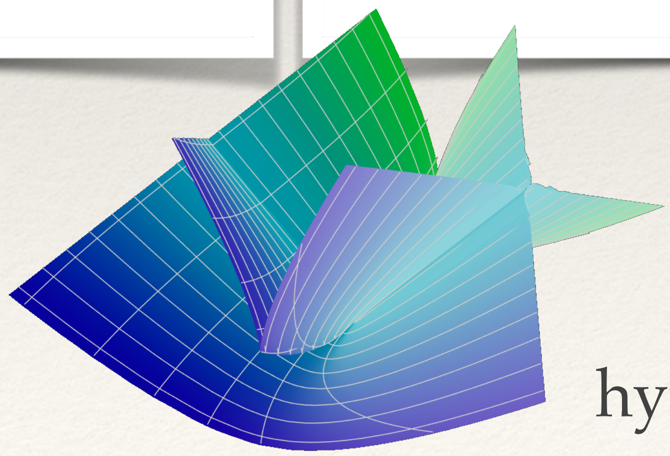
$Jt/\hbar = 0.24$



$Jt/\hbar = 0.34$

N=150
$K_L = K_R = J = 4U$
$K_X = 0$

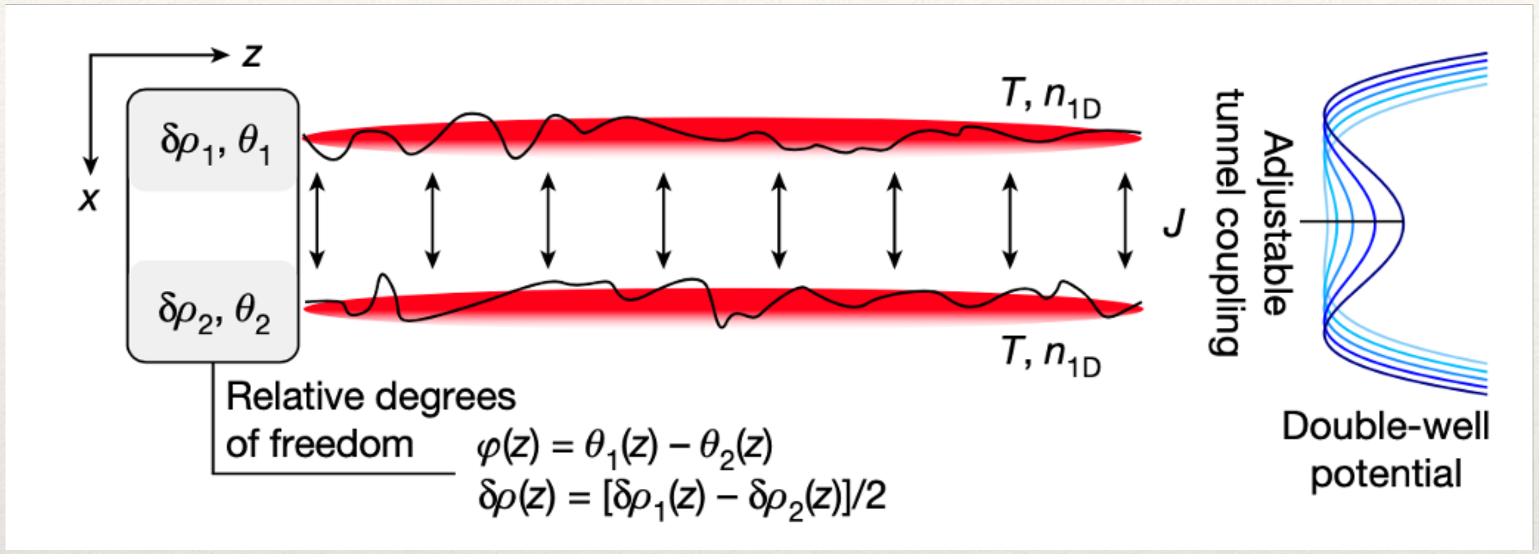
initial condition: single Fock state centered at 0,0



hyperbolic umbilic

Quantum caustic example 3: coupled 1D superfluids

Two coupled 1D superfluids



T. Schweigler, V. Kasper, S. Erne, I. Mazets, B. Rauer, F. Cataldini, T. Langen, T. Gasenzer, J. Berges & J. Schmiedmayer, Nature **545**, 323 (2017)

