Large Q@Diablerets

NRCFTs at large charge

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Based on work with Simeon, Domenico, Susanne & Ian

References

[Hellerman, Orlando, Reffert, Watanabe '15]
[Monin, Pirtskhalava, Rattazzi, Seibold '16]
[Álvarez Gaumé, Orlando, Reffert '20]

[Pal '18] [Kravec, Pal '18 & '19]
[Favrod, Orlando, Reffert '18]
[Orlando, VP, Reffert '20]
[Hellerman, Swanson '20]
[VP '21]
[Hellerman, Orlando, VP, Reffert, Swanson '21]

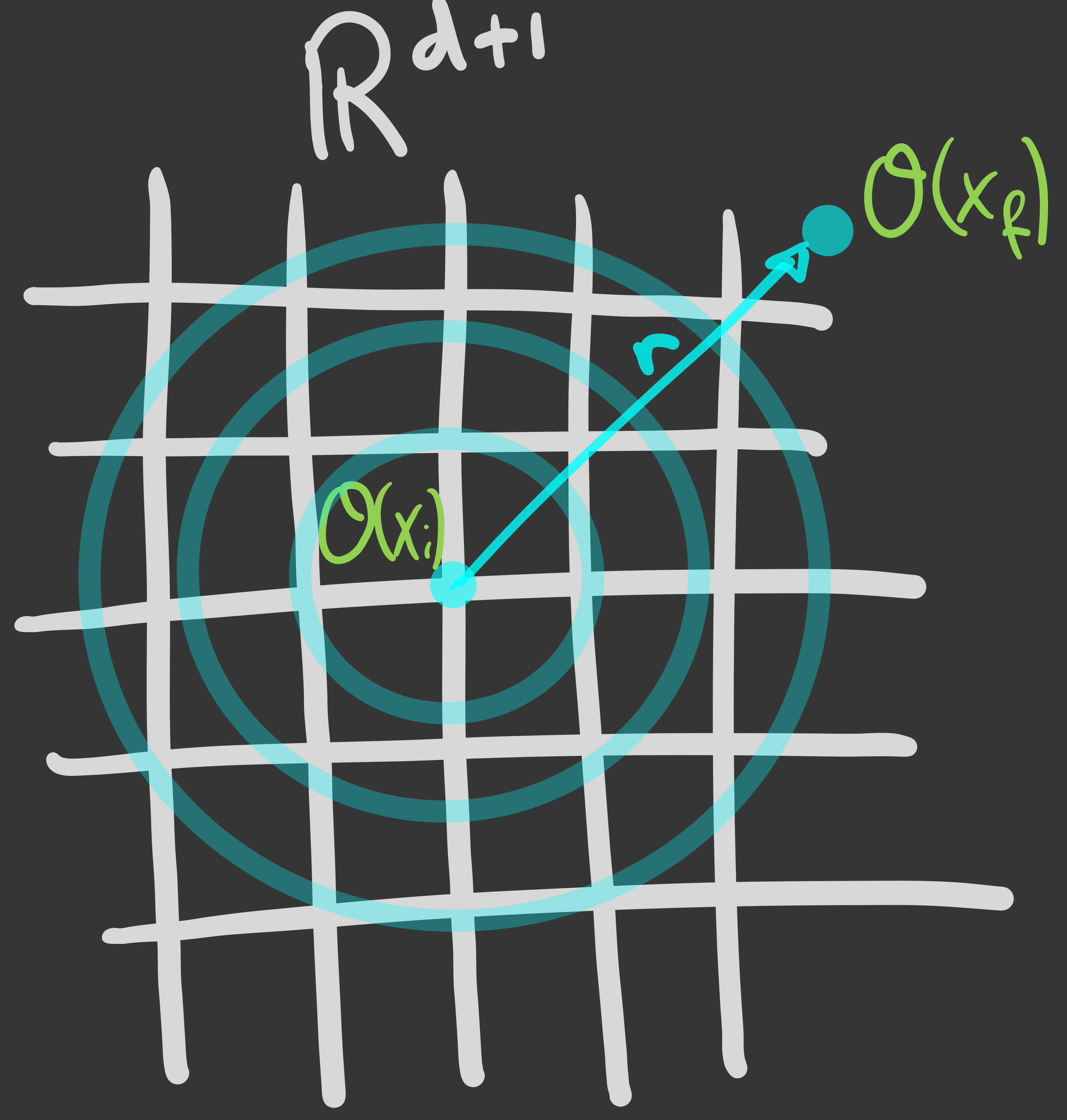
Primary Goal

Show that NRCFTs are arguably one of the most natural (yet underappreciated) playground to run the large charge program.

