The large-charge expansion for multiple charges

Jahmall Bersini
Ruđer Bošković Institute, Croatia

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Based on:

O. Antipin, JB, F. Sannino, Z. Wang, C. Zhang, *Phys.Rev.D* 102 (2020) 12, 125033
O. Antipin, JB, F. Sannino, Z. Wang, C. Zhang, *Phys.Rev.D* 103 (2021) 12, 125024
O. Antipin, JB, P. Panopoulos, *to appear*

The model

$$\mathcal{L} = -\frac{1}{2} Tr(F^{\mu\nu} F_{\mu\nu}) + Tr(\bar{\Psi} i \gamma_\mu D^\mu \Psi) + y Tr(\bar{\Psi}_L \Phi \Psi_R + \bar{\Psi}_R \Phi^\dagger \Psi_L)$$
$$+ Tr(\partial_\mu \Phi^\dagger \partial^\mu \Phi) - u \left[Tr(\Phi^\dagger \Phi) \right]^2 - v Tr(\Phi \Phi^\dagger \Phi \Phi^\dagger)$$

[F. Sannino, D. Litim, *Asymptotic safety guaranteed*, 2014]

$SU(N_C)$ gauge fields, N_f Dirac fermions in the fundamental of $SU(N_C)$, one $N_f x N_f$ complex matrix scalar field in the $(N_f, \bar{N}_f)$ of $U(N_f)xU(N_f)$

Four dimensional perturbative UV fixed point in the Veneziano limit:

$$N_f \to \infty \,, \quad N_c \to \infty \,, \quad z \equiv \frac{N_f}{N_c} = \text{ fixed}$$

$$\alpha_g^* = \frac{26}{57}\epsilon \,, \quad \alpha_y^* = \frac{4}{19}\epsilon \,, \quad \alpha_h^* = \frac{\sqrt{23}-1}{19}\epsilon \,, \quad \alpha_v^* = \frac{1}{19}\left(\sqrt{20+6\sqrt{23}} - 2\sqrt{23}\right)\epsilon$$

$$\epsilon \equiv z - \frac{11}{2}$$

$$\alpha_g = \frac{g^2 N_c}{(4\pi)^2}, \quad \alpha_y = \frac{y^2 N_c}{(4\pi)^2}, \quad \alpha_h = \frac{u N_f}{(4\pi)^2}, \quad \alpha_v = \frac{v N_f^2}{(4\pi)^2}$$

Perturbative realization of the asymptotic safety scenario
UV-complete BSM model building

Motivation

“for studying the large-charge expansion in this model”

1. Structurally similar to the Standard Model

2. One-of-a-kind example of non-supersymmetric 4D CFT

3. Suited to study the impact of the non-abelian structure of the global symmetry group on the large-charge expansion.

Previously considered in: D. Orlando, S. Reffert, F. Sannino, *“A safe CFT at large charge,”* 2020

The large-charge expansion for multiple charges

We charge the global $U(N_f)xU(N_f)$ flavor symmetry.

The charges are encoded in two diagonal N_f x N_f traceless matrices:

$$\mathcal{Q}_L = -V\dot{\Phi}_0\Phi_0^\dagger \qquad \mathcal{Q}_R = V\Phi_0^\dagger\dot{\Phi}_0$$

We can find an homogeneous ground state for every diagonal charge matrices such that $\mathcal{Q}_R + \mathcal{Q}_L = 0$

$$\boxed{\Phi_0\left(\tau\right) = e^{2iM\tau}B} \qquad B_{ii} = b_i \quad M_{ii} = -i\mu_i$$

$$\mathcal{Q} = \mathcal{Q}_L = Q\text{diag}(q_1, q_2, \ldots, q_{N_f})$$

Q → large expansion parameter

{qi} → O(1) parameters specifying the charge configuration.

Varying the $\{q_i\}$ we have the scaling dimension of operators transforming according to a variety of irreducible representations.