Luttinger parameter

$$K = \sqrt{\frac{n_{1D}(\hbar\pi)^2}{4 g_{1D} m}}$$

sine-Gordon model

$$H_{\mathrm{SG}} = \int dz \left[\Gamma \rho^2 + \epsilon \left(\frac{\partial \phi}{\partial z} \right)^2 - 2\mathcal{J} \cos(\phi) \right]$$

$$\frac{d\phi(z)}{dt} = 2\Gamma \rho(z)$$

$$\frac{d\rho(z)}{dt} = 2\epsilon \frac{\partial^2 \phi(z)}{\partial z^2} - 2\mathcal{J} \sin[\phi(z)]$$

phase difference

number difference

Modified Josephson's equations

$$\Gamma = \frac{\pi}{2K}$$

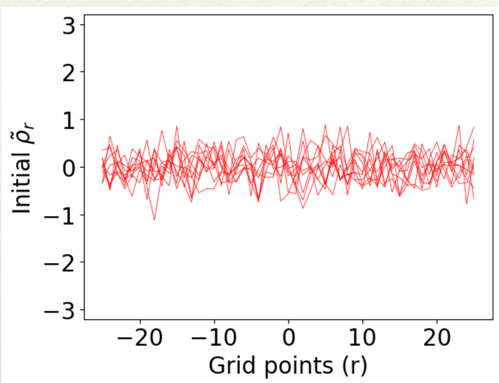
$$\epsilon = \frac{K}{2\pi}$$

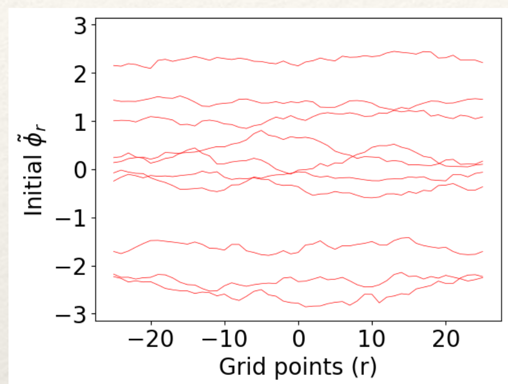
$$\mathcal{J} = \frac{K}{2\pi} \frac{\xi_h^2}{\xi_s^2}$$