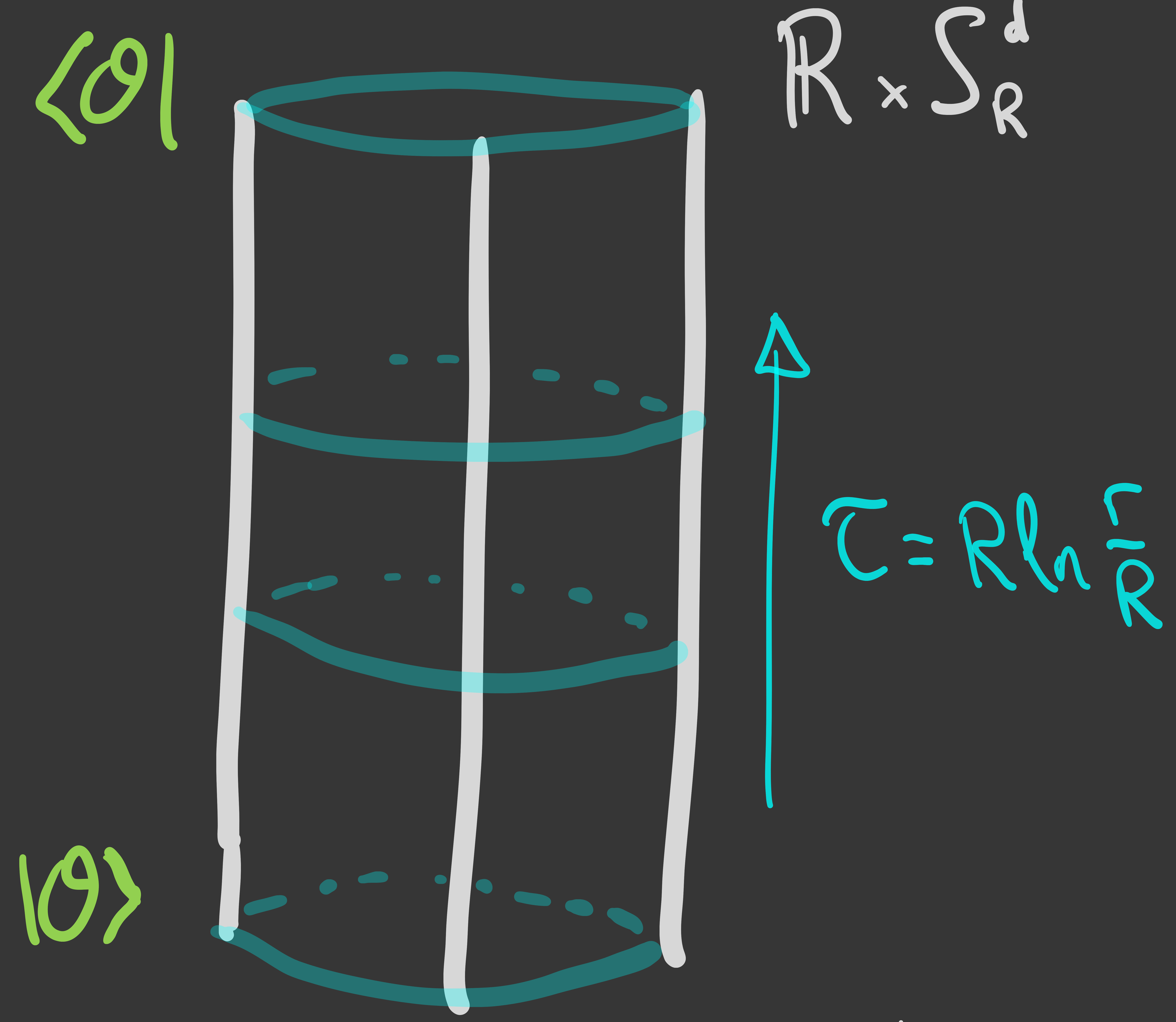
Descendant Goals

- Give heuristic argument for (part of) the final result

- Give more details



State/op. $\longleftrightarrow$ map



Vacuum correlators of charged operators

Finite density correlators (condmat phase)

$$\Delta = E \cdot R$$

$NRCFT_{d+1}$ algebra

(aka Shrödinger)