Double-scaling limit

$$Q \to \infty, \quad \kappa_I \to 0 \quad \text{with} \quad Q\kappa_I = (\text{fixed}) \qquad\qquad \kappa_I = \{\alpha_h, \alpha_v, \alpha_y, \alpha_g\}$$

We compute the scaling dimension of the lowest-lying operator with charge

$$\mathcal{Q} = \mathcal{Q}_L = Q\text{diag}(q_1, q_2, \ldots, q_{N_f})$$

Semiclassical expansion

$$\Delta_Q = \sum_{j=-1} \frac{1}{Q^j} \Delta_j \left(Q\kappa_I, \{q_i\}\right)$$

Δ_{-1} : classical term

Δ_0 : leading quantum correction

- $Q\kappa_I \ll 1$ ⇨ Perturbation theory
- $Q\kappa_I \gg 1$ ⇨ Large-charge limit – superfluid phase (?)

$$\Delta_Q = Q^{\frac{d}{d-1}} \left[\alpha_1 + \alpha_2 Q^{\frac{-2}{d-1}} + \alpha_3 Q^{\frac{-4}{d-1}} + \ldots\right] + Q^0 \left[\beta_0 + \beta_1 Q^{\frac{-2}{d-1}} + \ldots\right] + \ldots$$

Semiclassics: the LO

We consider a two-parameters family of charge configurations

$$Q_{J,s} = \mathrm{diag}(\underbrace{J, J, \ldots,}_{s} \underbrace{-J, -J, \ldots,}_{s} \underbrace{0, 0, \ldots}_{N_f - 2s})$$

Ground state ansatz

$$\Phi_0(\tau) = e^{2iM\tau} B$$

$$B_{ii} = b_i \qquad b_i = \begin{cases} b & i = 1, \ldots, 2s\,, \\ 0 & i = 2s+1, \ldots, N_f \end{cases}$$

$$M_{ii} = -i\mu_i \qquad \mu_i = \begin{cases} \mu & i = 1, \ldots, s\,, \\ -\mu & i = s+1, \ldots, 2s\,, \\ 0 & i = 2s+1, \ldots, N_f\,, \end{cases}$$

EOM

$$J = 2V\mu\, b^2\,, \qquad 2\mu^2 = (u + 2sv)b^2 + \frac{m^2}{2}$$

Conformal coupling to the Ricci scalar on the cylinder

Solution

$$J\Delta_{-1} = \frac{N_f^2}{72(\alpha_h N_f + 2s\alpha_v)} \frac{s}{2x^{\frac{4}{3}}} \left(\sqrt[3]{3}x^{8/3} - 3x^{4/3} + 6\sqrt[3]{3}x^{2/3} + 2\ 3^{2/3}x^2 + 3^{5/3}\right)$$

$$x \equiv \frac{72J}{N_f^2}(\alpha_h N_f + 2s\alpha_v) + \sqrt{-3 + \left(\frac{72J}{N_f^2}(\alpha_h N_f + 2s\alpha_v)\right)^2}$$

No contribution from fermions and gauge bosons

Fluctuation spectrum

# of DOF	Dispersion relation
$2sN_c$	$\omega_{f\pm}(\ell) = \sqrt{(\mu+\lambda_{f\pm})^2 - \frac{y^2 N_f^2 (m^2-4\mu^2)}{32\pi^2 (N_f\alpha_h + 2s\alpha_v)}}$
$4s(N_f-2s)$	$\omega_1 = \sqrt{J_\ell^2 + 4\mu^2}$
$2(N_f-2s)^2$	$\omega_2 = \sqrt{J_\ell^2 + m_2^2}$
$2s(2N_f-3s)$	$\omega_{3,4} = \sqrt{J_\ell^2 + 4\mu^2} \mp 2\mu$
$2s^2$	$\omega_{5,6} = \sqrt{J_\ell^2 + 4\mu^2 + m_1^2} \pm 2\mu$
$4s^2-2$	$\omega_{7,8} = \frac{1}{\sqrt{2}}\sqrt{2J_\ell^2 + m_1^2 + 16\mu^2 \pm \sqrt{(2J_\ell^2 + m_1^2 + 16\mu^2)^2 - 4J_\ell^2 (J_\ell^2 + m_1^2)}}$
2	$\omega_{9,10} = \frac{1}{\sqrt{2}}\sqrt{2J_\ell^2 + m_0^2 + 16\mu^2 \pm \sqrt{(2J_\ell^2 + m_0^2 + 16\mu^2)^2 - 4J_\ell^2 (J_\ell^2 + m_0^2)}}$