Thermal initial conditions

density

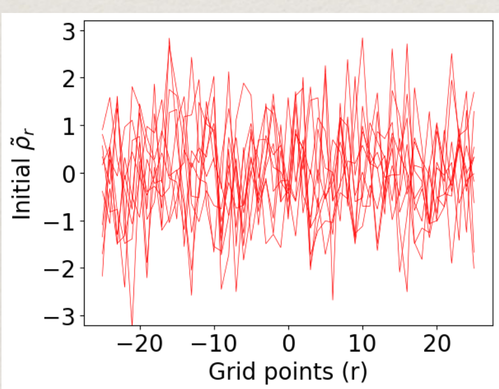
phase

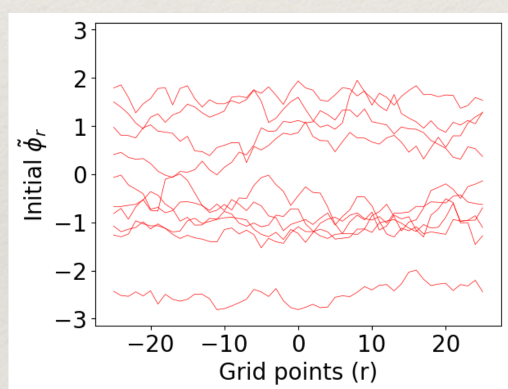




T=2 x 10^{-9} K

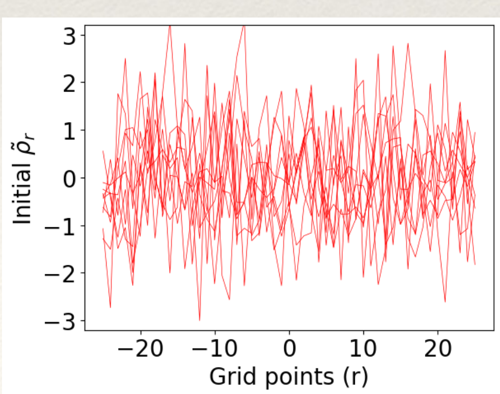
K=25

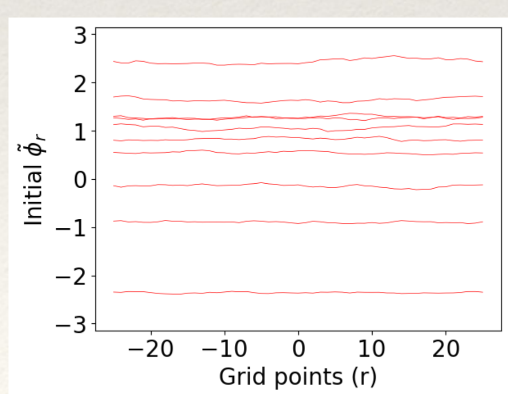




T=2 x 10^{-8} K

K=25





T=2 x 10^{-9} K

K=250

Tomonaga-Luttinger theory

$$H_{\mathrm{TL}} = \frac{ac\hbar}{2} \sum_{k=-N_L/2}^{N_L/2} \left[\frac{K}{\pi} \frac{4\pi^2 k^2}{L^2} |\varphi_k|^2 + \frac{\pi}{K} |\varrho_k|^2 \right]$$

(Fourier space)