$(\hbar = m = 1)$

Rotations	J_{ij}	Galilean
Translations	H, P_i	
Boosts	K_i	
Particle number	Q	$\to$ central ext.
Scaling	D	$(t, \vec{x}) \to (\lambda^2 t, \lambda \vec{x})$
Special conf. transf.	C	$(t, \vec{x}) \to \frac{1}{1+\lambda t}(t, \vec{x})$

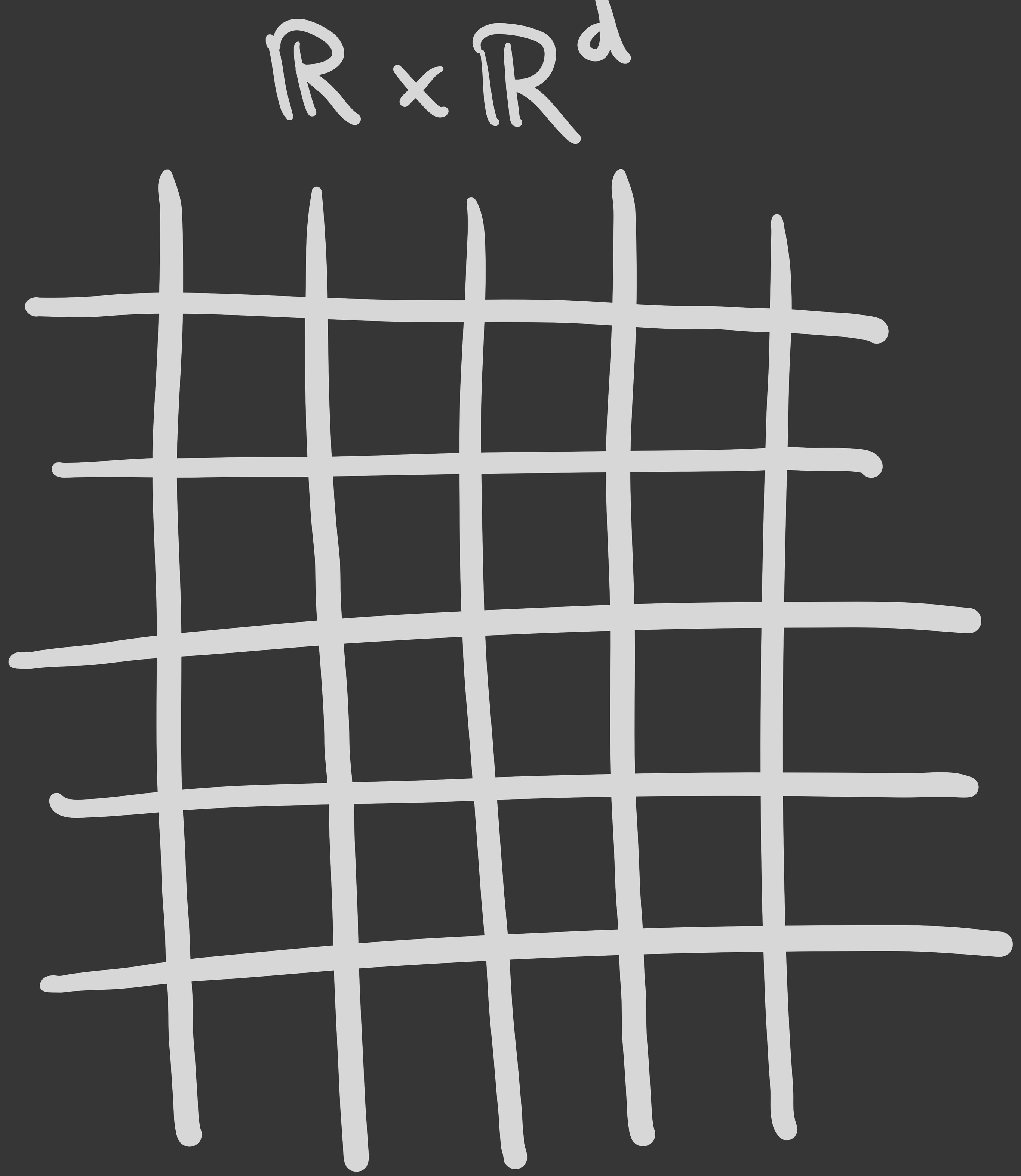
We learned yesterday that a state/op. map also exists in NRCFT via the N-S quantization of its $SL(2,\mathbb{R})$ subgroup, which maps flat space to flat space, but foliating along $H+C$.