$$m_0^2 = 8\mu^2 - 2m^2\,, \quad m_1^2 = \left(8\mu^2 - 2m^2\right)\frac{u_0}{u_0 + 2sv_0}\,, \quad m_2^2 = 4sv_0b^2 + m^2$$

Fluctuation spectrum

# of DOF	Dispersion relation	
$2sN_c$	$\omega_{f\pm}(\ell)=\sqrt{(\mu+\lambda_{f\pm})^2-\frac{y^2N_f^2\,(m^2-4\mu^2)}{32\pi^2\,(N_f\alpha_h+2s\alpha_v)}}$	→ Fermions; $\lambda_{f\pm}$ → Eigenvalues Dirac operator
$4s(N_f-2s)$	$\omega_1=\sqrt{J_\ell^2+4\mu^2}$	J_ℓ^2 → Eigenvalues of the Laplacian
$2(N_f-2s)^2$	$\omega_2=\sqrt{J_\ell^2+m_2^2}$	
$2s(2N_f-3s)$	$\omega_{3,4}=\sqrt{J_\ell^2+4\mu^2}\mp 2\mu$	→ Type II Goldstone bosons
$2s^2$	$\omega_{5,6}=\sqrt{J_\ell^2+4\mu^2+m_1^2}\pm 2\mu$	→ Type I Goldstone bosons
$4s^2-2$	$\omega_{7,8}=\frac{1}{\sqrt{2}}\sqrt{2J_\ell^2+m_1^2+16\mu^2\pm\sqrt{(2J_\ell^2+m_1^2+16\mu^2)^2-4J_\ell^2\,(J_\ell^2+m_1^2)}}$	
2	$\omega_{9,10}=\frac{1}{\sqrt{2}}\sqrt{2J_\ell^2+m_0^2+16\mu^2\pm\sqrt{(2J_\ell^2+m_0^2+16\mu^2)^2-4J_\ell^2\,(J_\ell^2+m_0^2)}}$	→ Phonon $v=\frac{1}{\sqrt{d-1}}$

$$m_0^2=8\mu^2-2m^2\,,\quad m_1^2=\left(8\mu^2-2m^2\right)\frac{u_0}{u_0+2sv_0}\,,\quad m_2^2=4sv_0b^2+m^2$$

Semiclassics: the NLO

It is given by the sum of the zero-point energy of the fluctuations:

$$\Delta_0 \approx \sum_{\ell} n_l \left(\sum_i \omega_i(\ell) \right)$$

ℓ labels the eigenvalues of the "momentum" that have multiplicity n_ℓ

$$\Delta_0 = \Delta_0^{(scalars)} - 2sN_c\Delta_0^{(fermions)}$$

$$\Delta_0^{(fermions)} = \rho_2(N_f, N_c, s, \mu) + \frac{1}{2}\sum_{\ell=1}^{\infty} \left[2(\ell+1)(\ell+2)\left(\omega_{f+}(\ell,\mu) + \omega_{f-}(\ell,\mu)\right)|_{d=4} + \sigma_2(\ell, N_f, N_c, s, \mu)\right]$$

$$\Delta_0^{(scalars)} = \rho(N_f, s, \mu) + \frac{1}{2}\sum_{\ell=1}^{\infty}\left[(1+\ell)^2 \left(\sum_i g_i(N_f, s)\omega_i(\ell,\mu)\right)|_{d=4} + \sigma(\ell, N_f, s, \mu)\right]$$

Highest-weight representation

There is a family of charge configurations such that the operators can be identified with group theory alone.