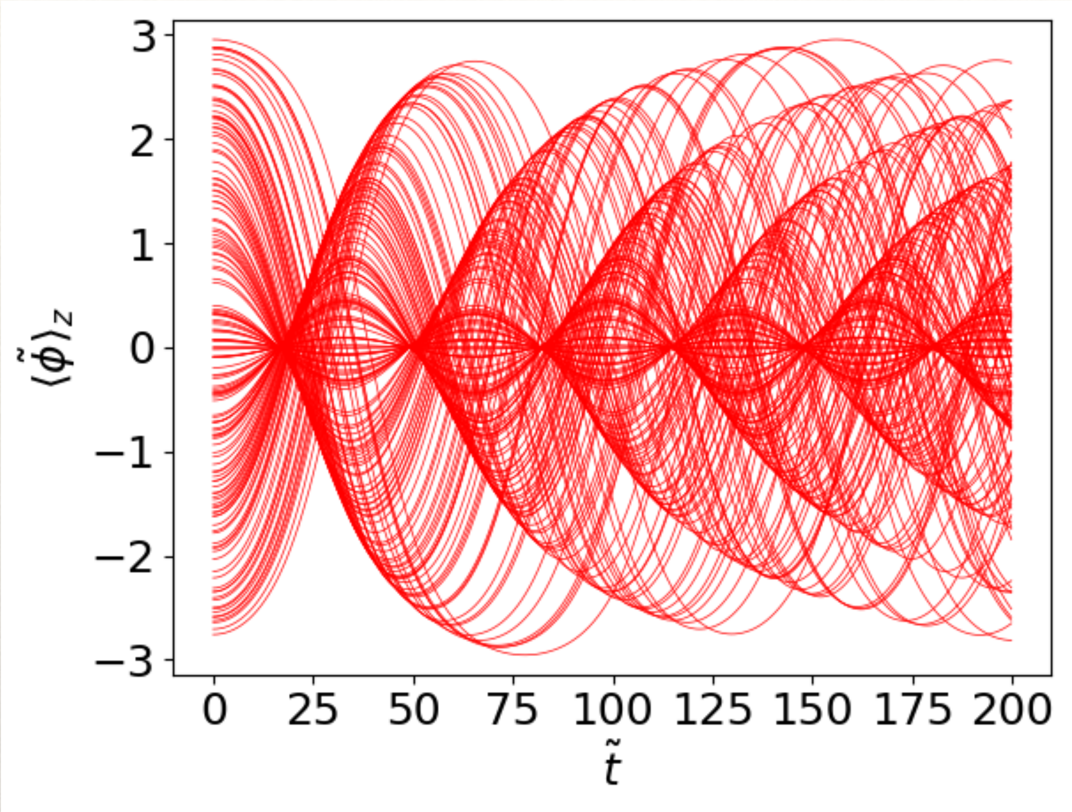
quasi-condensate regime:
K>1

Symbol	Parameter	Value
$\omega_\perp$	trapping frequency	$2\pi \times 3$ KHz
m	mass of Rb atom	1.41×10^{-25} Kg
a_{scat}	scattering length	98×0.52 [91]
N	Number of atoms	1200
L	System Length	18 μ m
n_{1D}	Average Density	$6.7 \times 10^7 m^{-1}$
g_{1D}	$2\ \hbar a_{\mathrm{scat}} \omega_\perp$	2×10^{-38} J/m
K	Luttinger parameter	25
T$*$	Temperature	$10^{-7}\ -\ 10^{-9} K$
$J*$	J-quench	0 - 50 Hz
N_L*	Number of grid points	50 - 100

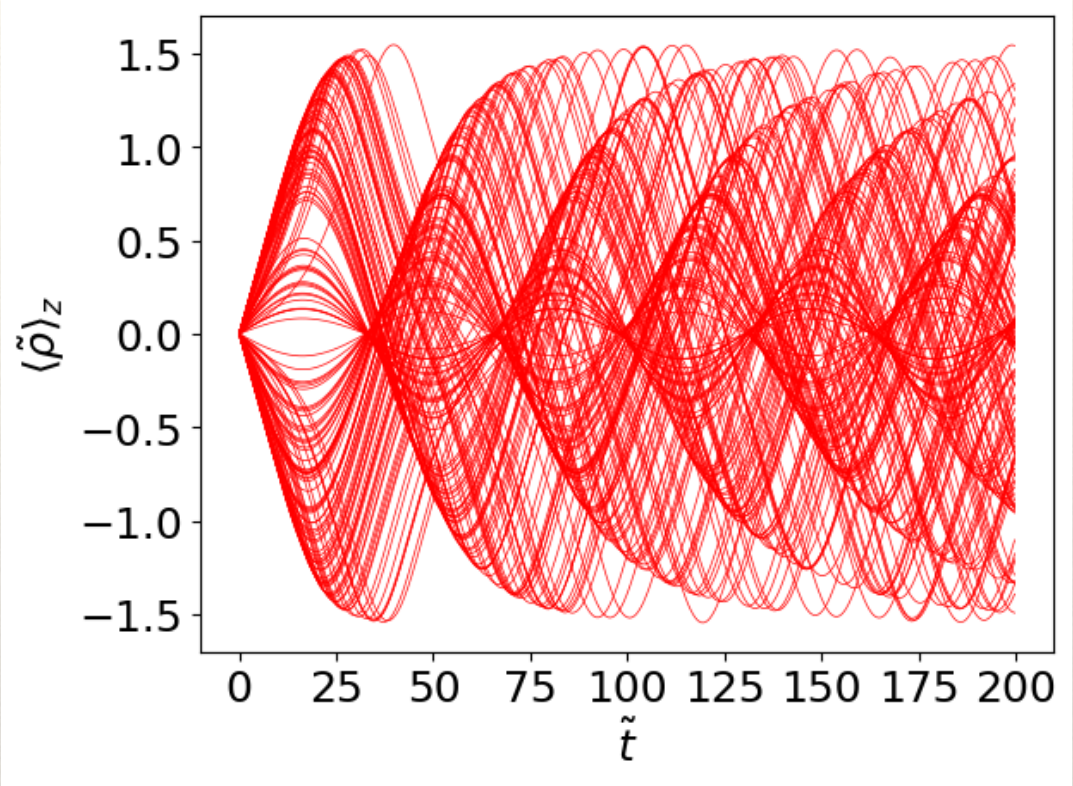
$$K = \sqrt{\frac{n_{1D}(\hbar\pi)^2}{4 g_{1D} m}}$$