A striking feature of NRCFTs is that

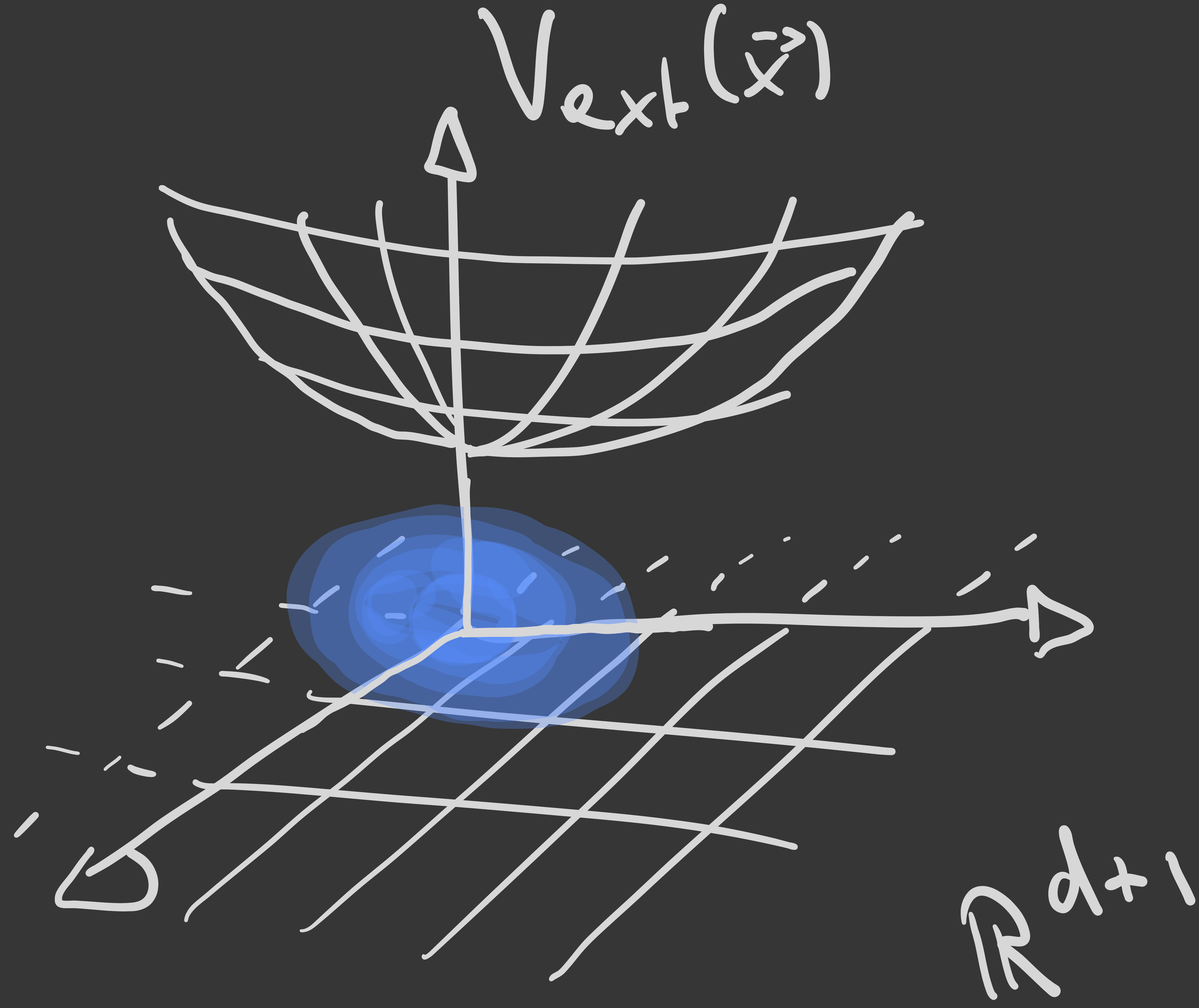
$$C = \int d^d x \; \frac{1}{2} x^2 \rho(x)$$

U(1) density

H+C describes a system coupled to a harmonic trap!



State/op. map



Vacuum correlators of charged operators

Finite density correlators (condmat phase)

$$\Delta = \frac{E}{\omega}$$

$CFT_{Q\gg 1}$

A

B

C

Swampland

Others?