This charge configuration corresponds to the highest-weight representation in the 2J-tensor power of Adjoint representation.

This has s=1.

$$\mathcal{Q}_J = \mathrm{diag}\,\{-\mathrm{J}, \mathrm{J}, 0, \cdots, 0\} \qquad \mathrm{J} = \mathrm{semi/integer}$$

$$\text{Irrep: } (\Gamma_J, \Gamma_J) \text{ of } U(N_f) \times U(N_f) \qquad \Gamma_J = (2J, 0, \cdots, 0, 2J)$$

Example: $J = 1/2$

$$(\mathbf{Adj}, \mathbf{Adj}) = \left(N_f^2 - 1, N_f^2 - 1\right)$$

$$\boxed{Tr[T^a \Phi T^b \Phi^\dagger]}$$

Perturbation theory

For s=1: complete 2-loop scaling dimension obtained by combining our results with the known perturbative results for Q=2 where Q=4 s J is the classical scaling dimension of the operator.

$$\Delta^{(2-\text{loop})}_{Q,s=1} = Q\left(\frac{d-2}{2}\right) + \frac{(Q-2)Q\alpha_h}{N_f} + \frac{2(Q-1)Q\alpha_v}{N_f^2} + Q\alpha_y - Q\left[2\left(\frac{3}{N_f^2} - \frac{4}{N_f} - 1\right)\alpha_h^2\right.$$
$$+8\left(\frac{2}{N_f^3} - \frac{3}{N_f^2}\right)\alpha_h\alpha_v + 2\left(\frac{1}{N_f^4} - \frac{3}{N_f^2}\right)\alpha_v^2 - \frac{4\alpha_h\alpha_y}{N_f} - \frac{4\alpha_v\alpha_y}{N_f^2}$$
$$\left.+z\left(\frac{3}{2} + \frac{2}{N_f}\right)\alpha_y^2 - \frac{5}{2}\left(1 - \frac{z^2}{N_f^2}\right)\alpha_g\alpha_y\right] + Q^2\left[2\left(\frac{1}{N_f^2} - \frac{2}{N_f}\right)\alpha_h^2\right.$$
$$\left.+8\left(\frac{3}{N_f^3} - \frac{2}{N_f^2}\right)\alpha_h\alpha_v + 4\left(\frac{3}{N_f^4} - \frac{1}{N_f^2}\right)\alpha_v^2 - \frac{2\alpha_h\alpha_y}{N_f} - \frac{4\alpha_v\alpha_y}{N_f^2} + \frac{z\alpha_y^2}{N_f}\right]$$
$$-\frac{2Q^3}{N_f^4}\left(N_f\alpha_h + 2\alpha_v\right)^2 \qquad (4.21)$$

It includes all the interactions (gauge, Yukawa, quartic)

Perturbation theory

1-loop scaling dimension as a function of the s-parameter

$$\Delta_Q = Q + Q^2 \frac{N_f \alpha_h + 2s\alpha_v}{sN_f^2} + Q\left(\alpha_y - \frac{2s\alpha_h}{N_f} - \frac{2\alpha_v}{N_f^2}\right)$$

Higher loop terms provide an Infinite number of checks for future diagrammatic computations.

Four loop check performed in [I. Jack and D.R.T. Jones, 2021]

Varying the charge configuration we have the scaling dimension of operators transforming according to a variety of irreducible representations.

"A lot of scaling dimensions with a single computation"

The Veneziano limit

$$N_f \to \infty\,, \quad N_c \to \infty\,, \quad z \equiv \frac{N_f}{N_c} = \text{ fixed}$$

This is the limit where the fixed point exists and is perturbative.