Dynamics following sudden connection of two 1D superfluids

phase difference (averaged over z)

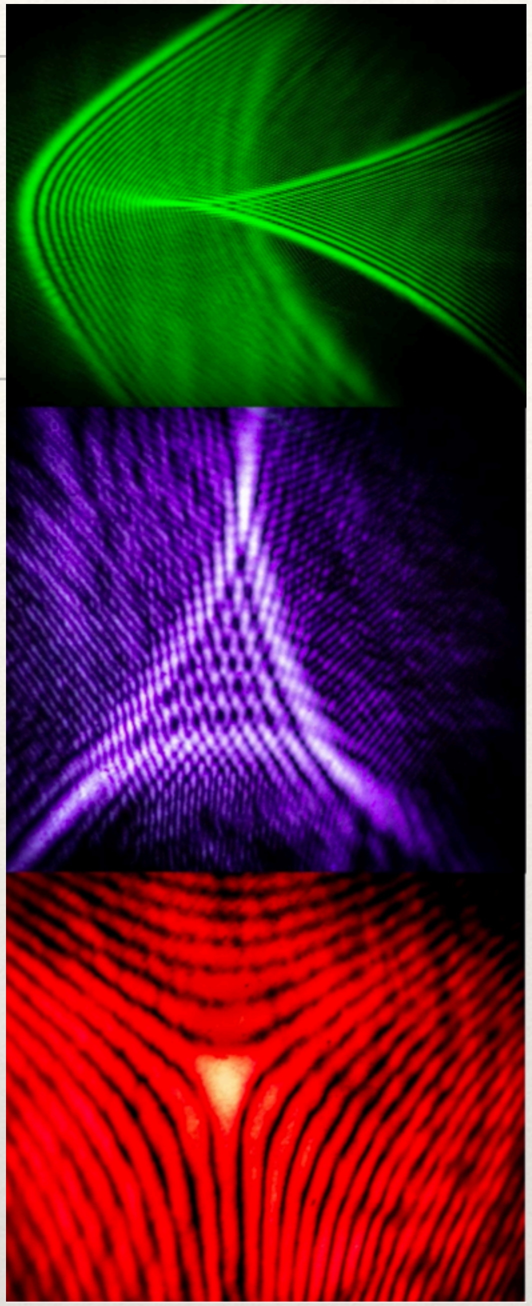


density difference (averaged over z)



Each trajectory is a different set of initial conditions (sampled from thermal distribution)