A: Commit to specific model

- Wilson-Fisher FP
  - ↳ O(2) ε-expansion
  - ↳ O(N) large-N
  - ↳ light-ray op.
- $\mathcal{N}=2$ SCFTs
- $\mathcal{N}=4$ SYM
- Asympt. safe CFTs
- Non-Abelian models

B: Study "large-charge universality classes"

Typical assumption:

- ↳ superfluid phase (with no other light d.o.f.)
- ↳ seperation of scales warranted by $Q\gg 1$
- ↳ Effective description

Achieved so far

CFTs

A

- Confirmed superfluid prediction in WF FP
- Resurgence
- Lattice
- (crazy things I don't understand [yet])

B (superfluid class)

- Conformal dimension for low-lying operators
- OPE for scalar and spinning op.
- Extensions: . parity-violating
  . boundary CFT
  . non-Abelian case

Achieved so far

NRCFTs

A

N/A

(in fact: lattice)

B (superfluid class)

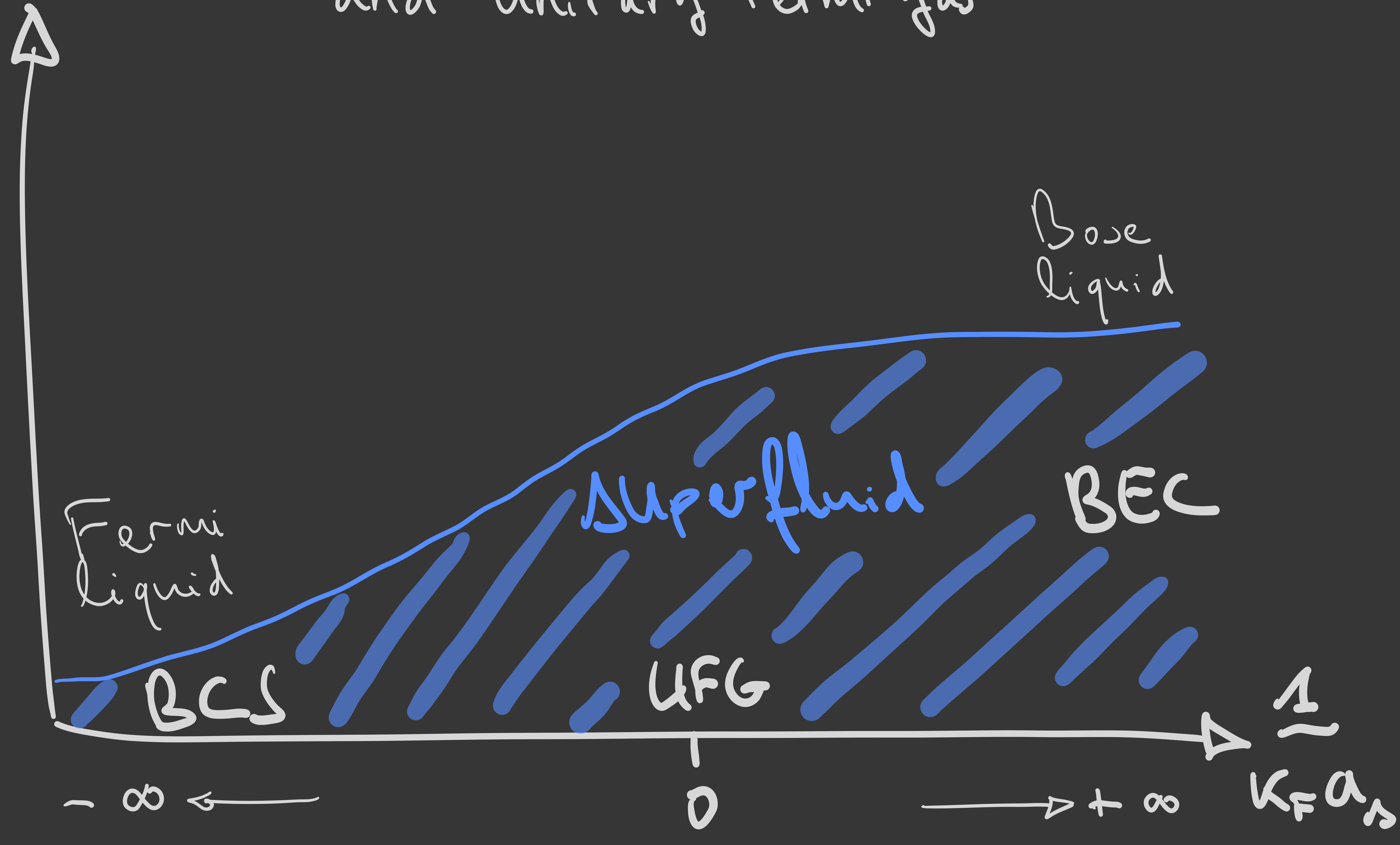
- Conformal dimension for low-lying operators
- OPE for scalar op.