We take the $N_f, N_c \to \infty$ limit while keeping Q finite. We have

$$\Delta_Q = \frac{Q}{4s}\Delta_{-1} + \Delta_0 + \mathcal{O}\left(\frac{4s}{Q}\Delta_1\right) = Q\Bigg[\underbrace{1}_{\Delta_{-1}}\underbrace{-4\alpha_h}_{\Delta_0^{(b)}}\underbrace{+\frac{z\alpha_y^2}{\alpha_h} - \alpha_y}_{-2sN_c\Delta_0^{(f)}}\Bigg] + \mathcal{O}\left(\frac{4s}{Q}\Delta_1\right)$$

Only the term linear in Q survive, i.e. $\Delta_Q = Q\Delta_{Q=1}$

This is the large-charge behavior of a free field theory

GENERALIZED FREE FIELD THEORY PHASE

Consequence of large-N factorization in the adjoint channel.

Single trace fixed-charge operators.

The large-charge limit

We take the $N_f, N_c \to \infty$ limit while keeping Q the largest parameter of our theory. This implies

$$Q \gg N_f^2/\epsilon$$

$$\mathcal{J} \equiv \frac{Q}{N_f^2}\left(\alpha_h + \alpha_v\right)$$ is kept fixed.

This is the limit considered in D. Orlando, S. Reffert, F. Sannino, 2020.
Taking for simplicity s= $N_f/2$, we have

$$\frac{\mathcal{J}^2}{\epsilon^2 J^2}\Delta_Q = \frac{\mathcal{J}^{4/3}}{\epsilon}\left[\frac{57}{88}\left(\sqrt{23}+\sqrt{46\sqrt{23}+189}+12\right)-3.3777(1)\epsilon+\mathcal{O}(\epsilon^2)\right] + \frac{\mathcal{J}^{2/3}}{\epsilon}\left[\frac{19}{176}\left(\sqrt{23}+\sqrt{46\sqrt{23}+189}+12\right)+4.5881(1)\epsilon+\mathcal{O}(\epsilon^2)\right]+\mathcal{O}\left(\mathcal{J}^0\right)$$

SUPERFLUID PHASE

On the logarithms of μ

or "technicalities matter"

Generally, in computing the large-charge behavior in the superfluid phase from the double-scaling limit, we have the appearance of Log(μ) terms.

For Wilson-Fisher fixed point these are crucial to obtain the correct scaling (i.e. $\Delta_Q \sim Q^{\frac{d}{d-1}}$) for a non-integer number of dimensions.

In our case, these terms cancel between scalars and fermions

Nice example of how different kinds of matter fields conspire to realize conformal dynamics.

On the logarithms of the charge

In O(N) invariant CFTs we expect the presence of an universal term stemming from the superfluid phonon scaling as

$$Q^0 \log Q$$

G. Cuomo, *"A note on the large charge expansion in 4d CFT,"* 2020

In our case, this term is sub-leading in the limit realizing the superfluid phase being suppressed by N_f.

Intuitively this is because we have only one superfluid phonon.

Phases

Our computation captures various regimes of the theory.
The free parameters are N_f, N_c, Q, ϵ ($\epsilon \equiv N_f/N_c - 11/2$)

$$Q \gg 1, \epsilon \ll 1, Q\epsilon \ll 1$$
$$N_f, N_c \text{ arbitrary}$$

Perturbation theory

$$N_f, N_c \to \infty$$
$$Q \ll N_f^2/\epsilon$$

Generalized free theory

$$\Delta_Q = Q\Delta_{Q=1}$$

$$N_f, N_c \to \infty$$
$$Q \gg N_f^2/\epsilon$$

Superfluid phase

$$\Delta_Q \sim Q^{\frac{d}{d-1}}$$

Conclusions

- We studied a Standard Model-like four dimensional CFT at large flavor charges.
- We compute the scaling dimension of the lowest-lying operators corresponding to a two-parameters family of charge configurations.
- We discussed the identification of fixed-charge operators from a group theoretical viewpoint.
- By varying the parameters of our theory, we identify three distinct regimes captured by our computation.
- For the superfluid phase, we discussed the cancellation of the Log(μ) terms between scalars and fermions and the absence of universal logarithmic contributions to Δ_Q.