Summary



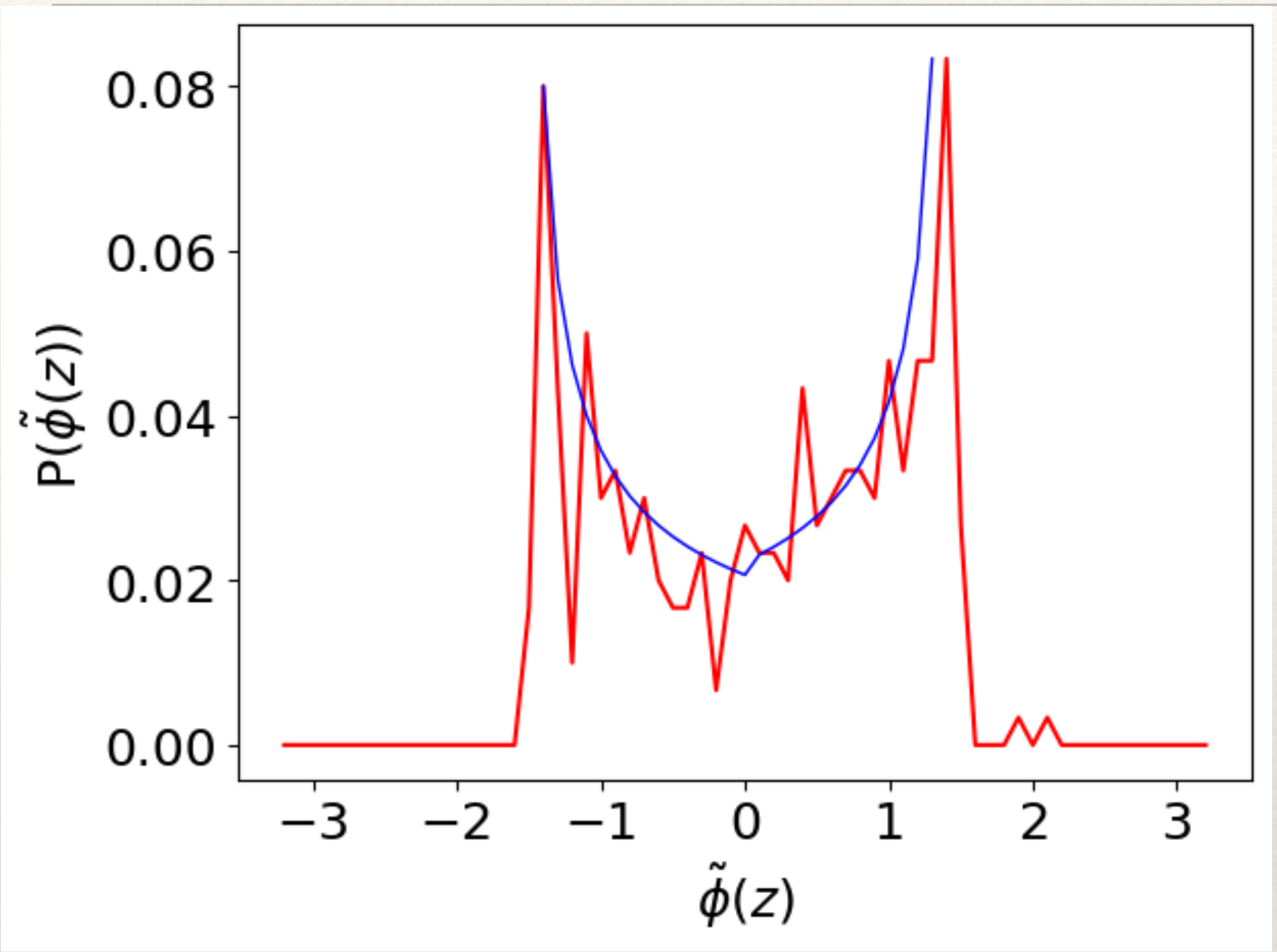
- Dynamics following quenches lead to **caustics** (in Fock or real space)
- Universality in quantum dynamics!
- Structural stability
- Strong fluctuations (nongaussian)
- Underlying mathematical description is catastrophe theory