Bucket list

- Unitary Fermi gas (ε-expansion, large-N)
- Anyons
- Resurgence(?)
- Lattice
- OPE for spinning op.
- Parity-violating (chiral superfluid)
- Boundary (?)

BCS - BEC crossover

and unitary Fermi gas

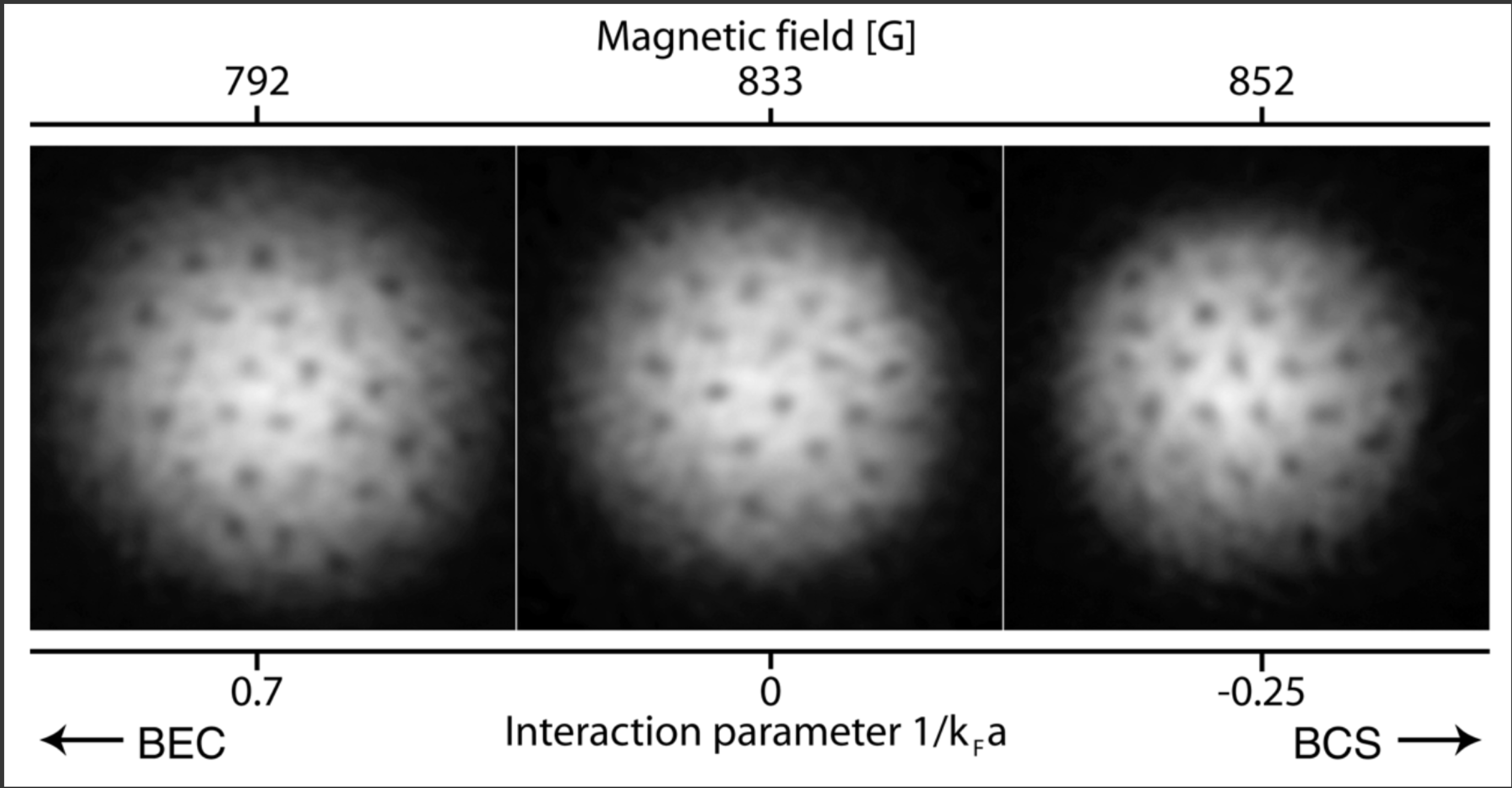


- Realized in the lab with ultracold atoms trapped in a harmonic potential.

- Interaction strength a_s tunable via so-called Feshbach magnetic field → diverges at "unitarity".

- Number of particles $\sim 10^5$.

When some angular momentum



is given to a superfluid, it develops vortices

Why do we care about NRCFTs?

Almost as constrained as relativistic CFTs

Underexplored

U(1) symmetry by default

Nobel Prize

Direct connection with experiments! ($Q \gg 1$ by default)

As we shall see, they have a richer large-charge structure

In fact, it is arguably one of the most natural playground for the large charge program.

Heuristics

$U(1)$ CFT$_{Q\gg 1}$

Densities $\begin{cases} \rho \sim \dfrac{Q}{\mathrm{Vol}} \sim \dfrac{Q}{R^d} \\ \varepsilon \sim \dfrac{E}{\mathrm{Vol}} \sim \dfrac{\Delta}{R^{d+1}} \end{cases} \xrightarrow{R\to\infty} \Delta \sim Q^{\frac{d+1}{d}}$

Scales $\begin{cases} \Lambda_{IR} = \dfrac{1}{R} \\ \Lambda_{UV} = \rho^{\frac{1}{d}} = \dfrac{Q^{\frac{1}{d}}}{R} \end{cases}$

Derivative expansion controlled by

$$\varepsilon^2 = \left(\frac{\Lambda_{IR}}{\Lambda_{UV}}\right)^2 = Q^{-\frac{2}{d}} \ll 1$$

Conclusion: $\Delta = Q^{\frac{d+1}{d}}\left[\alpha_1 + \frac{\alpha_2}{Q^{2/d}} + \dots\right] + Q^0\left[\beta_0 + \frac{\beta_1}{Q^{2/d}} + \dots\right]$

NRCFT

Densities (at the center) $\begin{cases} \rho \sim \dfrac{Q}{\mathrm{Vol}} \sim \omega^{\frac{d}{2}}\sqrt{Q} \\ \varepsilon \sim \dfrac{E}{\mathrm{Vol}} \sim \dfrac{\omega^{\frac{d}{2}+1}}{\sqrt{Q}}\Delta \end{cases} \xrightarrow{\omega\to 0} \Delta \sim Q^{\frac{d+1}{d}}$

Scales $\begin{cases} \Lambda_{IR} = \dfrac{1}{R_{cl}} \sim \sqrt{\omega}\, Q^{-\frac{1}{2d}} \\ \Lambda_{UV} = \rho^{\frac{1}{d}} \sim \sqrt{\omega}\, Q^{+\frac{1}{2d}} \end{cases}$

Derivative expansion controlled by

$$\varepsilon^2 = \left(\frac{\Lambda_{IR}}{\Lambda_{UV}}\right)^2 = Q^{-\frac{2}{d}} \ll 1$$

Conclusion (intermediate): $\Delta = Q^{\frac{d+1}{d}}\left[\alpha_1 + \frac{\alpha_2}{Q^{2/d}} + \dots\right] + Q^0\left[\beta_0 + \frac{\beta_1}{Q^{2/d}} + \dots\right]$

- But the charge density ρ is classically supported on a ball of radius R_{cl}.

- The EFT is not reliable close to the edge of the cloud

  ↳ tree-level divergences

- Dirichlet b.c. associated with vanishing of ρ defines a region (the edge) on which extra operators live and serve as counterterms

- The output is a whole new series of contributions to Δ, e.g.

$$\Delta^{(d=3)} = \alpha_1 Q^{\frac{4}{3}} + \alpha_2 Q^{\frac{2}{3}} + \alpha_3 Q^{\frac{5}{9}} + \alpha_4 Q^{\frac{1}{3}} + \alpha_5 Q^{\frac{1}{9}} + \frac{1}{3\sqrt{3}} \log Q + \ldots$$

- Moreover, the renormalization yields logarithmic enhancements in even d, e.g.

$$\Delta^{(d=2)} = \alpha_1 Q^{\frac{3}{2}} + \alpha_2 \sqrt{Q} \log Q + \alpha_3 \sqrt{Q} - 0.2942\ldots + \ldots$$

From here on...

conclude

Go back to "Bucket list"

↓

Thank the audience

↓

Standing ovation

continue

Dive into more details

questions

More details $(\omega=1)$

Leading-order Lagrangian:

$$\mathcal{L}_{LO} = c_0 U^{\frac{d}{2}+1} \qquad \text{where } U = \partial_t \chi - \frac{1}{2}r^2 - \frac{1}{2}(\partial_i \chi)^2$$