See posters at Tuesday's poster session 17:30-19:00

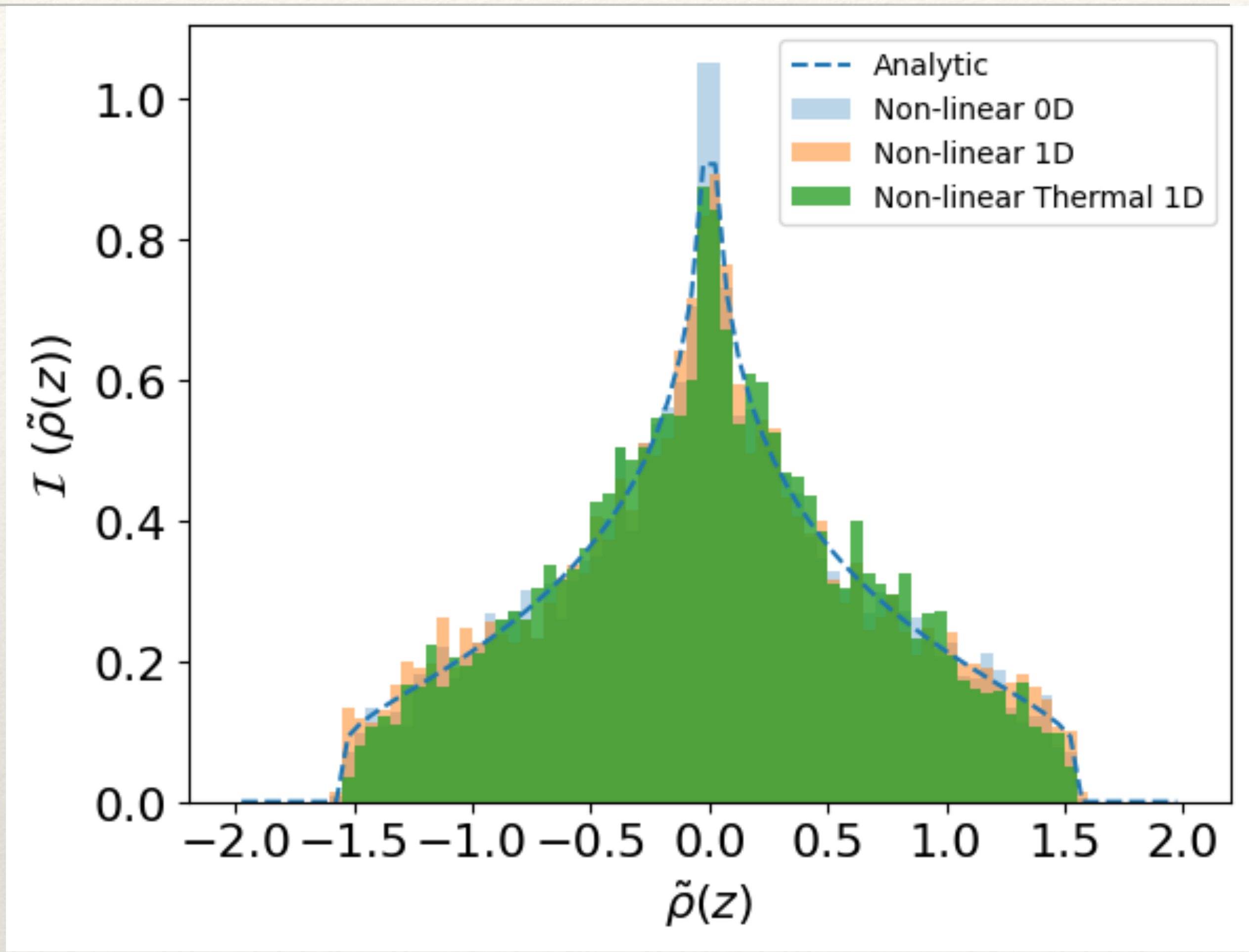
Denise Kamp "*Quantum catastrophes in a rotating BEC*" [poster 3]

Liam Farrell "*Logarithmic catastrophes and Stokes's phenomenon in waves at horizons: Hawking radiation*" [poster 23]

Signature of caustics in non-thermal probability distribution



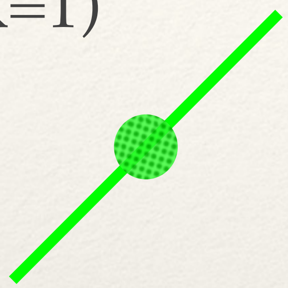
short time ($\tilde{t} = 32$)



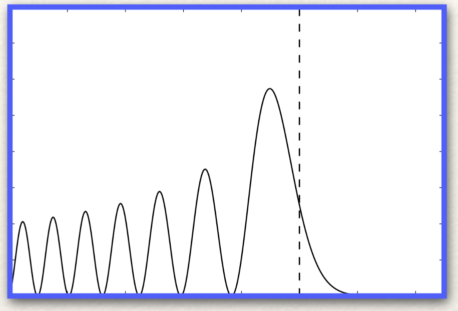
long time ($\tilde{t} = 500$)