[Relativistic: $\mathcal{L}_{LO} = c_0 (\partial\chi)^{d+1}$]. Superfluid ground-state given by $\langle \chi \rangle = \mu \cdot t$, i.e.

$$\langle U \rangle = \mu - \frac{1}{2}r^2 = \mu\left(1 - \frac{r^2}{2\mu}\right) \equiv \mu \cdot z$$

$$\langle \rho \rangle = \left\langle \frac{\partial \mathcal{L}_{LO}}{\partial \dot{\chi}} \right\rangle = \left\langle \frac{\partial \mathcal{L}_{LO}}{\partial U} \right\rangle \sim \langle U \rangle^{\frac{d}{2}} \sim (\mu \cdot z)^{\frac{d}{2}}$$

Charge density supported on ball of $R = \sqrt{2\mu}$.

Therefore, $\mu = \xi Q^{\frac{1}{d}}$ and $\Delta = \frac{d}{d+1} \xi Q^{\frac{d+1}{d}}$.

The $z = 1 - \frac{r^2}{R_{cl}^2}$ coordinate makes computation of global quantities very convenient:

$$\int_{\text{cloud}} d^d x \, f(\vec{x}) = \frac{(2\pi\mu)^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)} \int_0^1 dz \, (1-z)^{\frac{d-2}{2}} f(z)$$

for spherically inv. function f.

Only building blocks for corrections:

$$\begin{cases} U = \dot{x} - A_0(\vec{x}) - \frac{1}{2}(\partial_i x)^2 &, \quad \langle U \rangle \sim (\mu \cdot z)^{\frac{d}{2}} \\ z = \nabla^2 A_0 - \frac{1}{d}(\nabla^2 x)^2 &, \quad \langle z \rangle \sim 1 \end{cases}$$

Hence,

$$\mathcal{O}^{(m,n)} \equiv (\partial_i U)^{2m} z^n U^{\frac{d}{2}+1-(3m+2n)}$$

$$\Delta^{(m,n)} \sim \mu^{d+1-2(m+n)} \cdot \frac{\Gamma\left(\frac{d}{2}+m\right)\Gamma\left(\frac{d}{2}+2-(3m+2n)\right)}{\Gamma\left(d+2-2(m+n)\right)}$$

The edge is characterized by $U=0$.

Therefore, edge op. are of the form

$$\mathcal{Z}^{(p)} \equiv \delta(U) \cdot \mathcal{Z}^{p} \cdot (\partial_i U)^{\frac{d+4(1-p)}{3}}$$

giving

$$\Delta^{(p)} \sim Q^{\frac{2d-1-2p}{3}}.$$

Anytime a bulk operator $\mathcal{O}^{(min)}$ diverges, there exists $p \in \mathbb{N}$ such that $Z^{(p)}$ serves as a counterterm.

Only happens for positive μ-scaling in even spatial dimension.

Fluctuations

Spectrum of excitations:

$$E^d_{n,\ell} = \sqrt{\frac{4n}{d}(n+\ell+d-1)+\ell}$$

with multiplicity $M^{d-1}_\ell = (2\ell+d-2)\frac{\Gamma(\ell+d-2)}{\Gamma(\ell+1)\Gamma(d-1)}$

$$E^d_{\text{Casimir}} = \frac{1}{2}\sum_{n,\ell=0}^{\infty} M^{d-1}_\ell E^d_{n,\ell} = \begin{cases} -0{,}2942\ldots & (d=2) \\ -\frac{1}{2\sqrt{3}\,\varepsilon} & (d=3+2\varepsilon) \end{cases}$$

Upon renormalization, this yields $\Delta^{\text{fluct.}} = \frac{1}{3\sqrt{3}}\log Q$.

Final Result

$$\Delta = Q^{\frac{d+1}{d}}\left[a_1 + \frac{a_2}{Q^{2/d}} + \frac{a_3}{Q^{4/d}} + \cdots\right]$$

$$+\, Q^{\frac{2d-1}{3d}}\left[b_1 + \frac{b_2}{Q^{\frac{2}{3d}}} + \frac{b_3}{Q^{\frac{2}{3d}}} + \cdots\right]$$

$$+\, Q^{\frac{d-5}{3d}}\left[c_1 + \frac{c_2}{Q^{\frac{2}{3d}}} + \frac{c_3}{Q^{\frac{2}{3d}}} + \cdots\right]$$

$+ \log(Q)$ enhancements

Achieved so far

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