Wave Catastrophes

Fold (K=1)

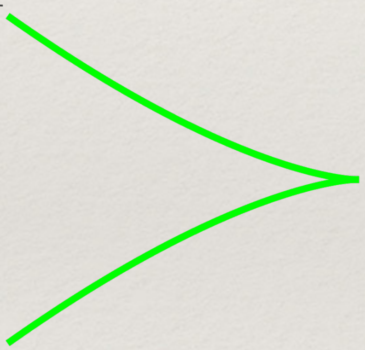


$$\Psi_{\text{fold}}(C) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(s^3/3 + Cs)}\, ds$$
$$= \sqrt{2\pi}\,\text{Ai}[C]$$

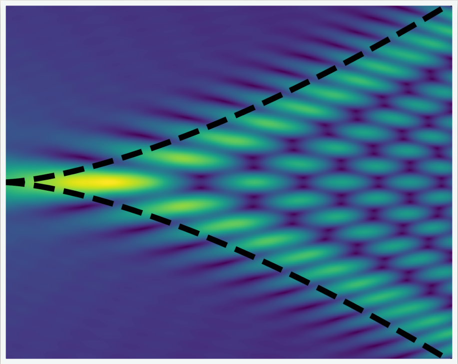


G.B. Airy, Trans. Camb. Phil. Soc. **6**, 379 (1838)

Cusp (K=2)



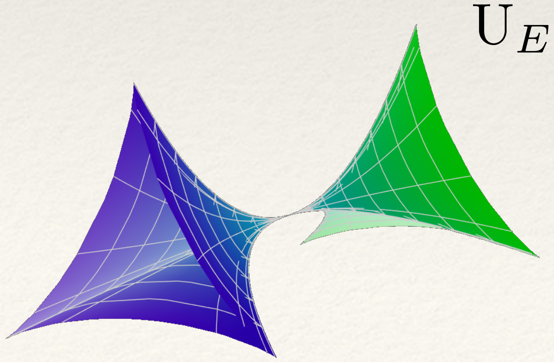
$$\Psi_{\text{cusp}}(C_1, C_2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i(s^4/4 + C_2 s^2/2 + C_1 s)} ds$$



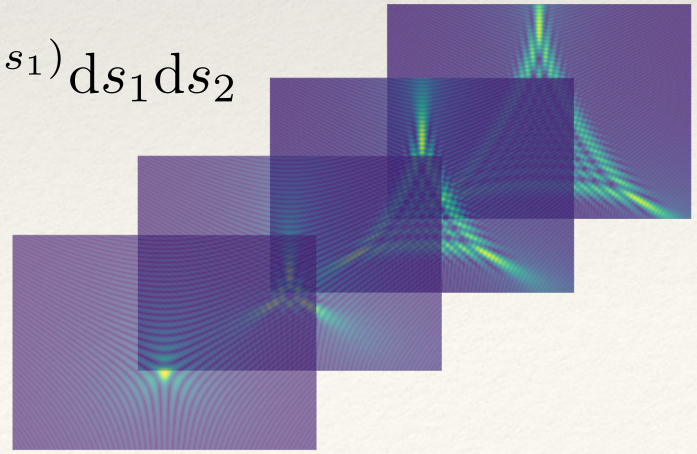
T. Pearcey, Phil. Mag. **37**, 311 (1946)

Elliptic Umbilic (K=3)

$$\mathrm{U}_E(C_1, C_2, C_3) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}(s_1^3 - 3 s_1 s_2^2 - C_3(s_1^2 + s_2^2) - C_2 s_2 - C_1 s_1)} \mathrm{d}s_1 \mathrm{d}s_2$$



Berry, Nye & Wright, Phil. Trans. R. Soc. A. **291**, 453 (1979)



Branched flow

Josh Hainge

Kicked rotor